AIR-COOLED HEAT EXCHANGER DESIGN

USING SUCCESSIVE QUADRATIC PROGRAMMING (SQP)

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A non-linear optimization algorithm is applied to the design of air cooled heat exchangers. In such equipment, the cold fluid (air) is impelled across banks of finned tubes by means of fans in forced or induced draft. The hot stream flows inside the tubes in one or more passes, and the process that takes place may be cooling of either a gas or a liquid, or condensation of either a pure vapor or a mixture.

The objective function is the minimum cost of the unit (investment and operation), subject to certain geometric and thermohydraulic constraints.

The optimization algorithm used is that developed by Biegler and Cuthrell [1], and programmed by them in the OPT package. The problem posed in this case is made of ten optimization variables, subject to five constraints related to geometric and operational parameters of the heat exchanger.

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Introduction

The purpose of the work is to design an air-cooled heat exchanger able to remove the required amount of heat from a process stream with a minimum cost. Physical properties and mass flow rate of both fluids, air inlet temperature, as well as inlet and outlet temperature of the process stream, are known parameters.

This problem was previously solved using a design algorithm developed by González and Echarte [2]. Given the design parameters: outside diameter of tubes, fin height and frequency, fin tip clearance, and air temperature increment, the algorithm found all the heat exchangers that did not exceed the allowed pressure drop of both streams nor the maximum allowed total width. To do this, every possible combination of the design variables was analyzed, giving them stepwise values. When the run finished, only the values of the parameters corresponding to the three best solutions remained stored.

Although the results obtained with this algorithm have been satisfactory and adopted by the process industries, it was considered that it should be better to pose the design problem as an optimization one, using a non-linear optimization algorithm for this purpose.

Posing the problem

Mathematical formulation

The objective of the optimal design of an air-cooled heat exchanger is to find the unit capable to accomplish the prescribed thermal duty, that is to transfer the required heat flux, with the minimum combined investment and operating cost.

The use of mathematical programming techniques for processes optimization has been increasing in the last forty years [3], giving rise to algorithms that solve bigger and more difficult problems.

A non-linear optimization problem is as follows:

Min $\boldsymbol{F(x)}$

Subject to: $\boldsymbol{g_i(x) = 0}$ i=1,…,MEQ

$\boldsymbol{hi(x) \leq 0}$ i=MEQ+1,…,M

$\boldsymbol{x_{Lj} \leq x_j \leq x_{Uj}}$ j=1,…,N (1)

Where:

$\boldsymbol{F}$ ($\mathbf{x}$) is the objective function;

$\boldsymbol{g_i(x) = 0}$ are equality constraints that represent the equipment model;

$\boldsymbol{h_i(x) \leq 0}$ are inequality constraints that represent design specifications, physical restrictions, etc.

N: number of independent variables, or optimization variables;

M: total number of constraints;

MEQ: number of equality constraints.

Subscripts L and U are, respectively, lower and upper bounds of optimization variables.

The design variables $\boldsymbol{(x)}$ considered were the following physical and geometrical parameters:

- outside tube diameter (D_o),
- fin frequency (N_F),
- fin height (H_F),
- fin tip clearance (S_F),
- air temperature increment (Δt_a),
- tube length (L),

- fin thickness (e_F),
- number of rows of the exchanger (N_R),
- number of passes of the hot fluid inside the tubes (N_P),
- number of sections of the unit (N_S).

The restrictions are related to the following geometric and operating parameters,:

- pressure drop in the hot fluid (ΔP_T),
- pressure drop in the air stream (ΔP_a),
- width of the unit (W),
- aspect ratio of a cell (R_A),
- fraction of tube length that can be covered with fins (F_F).

The design variables have upper and lower bounds, given by mathematical or operational reasons, or simply from heuristics. For example, although outside tube diameter may vary from zero to infinity, its practical values span between ½” and 2”; the rise in air temperature must be such that it does not lead to any temperature cross between hot and cold fluid, etc.

