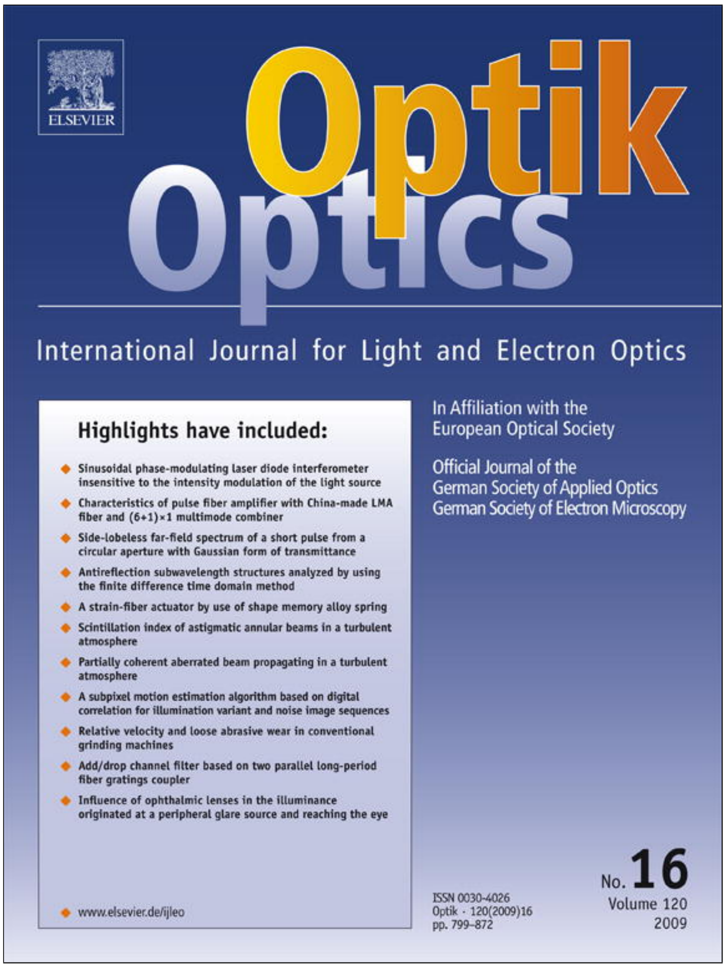
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Influence of ophthalmic lenses in the illuminance originated at a peripheral glare source and reaching the eye

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Abstract

To study the influence of glare on the visual performance of a subject wearing an ophthalmic lens, it is useful to know how the lens affects the illuminance reaching the eye. In this paper, considering spherical standard ophthalmic lenses and defining the relative illuminance, $\boldsymbol{E}_{\mathrm{r}}$, as the quotient between the illuminance at the cornea with and without lens, a methodology to evaluate $\boldsymbol{E}_{\mathrm{r}}$ in terms of easily determined parameters is developed. Three effects are considered, pupil size variation of the system with and without lens; lateral shifts of rays transmitted through the lens and reflections at the lens. Calculations are experimentally verified employing 5 organic ophthalmic lenses of ± 6; ± 4 and 0.12 dioptres and 2 glass plane parallel plates 1.95 and 6.6 mm thick. Using a photometer whose sensor is 12 mm apart from the lens and 740 mm apart from a glare source subtending an eccentricity angle of 9.6°, it results $\boldsymbol{E}_{\mathrm{r}} = 1.204$ for the 6 dioptres lens and $\boldsymbol{E}_{\mathrm{r}} = 0.803$ for the -6 dioptres one if sensor diameter is 10 mm while, for a 719 mm distance and a 10° angle, $\boldsymbol{E}_{\mathrm{r}} = 0.922$ for the thin plate and a 30 mm sensor and $\boldsymbol{E}_{\mathrm{r}} = 1.006$ for the thick plate and a 10 mm sensor. Experimental and theoretical results differ in less than 3%.

Keywords: Glare; Ophthalmic lenses; Ocular pupil; Lateral shifts; Reflections

1. Introduction

In certain circumstances of everyday life [1–5] a foveally fixated subject is exposed to the effects of a glare source lying in the periphery of his field of view (for example headlights of oncoming vehicles when driving in routes at night or brief light exposures in some psycho-physical tests). The peripheral glare source causes a decrease in the perceived brightness of the foveal object, which can be accounted for in terms of a veiling luminance produced by stray light falling on the fovea [6]. When the source is too close, the subject can be completely blinded but when it is further away, he can distinguish the foveal object and is

