Modern Physics Letters A
Vol. 30, No. 39 (2015) 1550211 (13 pages)

DOI: 10.1142/S0217732315502119

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Galilean symmetry in generalized Abelian Schrödinger–Higgs models with and without gauge field interaction

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Received 14 June 2015
Revised 23 September 2015
Accepted 29 September 2015
Published 17 November 2015

We consider a generalization of non-relativistic Schrödinger–Higgs Lagrangian by introducing a nonstandard kinetic term. We show that this model is Galilean invariant, we construct the conserved charges associated to the symmetries and realize the algebra of the Galilean group. In addition, we study the model in the presence of a gauge field. We also show that the gauged model is Galilean invariant. Finally, we explore relations between the twin models and their solutions.

Keywords: Galilean symmetry; gauge theories; Chern–Simons gauge theory; symmetries in theory of fields and particles.

PACS Nos.: 11.30.-j, 11.10.-z, 11.15.-q

1. Introduction

A new type of classical field theories has been intensively investigated during the several last years. These theories, named k-field models, are usually endowed with nonstandard kinetic terms that change the dynamics of the model under investigation. The k-field models have application in cosmology,[1–5] strong gravitational waves,[12] dark matter[13] and ghost condensates[7–11] and others. In particular, an interesting issue concerns the study of topological structures, where topologically nontrivial configurations, named topological k-solutions, can exist.[14–26]

In the recent years, theories with nonstandard kinetic term, named k-field models, have received much attention. The k-field models are mainly in connection with effective cosmological models[1–5] as well as the tachyon matter[6] and the ghost condensates.[7–11] The strong gravitational waves[12] and dark matter,[13] are also examples of non-canonical fields in cosmology. Also, topological structure of these models was analyzed,[14–26] showing that the k-theories can support topological soliton solutions both in models of matter as in gauged models.

In this paper, we propose to study a non-relativistic Higgs k-model. Here, the nonstandard kinetic terms are introduced by a function ω, which depend on the Higgs field. In particular, we show that $\omega(\rho)$, where $\rho = \phi^\dagger\phi$, is Galilean invariant and we will construct the conserved charges associated with this invariance. We also show that the model realizes the algebra of the Galilean group, if we choose a particular ω, i.e.

$$\omega(\rho) = \rho^n \,. \tag{1}$$

Finally, we analyze a non-relativistic gauge model with nonstandard kinetic term. In particular, we will concentrate on the Jackiw–Pi model[27,28] with nonstandard kinetic terms. We also show that this model is Galilean invariant, realizing the algebra of the group.

2. The Model and Its Symmetries

Let us start by considering the $(2+1)$-dimensional Schrödinger model governed by the action,

$$S = \int d^3x \left(i\phi^\dagger \partial_0 \phi - \frac{1}{2m}|\partial_i\phi|^2 + \lambda|\phi|^2 \right). \tag{2}$$

Here, $\phi(x)$ is a complex scalar field and λ is a strength coupling constant. Also, the metric tensor is $g^{\mu\nu} = (1, -1, -1)$.

It is well known that the model (2) presents Galilean invariance.[29] This means that action (2) is invariant under time and space translation, rotations, Galilean boost and the U(1) symmetry. More precisely, the Schrödinger model remains invariant under the following symmetry transformations:

1. The infinitesimal time translation of the field

$$\delta\phi = a\partial_0\phi \,, \tag{3}$$

where the Hamiltonian is the conserved charge associated to this symmetry

$$H = \int d^2x \left(\frac{1}{2m}|\partial_i\phi|^2 + \lambda|\phi|^2 \right). \tag{4}$$

2. The infinitesimal translation of the field

$$\delta\phi = a_i\partial_i\phi \,, \tag{5}$$

which leads to the conservation of linear momentum

$$P_i = \frac{i}{2}\int d^2x (\phi^\dagger\partial_i\phi - \partial_i\phi^\dagger\phi) \,. \tag{6}$$

3. The infinitesimal field transformation due to a rotation

$$\delta\phi = \theta\mathbf{r} \times \partial\phi \,, \tag{7}$$

where θ is the rotation angle. Here, the conserved charge obtained from the Noether theorem is angular momentum,

$$J = \int d^2x(-\mathcal{P}_1 x_2 + \mathcal{P}_2 x_1)\,, \tag{8}$$

where

$$\mathcal{P}_i = \frac{i}{2}(\phi^\dagger \partial_i \phi - \partial_i \phi^\dagger \phi)\,. \tag{9}$$