Although some design variables are discrete (outside diameter of tubes) or integer (number of rows, passes, sections), they were considered as continuous for optimization purposes.

Calculation of dependent variables

In order to reduce the problem size, dependent variables may be not explicitly written in terms of decision variables, but calculated by means of a simulator and then used to get the values of the nonlinear functions $\boldsymbol{h_i(x)}$ [4]. The simulation package solves the non-linear equations $\boldsymbol{g_i(x)}$ and calculates the objective function for a given set of independent variables

$\boldsymbol{x}$ [2]. An important reduction in the optimization problem can be achieved, thus resulting in the following:

$$\begin{aligned} &\text{Min } F(\boldsymbol{x}) \\ &\text{Subject to: } \boldsymbol{h_i(x) \le 0} \qquad i=1,\ldots,M \\ &\qquad \boldsymbol{x_{Lj} \le x_j \le x_{Uj}} \qquad j=1,\ldots,N \end{aligned} \tag{2}$$

As it was stated previously, the dependent variables, such as pressure drop, heat transfer coefficient and outlet temperature of both fluids, are calculated by means of a simulation package.

The design of an air-cooled exchanger reduces to solve the problem of finding the right value of ten independent variables (N = 10) that lead to a minimum cost of the unit, subject to the five inequality constraints stated in eq. (2). So, for this problem, M = 5 and MEQ = 0.

Mathematically, problem in eq. (2) may be written as:

$$\text{Min } C = C\,(D_O,\ N_F,\ H_F,\ S_F,\ \Delta t_a,\ L,\ N_R,\ N_P,\ N_S,\ e_F)$$

$$\begin{aligned} \text{Subject to: } & 0 \le F_F \le 1 \\ & 0 \le W \le W_{max} \\ & R_{A,min} \le R_A \le R_{A,max} \\ & 0 \le \Delta P_T \le \Delta P_{T,max} \\ & \Delta P_{a,min} \le \Delta P_a \le \Delta P_{a,max} \qquad (M = 5 \text{ inequality constraints}) \\ & D_{o,min} \le D_o \le D_{o,max} \\ & N_{F,min} \le N_F \le N_{F,max} \\ & H_{F,min} \le H_F \le H_{F,max} \\ & S_{F,min} \le S_F \le S_{F,max} \\ & \Delta t_{a,min} \le \Delta t_a \le \Delta t_{a,max} \end{aligned}$$

$L_{min} \leq L \leq L_{max}$

$1 \leq N_R \leq N_{R,max}$

$1 \leq N_P \leq N_{P,max}$

$1 \leq N_S \leq N_{S,max}$

$e_{F,min} \leq e_F \leq e_{F,max}$ (N = 10 design variables) (3)

Equality constraints have been eliminated, as they are calculated by means of a simulation program. The simulator consists of a main subroutine written in Fortran language. With known inlet temperatures and pressures, and the overall heat transfer coefficient assumed, the exit values of operating conditions are calculated. These data allow the correction of pressure drop and individual film coefficients leading to the overall coefficient updating. The procedure is repeated until convergence is achieved [2, 5]. The calculation of some variables, such as pressure drop of both fluids, film coefficients, fin efficiency, etc., are carried out by means of standard subroutines developed at PLAPIQUI. These subroutines are written under certain rules (error checking, SI units, etc.) and are available as components of a Library. The simulation program also checks that there are no temperature crosses, and that all geometrical variables are non-negative.

This Non Linear Programming problem can be efficiently solved by means of a successive quadratic programming (SQP) code. The OPT code, developed by Biegler and Cuthrell [1] was used for this purpose.

Numerical example

Numerical data

As an application of the proposed algorithm, equations (3) were applied to the design of a gas cooler to process 13.49 kg/s of a hydrocarbon gas mixture from 288.0 to 48.5 °C, by means of an air stream at 38 °C [5].

The process stream properties are shown in Table 1, and numerical values of independent and dependent variables in Table 2.