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said to be under the presence of disability glare, this causing more pronounced problems if he is old than if he is young [7]. If the subject wears refractive corrections, it is useful to analyze their influence on the amount of light originated at the glare source that reaches the cornea and traverses the pupil since this is the light that, propagating through the ocular media, arrives at the retina. A refractive correction can be a contact, a progressive, a monofocal or another type of lens [8–10]. The system contact lens – tear film – cornea is such that the intervening refraction indices are similar to each other and the vertex distance is small so, as a first approximation, it can be regarded as a new cornea and the contact lens influence on illumination can be neglected in comparison to that appearing with spectacles. In the present paper we address the problem of a system constituted by an eye and a monofocal standard ophthalmic lens of spherical surfaces. We take into account refraction and reflection at the lens [11–13] leaving aside scattering, absorption, lateral displacements of beams related to the Goos–Haenchen effect [14] and also interference phenomena [12]. We assume that the subject fixates his view in a forward object and that the glare source is small, far from his eyes and forming with the visual axis an eccentricity angle of about 10°, which corresponds to the border of the macula [8]. We define the relative illuminance at the eye, $\boldsymbol{E}_r$, as the quotient between the illuminance at the corneal vertex with and without ophthalmic lens. We consider 3 effects that modify the illuminance reaching the corneal vertex: pupil size variation of the optical system with and without ophthalmic lens [15]; lateral shifts of rays transmitted through the lens and reflections at both lens surfaces. In Section 2, we describe the notation employed; some parameters of the ophthalmic lens (power, refraction index, curvature radii and axial thickness) and some characteristics of the setup (glare source position; vertex distance and sensor parameters). In Section 3, based in simple concepts of paraxial geometrical and physical optics and under certain assumptions, we derive analytical formulas for the relative illuminance corresponding to each of the 3 considered effects and to the total one as a function of magnitudes that are either easily determined or fabrication data. In Section 4, we measure the relative illuminance for 5 organic ophthalmic lenses (powers ±6; ±4 and 0.12 dioptres) and for 2 glass plane parallel plates (thicknesses 1.95 and 6.6 mm) and these experimental results are found to be in very close agreement with those predicted by our formulas.

2. Ophthalmic lenses, setup and notation

We analyze monofocal standard ophthalmic lenses that correct hyperopia and myopia such that vertex distance [9,15] is denoted D_G (its typical value being between 12 and 14 mm). We consider the reflection and refraction of light at both lens surfaces [11–13] and, if the incidence and refraction angles, respectively indicated β_A and β_T, are measured from the normal to the interface between two media of refraction indices n and n', considering the scalar Snell law, it results

$$\cos(\beta_T) = \sqrt{1 - \left(\frac{n}{n'}\right)^2 \sin^2(\beta_A)} \tag{1}$$

The ophthalmic lenses parameters are the power, Φ_G; the refraction index, n_G; the curvature radii of its anterior and posterior surfaces, R_1 and R_2, associated to powers Φ_1 and Φ_2 and the axial thickness, e_{az} (Fig. 1). Under paraxial approximation and henceforth measuring powers in dioptres and distances in millimetres, if $\Phi_{G,t} = \Phi_1 + \Phi_2$ is the power assuming the lens to be thin, for a thick lens we have [12]

$$\begin{aligned} \Phi_G &= \Phi_{G,t} - \Phi_{Ad} \\ &= 1000\left[(n_G - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \left(\frac{(n_G-1)^2}{R_1 R_2}\right)\left(\frac{e_{ax}}{n_G}\right)\right] \\ \Phi_{Ad} &= \frac{\Phi_1 \Phi_2 e_{ax}}{1000 n_G} \end{aligned} \tag{2}$$

Concerning axial thickness, in diverging ophthalmic lenses we usually have $e_{ax} \leqslant 2$ mm while in converging ones e_{ax} varies with power and we estimate typical values considering a fixed value of R_1 and finding the value of R_2 that corresponds to a thin lens, this is $R_2 = R_1(1-n_G)/(1-n_G+0.001R_1\Phi_{G,t})$. Assuming the lens diameter, H, and the border thickness, e_{bor}, are known, if $h_1 = H/2$ then the sagitta a_1 and similarly a_2 (measured from the tangent plane and positive from left to right) and e_{ax} are such that

$$\begin{aligned} a_1 &= R_1(1 - \sqrt{b_1}) \\ b_1 &= \left(1 - \left(\frac{h_1}{R_1}\right)^2\right) \\ e_{ax} &= e_{bor} + a_1 - a_2 \end{aligned} \tag{3}$$

Once e_{ax} is calculated, we find the real lens power using Eq. (2). In Fig. 2 we plot e_{ax} as a function of Φ_G for converging ophthalmic lenses of up to 10 dioptres;

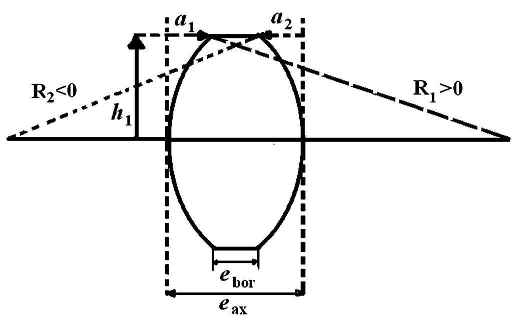

Fig. 1. Sagittas and axial thickness in a converging ophthalmic lens.

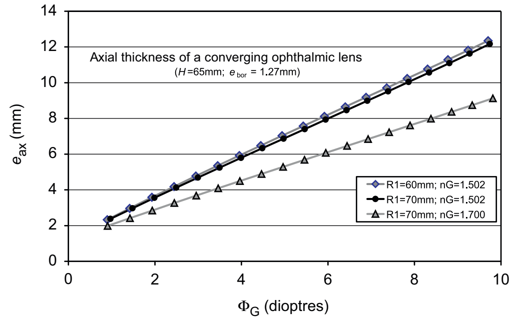

Fig. 2. Axial thickness of a converging ophthalmic lens as a function of its power for different values of R_1 and n_G ($H = 65$ mm; $e_{bor} = 1.27$ mm).