4. The infinitesimal field transformation due to Galilean boost

$$\delta\phi = imv_i r_i \phi - t v_i \partial_i \phi\,, \tag{10}$$

which leads to the conservation of the following charge:

$$G_i = \int d^2x(\mathcal{P}_i t - m x_i \rho)\,, \tag{11}$$

$$\rho = \phi^\dagger \phi\,. \tag{12}$$

5. The Galilean invariance is completed with the inclusion of U(1) symmetry

$$\delta\phi = i\alpha\phi\,. \tag{13}$$

Here, the mass operator $M = m\int d^2x\,\rho$ is the conserved charge associated to this transformation.

The algebra of the Galilean group may be realized by using the Poisson brackets for functions of the matter fields, which are defined from the symplectic structure of the Lagrangian at fixed time to be

$$\{F, G\}_{\mathrm{PB}} = i\int d^2x\left(\frac{\delta F}{\delta\phi^\dagger(r)}\frac{\delta G}{\delta\phi(r)} - \frac{\delta F}{\delta\phi(r)}\frac{\delta G}{\delta\phi^\dagger(r)}\right). \tag{14}$$

In the particular case in which $F = \phi$ and $G = \phi^\dagger$, we have

$$[\phi(x), \phi(x')^\dagger] = -i\delta^2(x - x')\,. \tag{15}$$

Using the Poisson bracket relations the above conserved charges can be shown to realize the algebra of the Galilean group

$$\begin{aligned} [P_i, P_j] &= [P_i, H] = [J, H] = [G_i, G_j] = 0\,, \\ [J, P_i] &= \epsilon^{ij} P_j\,, \\ [J, G_i] &= \epsilon^{ij} G_j\,, \\ [P_i, G_j] &= \delta^{ij} mN\,, \\ [H, G_i] &= P_i\,. \end{aligned} \tag{16}$$

In this section, we are interested in exploring a generalization of the model (2). Following the same idea of the works cited in Refs. 14–16 and 23–25, we modify the model (2) by changing both the canonical kinetic term of the scalar field and the potential term, so that the proposed model is described by the action

$$\begin{aligned} S &= \int d^3x\, \omega(\rho)\mathcal{L}_{\rm NR} = \int d^3x\, \omega(\rho)\left(i\phi^\dagger\partial_0\phi - \frac{1}{2m}|\partial_i\phi|^2 + \lambda|\phi|^2\right) \\ &= S_1 + S_2 + S_3\,, \end{aligned} \tag{17}$$

where

$$\begin{aligned} S_1 &= \int d^3x\, \omega(\rho) i\phi^\dagger\partial_0\phi\,, \\ S_2 &= -\int d^3x\, \omega(\rho)\frac{1}{2m}|\partial_i\phi|^2\,, \\ S_3 &= \int d^3x\, \omega(\rho)\lambda|\phi|^2\,. \end{aligned} \tag{18}$$

Here, the function $\omega(\rho)$ is in principle an arbitrary dielectric function of the complex scalar field ϕ, and ρ is related ϕ by

$$\rho = \phi^\dagger\phi\,, \tag{19}$$

where n is a positive integer.

Next, we will calculate the variation of the action (17) under time and space translation, angular rotation, Galilean boost and U(1) transformation. We begin to calculate the variation of the action (17) under time and space translation

$$\delta\phi = a\partial_0\phi\,, \tag{20}$$

$$\delta\phi = a_i\partial_i\phi\,. \tag{21}$$

The variation (20) implies

$$\delta\omega(\rho) = \frac{\delta\omega}{\delta\rho}\delta\rho = a\frac{\partial\omega}{\partial\rho}\partial_0\rho = a\partial_0\omega\,, \tag{22}$$

so that,

$$\begin{aligned} \delta S_1 &= \int d^3x[i\delta\omega(\rho)\phi^\dagger\partial_0\phi + i\omega(\rho)\delta(\phi^\dagger\partial_0\phi)] \\ &= \int d^3x[ia\partial_0\omega(\rho)\phi^\dagger\partial_0\phi + i\omega(\rho)\delta(\phi^\dagger\partial_0\phi)] \\ &= \int d^3x[ia\partial_0\omega(\rho)\phi^\dagger\partial_0\phi + i\omega(\rho)(a\partial_0\phi^\dagger\partial_0\phi + a\phi^\dagger\partial_0^2\phi)]\,. \end{aligned} \tag{23}$$

