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# Forced vibrations of a beam elastically restrained against rotation and carrying a spring–mass system

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## Abstract

An exact solution for the title problem is obtained in closed form fashion in the case of a Bernoulli–Euler beam. It is assumed that the exciting force is applied to the mass which is elastically mounted on the beam. The mathematical model constitutes a first order approximation to a motor or engine elastically mounted on a structural element. The operation of the machine generates a transverse, sinusoidally varying force. The problem is of basic interest in mechanical, naval and ocean engineering systems from the point of view of the determination of dynamic displacements and stresses; sound radiation calculations, etc. The present problem arose in connection with the mounting of an engine on a structural beam in a small naval vessel and when excessive vibrational level was noted. This study was undertaken in order to understand the physical problem and to correct the mechanical situation © 2002 Elsevier Science Ltd. All rights reserved.

*Keywords:* Bernoulli–Euler beam; Elastically mounted mass; Mechanical; Naval and ocean engineering; Dynamic displacements and stresses; Sound radiation calculations

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## 1. Introduction

Consider the structural–mechanical system shown in Fig. 1. Neglecting shear and rotatory inertia effect, the transverse dynamic behavior of the system is described by the partial differential equation (Laura et al., 1977; Jen and Magrab, 1993)

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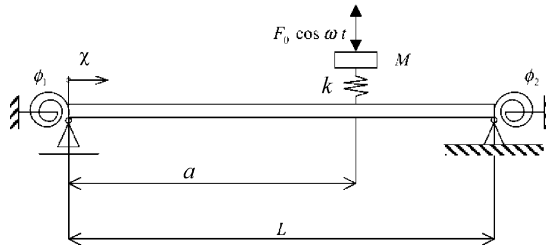


Fig. 1. Mechanical system under study subjected to forced excitation.

$$EI \frac{\partial^4 w}{\partial \bar{x}^4} + \rho A_0 \frac{\partial^2 w}{\partial t^2} + \frac{\omega^2 M W(a) + F_0}{\omega^2 \frac{M}{k} - 1} \cos \omega t \delta(\bar{x} - a) = 0 \tag{1}$$

and appropriate boundary conditions.

From a mathematical viewpoint one can express the displacement function  $w(x, t)$  in terms of the eigenfunctions of the system. Accordingly

$$w(\bar{x}, t) = W(\bar{x}) \cos \omega t = \cos \omega t \sum_{n=1}^{\infty} \Psi_n(\bar{x}) \tag{2}$$

where  $\Psi_n$  define the normal modes of the structure which for the present case are well known. However, the presence of Dirac's delta function on the left-hand side of Eq. (1) considerably complicates the solution. It will be shown in the next section that this difficulty can be circumvented in a straightforward fashion if one splits the structural element into two segments ( $0 \leq \bar{x} \leq a$ ) and ( $a \leq \bar{x} \leq L$ ), see Fig. 1.

## 2. Determination of the closed-form solution

The effect of the concentrated force transmitted through the spring will be treated in terms of a shear compatibility condition at  $\bar{x}=a$ . Accordingly, the dynamic behavior of the segmented portions of the beam is described by

$$\frac{d^4 W_i}{d\bar{x}^4} - \frac{\rho A_0}{EI} \omega^2 W_i = 0 \quad (i=1,2) \tag{3}$$

where  $W_1(\bar{x})$  and  $W_2(\bar{x})$  constitute the displacement amplitudes for ( $0 \leq \bar{x} \leq a$ ) and ( $a \leq \bar{x} \leq L$ ), respectively, and they must comply with the boundary conditions

$$W_1(0) = 0 \tag{4a}$$

$$\left. \frac{dW_1}{d\bar{x}} \right|_{\bar{x}=0} - \phi_1 EI \left. \frac{d^2 W_1}{d\bar{x}^2} \right|_{\bar{x}=0} = 0 \tag{4b}$$

$$W_1(a) = W_2(a) \tag{4c}$$

Table 1  
Simply supported beam;  $m=1, K=1$

$a/L$ ( $\omega_s/\omega_1$ )	$x$	$EIW(x)/F_0L^3$ $\omega/\omega_1$						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.10 (0.10118)	0	0	0	0	0	0	0	0
	0.10	0.1169460	0.0003717	0.0001434	0.0000980	0.0001407	0.0000739	0.0000125
	0.20	0.2022750	0.0006468	0.0002527	0.0001762	0.0002616	0.0001447	0.0000268
	0.30	0.2530430	0.0008143	0.0003224	0.0002298	0.0003523	0.0002044	0.0000408
	0.40	0.2734890	0.0008854	0.0003550	0.0002580	0.0004068	0.0002454	0.0000516
	0.50	0.2678790	0.0008719	0.0003534	0.0002612	0.0004216	0.0002623	0.0000574
	0.60	0.2405040	0.0007864	0.0003216	0.0002410	0.0003965	0.0002527	0.0000570
	0.70	0.1956800	0.0006421	0.0002646	0.0002004	0.0003344	0.0002169	0.0000500
	0.80	0.1377470	0.0004532	0.0001877	0.0001433	0.0002416	0.0001586	0.0000370
	0.90	0.0710626	0.0002342	0.0000973	0.0000746	0.0001266	0.0000837	0.0000197
1	0	0	0	0	0	0	0	
0.30 (0.10058)	0	0	0	0	0	0	0	0
	0.10	0.5137140	0.0008024	0.0003172	0.0002245	0.0003321	0.0002163	0.0000414
	0.20	0.9658730	0.0015098	0.0005978	0.0004242	0.0006299	0.0004128	0.0000797
	0.30	1.2949700	0.0020279	0.0008060	0.0005754	0.0008620	0.0005721	0.0001125
	0.40	1.4541300	0.0022848	0.0009143	0.0006597	0.0010036	0.0006802	0.0001376
	0.50	1.4548100	0.0022944	0.0009251	0.0006755	0.0010446	0.0007235	0.0001506
	0.60	1.3231300	0.0020940	0.0008503	0.0006276	0.0009847	0.0006950	0.0001480
	0.70	1.0853600	0.0017228	0.0007037	0.0005239	0.0008318	0.0005958	0.0001292
	0.80	1.0853600	0.0012215	0.0005011	0.0003755	0.0006013	0.0004352	0.0000956
	0.90	0.3972590	0.0006328	0.0002603	0.0001958	0.0003152	0.0002296	0.0000508
1	0	0	0	0	0	0	0	
0.50 (0.10027)	0	0	0	0	0	0	0	0
	0.10	1.1509700	0.0008527	0.0003447	0.0002520	0.0003842	0.0002884	0.0000591
	0.20	2.2084200	0.0016350	0.0006599	0.0004814	0.0007320	0.0005476	0.0001117
	0.30	3.0789400	0.0022772	0.0009172	0.0006670	0.0010100	0.0007517	0.0001523
	0.40	3.6693500	0.0027109	0.0010895	0.0007896	0.0011904	0.0008811	0.0001773
	0.50	3.8867400	0.0028698	0.0011520	0.0008335	0.0012535	0.0009249	0.0001855
	0.60	3.6693500	0.0027109	0.0010895	0.0007896	0.0011904	0.0008811	0.0001773
	0.70	3.0789400	0.0022772	0.0009172	0.0006670	0.0010100	0.0007517	0.0001523
	0.80	2.2084200	0.0016350	0.0006599	0.0004814	0.0007320	0.0005476	0.0001117
	0.90	1.1509700	0.0008527	0.0003447	0.0002520	0.0003842	0.0002884	0.0000591
1	0	0	0	0	0	0	0	

Table 2

Simply supported beam;  $m=1, K=1$ 

$a/L$ ( $\omega_s/\omega_1$ )	$x$	$M(x)/F_0L$ $\omega/\omega_1$						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.10 (0.10118)	0	0	0	0	0	0	0	0
	0.10	3.8783700	0.0118111	0.0041303	0.0023273	0.0022041	0.0001706	0.0002762
	0.20	3.4563100	0.0107571	0.0039591	0.0024787	0.0030291	0.0011175	0.0000392
	0.30	3.0323000	0.0096472	0.0037272	0.0025470	0.0036497	0.0018950	0.0003104
	0.40	2.6058600	0.0084666	0.0034175	0.0025067	0.0039947	0.0024335	0.0005150
	0.50	2.1767700	0.0072090	0.0030221	0.0023442	0.0040213	0.0026850	0.0006353
	0.60	1.7450800	0.0058755	0.0025412	0.0020580	0.0037180	0.0026299	0.0006620
	0.70	1.3110700	0.0044734	0.0019825	0.0016576	0.0031042	0.0022797	0.0005958
	0.80	0.8751690	0.0030154	0.0013598	0.0011621	0.0022286	0.0016759	0.0004481
	0.90	0.4379290	0.0015179	0.0006917	0.0005988	0.0011638	0.0008869	0.0002401
1	0	0	0	0	0	0	0	
0.30 (0.10058)	0.	0	0	0	0	0	0	0
	0.10	6.1560300	0.0095067	0.0036668	0.0024887	0.0034396	0.0020054	0.0003155
	0.20	12.3071000	0.0189437	0.0072572	0.0048712	0.0066195	0.0037577	0.0005633
	0.30	18.4489000	0.0282497	0.0107034	0.0070532	0.0093068	0.0050272	0.0006806
	0.40	15.8499000	0.0247336	0.0097572	0.0068821	0.0101235	0.0065407	0.0012362
	0.50	13.2369000	0.0210189	0.0085904	0.0063988	0.0101551	0.0072590	0.0015669
	0.60	10.6098000	0.0171047	0.0072001	0.0055956	0.0093691	0.0071316	0.0016518
	0.70	7.9699000	0.0130081	0.0056041	0.0044952	0.0078123	0.0061921	0.0014954
	0.80	5.3195100	0.0087614	0.0038379	0.0031464	0.0056043	0.0045565	0.0011283
	0.90	2.6616800	0.0044083	0.0019504	0.0016196	0.0029255	0.0024124	0.0006055
1	0	0	0	0	0	0	0	
0.50 (0.10027)	0.	0	0	0	0	0	0	0
	0.10	9.3526600	0.0070506	0.0029476	0.0022615	0.0036643	0.0029520	0.0006534
	0.20	18.6942000	0.0140270	0.0058119	0.0044036	0.0070279	0.0055669	0.0012105
	0.30	28.0143000	0.0208610	0.0085166	0.0063178	0.0098185	0.0075418	0.0015853
	0.40	37.3047000	0.0274969	0.0109997	0.0079161	0.0118187	0.0086381	0.0017116
	0.50	46.5597000	0.0338972	0.0132197	0.0091406	0.0128874	0.0087044	0.0015483
	0.60	37.3047000	0.0274969	0.0109997	0.0079161	0.0118187	0.0086381	0.0017116
	0.70	28.0143000	0.0208610	0.0085166	0.0063178	0.0098185	0.0075418	0.0015853
	0.80	18.6942000	0.0140270	0.0058119	0.0044036	0.0070279	0.0055669	0.0012105
	0.90	9.3526600	0.0070506	0.0029476	0.0022615	0.0036643	0.0029520	0.0006534
1	0	0	0	0	0	0	0	