This problem consists of ten variables, and ten inequality constraints (two per each constrained variable).

Sensitivity analysis of the objective function

In order to get a better understanding of the problem, the change in the cost of the unit with each one of the design variables was studied prior to the utilization of any optimization technique. The most relevant results are shown in Figs. 1 to 4. The influence of N_S was not displayed because it is important in the change of the aspect ratio but not in the cost; e_F is not of great importance in the range considered.

It can be seen that the behavior of all variables is appropriate for a SQP code, as they exhibit a minimum (see Figs. 1 to 4), or the cost is decreasing with increasing values of the variable (S_F, L and H_F).

Results

As a result of the application of the SQP algorithm together with the simulation package, an optimum in the objective function of the continuous problem was found. The value of the cost was about US$ 83,000. The way the algorithm approaches to the optimum and the values the independent variables take in successive iterations are shown in Table 3. The computed error is the Kuhn Tucker error, which was set to a maximum of 0.1 for the solution to be satisfactory. The values taken by the optimization and constrained variables at the optimum are in last column of Table 2. In Table 4 other variables of interest in the heat transfer equipment are shown.

Conclusions and Remarks

The algorithm that uses the SQP code has proved to be useful for the optimal design of different air-cooled heat exchangers, leading to units of lower cost than those already installed in plant for the same purpose. It has been tested with different process streams, and the results have been very good.

Although some integer variables give non-integer values at the optimum, the results obtained with this method can be used as a good starting point. Besides, it must be taken into account that, despite the layout obtained, the supplier will adapt the required unit to what he usually manufactures and complies with the required service.

The optimization program allows to fix any of the design variables (eventually all of them) [6], so the feasibility of any solution can be checked, when continuous values of some variables are rounded to integer ones.

Nomenclature

D_o: outside diameter of bare tube [mm]

e_F: fin thickness [mm]

F_F: fraction of tube length occupied by fins [-]

H_F: fin height [mm]

L: tube length [m]

N_F: fin frequency [fins / m]

N_R: number of tube rows in crossflow [-]

N_P: number of tube passes [-]

N_S: number of sections that contains the unit [-].A section is a part of the equipment that contains at least one cell and covers the tube length.

N_T: number of tubes per section [-]

R_A: aspect ratio (length / width) of a cell [-]. A cell is defined as the rectangular surface in which the circumference of a fan is contained.

S_F: fin tip clearance [mm]

U: overall heat transfer coefficient [W/m^2°C]

v_a: air face velocity, (based on the unit projected area = L x W) [m/s]

W: total width of the unit [m]

w_a: total mass flow rate of air [kg/s]

ΔP_a: pressure loss of air [Pa]

ΔP_T: pressure loss of tubeside fluid [kPa]

Δt_a: air temperature increase [°C]

Δt_m: weighed mean temperature difference between hot and cold stream [°C].