By integration by parts, the last term of this integral, we have,

$$\delta S_1 = 0\,. \tag{24}$$

The variation of S_2 under (20) leads to

$$\delta S_2 = -\frac{a}{2m}\int d^3x[(\partial_i\partial_0\phi^\dagger\partial_i\phi + \partial_i\phi^\dagger\partial_i\partial_0\phi)\omega(\rho) + |\partial_i\phi|^2\partial_0\omega(\rho)]\,. \tag{25}$$

Then, integrating by parts the first term of this integral, we immediately arrive at

$$\delta S_2 = 0\,. \tag{26}$$

Finally, it is easy to check that,

$$\delta S_3 = \lambda\int d^3x[\delta\omega(\rho)\rho + \omega(\rho)\delta\rho] = a\lambda\int d^3x\,\partial_0[\omega(\rho)\rho] = 0\,, \tag{27}$$

where we have supposed the boundary condition

$$\lim_{t,x\to\infty}\phi = 0\,. \tag{28}$$

Thus, the model is invariant under time translation. Space translation involves

$$\delta\omega(\rho) = a_i\partial_i\omega(\rho)\,. \tag{29}$$

Then we have,

$$\delta S_1 = \int d^3x[ia_i\partial_i\omega(\rho)\phi^\dagger\partial_0\phi - ia_i\omega(\rho)(\partial_i\phi^\dagger\partial_0\phi + \phi^\dagger\partial_0\partial_i\phi)]\,. \tag{30}$$

Integrating by parts the last term of this integral, we get

$$\delta S_1 = 0\,. \tag{31}$$

The variation with respect to S_2 is

$$\delta S_2 = -\,\frac{1}{2m}\int d^3x[(a_i\partial_i^2\phi^\dagger\partial_i\phi + a_i\partial_i\phi^\dagger\partial_i^2\phi)\omega(\rho) + |\partial_i\phi|^2a_i\partial_i\omega(\rho)]\,. \tag{32}$$

It can be easily seen that integrating by parts the first term, the variation becomes zero.

For S_3, we have

$$\delta S_3 = \lambda\int d^3x[\delta\omega(\rho)\rho + \omega(\rho)\delta\rho] = a_i\lambda\int d^3x\,\partial_i[\omega(\rho)\rho] = 0\,. \tag{33}$$

The model is also invariant under rotations. Indeed, we have from (7)

$$\delta\phi = x_1\partial_2\phi - x_2\partial_1\phi\,, \tag{34}$$

such that,

$$\delta\omega(\rho) = \frac{\partial_\omega}{\partial\rho}(x_1\partial_2\rho - x_2\partial_1\rho) = x_1\partial_2\omega(\rho) - x_2\partial_1\omega(\rho)\,, \tag{35}$$

$$\begin{aligned}\delta S_1 = i\int d^3x[[x_1\partial_2\omega(\rho) - x_2\partial_1\omega(\rho)]\phi^\dagger\partial_0\phi + \omega(\rho)\partial_0\phi[x_1\partial_2\phi^\dagger - x_2\partial_1\phi^\dagger]\\ + \omega(\rho)\phi^\dagger\partial_0[x_1\partial_2\phi - x_2\partial_1\phi]]\,.\end{aligned} \tag{36}$$

Integrating by parts, in x_1 and x_2, the last term of this variation, we easily arrive at

$$\delta S_1 = 0 \,. \tag{37}$$

Variation with respect to S_2 requires a bit more attention. By using (34) and (35), we have

$$\delta S_2 = -\frac{1}{2m}\int d^3x[\partial_i(x_1\partial_2\phi^\dagger - x_2\partial_1\phi^\dagger)\partial_i\phi\omega(\rho) + \omega(\rho)\partial_i\phi^\dagger\partial_i(x_1\partial_2\phi - x_2\partial_1\phi) + |\partial_i\phi|^2[x_1\partial_2\omega(\rho) - x_2\partial_1\omega(\rho)]] \,. \tag{38}$$

Developing the first two terms of this integral and after some algebra we can check that $\delta S_2 = 0$,