$$\left. \frac{dW_1}{d\bar{x}} \right|_{\bar{x}=a} = \left. \frac{dW_2}{d\bar{x}} \right|_{\bar{x}=a} \tag{4d}$$

$$\left. \frac{d^2W_1}{d\bar{x}^2} \right|_{\bar{x}=a} = \left. \frac{d^2W_2}{d\bar{x}^2} \right|_{\bar{x}=a} \tag{4e}$$

$$-EI \left. \frac{d^3W_1}{d\bar{x}^3} \right|_{\bar{x}=a} + EI \left. \frac{d^3W_2}{d\bar{x}^3} \right|_{\bar{x}=a} + \frac{\omega^2 MW(a) + F_0}{\omega^2 \frac{M}{k} - 1} = 0 \tag{4f}$$

$$W_2(L) = 0 \tag{4g}$$

$$\left. \frac{dW_2}{d\bar{x}} \right|_{\bar{x}=L} + \phi_2 EI \left. \frac{d^2W_2}{d\bar{x}^2} \right|_{\bar{x}=L} = 0 \tag{4h}$$

Eqs. (4a) and (4h) are the constitutive relations which define  $\phi_1$  and  $\phi_2$ , respectively. Finally, the solution of Eq. (3) results

$$W_i(x) = A_i \cos \beta x + B_i \sin \beta x + C_i \cosh \beta x + D_i \sinh \beta x \tag{5}$$

where  $x = \bar{x}/L$  and  $\beta^2 = \sqrt{(\rho A_0/EI)\omega L^2}$ . Substituting Eq. (5) into the boundary conditions Eq. (4a–h) one determines  $A_i$ ,  $B_i$ ,  $C_i$  and  $D_i$  as a function of all the participating parameters.

### 3. Numerical results

Values of dimensionless vibration amplitudes  $\left( \frac{W}{F_0 L^3/EI} \right)$  and bending moments  $(M/F_0 L)$  have been obtained as a function of  $a/L$ ,  $m = M/\rho A_0 L = M/M_b$ ,  $K = k/EI/L^3$  and  $\omega/\omega_1$ , where  $\omega_1$  is the fundamental circular frequency of the “bare” structural element; see Tables 1–18. Values of  $\omega_{1s}/\omega_1$  are indicated in each case, where  $\omega_{1s}$  is the fundamental natural frequency of the whole system (structure–spring–mass). Clearly, when  $\omega/\omega_1 = 1$  one does not have the resonance condition since the natural frequencies of the overall system do not coincide with the natural frequencies of the “bare” structure. This will become obvious from the analysis of the data obtained for the following:

- simply supported

$$\left( \Omega_1 = \sqrt{\frac{\rho h}{EI}} \omega_1 L^2 = (\pi)^2 \right)$$

Table 3  
Simply supported beam;  $m=0.10$ ,  $K=2$

$a/L$ ( $\omega_s/\omega_1$ )	$x$	$EIW(x)/F_0L^3$ $\omega/\omega_1$						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.10 (0.45167)	0	0	0	0	0	0	0	0
	0.10	0.0028602	0.0051807	0.1488573	0.0032652	0.0036551	0.0017857	0.0002842
	0.20	0.0049471	0.0090138	0.0262260	0.0058742	0.0067949	0.0034971	0.0006083
	0.30	0.0061887	0.0113484	0.0334628	0.0076593	0.0091524	0.0049418	0.0009244
	0.40	0.0066888	0.0123393	0.0368381	0.0085985	0.0105677	0.0059332	0.0011713
	0.50	0.0065516	0.0121511	0.0366740	0.0087054	0.0109523	0.0063422	0.0013029
	0.60	0.0058820	0.0109592	0.0333810	0.0080344	0.0102995	0.0061082	0.0012922
	0.70	0.0047858	0.0089492	0.0274574	0.0066806	0.0086876	0.0052444	0.0011331
	0.80	0.0033689	0.0063164	0.0194817	0.0047769	0.0062750	0.0038347	0.0008404
	0.90	0.0017380	0.0032639	0.0100988	0.0024878	0.0032876	0.0020236	0.0004471
1	0	0	0	0	0	0	0	
0.30 (0.44516)	0	0	0	0	0	0	0	0
	0.10	0.0061964	0.0115871	0.0285811	0.0071044	0.0078993	0.0056999	0.0009596
	0.20	0.0116503	0.0218022	0.0538642	0.0134241	0.0149855	0.0108770	0.0018473
	0.30	0.0156198	0.0292835	0.0726192	0.0182077	0.0205068	0.0150746	0.0026071
	0.40	0.0175396	0.0329920	0.0823754	0.0208765	0.0238749	0.0179222	0.0031887
	0.50	0.0175478	0.0331313	0.0833569	0.0213757	0.0248503	0.0190639	0.0034883
	0.60	0.0159595	0.0302379	0.0766150	0.0198587	0.0234261	0.0183116	0.0034296
	0.70	0.0130916	0.0248767	0.0634013	0.0165788	0.0197885	0.0156980	0.0029929
	0.80	0.0092624	0.0176390	0.0451510	0.0118832	0.0143052	0.0114682	0.0022137
	0.90	0.0047917	0.0091375	0.0234518	0.0061966	0.0074981	0.0060489	0.0011762
1	0	0	0	0	0	0	0	
0.50 (0.44186)	0	0	0	0	0	0	0	0
	0.10	0.0065639	0.0125439	0.0290543	0.0077597	0.0087401	0.0081171	0.0013900
	0.20	0.0125944	0.0240515	0.0556298	0.0148260	0.0166523	0.0154113	0.0026282
	0.30	0.0175590	0.0334975	0.0773172	0.0205419	0.0229766	0.0211540	0.0035851
	0.40	0.0209260	0.0398772	0.0918409	0.0243201	0.0270822	0.0247950	0.0041738
	0.50	0.0221658	0.0422150	0.0971107	0.0256699	0.0285169	0.0260293	0.0043653
	0.60	0.0209260	0.0398772	0.0918409	0.0243201	0.0270822	0.0247950	0.0041738
	0.70	0.0175590	0.0334975	0.0773172	0.0205419	0.0229766	0.0211540	0.0035851
	0.80	0.0125944	0.0240515	0.0556298	0.0148260	0.0166523	0.0154113	0.0026282
	0.90	0.0065639	0.0125439	0.0290543	0.0077597	0.0087401	0.0081171	0.0013900
1	0	0	0	0	0	0	0	