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FIGURE CAPTIONS

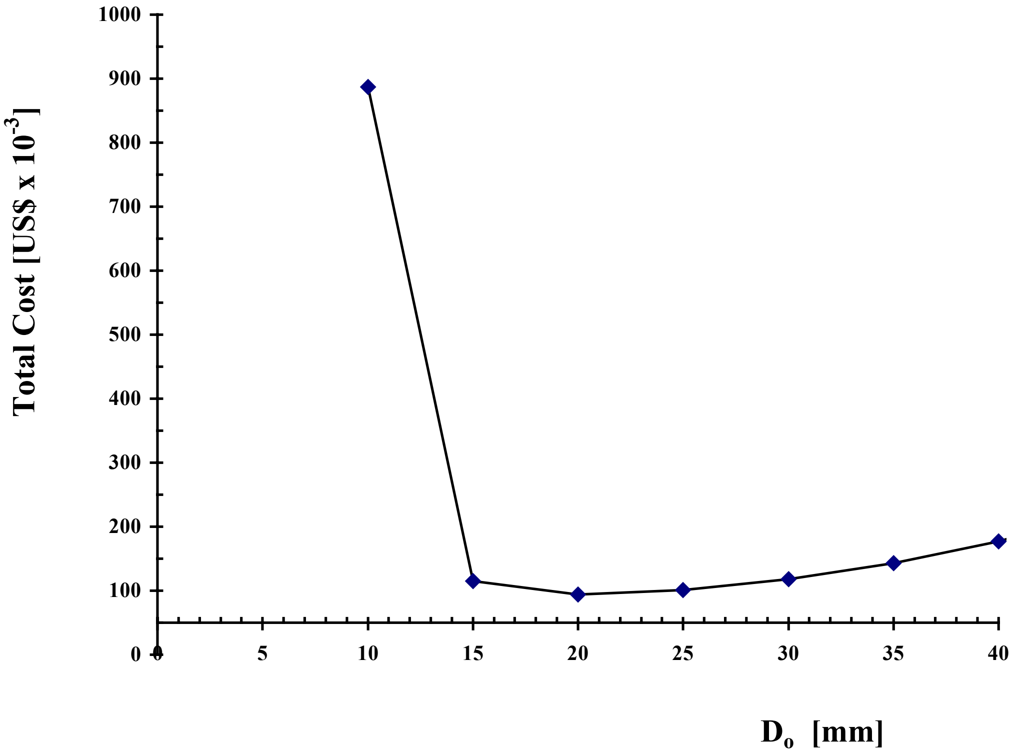
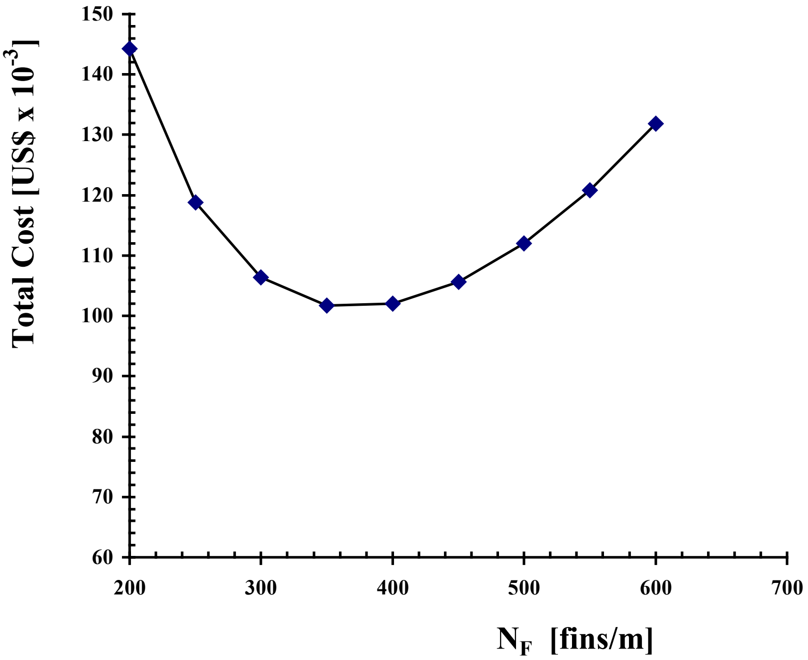

1000
900
800
700
600
500
400
300
200
100
0
0
5
10
15
20
25
30
35
40
Total Cost [US$ x 10^-3]
D_o [mm]

Figure 1.



Total Cost [US$ x 10-3]
150
140
130
120
110
100
90
80
70
60
200
300
400
500
600
700
NF [fins/m]

Figure 2.