The invariance under rotation of the model is completed by the variation of S_3,

$$\begin{aligned}\delta S_3 = \lambda\int d^3x\,\delta[\omega(\rho)\rho] &= \lambda\int d^3x[x_1\partial_2\omega(\rho) - x_2\partial_1\omega(\rho)]\rho + \omega(\rho)[x_1\partial_2\rho - x_2\partial_1\rho] \\ &= \lambda\int d^3x[\partial_2[\omega(\rho)\rho x_1] - \partial_1[\omega(\rho)\rho x_2]] = 0 \,.\end{aligned} \tag{39}$$

Let us concentrate on the Galilean boost,

$$\delta\phi = (imv_ix_i - tv_i\partial_i)\phi \,. \tag{40}$$

Under this transformation, the variation of $\omega(\rho)$ is

$$\delta\omega(\rho) = \frac{\partial\omega}{\partial\rho}\delta\rho = -tv_i\partial_i\rho\frac{\partial\omega}{\partial\rho} = -tv_i\partial_i\omega(\rho) \,. \tag{41}$$

Thus, we have for S_1 the following variation:

$$\delta S_1 = i\int d^3x[-tv_i\partial_i\omega(\rho)\phi^\dagger\partial_0\phi - \omega(\rho)tv_i\partial_i\phi^\dagger\partial_0\phi - \omega(\rho)\phi^\dagger tv_i\partial_i\partial_0\phi] \,. \tag{42}$$

Integrating by parts the last term, it is easy to check that $\delta S_1 = 0$.

The variation of S_2 may be evaluated by using (40) and (41), so that

$$\delta S_2 = -\frac{1}{2m}\int d^3x[-tv_i\partial_i\omega(\rho)|\partial_i\phi|^2 + (-tv_i\partial_i^2\phi^\dagger\partial_i\phi - tv_i\partial_i\phi^\dagger\partial_i^2\phi)\omega(\rho)] \,, \tag{43}$$

which vanish after integrating by parts the last term of this integral.

The S_3 is also invariant under Galilean boost. Indeed, we have

$$\delta S_3 = \lambda\int d^3x[-tv_i\rho\partial_i\omega(\rho) - tv_i\omega(\rho)\partial_i\phi^\dagger\phi + tv_i\omega(\rho)\phi^\dagger\partial_i\phi] \,, \tag{44}$$

where the last term may be integrated by parts, arriving to $\delta S_3 = 0$.

Finally, the U(1) invariance of (17) is automatically satisfied, since $\omega(\rho)$ and $\mathcal{L}_{\rm NR}$ are U(1) invariant, and then

$$\delta S = \int d^3x(\delta\omega(\rho)\mathcal{L}_{\rm NR} + \omega(\rho)\delta\mathcal{L}_{\rm NR}) = 0\,. \tag{45}$$

Using the Noether, it is not difficult to obtain the conserved charges associated to the Galilean symmetries. In particular, we arrive at the following quantities:

$$\begin{aligned} H &= \int d^2x\, j_0\, dx^2 = \int d^2x\, i\phi^\dagger \partial_0\phi\omega(\rho) - \mathcal{L} \\ &= \int d^2x\left(\frac{1}{2m}|\partial_i\phi|^2 + \lambda|\phi|^2\right)\omega(\rho)\,, \end{aligned} \tag{46}$$

which is the Hamiltonian of the model (17).

$$P_i = \frac{i}{2}\int d^2x[(\phi^\dagger\partial_i\phi - \partial_i\phi^\dagger\phi)\omega(\rho) - \rho\partial_i\omega(\rho)]\,. \tag{47}$$

This is the conserved charge associated to space translations, which differs from the usual non-relativistic P_i in the fact that here, we have the function $\omega(\rho)$ multiplying the term $\phi^\dagger\partial_i\phi - \partial_i\phi^\dagger\phi$.

$$J = \int d^2x(-\mathcal{P}_1x_2 + \mathcal{P}_2x_1) \tag{48}$$

is the usual expression of the angular momentum.

$$G = \int d^2x(-mx_i\rho + \mathcal{P}_i t)\omega(\rho)\,, \tag{49}$$

which differs from (11) only on the factor $\omega(\rho)$.

$$N = -\alpha\int d^2x\,\omega(\rho)\rho \tag{50}$$

is a generalization of the usual mass operator.