Table 4

Simply supported beam;  $m=0.10$ ,  $K=2$ 

$a/L$ ( $\omega_s/\omega_1$ )	$x$	$M(x)/F_0L$ $\omega/\omega_1$						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.10 (0.45167)	0	0	0	0	0	0	0	0
	0.10	0.0948538	0.1646068	0.4286535	0.0775772	0.0572554	0.0041241	0.0062653
	0.20	0.0845313	0.1499175	0.4108888	0.0826251	0.0786865	0.0270160	0.0008888
	0.30	0.0741613	0.1344490	0.3868190	0.0849020	0.0948078	0.0458123	0.0070426
	0.40	0.0063732	0.1179954	0.3546796	0.0835568	0.1037701	0.0588288	0.0116844
	0.50	0.0532374	0.1004686	0.3136413	0.0781407	0.1044622	0.0649097	0.0144138
	0.60	0.0426797	0.0818839	0.2637355	0.0686005	0.0965813	0.0635790	0.0150181
	0.70	0.0320651	0.0623444	0.2057542	0.0552527	0.0806374	0.0551108	0.0135156
	0.80	0.0214041	0.0420249	0.1411282	0.0387381	0.0578917	0.0405153	0.0101663
	0.90	0.0107105	0.0211548	0.0717864	0.0199588	0.0302329	0.0214397	0.0054474
1	0	0	0	0	0	0	0	
0.30 (0.44516)	0	0	0	0	0	0	0	0
	0.10	0.0742535	0.1372771	0.3303889	0.0787508	0.0818276	0.0528400	0.0073107
	0.20	0.1484473	0.2735484	0.6538846	0.1541419	0.1574762	0.0990135	0.0130517
	0.30	0.2225297	0.4079283	0.9643959	0.2231869	0.2214044	0.1324636	0.0157694
	0.40	0.1911800	0.3571548	0.8791458	0.2177723	0.2408346	0.1723454	0.0286414
	0.50	0.1596620	0.3035149	0.7740137	0.2024797	0.2415851	0.1912717	0.0363031
	0.60	0.1279743	0.2469927	0.6487393	0.1770649	0.2228869	0.1879160	0.0382720
	0.70	0.0961323	0.1878376	0.5049389	0.1422417	0.1858512	0.1631551	0.0346472
	0.80	0.0641635	0.1265153	0.3458012	0.0995618	0.1333240	0.1200623	0.0261429
	0.90	0.0321049	0.0636558	0.1757383	0.0512496	0.0695973	0.0635648	0.0140300
1	0	0	0	0	0	0	0	
0.50 (0.44186)	0	0	0	0	0	0	0	0
	0.10	0.0533349	0.1037158	0.2484766	0.0696521	0.0833621	0.0830751	0.0153777
	0.20	0.1066115	0.2063395	0.4899282	0.1356282	0.1598828	0.1566644	0.0284880
	0.30	0.1597641	0.3068697	0.7179320	0.1945816	0.2233695	0.2122424	0.0373106
	0.40	0.2127461	0.4044856	0.9272529	0.2438074	0.2688738	0.2430948	0.0402824
	0.50	0.2655264	0.4986351	1.1143964	0.2815220	0.2931862	0.2449605	0.0364381
	0.60	0.2127461	0.4044856	0.9272529	0.2438074	0.2688738	0.2430948	0.0402824
	0.70	0.1597641	0.3068697	0.7179320	0.1945816	0.2233695	0.2122424	0.0373106
	0.80	0.1066115	0.2063395	0.4899282	0.1356282	0.1598828	0.1566644	0.0284880
	0.90	0.0533375	0.1037158	0.2484766	0.0696521	0.0833621	0.0830751	0.0153777
1	0	0	0	0	0	0	0	

Table 5

Simply supported beam;  $m=0.20$ ,  $K=1$ 

$a/L$ ( $\omega_s/\omega_1$ )	$x$	$EIW(x)/F_0L^3$ $\omega/\omega_1$						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.10 (0.22624)	0	0	0	0	0	0	0	0
	0.10	0.00338047	0.00381919	0.00086476	0.00053538	0.00074115	0.00038253	0.00006421
	0.20	0.00584701	0.00664492	0.00152356	0.00096318	0.00137783	0.00074916	0.00013744
	0.30	0.00731451	0.00836596	0.00194397	0.00125588	0.00185586	0.00105864	0.00020884
	0.40	0.00790554	0.00909645	0.00214005	0.00140986	0.00214286	0.00127102	0.00026462
	0.50	0.00774339	0.00895776	0.00213052	0.00142740	0.00222085	0.00135862	0.00029437
	0.60	0.00695207	0.00807906	0.00193921	0.00131738	0.00208847	0.00130850	0.00029195
	0.70	0.00565637	0.00659731	0.00159509	0.00109540	0.00176161	0.00112347	0.00025600
	0.80	0.00398173	0.00465645	0.00113176	0.00078326	0.00127240	0.00082147	0.00018986
	0.90	0.00205415	0.00240611	0.00058667	0.00040791	0.00066663	0.00043349	0.00010102
1	0	0	0	0	0	0	0	
0.30 (0.22483)	0	0	0	0	0	0	0	0
	0.10	0.00733603	0.00811934	0.00190614	0.00122451	0.00174369	0.00112261	0.00021236
	0.20	0.01379300	0.01527730	0.00359232	0.00231377	0.00330790	0.00214225	0.00040883
	0.30	0.01849260	0.02051960	0.00484314	0.00313827	0.00452666	0.00296897	0.00057698
	0.40	0.02076550	0.02311820	0.00549380	0.00359825	0.00527013	0.00352982	0.00070570
	0.50	0.02077520	0.02321590	0.00555925	0.00368430	0.00548544	0.00375468	0.00077200
	0.60	0.01889480	0.02118840	0.00510963	0.00342283	0.00517107	0.00360650	0.00075901
	0.70	0.01549940	0.01743170	0.00422837	0.00285752	0.00436809	0.00309175	0.00066235
	0.80	0.01096600	0.01236010	0.00301122	0.00204818	0.00315773	0.00225868	0.00048992
	0.90	0.00567301	0.00640283	0.00156405	0.00106804	0.00165514	0.00119135	0.00026031
1	0	0	0	0	0	0	0	
0.50 (0.22412)	0	0	0	0	0	0	0	0
	0.10	0.00777789	0.00856250	0.00206752	0.00137255	0.00201410	0.00149906	0.00030295
	0.20	0.01492380	0.01641770	0.00395864	0.00262243	0.00383742	0.00284616	0.00057279
	0.30	0.02080660	0.02286550	0.00550193	0.00363346	0.00529481	0.00390673	0.00078134
	0.40	0.02479640	0.02722040	0.00653544	0.00430176	0.00624094	0.00457914	0.00090963
	0.50	0.02626540	0.02881610	0.00691044	0.00454051	0.00657154	0.00480710	0.00095138
	0.60	0.02479640	0.02722040	0.00653544	0.00430176	0.00624094	0.00457914	0.00090963
	0.70	0.02080660	0.02286550	0.00550193	0.00363346	0.00529481	0.00390673	0.00078134
	0.80	0.01492380	0.01641770	0.00395864	0.00262243	0.00383742	0.00284616	0.00057279
	0.90	0.00777789	0.00856250	0.00206752	0.00137255	0.00201410	0.00149906	0.00030295
1	0	0	0	0	0	0	0	



Table 6  
Simply supported beam;  $m=0.20$ ,  $K=1$

$a/L$ ( $\omega_s/\omega_1$ )	$x$	$M(x)/F_0L$ $\omega/\omega_1$						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.10 (0.22624)	0	0	0	0	0	0	0	0
	0.10	0.1121090	0.1213470	0.0249019	0.0127201	0.0116099	0.0008835	-0.0014155
	0.20	0.0999089	0.1105180	0.0238699	0.0135478	0.0159556	0.0057874	0.0002008
	0.30	0.0876524	0.0991151	0.0224716	0.0139211	0.0192246	0.0098139	0.0015911
	0.40	0.0753255	0.0869855	0.0206045	0.0137005	0.0210419	0.0126023	0.0026398
	0.50	0.0629221	0.0740649	0.0182205	0.0128125	0.0211822	0.0139050	0.0032565
	0.60	0.0504438	0.0603644	0.0153213	0.0112482	0.0195842	0.0136199	0.0033930
	0.70	0.0378982	0.0459600	0.0119530	0.0090596	0.0163512	0.0118059	0.0030536
	0.80	0.0252978	0.0309806	0.0081986	0.0063518	0.0117389	0.0086792	0.0022969
	0.90	0.0126589	0.0155952	0.0041703	0.0032726	0.0061305	0.0045928	0.0012307
1	0	0	0	0	0	0	0	
0.30 (0.22483)	0	0	0	0	0	0	0	0
	0.10	0.0879103	0.0961932	0.0220344	0.0135734	0.0180626	0.0104069	0.0016179
	0.20	0.1757500	0.1916820	0.0436090	0.0265678	0.0347612	0.0195009	0.0028885
	0.30	0.2634570	0.2858450	0.0643177	0.0384683	0.0488727	0.0260890	0.0034900
	0.40	0.2263420	0.2502660	0.0586322	0.0375351	0.0531617	0.0339438	0.0063387
	0.50	0.1890270	0.2126800	0.0516207	0.0348993	0.0533273	0.0376714	0.0080343
	0.60	0.1515120	0.1730730	0.0432659	0.0305188	0.0491999	0.0370104	0.0084700
	0.70	0.1138130	0.1316220	0.0336755	0.0245167	0.0410247	0.0321348	0.0076678
	0.80	0.0759645	0.0886521	0.0230622	0.0171604	0.0294299	0.0236465	0.0057857
	0.90	0.0380097	0.0446051	0.0117204	0.0088333	0.0153629	0.0125192	0.0031050
1	0	0	0	0	0	0	0	
0.50 (0.22412)	0	0	0	0	0	0	0	0
	0.10	0.0632025	0.0707968	0.0176817	0.0123201	0.0192103	0.0153423	0.0033514
	0.20	0.1263300	0.1408480	0.0348635	0.0239900	0.0368440	0.0289328	0.0062087
	0.30	0.1893130	0.2094710	0.0510884	0.0344177	0.0514742	0.0391969	0.0081315
	0.40	0.2520940	0.2761040	0.0659838	0.0431248	0.0619603	0.0448947	0.0087792
	0.50	0.3146360	0.3403700	0.0793010	0.0497957	0.0675630	0.0452393	0.0079413
	0.60	0.2520940	0.2761040	0.0659838	0.0431248	0.0619603	0.0448947	0.0087792
	0.70	0.1893130	0.2094710	0.0510884	0.0344177	0.0514742	0.0391969	0.0081315
	0.80	0.1263300	0.1408480	0.0348635	0.0239900	0.0368440	0.0289328	0.0062087
	0.90	0.0632025	0.0707968	0.0176817	0.0123201	0.0192103	0.0153423	0.0033514
1	0	0	0	0	0	0	0	