$H = 65$ mm; $e_{bor} = 1.27$ mm and different values of R_1 and n_G. We obtain that e_{ax} increases as power increases and that, for a given power, e_{ax} increases if n_G and/or R_1 decrease. Moreover, it results $|\Phi_{Ad}|<0.2$ dioptres in all the considered cases so, when calculating power, the error arising from the assumption that the lens is thin, is small.

The ophthalmic lens affects the illuminance originated at the glare source that reaches the eye and, if E_G and E, respectively indicate the illuminance with and without the lens, the relative illuminance is $\boldsymbol{E}_r = E_G/E$. To derive an analytical formula for $\boldsymbol{E}_r$ and to verify it experimentally, we use the setups of Fig. 3(a) and (c), respectively. In Fig. 3(a) we consider an eye such that its corneal vertex, VC, is at a distance D_G from the lens. The ocular entrance pupil (Fig. 3(b)) has a diameter termed $d_P$and is at a distance from VC, termed s'_I, which is small in comparison to other intervening distances (for example $|s'_I| = 3.04$ mm in Le Grand's simplified eye [15]). In Fig. 3(c), we consider a photometer sensor at a distance D_S from the point VC where the cornea would be placed if the sensor were an eye. We assume that, besides containing VC, the anterior sensor plane contains the entrance pupil when there is no lens (Fig. 3(d)) so that the distance VC-pupil is $s'_{I,S} = 0$ and the sensor diameter (which is also the pupil diameter) is termed d_S. In Fig. 3(a) and (c), the source consists in a lamp and a stop of diameter d_{gl} centred at a point J, which is at a distance B' from VC and at a lateral distance g subtending at VC an eccentricity angle $\theta = (180/\pi)\tan^{-1}(g/B)$ (with $B = (B'^2 - g^2)^{1/2}$). We consider distances, angles and illuminances values that correspond to field measurements performed in a route 7×10^3 mm wide in Tucumán, Argentina. For example, a car headlight with a diameter of about 270 mm placed at 20×10^3 mm and forming 10° with the direction in which the driver travels, gives rise to about 50 lx at the driver's eye. If distances were divided by a factor 27, the distance source-sensor would be 750 mm and the stop diameter 10 mm though the stop in our experimental setup is

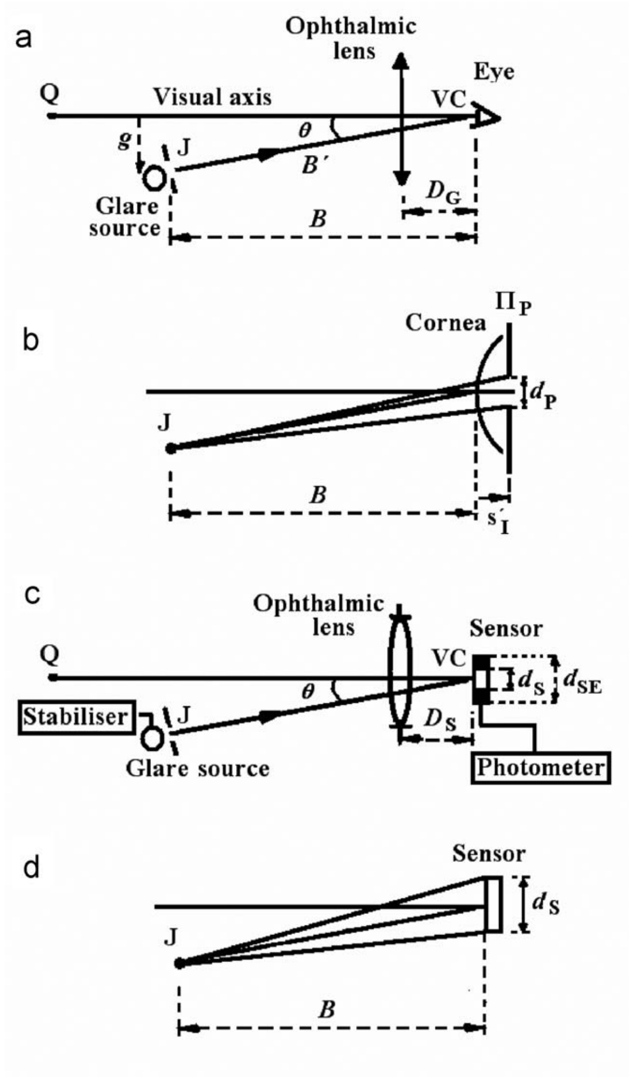

Fig. 3. (a) Setup for theoretical calculations. J: central point of the glare source; Q: fixation point of the subject; VC: corneal vertex; θ: eccentricity angle. (b) Entrance pupil in the naked eye (Π_P) and cone of light emitted by J. (c) Experimental setup. d_S and d_{SE}: diameters of the sensor and of its external border. (d) Sensor and cone of light emitted by J.

approximately twice as large as this and, as in previous articles [4,5], the illuminance E is 60 lx.