3. The Galilean Algebra

In this section, we shall study the algebra of the generators associate to the symmetry transformations studied in Sec. 2. We have seen in Sec. 2 that the algebra of the Galilean group is realized by the Poisson bracket (14). Also, the Poisson bracket (14) implies the commutation relation (15), which is the fundamental relation to construct the algebra (16). The commutator (15) is the usual commutator between the fundamental field of the theory and its canonical conjugate, which is usually defined as

$$\pi = \frac{\partial\mathcal{L}}{\partial(\partial_0\phi)} = i\phi^\dagger\,, \tag{51}$$

so that,

$$[\phi(x), \pi(x')] = \delta^2(x - x')\,. \tag{52}$$

However, if we apply this commutation relation to construct the Galilean algebra of the model (17), it is not difficult to see that we cannot construct the Galilean algebra (16). For instance, we can check easily that,

$$[P_i, H] \neq 0\,, \tag{53}$$

where P_i and H are given by the expressions (46) and (47). The problem lies in the fact that, here, π is not $i\phi^\dagger$. Indeed,

$$\pi = \frac{\partial \mathcal{L}}{\partial(\partial_0 \phi)} = i\phi^\dagger \omega(\rho)\,, \tag{54}$$

so that π is a function of $\phi^\dagger$ and ϕ and therefore the definition (14) of the Poisson bracket does not apply. For this reason, we must redefine the theory in terms of new fundamental fields. In general, this is difficult due to the arbitrariness of the function $\omega(\rho)$. However, if we choose

$$\omega(\rho) = \rho^n\,, \tag{55}$$

where n is an arbitrary positive real number, we can rewrite the model (17) as follows

$$\begin{aligned} S &= \int d^3x \left(i\phi^\dagger \partial_0 \phi - \frac{1}{2m}|\partial_i \phi|^2 + \lambda\rho \right)\omega(\rho) \\ &= \int d^2x \left(i\phi^\dagger \partial_0 \phi \rho^n - \frac{1}{2m}|\partial_i \phi|^2 \rho^n + \lambda \rho^{n+1} \right) \\ &= \int d^3x \left(i(\phi^{n+1})^\dagger \phi^n \partial_0 \phi - \frac{1}{2m}\phi^n \partial_i \phi (\phi^n)^\dagger \partial_i \phi^\dagger + \lambda (\phi^{n+1})^\dagger \phi^{n+1} \right) \\ &= \int d^3x \left(\frac{i}{n+1}(\phi^{n+1})^\dagger \partial_0 \phi^{n+1} - \frac{1}{2m(n+1)^2}\partial_i(\phi^{n+1})\partial_i(\phi^{n+1})^\dagger \right. \\ &\qquad \left. + \lambda (\phi^{n+1})^\dagger \phi^{n+1} \right). \end{aligned} \tag{56}$$

From (56), it is natural to define new fields, such that

$$\psi = \phi^{n+1}\,, \qquad \psi^\dagger = (\phi^\dagger)^{n+1}\,. \tag{57}$$

Thus, the action (56) is rewritten as

$$S = \int d^3x \left(\frac{i}{n+1}\psi^\dagger \partial_0 \psi - \frac{1}{2m(n+1)^2}\partial_i(\psi)\partial_i(\psi)^\dagger + \lambda \psi^\dagger \psi \right). \tag{58}$$

Comparing this action with the non-relativistic action (2), we see immediately that both are very similar. Then, the canonical conjugate field is

$$\pi = \frac{\partial \mathcal{L}}{\partial(\partial_0 \psi)} = \frac{i}{n+1}\psi^\dagger \tag{59}$$

and we can define the Poisson bracket, in terms of the new fields, following the definition (14),

$$\{F,G\}_{\rm PB} = i\int d^2x\left(\frac{\delta F}{\delta\psi^\dagger}\frac{\delta G}{\delta\psi} - \frac{\delta F}{\delta\psi}\frac{\delta G}{\delta\psi^\dagger}\right). \qquad (60)$$

In particular, if $F=\psi$ and $G=\psi^\dagger$, we recover the usual commutation relation between the fundamental field and its canonical conjugate,

$$\{\psi,\psi^\dagger\}_{\rm PB} = [\psi,\psi^\dagger] = i\int d^2x(-\delta^2(x-x')\delta^2(x-x')) = -i\delta^2(x-x')\,. \qquad (61)$$

We can proceed in the same way as with the action and check that the conserved charges (46)–(50), written in terms of the fields ψ and $\psi^\dagger$, are identical to the conserved charges of the model (2). Thus, the conserved charges written in terms of ψ and $\psi^\dagger$ as well as the commutation relation between ψ and $\psi^\dagger$ lead us to the similar context of the non-relativistic case analyzed in Sec. 2. Therefore, it is easy to understand that the generalized model (17), with $\omega(\rho)=\rho^n$ satisfies the algebra of the Galilean group expressed in (16).