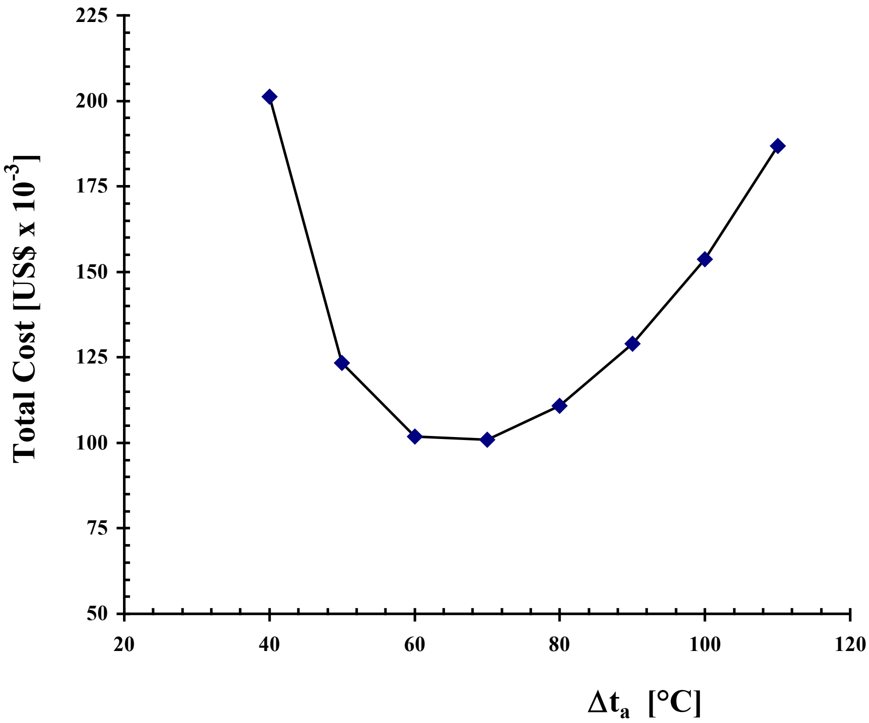

225
200
175
150
125
100
75
50
Total Cost [US$ x 10^-3]
20
40
60
80
100
120
Δt_a [°C]

Figure 3.

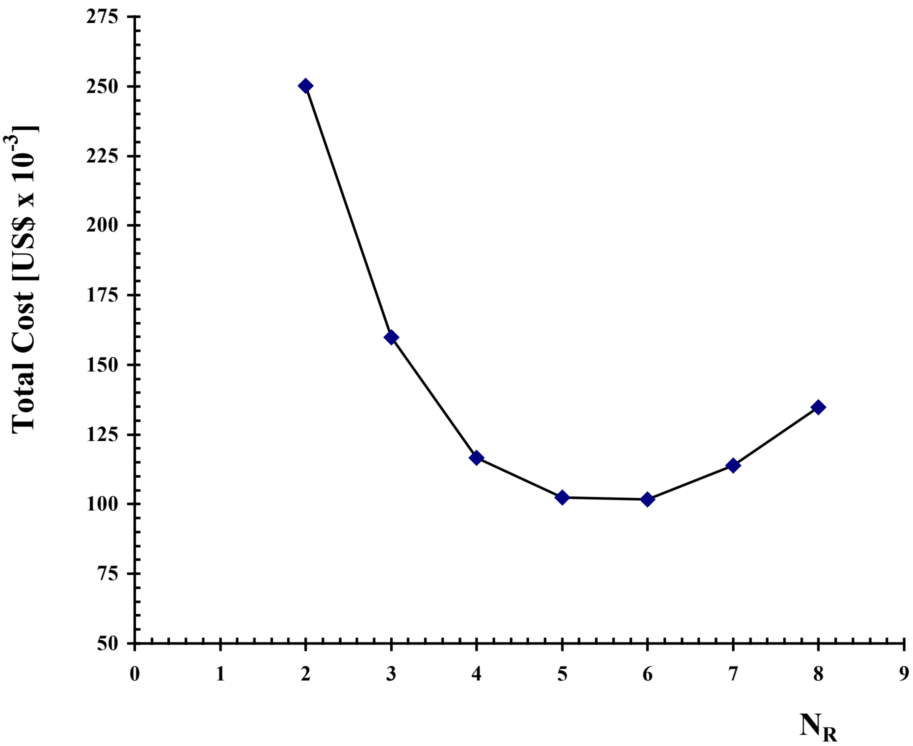

Total Cost [US$ x 10^-3]
275
250
225
200
175
150
125
100
75
50
0
1
2
3
4
5
6
7
8
9
N_R

Figure 4.