3. Analytical formulas for the relative illuminance

We obtain analytical formulas for the relative illuminances $\boldsymbol{E}_{r,m}$, $\boldsymbol{E}_{r,g}$ and $\boldsymbol{E}_{r,p}$ corresponding to the 3 considered effects introduced by the ophthalmic lens (pupil size variation of the system with and without lens, lateral shifts of rays transmitted through the lens and reflections at the lens) and also for the total relative illuminance, $\boldsymbol{E}_r^{[C]}$. We define the perceptual illuminance variation when an ophthalmic lens is introduced as $\varepsilon_{\%} = 100\ (\boldsymbol{E}_{\mathrm{r}}^{[C]}-1)$ so $\varepsilon_{\%}>0$ if there is gain and $\varepsilon_{\%}<0$ if there is loss. We principally regard as our system one composed by an ophthalmic lens and an eye though, for convenience, we also consider the systems lens-photometer sensor and plane parallel plate-sensor.

3.1. Relative illuminance due to pupil size variation of the system with and without lens

The formula for $\boldsymbol{E}_{\mathrm{r,m}}$ can be derived considering how the ophthalmic lens affects the convergence of the beam originated at point J and refracted at the lens or, alternatively, the system entrance pupil [12,13]. Taking into account the pupils, we consider the systems lens-eye (indicating associated magnitudes with suffix P) and lens-photometer sensor (indicating related magnitudes with suffix S).

For the system ophthalmic lens-eye, we assume the paraxial approximation holds; the system is free from vignetting, that pupils with and without lens are circular of diameters $d_{\mathrm{P,G}}$ and d_{P}, respectively and that neither the iris diameter nor the distance from VC to the ocular entrance pupil vary when the lens is introduced [15]. The entrance pupil of the system lens-eye, termed $\Pi_{\mathrm{P,G}}$ for brevity, is the reverse image [15] through the lens of the ocular entrance pupil, termed Π_{P}, and we calculate the lateral magnification for the conjugate planes containing both pupils, m_{G}, assuming the lens to be thin. If Π_{P} is at a distance $s_{\mathrm{P,G}} = -(D_{\mathrm{G}} + |s'_{\mathrm{I}}|)$from the lens, its reverse image is at a distance $s'_{\mathrm{P,G}}$ and, according to the DIN norms [12,13], we have $1/s'_{\mathrm{P,G}} - 1/s_{\mathrm{P,G}} = \Phi_{\mathrm{G}}$ and $m_{\mathrm{G}} = d_{\mathrm{P,G}}/d_{\mathrm{P}} = s'_{\mathrm{P,G}}/s_{\mathrm{P,G}}$, so it results

$$m_{\mathrm{G}} = \frac{1}{1 - 0.001(D_{\mathrm{G}} + |s'_{\mathrm{I}}|)\ \Phi_{\mathrm{G}}(\text{dioptres mm})^{-1}} \tag{4}$$

We have $\Phi_{\mathrm{G}}>0$ and $m_{\mathrm{G}}>1$ if the lens is converging and $\Phi_{\mathrm{G}}<0$ and $m_{\mathrm{G}}<1$ if it is diverging so the cone of light emitted by J that passes through the pupil of a given eye (Fig. 3(b)) has a larger aperture interposing a converging lens than a diverging one. We relate m_{G} to $\boldsymbol{E}_{\mathrm{r,m}}$ assuming J emits a luminous intensity I_0 (measured as usual in lumen/st) within this cone of light and that J is far enough so that, if r is the distance from J to any pupil point, I_0/r^2 is the same for every pupil zone. The areas of $\Pi_{\mathrm{P,G}}$ and Π_{P} are termed $A_{\mathrm{P,G}}$ and A_{P}, respectively and the corresponding luminous fluxes are $\varphi_{\mathrm{P,G}}$ and φ_{P}. Since the paraxial approximation is assumed to hold for the inclination angle, the flux arriving at Π_{P} is $\varphi_{\mathrm{P}} = (st)A_{\mathrm{P}}I_0/r^2$ and the illuminance [12,13] is $\varphi_{\mathrm{P}}/A_{\mathrm{P}}$, while the illuminance at $\Pi_{\mathrm{P,G}}$ is $\varphi_{\mathrm{P,G}}/A_{\mathrm{P,G}}$ and, considering $\varphi_{\mathrm{P,G}}/A_{\mathrm{P,G}} = \varphi_{\mathrm{P}}/A_{\mathrm{P}}$, it results $\varphi_{\mathrm{P,G}}/\varphi_{\mathrm{P}} = d^2_{\mathrm{P,G}}/d^2_{\mathrm{P}} = (m_{\mathrm{G}})^2$. If instead of regarding the illuminances at $\Pi_{\mathrm{P,G}}$ and Π_{P}, we regard the illuminance at Π_{P} not only when there is no lens (termed E_{P}) but also when there is lens (termed $E_{\mathrm{P,G}}$), since Π_{P} has a fixed area and the flux of interest is that traversing the pupil, we have $E_{\mathrm{P,G}}/E_{\mathrm{P}} = \varphi_{\mathrm{P,G}}/\varphi_{\mathrm{P}}$. Thus, $\boldsymbol{E}_{\mathrm{r,m}} = E_{\mathrm{P,G}}/E_{\mathrm{P}}$ is such that