4. Gauged Model

Let us consider the model, in which Higgs field is coupled to a gauge field $A_\mu(x)$,

$$\begin{aligned} S &= S_A + \int d^3x\,\omega(\rho)\mathcal{L}_{\rm NR} \\ &= S_A + \int d^3x\,\omega(\rho)\left(i\phi^\dagger D_0\phi - \frac{1}{2m}|D_i\phi|^2 + \lambda|\phi|^4\right), \end{aligned} \qquad (62)$$

where the covariant derivative is

$$D_\mu = \partial_\mu + ieA_\mu\,, \qquad \mu = 0,1,2\,, \qquad (63)$$

and S_A denotes the dynamics of the gauge field. In particular, we will assume that S_A is a $(2+1)$-dimensional Chern–Simons action, given by,

$$S_{\rm CS} = \frac{\kappa}{4}\int d^3x\,\epsilon^{\mu\nu\alpha}A_\mu F_{\nu\alpha} = \kappa\int d^3x(A_0F_{12} + A_2\partial_0A_1)\,. \qquad (64)$$

In the same form as that in the model (17), it is not difficult to see that the model (62) is invariant under time and space translations, angular rotation, Galilean boost and U(1) transformation. In addition, if we choose, $\omega(\rho)=\rho^n$, the model (62) may be rewritten as

$$\begin{aligned} S = \int d^3x\bigg(& i(\phi^{n+1})^\dagger\left[\frac{1}{n+1}\partial_0\phi^{n+1} + ieA_0\phi^{n+1}\right] - \frac{1}{2m}\left(\frac{1}{n+1}\partial_i(\phi^\dagger)^{n+1}\right. \\ & \left. - ieA_i(\phi^\dagger)^{n+1}\right)\left(\frac{1}{n+1}\partial_i\phi^{n+1} + ieA_i\phi^{n+1}\right) + \lambda\rho^{n+2}\bigg) + S_{\rm CS}\,. \end{aligned} \qquad (65)$$

In terms of the fields ψ and $\psi^\dagger$,

$$S = \int d^3x \left(i\psi^\dagger \left[\frac{1}{n+1} \partial_0 \psi + ieA_0\psi \right] - \frac{1}{2m} \left(\frac{1}{n+1} \partial_i \psi^\dagger - ieA_i\psi^\dagger \right) \left(\frac{1}{n+1} \partial_i \psi + ieA_i\psi \right) + \lambda(\psi^\dagger\psi)^2 \right) + S_{\text{CS}} \,. \tag{66}$$

Let us, now, define the action S', such that $S' = (n+1)S$,

$$S' = \int d^3x \left(i\psi^\dagger D_0'\psi - \frac{1}{2m} |D_i'\psi|^2 + \lambda_1(\psi^\dagger\psi)^2 \right) + S'_{\text{CS}} \,, \tag{67}$$

where here, the covariant derivative is defined as

$$D'_\mu \psi = \partial_\mu \psi + ie_1 A_\mu \psi \,, \qquad \mu = 0, 1, 2 \,, \tag{68}$$

the S'_{CS} is

$$S'_{\text{CS}} = \frac{\kappa_1}{4} \int d^3x \, \epsilon^{\mu\nu\alpha} A_\mu F_{\nu\alpha} = \kappa_1 \int d^3x (A_0 F_{12} + A_2 \partial_0 A_1) \tag{69}$$

and the coupling constants e_1, κ_1 and λ_1 are

$$e_1 = e(n+1) \,, \qquad \kappa_1 = \kappa(n+1) \,, \qquad \lambda_1 = \lambda(n+1) \,. \tag{70}$$

Thus, the model (62) may be rewritten in terms of fields ψ and $\psi^\dagger$ as

$$S = \int d^3x \frac{1}{n+1} \left(i\psi^\dagger D_0'\psi - \frac{1}{2m} |D_i'\psi|^2 + \lambda_1(\psi^\dagger\psi)^2 \right) + S_{\text{CS}} \,. \tag{71}$$

This is the well-known Jackiw–Pi model,[27,28] which is Galilean invariant and satisfies the algebra of the formula (14) inherent to the Galilean group. So, the model (62) realizes the Galilean algebra.