Table 1: Process Stream Properties

Inlet pressure [bar eff.]	59.3
Inlet Specific heat [J/kg K]	3141
Outlet Specific heat [J/kg K]	2320
Thermal conductivity [W/m K]	0.06
Inlet Density [kg/m^3]	21.1
Inlet viscosity [cp]	0.019
Outlet viscosity [cp]	0.015

Table 2: Optimization and Constrained Variables

Optimization Variables			
Variable	**Min.**	**Max.**	**Value at optimum**
D_o [mm]	15.9	60.0	15.9
N_F [fins/m]	197	433	433
H_F [mm]	10	25	25
S_F [mm]	0.05	10.0	10.0
Δt_a [°C]	30.0	130.0	62.95
L [m]	1.0	10.0	10.0
N_R [-]	2	8	8
N_P [-]	1	4	1.14
N_S [-]	1	5	1
e_F [mm]	0.1	0.5	0.5
Constrained Variables			
Variable	**Min.**	**Max.**	**Value at optimum**
F_F [-]	0.0001	1.0	0.22
W [m]	0.001	20.0	6.10
R_A	0.588	1.7	0.82
ΔP_T [kPa]	1.0	34.0	34.0
ΔP_a [Pa]	80	160	104.9

Table 3: Approach to the Optimum Point

IT	ERROR	O.F.	D_o	N_F	H_F	S_F	Δt_a	L	N_R	N_p	N_s	e_F
		(US$)	(mm)	(fins/m)	(mm)	(mm)	(°C)	(m)	(-)	(-)	(-)	(mm)
1	.397572E+06	200985.6	25.4	354.0	15.9	6.36	50.00	9.14	4.0	2.0	1.0	.34
2	.113864E+06	128111.1	15.9	433.0	25.0	10.00	30.00	3.08	4.5	4.0	5.0	.50
3	.682411E+05	145162.1	17.3	433.0	10.0	.05	78.15	3.72	8.0	4.0	5.0	.50
4	.534536E+05	126652.2	27.1	433.0	25.0	.05	68.53	6.30	8.0	4.0	1.0	.50
5	.404433E+05	113353.3	35.3	433.0	25.0	.05	102.31	10.00	8.0	4.0	1.6	.50
6	.137666E+06	122284.5	18.9	433.0	25.0	10.00	72.46	10.00	8.0	1.0	1.6	.50
7	.343663E+05	92573.58	17.3	410.9	25.0	10.00	49.78	10.00	4.8	1.4	1.3	.50
8	132902E+05	89277.29	17.5	413.7	25.0	10.00	47.22	9.93	5.1	1.4	1.4	.50
9	.103990E+05	85152.89	15.9	433.0	25.0	10.00	71.97	10.00	8.0	1.2	1.0	.50
10	.104041E+04	83935.15	15.9	433.0	25.0	10.00	67.31	10.00	8.0	1.2	1.1	.50
11	.432364E+02	83444.26	15.9	433.0	25.0	10.00	62.24	10.00	8.0	1.1	1.0	.50
12	.876407E+00	83417.17	15.9	433.0	25.0	10.00	63.04	10.00	8.0	1.1	1.0	.50
13	**.422465E-02**	**83416.53**	**15.9**	**433.0**	**25.0**	**10.00**	**62.95**	**10.00**	**8.0**	**1.1**	**1.0**	**.50**

Table 4: Results at Optimum Point

Other Dependent Variables	
Variable	**Value at Optimum**
N_T [-]	642.4
Area [m^2]	320.90
Δt_m [°C]	60.33
U [W/m^2°C]	469
v_a [m/s]	2.07
w_a [kg/s]	143.2