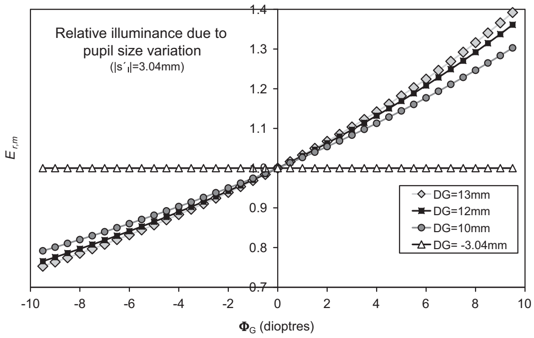

Fig. 4. Relative illuminance due to pupil size variation in the presence of an ophthalmic lens as a function of its power in the cases $D_{\mathrm{G}} = 10\,\mathrm{mm}$; $D_{\mathrm{G}} = 12\,\mathrm{mm}$; $D_{\mathrm{G}} = 13\,\mathrm{mm}$ and $D_{\mathrm{G}} = -3.04\,\mathrm{mm}$.

$$\boldsymbol{E}_{\mathrm{r,m}} = (m_{\mathrm{G}})^2 \tag{5}$$

On the other hand, for the system ophthalmic lens-photometer sensor, since $|s'_{\mathrm{I}}|$ is different from zero whereas $|s'_{\mathrm{I,S}}| = 0$ (Fig. 3), in Eq. (4) we replace D_{G} by $D'_{\mathrm{G}} = D_{\mathrm{S}} - |s'_{\mathrm{I}}|$. The entrance pupils with and without the lens are termed $\Pi_{\mathrm{S,G}}$ and Π_{S} and considerations concerning illuminances are similar to those stated above for the eye so, if $E_{\mathrm{S,G}}$ and E_{S} are the illuminances with and without lens, at the anterior sensor plane (which is also the plane of Π_{S}), we have $\boldsymbol{E}_{\mathrm{r,m}} = E_{\mathrm{S,G}}/E_{\mathrm{S}} = (m_{\mathrm{G}})^2$ and Eq. (5) holds.

According to Eqs. (4) and (5), an ophthalmic lens introduces an illuminance gain if it is converging and a loss if it is diverging. Furthermore, $\boldsymbol{E}_{\mathrm{r,m}}$ is independent of pupil diameter (d_{P} for the eye or d_{S} for the sensor), it only depends on the lens power and on the distance between the lens and the entrance pupil of the system without lens ($D_{\mathrm{G}} - |s'_{\mathrm{I}}|$ for the eye or D_{S} for the sensor). It results $\boldsymbol{E}_{\mathrm{r,m}} = 1$ if $\Phi_{\mathrm{G}} = 0$ (which occurs if instead of an ophthalmic lens there is a plane parallel plate) and/or if $D_{\mathrm{G}} + |s'_{\mathrm{I}}| = 0$ (which lacks sense for the system lens-eye but may be valid for the system lens-sensor). Considering $|s'_{\mathrm{I}}| = 3.04\,\mathrm{mm}$, in Fig. 4 we plot $\boldsymbol{E}_{\mathrm{r,m}}$ as a function of Φ_{G} for values of D_{G} equal to 10, 12, 13 and $-3.04\,\mathrm{mm}$. For lenses of up to ± 6 dioptres, $\boldsymbol{E}_{\mathrm{r,m}}$ varies at most 0.04 when D_{G} varies from 10 to 13 mm and, for $D_{\mathrm{G}} = 12\,\mathrm{mm}$, we get $\boldsymbol{E}_{\mathrm{r,m}} = 1.208$ for the 6 dioptres lens and $\boldsymbol{E}_{\mathrm{r,m}} = 0.841$ for the -6 dioptres one.

3.2. Relative illuminance due to lateral shifts of rays transmitted through the lens

We derive the formula for $\boldsymbol{E}_{\mathrm{r,g}}$ considering the lateral shifts of rays transmitted through an ophthalmic lens (these shifts not being related to the Goos–Haenchen

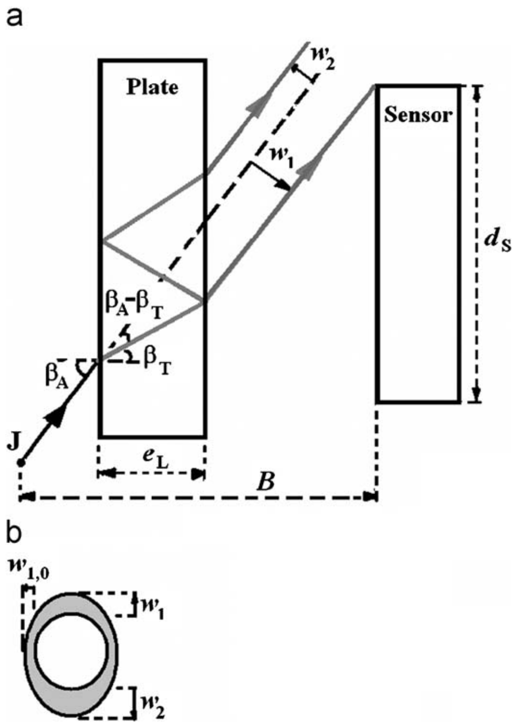

Fig. 5. (a) Lateral shifts w_1 and w_2 in a system plane parallel plate-photometer sensor. (b) Widening of the beam reaching the sensor.