Table 7  
Clamped–simply supported beam;  $m=1$ ,  $K=1$

$a/L$ ( $\omega_s/\omega_1$ )	$x$	$EIW(x)/F_0L^3$ $\omega/\omega_1$						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.10 (0.06485)	0	0	0	0	0	0	0	0
	0.10	0.00019166	0.00001330	0.00000496	0.00000294	0.00000308	0.00000057	-0.00000023
	0.20	0.00041038	0.00002891	0.00001117	0.00000509	0.00000873	0.00000312	0.00000008
	0.30	0.00054516	0.00003899	0.00001559	0.00001054	0.00001464	0.00000690	0.00000095
	0.40	0.00060606	0.00004397	0.00001813	0.00001291	0.00001953	0.00001070	0.00000200
	0.50	0.00060327	0.00004431	0.00001875	0.00001393	0.00002244	0.00001347	0.00000289
	0.60	0.00054711	0.00004062	0.00001756	0.00001348	0.00002273	0.00001450	0.00000336
	0.70	0.00044806	0.00003354	0.00001475	0.00001160	0.00002022	0.00001342	0.00000327
	0.80	0.00031671	0.00002385	0.00001061	0.00000849	0.00001513	0.00001031	0.00000258
	0.90	0.00016376	0.00001238	0.00000555	0.00000449	0.00000809	0.00000560	0.00000143
1	0	0	0	0	0	0	0	
0.50 (0.06456)	0	0	0	0	0	0	0	0
	0.10	0.00059411	0.00004387	0.00001853	0.00001369	0.00002145	0.00001405	0.00000293
	0.20	0.00204526	0.00015086	0.00006360	0.00004684	0.00007311	0.00004769	0.00000988
	0.30	0.00385689	0.00028418	0.00011954	0.00008776	0.00013642	0.00008857	0.00001826
	0.40	0.00553292	0.00040734	0.00017109	0.00012532	0.00019430	0.00012578	0.00002587
	0.50	0.00657817	0.00048449	0.00020366	0.00014941	0.00023223	0.00015093	0.00003124
	0.60	0.00661790	0.00048868	0.00020652	0.00015277	0.00024023	0.00015859	0.00003350
	0.70	0.00575538	0.00042637	0.00018137	0.00013551	0.00021601	0.00014513	0.00003134
	0.80	0.00421454	0.00031308	0.00013392	0.00010089	0.00016263	0.00011082	0.00002434
	0.90	0.00222066	0.00016526	0.00007094	0.00005373	0.00008723	0.00005996	0.00001331
1	0	0	0	0	0	0	0	
0.90 (0.06480)	0	0	0	0	0	0	0	0
	0.10	0.00016330	0.00001236	0.00000554	0.00000448	0.00000804	0.00000563	0.00000143
	0.20	0.00057979	0.00004370	0.00001943	0.00001553	0.00002754	0.00001899	0.00000474
	0.30	0.00113941	0.00008548	0.00003765	0.00002970	0.00005177	0.00003499	0.00000854
	0.40	0.00173222	0.00012922	0.00005628	0.00004367	0.00007453	0.00004911	0.00001163
	0.50	0.00224857	0.00016664	0.00007164	0.00005451	0.00009063	0.00005779	0.00001316
	0.60	0.00257922	0.00018975	0.00008037	0.00005977	0.00009629	0.00005890	0.00001272
	0.70	0.00261545	0.00019089	0.00007954	0.00005763	0.00008942	0.00005192	0.00001042
	0.80	0.00224914	0.00016280	0.00006666	0.00004695	0.00006978	0.00003796	0.00000688
	0.90	0.00137278	0.00009862	0.00003974	0.00002725	0.00003877	0.00001963	0.00000311
1	0	0	0	0	0	0	0	

Table 8  
Clamped–simply supported beam;  $m=1, K=1$

$a/L$ ( $\omega_s/\omega_1$ )	$x$	$M(x)/F_0L$ $\omega/\omega_1$						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.10 (0.06485)	0	-0.06219481	-0.00428366	-0.00157019	-0.00089297	-0.00084118	-0.00004234	-0.00011627
	0.10	0.00939100	0.00058668	0.00016126	0.00002330	-0.00016879	-0.00256061	-0.00009510
	0.20	0.00839484	0.00055240	0.00017882	0.00007046	-0.00002443	-0.00012302	-0.00005628
	0.30	0.00738909	0.00051203	0.00018982	0.00010942	0.00010308	0.00000077	-0.00001796
	0.40	0.00637053	0.00046340	0.00019166	0.00013622	0.00020257	0.00010469	0.00001650
	0.50	0.00533768	0.00040546	0.00018281	0.00014812	0.00026477	0.00017810	0.00004300
	0.60	0.00429060	0.00033810	0.00016291	0.00014394	0.00028418	0.00021319	0.00005802
	0.70	0.00323060	0.00026212	0.00013265	0.00012419	0.00026028	0.00020708	0.00005973
	0.80	0.00216002	0.00017900	0.00009370	0.00009106	0.00019787	0.00016286	0.00004851
	0.90	0.00108194	0.00009082	0.00004849	0.00004812	0.00010664	0.00008937	0.00002707
1	0	0	0	0	0	0	0	
0.50 (0.06456)	0	-0.13538045	-0.01000475	-0.00423341	-0.00313466	-0.00492696	-0.00324019	-0.00067753
	0.10	-0.08570390	-0.00631184	-0.00265269	-0.00194469	-0.00301751	-0.00195436	-0.00040180
	0.20	-0.03604318	-0.00262944	-0.00108429	-0.00077255	-0.00115421	-0.00071363	-0.00013917
	0.30	0.01356820	0.00102021	0.00044576	0.00034428	0.00056646	0.00038821	0.00008329
	0.40	0.06308817	0.00460925	0.00190500	0.00135921	0.00202534	0.00123619	0.00023265
	0.50	0.11247786	0.00811195	0.00326350	0.00222949	0.00311332	0.00172532	0.00027885
	0.60	0.09023818	0.00666000	0.00281177	0.00207694	0.00326032	0.00214849	0.00045322
	0.70	0.06784297	0.00510468	0.00223869	0.00174846	0.00294996	0.00212062	0.00049451
	0.80	0.04531229	0.00345905	0.00155893	0.00126382	0.00222839	0.00168034	0.00041153
	0.90	0.02268231	0.00174705	0.00080035	0.00066288	0.00119722	0.00092531	0.00023211
1	0	0	0	0	0	0	0	
0.90 (0.06480)	0	-0.03633103	-0.00275805	-0.00124305	-0.00101301	-0.00183907	-0.00130249	-0.00033442
	0.10	-0.02531886	-0.00189855	-0.00083550	-0.00065809	-0.00114504	-0.00077208	-0.00018780
	0.20	-0.01431106	-0.00104203	-0.00043165	-0.00030902	-0.00046831	-0.00025970	-0.00004754
	0.30	-0.00331734	-0.00019504	-0.00003956	0.00002165	0.00015463	0.00019743	0.00007352
	0.40	0.00764924	0.00063362	0.00033014	0.00031776	0.00067814	0.00055437	0.00016052
	0.50	0.01857479	0.00143475	0.00066654	0.00056330	0.00105922	0.00077139	0.00020132
	0.60	0.02944725	0.00220048	0.00096071	0.00074590	0.00126747	0.00082400	0.00018991
	0.70	0.04025898	0.00292601	0.00120758	0.00085958	0.00129233	0.00070914	0.00012803
	0.80	0.05100934	0.00361122	0.00140777	0.00090697	0.00114707	0.00044663	0.00002470
	0.90	0.06170724	0.00426225	0.00156905	0.00090055	0.00086929	0.00007600	0.00010619
1	0	0	0	0	0	0	0	