5. Twin Models

We can also modify the model (2) by introducing two different dielectric functions

$$S = \int d^3x \left[\omega_1(\rho) \left(i\phi^\dagger \partial_0 \phi - \frac{1}{2m} |\partial_i \phi|^2 \right) + \lambda \omega_2(\rho) |\phi|^2 \right]. \tag{72}$$

Similar to the model (17), it is easy to check that (72) is also Galilean invariant. Indeed, we can rewrite (72) in three separate actions as in formula (18)

$$\begin{aligned} S_1 &= \int d^3x \, \omega_1(\rho) i\phi^\dagger \partial_0 \phi \,, \\ S_2 &= -\int d^3x \, \omega_1(\rho) \frac{1}{2m} |\partial_i \phi|^2 \,, \\ S_3 &= \int d^3x \, \omega_2(\rho) \lambda |\phi|^2 \end{aligned} \tag{73}$$

and as we showed in Sec. 2 each of the three actions are Galilean invariant for an arbitrary dielectric function. So, the actions written in formula (73) are Galilean invariant for arbitrary ω_1 and ω_2.

Again, the model does not satisfy the Galilean algebra for an arbitrary ω_1 and ω_2. However, if we choose $\omega_1 = \rho^n$ and $\omega_2 = \rho^h$, with n and h arbitrary positive real numbers, we can rewrite (72) as

$$S = \int d^3x \left(\frac{i}{n+1} (\phi^\dagger)^{n+1} \partial_0 \phi^{n+1} - \frac{1}{2m(n+1)^2} |\partial_i \phi^{n+1}|^2 + \lambda |\phi|^{2(h+1)} \right). \tag{74}$$

If we define ψ as in (57), we have

$$\phi = \psi^{\frac{1}{n+1}}\,, \tag{75}$$

so that

$$S = \int d^3x \left(\frac{i}{n+1} \psi^\dagger \partial_0 \psi - \frac{1}{2m(n+1)^2} |\partial_i \phi^{n+1}|^2 + \lambda |\phi|^{2\frac{h+1}{n+1}} \right). \tag{76}$$

Writing in this form it is evident that the model (72) satisfies the Galilean algebra. We can proceed in the same way for the gauged model.

Finally, let us concentrate on the solutions of the deformed model (17) . Here, we are interested on the static field configurations that minimize the energy functional associated to the model (17). Thus, for the model (17), we have

$$E = \int d^2x \left(\frac{1}{2m} |\partial_i \phi|^2 + \lambda |\phi|^2 \right) \omega(\rho)\,. \tag{77}$$

In the particular case that we choose the coupling constant to be

$$\lambda = \frac{1}{2m}\,, \tag{78}$$

the theory is governed by the Hamiltonian

$$E = \int d^2x \left(|\partial_i \phi - \phi|^2 + \partial_i \rho \right) \frac{\omega(\rho)}{2m}\,. \tag{79}$$

The last term of this expression may be written as a total derivative if

$$\omega(\rho) = \frac{\partial f}{\partial \rho}\,, \tag{80}$$

which may be assumed without loss of generality. In this way, we have

$$E = \int d^2x \left(|\partial_i \phi - \phi|^2 \frac{\omega(\rho)}{2m} + \partial_i f(\rho) \right). \tag{81}$$

The total derivative may be dropped with the hypothesis that $f(\rho)$ is well-behaved. Then, the energy is bounded below by zero, and this lower bound is saturated by solutions to the first-order self-duality equation

$$\partial_i \phi = \phi\,. \tag{82}$$

The solution of this equation satisfies not only the Euler–Lagrange equation of (77), but also Euler–Lagrange equation of the model (2), i.e.

$$\partial_i^2 \phi = \phi \,. \tag{83}$$

Here, it is important to note that the Euler–Lagrange equation following from the functional (77) is different from that following from the model (2). Thus, we conclude that the solution of Eq. (82) solves the Euler–Lagrange equations of an infinitely large family of theories parametrized by the functional $\omega(\rho)$. This deformation procedure has been used recently by many authors[30–36] to obtain relations between similar models and their solutions.

In summary, we have proposed a generalization of the non-relativistic Schrödinger–Higgs model. We have shown that this generalized model admits Galilean invariance and we have also explored its twin models and their solutions. In addition, we show the Galilean invariance of a generalization of the Jackiw–Pi model.

Acknowledgments

I would like to thank Department of Physics at Univesidad de Buenos Aires for their hospitality. This work is supported by CONICET.

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