effect [14]). For simplicity, first we consider a system constituted by a plane parallel plate of thickness e_L and refraction index n_L and a photometer sensor. A ray emitted by J and incident upon the plate with incidence angle β_A, gives rise to a 1st transmitted ray (resulting from 2 refractions at the plate) with lateral shift [13] w_1 and to a 2nd transmitted ray (resulting from 4 refractions) with lateral shift w_2 (w_1 and w_2 being measured from the incident ray prolongation and such that $w_1<0$ and $w_2>0$ in Fig. 5(a)). We leave aside all other transmitted rays and assume the paraxial approximation holds for the eccentricity angle and for the incidence angles of all rays constituting the cone of light emitted by J that reaches the sensor. We have $w_1 = e_L\beta_A(-1+(1/n_L))$ and $w_2 = e_L\beta_A(-1+(3/n_L))$ and we estimate $\boldsymbol{E}_{r,g}$ in the cases $\theta = 0$ and $\theta \neq 0$.

If $\theta = 0$, the maximum incidence angle for a ray to reach the sensor when the plate is absent is $\beta_{A,0}|_{max} = (d_S/2)/B$ (suffix 0 indicating that it corresponds to $\theta = 0$) and this angle slightly increases when the plate is introduced. Neglecting the variation of $w_{1,0}$ with the incidence angle for rays near the boundaries of the cone of light, we calculate $w_{1,0}$ considering $\beta_A = \beta_{A,0}|_{max}$ and we have

$$w_{1,0} = \left(-1+\frac{1}{n_L}\right)\left(\frac{e_L}{B}\right)\left(\frac{d_S}{2}\right) \tag{6}$$

The cross-section of the cone of light is circular and we regard the system plate-sensor as a system constituted by a fictitious sensor of diameter $d_S + 2|w_{1,0}|$ and area A_{fic} whose illuminance is the same as that in the real sensor of diameter d_S and area A_{real}. Assuming uniform flux at the sensor and if $E_{fic} = \varphi_{fic}/A_{fic}$ and $E_{real} = \varphi_{real}/A_{real}$ are the illuminances in the fictitious and real sensors respectively, we get $\varphi_{fic}/\varphi_{real} = A_{fic}/A_{real}$. In the real case, the sensor area does not vary when the plate is introduced and all the flux φ_{fic} arrives at it, hence $\boldsymbol{E}_{r,g,0} = E_{S,L}/E_S = \varphi_{fic}/\varphi_{real}$ and we have

$$\boldsymbol{E}_{r,g,0} = \left(1+|w_{1,0}|\left(\frac{2}{d_S}\right)\right)^2 \tag{7}$$

If $\theta \neq 0$, we consider the situation at the meridian plane and at the sagittal plane [12]. For a sagittal fan of rays, the lateral shifts are similar to those considered for $\theta = 0$ and for the front and back sensor borders we have $w_{1,0}$. For a meridian fan of rays, when the plate is absent, the minimum and maximum incidence angles for the ray to arrive at the lower and upper sensor borders, respectively are $\beta_A|_{min} = (g-(d_S/2))/B$ and $\beta_A|_{max} = (g+(d_S/2))/B$. When the plate is present, rays with lateral shifts w_2 arrive at the sensor lower border while rays with lateral shifts w_1 and higher intensities than the former arrive at the upper border. Leaving aside rays that undergo more than 2 refractions at the plate, we only take into account the upper border and in it we assume that the lateral shift varies so little with the incidence angle that we can approximate it to that corresponding to $\beta_A|_{max}$ and we get

$$w_1 = \left(-1+\frac{1}{n_L}\right)\left(\frac{e_L}{B}\right)\left(\frac{d_S}{2}+g\right) \tag{8}$$

The complete cone of light emerging from J and reaching the sensor has a larger aperture with plate than without it and in Fig. 5(b) we depict a cross-section of this cone in the absence (white) and presence (grey) of a plate. The exact theoretical calculation of the relative illuminance is beyond the scope of the present paper but we estimate a superior limit, $\boldsymbol{E}_{r,g,s}$ and an inferior one, $\boldsymbol{E}_{r,g,i}$, which we assume correspond respectively to considering shifts w_1 and $w_{1,0}$ for the whole cone of light

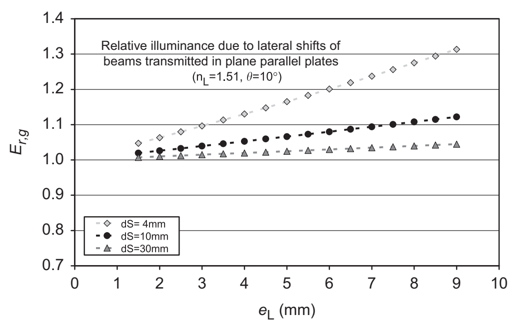

Fig. 6. Relative illuminance due to lateral shifts occurring in a plane parallel plate as a function of its thickness ($n_L = 1.51$, $\theta = 10°$) for different sensor diameters.