Table 9  
Clamped–simply supported beam;  $m=0.10, K=2$

$a/L$ ( $\omega_s/\omega_1$ )	$x$	$EIW(x)/F_0L^3$ $\omega/\omega_1$						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.10 (0.28998)	0	0	0	0	0	0	0	0
	0.10	0.0002997	0.0038584	0.0001471	0.0000703	0.0000684	0.0000122	0.0000048
	0.20	0.0006418	0.0083855	0.0003309	0.0001696	0.0001937	0.0000668	0.0000016
	0.30	0.0008525	0.0113117	0.0004619	0.0002522	0.0003247	0.0001479	0.0000199
	0.40	0.0009478	0.0127545	0.0005369	0.0003088	0.0004334	0.0002292	0.0000419
	0.50	0.0009434	0.0128553	0.0005555	0.0003332	0.0004978	0.0002886	0.0000607
	0.60	0.0008556	0.0117822	0.0005201	0.0003225	0.0005043	0.0003106	0.0000706
	0.70	0.0007007	0.0097303	0.0004368	0.0002776	0.0004485	0.0002876	0.0000686
	0.80	0.0004953	0.0069199	0.0003143	0.0002033	0.0003357	0.0002209	0.0000543
	0.90	0.0002561	0.0035913	0.0001073	0.0001073	0.0001795	0.0001199	0.0000299
1	0	0	0	0	0	0	0	
0.50 (0.28722)	0	0	0	0	0	0	0	0
	0.10	0.0009457	0.0099230	0.0005365	0.0003199	0.0004515	0.0003167	0.0000623
	0.20	0.0032556	0.0341254	0.0018411	0.0010947	0.0015391	0.0010747	0.0002104
	0.30	0.0061393	0.0642817	0.0034606	0.0020510	0.0028722	0.0019961	0.0003888
	0.40	0.0088071	0.0921418	0.0049528	0.0029289	0.0040906	0.0028348	0.0005507
	0.50	0.0104709	0.1095920	0.0058959	0.0034920	0.0048892	0.0034016	0.0006650
	0.60	0.0105342	0.1105416	0.0059787	0.0035705	0.0050576	0.0035741	0.0007132
	0.70	0.0091612	0.0964458	0.0052505	0.0031670	0.0045477	0.0032709	0.0006672
	0.80	0.0067086	0.0708201	0.0038768	0.0023580	0.0034240	0.0024975	0.0005182
	0.90	0.0035348	0.0373827	0.0020538	0.0012559	0.0001837	0.0013514	0.0002834
1	0	0	0	0	0	0	0	
0.90 (0.28948)	0	0	0	0	0	0	0	0
	0.10	0.0002562	0.0034104	0.0001633	0.0001066	0.0001768	0.0001213	0.0000300
	0.20	0.0009096	0.0120609	0.0005732	0.0003702	0.0006059	0.0004094	0.0000996
	0.30	0.0017876	0.0235893	0.0011107	0.0007077	0.0011389	0.0007544	0.0001795
	0.40	0.0027177	0.0356602	0.0016604	0.0010408	0.0016398	0.0010587	0.0002446
	0.50	0.0035278	0.0459889	0.0021135	0.0012992	0.0019941	0.0012460	0.0002767
	0.60	0.0040466	0.0523661	0.0023711	0.0014246	0.0021185	0.0012699	0.0002674
	0.70	0.0041034	0.0526798	0.0023464	0.0013736	0.0019675	0.0011193	0.0002191
	0.80	0.0035287	0.0449278	0.0019666	0.0011190	0.0015352	0.0008184	0.0001446
	0.90	0.0021538	0.0272176	0.0011725	0.0006494	0.0008531	0.0004232	0.0000654
1	0	0	0	0	0	0	0	

Table 10  
Clamped–simply supported beam;  $m=0.10, K=2$

$a/L$ ( $\omega_s/\omega_1$ )	$x$	$M(x)/F_0L$ $\omega/\omega_1$						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.10 (0.28998)	0	-0.0972610	-1.2426476	-0.0465091	-0.0213662	-0.0186625	-0.0009070	-0.0024410
	0.10	0.0146858	0.1701906	0.0047766	0.0005575	-0.0037448	-0.0054857	-0.0019965
	0.20	0.0131279	0.1602450	0.0052966	0.0016858	-0.0005419	-0.0026355	-0.0011816
	0.30	0.0115551	0.1485338	0.0056226	0.0026182	0.0022870	0.0000164	-0.0003771
	0.40	0.0099623	0.1344291	0.0056769	0.0032594	0.0044943	0.0022429	0.0003464
	0.50	0.0083471	0.1176194	0.0054149	0.0035441	0.0058742	0.0038156	0.0009026
	0.60	0.0067097	0.0980804	0.0048254	0.0034440	0.0063049	0.0045673	0.0012181
	0.70	0.0050521	0.0760380	0.0039292	0.0029716	0.0057747	0.0044365	0.0012540
	0.80	0.0033779	0.0519274	0.0027754	0.0021788	0.0043900	0.0034892	0.0010185
	0.90	0.0016920	0.0263455	0.0014362	0.0011514	0.0023660	0.0019147	0.0005684
1	0	0	0	0	0	0	0	
0.50 (0.28722)	0	-0.2154946	-2.2630967	-0.1225538	-0.0732618	-0.1037294	-0.0730248	-0.0144240
	0.10	-0.1364209	-1.4277524	-0.0767932	-0.0454503	-0.0635290	-0.0440457	-0.0085539
	0.20	-0.0573725	-0.5947858	-0.0313895	-0.0180557	-0.0243001	-0.0160833	-0.0029628
	0.30	0.0215975	0.2307737	0.0129045	0.0080463	0.0119260	0.0087491	0.0017731
	0.40	0.1004219	1.0426213	0.0551482	0.0317669	0.0426404	0.0278603	0.0049529
	0.50	0.1790389	1.8349410	0.0944758	0.0521066	0.0655460	0.0388838	0.0059365
	0.60	0.1436385	1.5065060	0.0813985	0.0485414	0.0686410	0.0484209	0.0096487
	0.70	0.1079904	1.1546891	0.0648082	0.0408642	0.0621069	0.0477928	0.0105278
	0.80	0.0721268	0.7824436	0.0451297	0.0295374	0.0469153	0.0378702	0.0087611
	0.90	0.0361050	0.3951856	0.0231694	0.0154927	0.0252057	0.0208539	0.0049415
1	0	0	0	0	0	0	0	
0.90 (0.28948)	0	-0.0570001	-0.7611556	-0.0366706	-0.0241432	-0.0404629	-0.0280802	-0.0070309
	0.10	-0.0397230	-0.5239534	-0.0246476	-0.0156844	-0.0251929	-0.0166451	-0.0039484
	0.20	-0.0224528	-0.2875743	-0.0127338	-0.0073650	-0.0103037	-0.0055989	-0.0009996
	0.30	-0.0052046	-0.0538268	-0.0011670	0.0005161	0.0034021	0.0042563	0.0015457
	0.40	0.0120010	0.1748641	0.0097392	0.0075732	0.0149203	0.0119516	0.0033749
	0.50	0.0291421	0.3959567	0.0196634	0.0134253	0.0233047	0.0166302	0.0042326
	0.60	0.0462000	0.6072805	0.0283412	0.0177770	0.0278865	0.0177644	0.0039928
	0.70	0.0631627	0.8075089	0.0356240	0.0204865	0.0284336	0.0152881	0.0026917
	0.80	0.0800290	0.9966104	0.0415299	0.0216159	0.0252376	0.0096288	0.0005193
	0.90	0.0968130	1.1762772	0.0462877	0.0214628	0.0191258	0.0016384	0.0022325
1	0	0	0	0	0	0	0	

Table 11  
Clamped–simply supported beam;  $m=0.20, K=1$

$a/L$ ( $\omega_s/\omega_1$ )	$x$	$EIW(x)/F_0L^3$ $\omega/\omega_1$						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.10 (0.14501)	0	0	0	0	0	0	0	0
	0.10	0.0005036	0.0000827	0.0000267	0.0000152	0.0000158	0.0000029	-0.0000012
	0.20	0.0010782	0.0001798	0.0000600	0.0000367	0.0000446	0.0000158	0.0000004
	0.30	0.0014323	0.0002425	0.0000837	0.0000546	0.0000747	0.0000350	0.0000048
	0.40	0.0015923	0.0002735	0.0000973	0.0000669	0.0000998	0.0000542	0.0000101
	0.50	0.0015850	0.0002756	0.0001007	0.0000721	0.0001146	0.0000683	0.0000146
	0.60	0.0014374	0.0002526	0.0000943	0.0000698	0.0001161	0.0000735	0.0000170
	0.70	0.0011772	0.0002086	0.0000791	0.0000601	0.0001032	0.0000681	0.0000165
	0.80	0.0008321	0.0001484	0.0000570	0.0000440	0.0000773	0.0000523	0.0000131
	0.90	0.0004303	0.0000770	0.0000298	0.0000232	0.0000413	0.0000284	0.0000072
1	0	0	0	0	0	0	0	
0.50 (0.14436)	0	0	0	0	0	0	0	0
	0.10	0.0015979	0.0002722	0.0000994	0.0000709	0.0001094	0.0000713	0.0000148
	0.20	0.0055010	0.0009360	0.0003411	0.0002424	0.0003730	0.0002420	0.0000499
	0.30	0.0103736	0.0017631	0.0006411	0.0004543	0.0006960	0.0004494	0.0000922
	0.40	0.0148814	0.0025272	0.0009175	0.0006487	0.0009913	0.0006382	0.0001307
	0.50	0.0176927	0.0030059	0.0010922	0.0007734	0.0011848	0.0007658	0.0001578
	0.60	0.0177996	0.0030319	0.0011075	0.0007908	0.0012256	0.0008046	0.0001692
	0.70	0.0154797	0.0026453	0.0009726	0.0007014	0.0011020	0.0007364	0.0001583
	0.80	0.0113355	0.0019424	0.0007182	0.0005222	0.0008297	0.0005623	0.0001230
	0.90	0.0059727	0.0010253	0.0003805	0.0002781	0.0004450	0.0003043	0.0000672
1	0	0	0	0	0	0	0	
0.90 (0.14489)	0	0	0	0	0	0	0	0
	0.10	0.0004309	0.0000768	0.0000297	0.0000232	0.0000410	0.0000285	0.0000071
	0.20	0.0015299	0.0002717	0.0001043	0.0000804	0.0001406	0.0000963	0.0000238
	0.30	0.0030065	0.0005314	0.0002020	0.0001538	0.0002643	0.0001774	0.0000430
	0.40	0.0045706	0.0008033	0.0003020	0.0002262	0.0003805	0.0002490	0.0000587
	0.50	0.0059331	0.0010360	0.0003845	0.0002823	0.0004628	0.0002931	0.0000665
	0.60	0.0068055	0.0011796	0.0004313	0.0003095	0.0004916	0.0002987	0.0000641
	0.70	0.0069011	0.0011867	0.0004268	0.0002985	0.0004566	0.0002633	0.0000525
	0.80	0.0059346	0.0010121	0.0003577	0.0002431	0.0003563	0.0001925	0.0000346
	0.90	0.0036222	0.0006131	0.0002133	0.0001411	0.0001980	0.0000995	0.0000156
1	0	0	0	0	0	0	0	