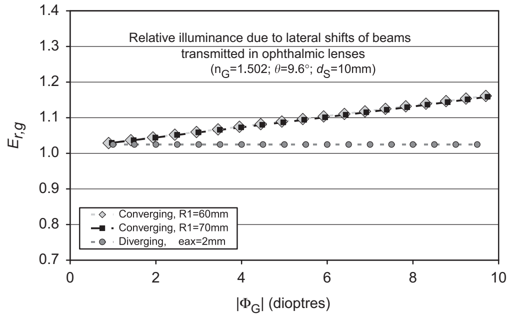

Fig. 7. Relative illuminance due to lateral shifts occurring in an ophthalmic lens as a function of the absolute value of its power ($n_G = 1.502$, $\theta = 9.6°$, $d_S = 10$ mm) for converging lenses ($R_1 = 60$ or 70 mm) and for diverging lenses ($e_{ax} = 2$ mm).

that reaches the sensor. Using the methodology employed in the case $\theta = 0$ and replacing $w_{1,0}$ in Eq. (7) by w_1 of Eq. (8) or by $w_{1,0}$ of Eq. (6), we obtain

$$\boldsymbol{E}_{r,g,s} = \left(1 + k\left(1 + \frac{2g}{d_S}\right)\right)^2 \quad \boldsymbol{E}_{r,g,i} = (1+k)^2 \tag{9}$$

where to simplify the formulas, we define a variable k such that

$$k = \left(1 - \frac{1}{n_L}\right)\left(\frac{e_L}{B}\right) \tag{10}$$

We consider that the value of the relative illuminance is the mean between both limits, this is

$$\boldsymbol{E}_{r,g} = \frac{(1 + k(1 + (2g/d_S)))^2 + (1+k)^2}{2} \tag{11}$$

In Fig. 6, for $\theta = 10°$ and distances similar to those of Section 4, we plot $\boldsymbol{E}_{r,g}$ as a function of thickness for plates of refraction index 1.51 and sensor diameters 4, 10 and 30 mm. The illuminance gain increases if the eccentricity angle increases; if the distance source-sensor decreases; if the thickness and/or refraction index of the plate increases and/or if the sensor diameter decreases.

In the case of the system ophthalmic lens-sensor, the exact theoretical calculation of the illuminance gain exceeds the scope of this article but we estimate it in the surroundings of the optical axis approximating the lens to a plane parallel plate of thickness equal to the lens axial one. We assume the approximations considered for the plate hold and use Eqs. (10)–(11) replacing n_L by n_G and e_L by e_{ax}. In Fig. 7, we plot $\boldsymbol{E}_{r,g}$ as a function of the power absolute value for $n_G = 1.502$ and $d_S = 10$ mm. The axial thickness is considered to be 2 mm for all diverging lenses and is calculated using Eq. (3) and a fixed value of R_1 for converging ones. For diverging lenses the gain is only 2.5% while for converging ones it increases as power increases and, for example, for a 6 dioptres lens (with R_1 either 60 or 70 mm), it is 10%.

In the case of the system ophthalmic lens-eye (Fig. 3(a)) the situation is similar, the approximation of the lens to a plate having thickness e_{ax} can be adequate if the subject looks forward at Q (though the appropriate thickness must be evaluated if he rotates his eyes) and d_S has to be replaced by d_P.

3.3. Relative illuminance due to reflections at the lens

We derive the formula for $\boldsymbol{E}_{r,p}$ taking into account the reflections in both ophthalmic lens surfaces and, for simplicity, we first consider the case of a monochromatic plane wavefront incident upon a plane interface between two media of refraction indices n and n'. The reflection coefficients C_r and C_l associated to the parallel and perpendicular components of the electric field are [11,12]

$$C_r = \left(\frac{n\cos(\beta_A) - n'\cos(\beta_T)}{n\cos(\beta_A) + n'\cos(\beta_T)}\right)^2$$
$$C_l = \left(\frac{n'\cos(\beta_A) - n\cos(\beta_T)}{n'\cos(\beta_A) + n\cos(\beta_T)}\right)^2 \tag{12}$$

and it results that if $\beta_A \leqslant 20°$ then the difference between C_r and C_l is small. If $\beta_A = 0$ (normal incidence) or if β_A verifies the paraxial approximation, the reflection coefficients for the perpendicular and parallel modes coincide and, indicating them as C_0, from Eq. (12) it follows that

$$C_0 = \left(\frac{n - n'}{n + n'}\right)^2 \tag{13}$$

For example, for $n = 1$ and $n' = 1.53$ we have $C_0 = 0.04$ so 4% of the incident light is reflected.

In the case in which light originated at point J is incident upon an ophthalmic lens, the incidence angle depends on its surfaces curvature radii and on the location of J. In the Appendix A we analyze whether the paraxial approximation is an adequate assumption for the evaluation of reflection coefficients (C) and we evaluate $\boldsymbol{\delta_{\%}C} = 100(C - C_0)/C$. Considering, for example $n_G = 1.502$, $B = 738.6$ mm, $\theta = 10°$ and an aperture of 10 mm, the largest value of $\boldsymbol{\delta_{\%}C}$ corresponds to the upper marginal ray, for the 1st surface we obtain $\boldsymbol{\delta_{\%}C} = -10.7\%$ for the ray incident with $\beta_A = 15.4°$ on a 6 dioptres lens and for the 2nd surface we have $\boldsymbol{\delta_{\%}C} = -13.6\%$ for the ray incident with $\beta_A = 11.3°$ on a −6 dioptres lens (these percentages reduce to −6.5% and −7.5%, respectively if the aperture is 4 mm). Tolerating these percentages, we calculate coefficients under paraxial approximation for all the rays constituting the cone of light that reaches the eye (Fig. 3(a)) or the photometer sensor (Fig. 3(c)). Thus the relative illuminance due to losses in reflections is $\boldsymbol{E}_{r,p} = (1 - C_{0,1})(1 - C_{0,2})$ and only depends on n_G since, from Eq. (13),