Table 12

Clamped–simply supported beam;  $m=0.20$ ,  $K=1$

$a/L$ ( $\omega_1/\omega_1$ )	$x$	$M(x)/F_0L$ $\omega/\omega_1$						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.10 (0.14501)	0	-0.1634041	-0.0266420	-0.0084277	-0.0046250	-0.0042956	-0.0002147	0.0000000
	0.10	0.0246729	0.0036488	0.0008656	0.0001207	-0.0008619	-0.0012984	-0.0000012
	0.20	0.0220557	0.0034356	0.0009598	0.0003649	-0.0001247	-0.0006238	0.0000004
	0.30	0.0194133	0.0031845	0.0010189	0.0005668	0.0005264	0.0000039	0.0000048
	0.40	0.0167373	0.0028821	0.0010287	0.0007056	0.0010344	0.0005309	0.0000101
	0.50	0.0140237	0.0025217	0.0009812	0.0007672	0.0013521	0.0009031	0.0000146
	0.60	0.0112727	0.0021028	0.0008744	0.0007455	0.0014512	0.0010810	0.0000170
	0.70	0.0084878	0.0016302	0.0007120	0.0006432	0.0013292	0.0010501	0.0000165
	0.80	0.0056750	0.0011133	0.0005029	0.0004716	0.0010105	0.0008259	0.0000131
	0.90	0.0028426	0.0005648	0.0002602	0.0002492	0.0005446	0.0004532	0.0000072
1	0	0	0	0	0	0	0	
0.50 (0.14436)	0	-0.3641211	-0.0620715	-0.0227024	-0.0162255	-0.0251362	-0.0164403	-0.0034222
	0.10	-0.2305104	-0.0391599	-0.0142255	-0.0100660	-0.0153947	-0.0099162	-0.0020295
	0.20	-0.0969422	-0.0163136	-0.0058147	-0.0039988	-0.0058885	-0.0036209	-0.0007029
	0.30	0.0364932	0.0063296	0.0023905	0.0017820	0.0028900	0.0019697	0.0004207
	0.40	0.1696828	0.0285967	0.0102159	0.0070355	0.0103328	0.0062723	0.0011751
	0.50	0.3025220	0.0503282	0.0175011	0.0115402	0.0158834	0.0087540	0.0014085
	0.60	0.2427059	0.0413200	0.0150786	0.0107506	0.0166334	0.0109012	0.0022892
	0.70	0.1824714	0.0316705	0.0120054	0.0090503	0.0150500	0.0107598	0.0024978
	0.80	0.1218726	0.0214606	0.0083600	0.0065417	0.0113687	0.0085258	0.0020786
	0.90	0.0610067	0.0108390	0.0042920	0.0034312	0.0061080	0.0046949	0.0011724
1	0	0	0	0	0	0	0	
0.90 (0.14489)	0	-0.0958631	-0.0171460	-0.0066709	-0.0052462	-0.0093900	-0.0066051	-0.0016890
	0.10	-0.0668064	-0.0118027	-0.0044837	-0.0034081	-0.0058464	-0.0039153	-0.0009485
	0.20	-0.0377612	-0.0064780	-0.0023164	-0.0016004	-0.0023911	-0.0013170	-0.0002401
	0.30	-0.0087531	-0.0012125	-0.0002123	0.0001121	0.0007895	0.0010012	0.0003713
	0.40	0.0201833	0.0039390	0.0017717	0.0016456	0.0034625	0.0028113	0.0008107
	0.50	0.0490115	0.0089194	0.0035770	0.0029172	0.0054082	0.0039118	0.0010168
	0.60	0.0776996	0.0136797	0.0051556	0.0038628	0.0064715	0.0041786	0.0009591
	0.70	0.1062274	0.0181901	0.0064805	0.0044516	0.0065984	0.0035961	0.0006466
	0.80	0.1345933	0.0224499	0.0075548	0.0046970	0.0058567	0.0022649	0.0001247
	0.90	0.1628208	0.0264971	0.0084203	0.0046638	0.0044384	0.0003854	0.0005363
1	0	0	0	0	0	0	0	

Table 13

Clamped–clamped beam;  $m=1, K=1$ 

$a/L$ ( $\omega_s/\omega_1$ )	$x$	$EIW(x)/F_0L^3$ $\omega/\omega_1$						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.10 (0.04469)	0	0	0	0	0	0	0	0
	0.10	0.0000609	0.0000057	0.0000022	0.0000013	0.0000014	0.0000003	0.0000001
	0.20	0.0001231	0.0000117	0.0000046	0.0000030	0.0000037	0.0000014	0.0000000
	0.30	0.0001519	0.0000147	0.0000060	0.0000042	0.0000059	0.0000029	0.0000004
	0.40	0.0001542	0.0000152	0.0000064	0.0000047	0.0000073	0.0000042	0.0000008
	0.50	0.0001367	0.0000136	0.0000060	0.0000046	0.0000077	0.0000048	0.0000011
	0.60	0.0001065	0.0000108	0.0000048	0.0000039	0.0000068	0.0000046	0.0000011
	0.70	0.0000706	0.0000072	0.0000033	0.0000027	0.0000050	0.0000035	0.0000009
	0.80	0.0000362	0.0000037	0.0000017	0.0000015	0.0000028	0.0000020	0.0000006
	0.90	0.0000102	0.0000011	0.0000005	0.0000004	0.0000008	0.0000006	0.0000002
1	0	0	0	0	0	0	0	
0.30 (0.04463)	0	0	0	0	0	0	0	0
	0.10	0.0001513	0.0000147	0.0000060	0.0000041	0.0000058	0.0000029	0.0000004
	0.20	0.0004744	0.0000460	0.0000190	0.0000132	0.0000189	0.0000097	0.0000015
	0.30	0.0007733	0.0000754	0.0000314	0.0000221	0.0000327	0.0000178	0.0000031
	0.40	0.0008937	0.0000879	0.0000372	0.0000271	0.0000420	0.0000246	0.0000049
	0.50	0.0008474	0.0000841	0.0000364	0.0000273	0.0000444	0.0000279	0.0000061
	0.60	0.0006877	0.0000689	0.0000304	0.0000235	0.0000397	0.0000263	0.0000062
	0.70	0.0004688	0.0000474	0.0000212	0.0000168	0.0000293	0.0000202	0.0000050
	0.80	0.0002448	0.0000249	0.0000113	0.0000091	0.0000163	0.0000116	0.0000029
	0.90	0.0000703	0.0000072	0.0000033	0.0000027	0.0000049	0.0000036	0.0000009
1	0	0	0	0	0	0	0	
0.50 (0.04458)	0	0	0	0	0	0	0	0
	0.10	0.0001357	0.0000136	0.0000059	0.0000045	0.0000075	0.0000049	0.0000011
	0.20	0.0004590	0.0000458	0.0000199	0.0000151	0.0000246	0.0000161	0.0000036
	0.30	0.0008447	0.0000839	0.0000363	0.0000272	0.0000438	0.0000282	0.0000061
	0.40	0.0011675	0.0001156	0.0000497	0.0000369	0.0000586	0.0000372	0.0000079
	0.50	0.0013027	0.0001288	0.0000551	0.0000407	0.0000643	0.0000404	0.0000085
	0.60	0.0011675	0.0001156	0.0000497	0.0000369	0.0000586	0.0000372	0.0000079
	0.70	0.0008447	0.0000839	0.0000363	0.0000272	0.0000438	0.0000282	0.0000061
	0.80	0.0004590	0.0000458	0.0000199	0.0000151	0.0000246	0.0000161	0.0000036
	0.90	0.0001357	0.0000136	0.0000059	0.0000045	0.0000075	0.0000049	0.0000011
1	0	0	0	0	0	0	0	



Table 14

Clamped–clamped beam;  $m=1, K=1$ 

$a/L$ ( $\omega_s/\omega_1$ )	$x$	$M(x)/F_0L$ $\omega/\omega_1$						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.10 (0.04469)	0	-0.0202632	-0.0018816	-0.0007031	-0.0004035	-0.0003839	-0.0000211	-0.0000532
	0.10	0.0040191	0.0003466	0.0001054	0.0000299	0.0000563	-0.0000113	-0.0000446
	0.20	0.0033427	0.0003028	0.0001058	0.0000507	0.0000182	-0.0000405	-0.0002340
	0.30	0.0026604	0.0002538	0.0001006	0.0000644	0.0000777	0.0000231	0.0000028
	0.40	0.0019705	0.0001982	0.0000879	0.0000681	0.0001135	0.0000692	0.0000143
	0.50	0.0012730	0.0001360	0.0000673	0.0000603	0.0001200	0.0000902	0.0000245
	0.60	0.0005687	0.0000676	0.0000392	0.0000414	0.0000959	0.0000826	0.0000255
	0.70	-0.0001409	-0.0000056	0.0000052	0.0000132	0.0000444	0.0000475	0.0000170
	0.80	-0.0008541	-0.0000820	-0.0000330	0.0000216	-0.0000270	-0.0000087	0.0000008
	0.90	-0.0015690	-0.0001602	-0.0000734	-0.0000601	-0.0001099	-0.0000773	-0.0000201
1	-0.0022846	-0.0002389	-0.0001144	-0.0000998	-0.0001967	-0.0001501	-0.0000426	
0.30 (0.04463)	0	-0.0367907	-0.0035587	-0.0014556	-0.0009962	-0.0013926	-0.0006816	-0.0000929
	0.10	-0.0171821	-0.0016727	-0.0006942	-0.0004883	-0.0007180	-0.0003887	-0.0000677
	0.20	0.0024182	0.0002061	0.0000589	0.0000084	-0.0000695	-0.0001153	-0.0000466
	0.30	0.0219948	0.0020642	0.0007883	0.0004729	0.0005023	0.0000985	-0.0000387
	0.40	0.0166847	0.0016268	0.0006769	0.0004769	0.0006992	0.0003724	-0.0000614
	0.50	0.0113305	0.0011505	0.0005196	0.0004154	0.0007284	0.0004992	0.0001205
	0.60	0.0059344	0.0006366	0.0003173	0.0002877	0.0005800	0.0004597	0.0001288
	0.70	0.0005041	0.0000920	0.0000773	0.0001029	0.0002726	0.0002635	0.0000859
	0.80	-0.0049496	-0.0004739	-0.0001891	-0.0001228	-0.0001529	-0.0000537	0.0000015
	0.90	-0.0104158	-0.0010513	-0.0004698	-0.0003713	-0.0006451	-0.0004412	-0.0001077
1	-0.0158860	-0.0016322	-0.0007552	-0.0006271	-0.0011595	-0.0008527	-0.0002255	
0.50 (0.04458)	0	-0.0313152	-0.0031361	-0.0013770	-0.0010579	-0.0017568	-0.0011757	-0.0002671
	0.10	-0.0187680	-0.0018627	-0.0008033	-0.0006009	-0.0000635	-0.0006174	-0.0001332
	0.20	-0.0062283	-0.0005961	-0.0002380	-0.0001563	-0.0002036	-0.0000920	-0.0000097
	0.30	0.0062881	0.0006496	0.0003022	0.0002510	0.0004559	0.0003354	0.0000837
	0.40	0.0187625	0.0018578	0.0007974	0.0005920	0.0009393	0.0005934	0.0001257
	0.50	0.0311793	0.0030146	0.0012313	0.0008438	0.0011880	0.0006293	0.0001016
	0.60	0.0187625	0.0018578	0.0007974	0.0005920	0.0009393	0.0005934	0.0001257
	0.70	0.0062881	0.0006496	0.0003022	0.0002510	0.0004559	0.0003354	0.0000837
	0.80	-0.0062283	-0.0005961	-0.0002380	-0.0001563	-0.0002036	-0.0000920	-0.0000097
	0.90	-0.0187680	-0.0018627	-0.0008033	-0.0006009	-0.0009635	-0.0006174	-0.0001332
1	-0.0313152	-0.0031361	-0.0013770	-0.0010579	-0.0017568	-0.0011757	-0.0002671	

Table 15

Clamped–clamped beam;  $m=0.1$ ,  $K=2$ 

$a/L$ ( $\omega_s/\omega_1$ )	$x$	$EIW(x)/F_0L^3$ $\omega/\omega_1$						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.10 (0.19984)	0	0	0	0	0	0	0	0
	0.10	0.0003254	0.0002001	0.0000512	0.0000281	0.0000288	0.0000053	-0.0000021
	0.20	0.0006582	0.0004114	0.0001092	0.0000645	0.0000782	0.0000282	0.0000009
	0.30	0.0008124	0.0005164	0.0001423	0.0000901	0.0001240	0.0000596	0.0000086
	0.40	0.0008243	0.0005325	0.0001519	0.0001019	0.0001539	0.0000866	0.0000170
	0.50	0.0007309	0.0004791	0.0001409	0.0000991	0.0001605	0.0000997	0.0000226
	0.60	0.0005695	0.0003782	0.0001141	0.0000835	0.0001452	0.0000947	0.0000233
	0.70	0.0003776	0.0002536	0.0000782	0.0000590	0.0001049	0.0000731	0.0000190
	0.80	0.0001933	0.0001310	0.0000411	0.0000318	0.0000583	0.0000420	0.0000113
	0.90	0.0000547	0.0000374	0.0000119	0.0000094	0.0000176	0.0000130	0.0000036
1	0	0	0	0	0	0	0	
0.30 (0.19925)	0	0	0	0	0	0	0	0
	0.10	0.0008171	0.0005095	0.0001410	0.0000890	0.0001206	0.0000610	0.0000087
	0.20	0.0025623	0.0016011	0.0004452	0.0002835	0.0003901	0.0002035	0.0000313
	0.30	0.0041770	0.0026215	0.0007359	0.0004766	0.0006753	0.0003711	0.0000637
	0.40	0.0048275	0.0030561	0.0008738	0.0005835	0.0008673	0.0005142	0.0001005
	0.50	0.0045770	0.0029260	0.0008538	0.0005889	0.0009177	0.0005821	0.0001253
	0.60	0.0037147	0.0023969	0.0007126	0.0005059	0.0008206	0.0005486	0.0001263
	0.70	0.0025320	0.0016472	0.0004978	0.0003621	0.0006064	0.0004218	0.0001018
	0.80	0.0013222	0.0008661	0.0002653	0.0001968	0.0003379	0.0002420	0.0000604
	0.90	0.0003798	0.0002502	0.0000775	0.0000583	0.0001021	0.0000747	0.0000191
1	0	0	0	0	0	0	0	
0.50 (0.19881)	0	0	0	0	0	0	0	0
	0.10	0.0007383	0.0004679	0.0001385	0.0000970	0.0001520	0.0001049	0.0000229
	0.20	0.0024981	0.0015789	0.0004649	0.0003228	0.0005008	0.0003414	0.0000735
	0.30	0.0045969	0.0028962	0.0008473	0.0005828	0.0008928	0.0005991	0.0001265
	0.40	0.0063537	0.0039904	0.0011601	0.0007903	0.0011949	0.0007886	0.0001632
	0.50	0.0070896	0.0044449	0.0012877	0.0008725	0.0013097	0.0008562	0.0001751
	0.60	0.0063537	0.0039904	0.0011601	0.0007903	0.0011949	0.0007886	0.0001632
	0.70	0.0045969	0.0028962	0.0008473	0.0005828	0.0008928	0.0005991	0.0001265
	0.80	0.0024981	0.0015789	0.0004649	0.0003228	0.0005008	0.0003414	0.0000735
	0.90	0.0007383	0.0004679	0.0001385	0.0000970	0.0001520	0.0001049	0.0000229
1	0	0	0	0	0	0	0	

Table 16  
Clamped–clamped beam;  $m=0.1, K=2$

$a/L$ ( $\omega_s/\omega_1$ )	$x$	$M(x)/F_0L$ $\omega/\omega_1$						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.10 (0.19984)	0	-0.10835598	-0.06611927	-0.01659530	-0.00874616	-0.00805208	-0.00043665	0.00108905
	0.10	0.02149181	0.01217895	0.00248752	0.00064740	-0.00118282	-0.00232344	-0.00091192
	0.20	0.01787505	0.01063863	0.00249799	0.00110001	0.00038148	-0.00083654	-0.00047877
	0.30	0.01422608	0.00891697	0.00237442	0.00139665	0.00163014	0.00047517	-0.00005625
	0.40	0.01053705	0.00696599	0.00207520	0.00147522	0.00238161	0.00142806	0.00029298
	0.50	0.00680719	0.00477773	0.00158805	0.00130697	0.00251708	0.00186367	0.00050046
	0.60	0.00304103	0.00237541	0.00092627	0.00089834	0.00201013	0.00170477	0.00052049
	0.70	-0.00075351	-0.00019638	0.00012267	0.00028684	0.00093207	0.00098103	0.00034710
	0.80	-0.00456698	-0.00288247	-0.00077865	-0.00046884	-0.00056819	-0.00018035	0.00001559
	0.90	-0.00839032	-0.00562866	-0.00173224	-0.00130352	-0.00230676	-0.00159717	-0.00041160
1	-0.01221675	-0.00839381	-0.00270252	-0.00216389	-0.00412443	-0.00310073	-0.00087206	
0.30 (0.19925)	0	-0.19872237	-0.12377976	-0.03416558	-0.02145890	-0.02878695	0.01424341	0.00190668
	0.10	-0.09280800	-0.05818018	-0.01629529	-0.01051867	-0.01484197	0.00812123	0.00138997
	0.20	0.01306159	0.00716800	0.00138153	0.00018164	-0.00143631	0.00240940	0.00095637
	0.30	0.11880348	0.07179757	0.00185026	0.01018706	0.01038257	-0.00205767	0.00079428
	0.40	0.09012117	0.05658438	0.01588732	0.01027243	0.01445349	-0.00778165	-0.00125952
	0.50	0.06120098	0.04001560	0.01219495	0.00894749	0.01505637	-0.01042995	-0.00247220
	0.60	0.03205423	0.02214362	0.00744686	0.00619643	0.01198892	-0.00960482	-0.00264194
	0.70	0.00272287	0.00320014	0.00181472	0.00221703	0.00563447	-0.00550511	-0.00176135
	0.80	-0.02673512	-0.01648422	-0.00443847	-0.00264596	-0.00315980	0.00112168	-0.00003169
	0.90	-0.05626040	-0.03656495	-0.01102828	-0.00799701	-0.01333472	0.00922017	0.00220877
1	-0.00858070	-0.05677211	-0.01772652	-0.01350750	-0.02396860	0.01781775	0.00462627	
0.50 (0.19881)	0	-0.17042607	-0.10823823	-0.03217359	-0.02265774	-0.03579692	-0.02494657	-0.00550701
	0.10	-0.10214105	-0.06428886	-0.01877054	-0.01287105	-0.01963153	-0.01309879	-0.00274638
	0.20	-0.03389618	-0.02057443	-0.00556040	-0.00334847	-0.00414871	-0.00195265	-0.00019897
	0.30	0.03422170	0.02242096	0.00706216	0.00537542	0.00928937	0.00711571	0.00172550
	0.40	0.10211109	0.06411997	0.01863170	0.01268066	0.01913855	0.01259095	0.00259173
	0.50	0.16968632	0.10404531	0.02876893	0.01807332	0.02420699	0.01335238	0.00209540
	0.60	0.10211109	0.06411997	0.01863170	0.01268066	0.01913855	0.01259095	0.00259173
	0.70	0.03422170	0.02242096	0.00706216	0.00537542	0.00928937	0.00711571	0.00172550
	0.80	-0.03389618	-0.02057443	-0.00556040	-0.00334847	-0.00414871	-0.00195265	-0.00019897
	0.90	-0.10214052	-0.06428886	-0.01877054	-0.01287105	-0.01963153	-0.01309879	-0.00274638
1	-0.17042607	-0.10823823	-0.03217359	-0.02265774	-0.03579692	-0.02494657	-0.00550701	

Table 17

Clamped–clamped beam;  $m=0.2$ ,  $K=1$ 

$a/L$ ( $\omega_s/\omega_1$ )	$x$	$EIW(x)/F_0L^3$ $\omega/\omega_1$						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.10 (0.09993)	0	0	0	0	0	0	0	0
	0.10	0.15090164	0.00003131	0.00001120	0.00000658	-0.00000693	0.00000130	-0.00000052
	0.20	0.30528620	0.00006438	0.00023908	0.00001512	0.00001882	0.00000688	0.00000021
	0.30	0.37678027	0.00008082	0.00003115	0.00002112	0.00002985	0.00001453	0.00000212
	0.40	0.38231048	0.00008333	0.00003325	0.00002388	0.00003706	0.00002110	0.00000417
	0.50	0.33898655	0.00007498	0.00003083	0.00002323	0.00003865	0.00002430	0.00000554
	0.60	0.26410549	0.00005918	0.00002497	0.00001957	0.00003432	0.00002307	0.00000572
	0.70	0.17513136	0.00003968	0.00001711	0.00001384	0.00002526	0.00001781	0.00000466
	0.80	0.08965926	0.00002051	0.00000900	0.00000746	0.00001404	0.00001024	0.00000278
	0.90	0.02537224	0.00000585	0.00000260	0.00000220	0.00000423	0.00000317	0.00000088
1	0	0	0	0	0	0	0	
0.30 (0.09979)	0	0	0	0	0	0	0	0
	0.10	0.08276445	0.00008051	0.00003103	0.00002101	0.00002946	0.00001469	0.00000212
	0.20	0.25954020	0.00025297	0.00009796	0.00006688	0.00009528	0.00004902	0.00000766
	0.30	0.42309630	0.00041420	0.00016194	0.00011246	0.00016492	0.00008940	0.00001560
	0.40	0.48898982	0.00048287	0.00019229	0.00013767	0.00021182	0.00012388	0.00002460
	0.50	0.46361751	0.00046232	0.00018787	0.00013894	0.00022412	0.00014023	0.00003068
	0.60	0.37627237	0.00037871	0.00015682	0.00011937	0.00020041	0.00013215	0.00003093
	0.70	0.25647432	0.00026025	0.00010954	0.00008543	0.00014809	0.00010160	0.00002493
	0.80	0.13392889	0.00013684	0.00005838	0.00004643	0.00008251	0.00005830	0.00001479
	0.90	0.03846978	0.00003953	0.00001704	0.00001376	0.00002493	0.00001800	0.00000467
1	0	0	0	0	0	0	0	
0.50 (0.09968)	0	0	0	0	0	0	0	0
	0.10	0.04700386	0.00007447	0.00003061	0.00002300	0.00003764	0.00002490	0.00000558
	0.20	0.15903604	0.00025129	0.00010274	0.00007659	0.00012405	0.00008100	0.00001790
	0.30	0.29265439	0.00046095	0.00018727	0.00013828	0.00022115	0.00014216	0.00003083
	0.40	0.40449817	0.00063510	0.00025638	0.00018749	0.00029599	0.00018714	0.00003976
	0.50	0.45135132	0.00070744	0.00028459	0.00020701	0.00032441	0.00020318	0.00004256
	0.60	0.40449817	0.00063510	0.00025638	0.00018749	0.00029599	0.00018714	0.00003976
	0.70	0.29265439	0.00046095	0.00018727	0.00013828	0.00022115	0.00014216	0.00003083
	0.80	0.15903604	0.00025129	0.00010274	0.00007659	0.00012405	0.00008100	0.00001790
	0.90	0.04700386	0.00007447	0.00003061	0.00002300	0.00003764	0.00002490	0.00000558
1	0	0	0	0	0	0	0	

Table 18

Clamped–clamped beam;  $m=0.2$ ,  $K=1$ 

$a/L$ ( $\omega_{1s}/\omega_1$ )	$x$	$M(x)/F_0L$ $\omega/\omega_1$						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.10 (0.09993)	0	-50.254672	-0.01034715	-0.00363228	-0.00205083	-0.00193884	-0.00010642	-0.00026743
	0.10	9.967736	0.00190591	0.00054445	0.00015181	-0.00028481	-0.00056626	-0.00022393
	0.20	8.290310	0.00166486	0.00054675	0.00025793	0.00009186	-0.00020388	-0.00011757
	0.30	6.597919	0.00139544	0.00051970	0.00032749	0.00039252	0.00011581	0.00001381
	0.40	4.887002	0.00109012	0.00045421	0.00034591	0.00057346	0.00034804	0.00007195
	0.50	3.157123	0.00074768	0.00034758	0.00030646	0.00060608	0.00045421	0.00012289
	0.60	1.410406	0.00037173	0.00020274	0.00021065	0.00048401	0.00041548	0.00012781
	0.70	-0.349472	-0.00003073	0.00002685	0.00006726	0.00022443	0.00023909	0.00008524
	0.80	-2.118131	-0.00045108	-0.00017043	-0.00010993	-0.00013681	-0.00004395	0.00000383
	0.90	-3.891366	-0.00088084	-0.00037914	-0.00030566	-0.00055544	-0.00038926	0.00010107
1	-5.666034	-0.00131356	-0.00059151	-0.00050740	-0.00099311	-0.00075570	0.00021415	
0.30 (0.09979)	0	-20.129110	-0.01955729	-0.00751824	-0.00506310	-0.00703063	-0.00343144	0.00046687
	0.10	-9.400766	-0.00919251	-0.00358583	-0.00248182	-0.00362485	-0.00195652	0.00034034
	0.20	1.323043	0.00113255	0.00030401	0.00004286	-0.00035079	-0.00058046	0.00023417
	0.30	12.033916	0.01134406	0.00407157	0.00240358	0.00253573	0.00049572	0.00019449
	0.40	9.128610	0.00894037	0.00349605	0.00242372	0.00352997	0.00187471	0.00030840
	0.50	6.199208	0.00632249	0.00268354	0.00211111	0.00367721	0.00251272	0.00060534
	0.60	3.246857	0.00349871	0.00163870	0.00146201	0.00292805	0.00231394	0.00064690
	0.70	-0.275807	0.00050562	0.00039933	0.00052309	0.00137610	0.00132626	0.00043128
	0.80	-2.708070	-0.00260452	-0.00097670	-0.00062430	-0.00077172	-0.00027023	0.00000776
	0.90	-5.698764	-0.00577729	-0.00242681	-0.00188683	-0.00325673	-0.00222127	0.00054083
1	-8.691621	-0.00897003	-0.00390077	-0.00318702	-0.00585384	-0.00429255	0.00113278	
0.50 (0.09968)	0	-10.849927	-0.01722676	-0.00711075	-0.00537552	-0.00886708	-0.00591974	0.00134190
	0.10	-6.502627	-0.01023196	-0.00414851	-0.00305364	-0.00486283	-0.00310830	0.00066921
	0.20	-2.157951	-0.00327454	-0.00122892	-0.00079442	-0.00102766	-0.00046336	0.00004848
	0.30	2.178674	0.00356843	0.00156082	0.00127531	0.00230103	0.00168853	0.00042046
	0.40	6.500742	0.01020508	0.00411783	0.00300847	0.00474072	0.00298779	0.0063153
	0.50	10.802832	0.01655943	0.00635828	0.00428787	0.00599619	0.00316847	0.0051059
	0.60	6.500742	0.01020508	0.00411783	0.00300847	0.00474072	0.00298779	0.00063153
	0.70	2.178674	-0.00356843	0.00156082	0.00127531	0.00230103	0.00168853	0.00042046
	0.80	-2.157951	-0.00327454	0.00122892	-0.00079440	-0.00102766	-0.00046336	0.0004848
	0.90	-6.502627	-0.01023196	0.00414851	-0.00305364	-0.00486283	-0.00310830	0.00066921
1	-10.849927	-0.01722676	0.00711075	-0.00537552	-0.00886708	-0.00591974	0.00134190	

- clamped–clamped

$$\left( \Omega_1 = \sqrt{\frac{\rho h}{EI}} \omega_1 L^2 = (4.73004)^2 \right)$$

- clamped–simply supported

$$\left( \Omega_1 = \sqrt{\frac{\rho h}{EI}} \omega_1 L^2 = (3.926602)^2 \right)$$

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