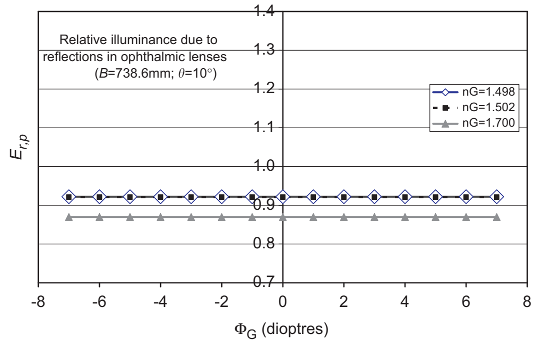

Fig. 8. Relative illuminance due to reflections at both surfaces of an ophthalmic lens as a function of its power for different refraction indices ($\theta = 10^\circ$ and the source is far away so reflection coefficients are computed under paraxial approximation).

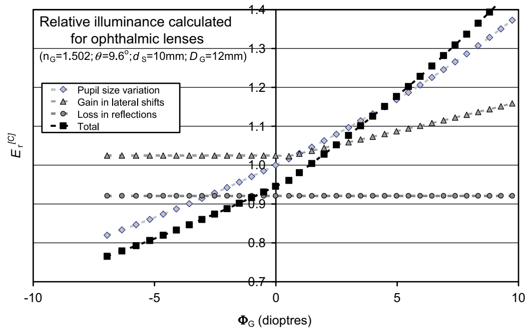

Fig. 9. Relative illuminance due to each of the 3 considered effects and to the 3 simultaneously in the presence of an ophthalmic lens as a function of its power ($n_G = 1.502$, $\theta = 9.6^\circ$, $d_S = 10\,\text{mm}$, $D_G = 12\,\text{mm}$).

it is

$$\boldsymbol{E}_{r,p} = \left(1 - \left(\frac{1-n_G}{1+n_G}\right)^2\right)^2 \qquad (14)$$

The plot $\boldsymbol{E}_{r,p}$ versus lens power is a constant, which tends to 1 as refraction index decreases (Fig. 8).

3.4. Total relative illuminance

The total relative illuminance theoretically calculated, $\boldsymbol{E}_r^{[C]}$, is

$$\boldsymbol{E}_r^{[C]} = 1 + (\boldsymbol{E}_{r,m} - 1) + (\boldsymbol{E}_{r,g} - 1) + (\boldsymbol{E}_{r,p} - 1) \qquad (15)$$

where the formula for $\boldsymbol{E}_{r,m}$ is given in Eqs. (4) and (5); $\boldsymbol{E}_{r,g}$ in Eqs. (10) and (11) and $\boldsymbol{E}_{r,p}$ in Eq. (14). In Fig. 9, we plot the relative illuminances corresponding to the 3 effects and to the total one as a function of lens power considering distances similar to those of Section 4, which are $D_G = 12\,\text{mm}$; $e_{ax} = 2\,\text{mm}$ for every diverging ophthalmic lens and $R_1 = 70\,\text{mm}$. If $d_S = 10\,\text{mm}$, the effects of lateral shifts and of reflections modify in 2% the relative illuminance, which would be obtained only taking into account the effect of pupil size variation introduced by a 6 dioptres lens. Considering the 3 effects, there is a critical lens power, $\Phi_G|_{crit}$ (which is 1.4 dioptres in Fig. 9), such that there is loss in diverging and converging lenses of power $\Phi_G < \Phi_G|_{crit}$ and gain in converging ones of power $\Phi_G > \Phi_G|_{crit}$. The loss is −16% for −4 dioptres and −21% for −6 dioptres, whereas the gain is 13% for 4 dioptres and 23% for 6 dioptres. Since $\boldsymbol{E}_{r,g}$ depends on d_S (Fig. 6), if the sensor is the eye, these percentages vary because pupil diameter depends on the subject and, under natural conditions, it is usually less than 10 mm [8,16]. For example if $d_S = 4\,\text{mm}$, the effects of lateral shifts and reflections modify in 15% the relative illuminance, which would be obtained only considering the effect of pupil size variation introduced by a 6 dioptres lens and we have $\varepsilon_{\%} = -18\%$ for a −6 dioptres lens and $\varepsilon_{\%} = 39\%$ for a 6 dioptres one.

For plane parallel plates, the second term in Eq. (15) is absent, the loss due to reflections is independent of the plate thickness and the gain due to lateral shifts increases when e_L increases and/or when d_S decreases. In Fig. 10, considering $n_L = 1.51$ and $d_S = 10\,\text{mm}$, we plot $\boldsymbol{E}_{r,g}$, $\boldsymbol{E}_{r,p}$ and $\boldsymbol{E}_r^{[C]}$ versus thickness and it results that for thicknesses lesser than 6 mm there is loss while for larger ones there is gain, for example $\varepsilon_{\%} = -5.5\%$ for $e_L = 2\,\text{mm}$ and $\varepsilon_{\%} = 4\%$ for $e_L = 9\,\text{mm}$.

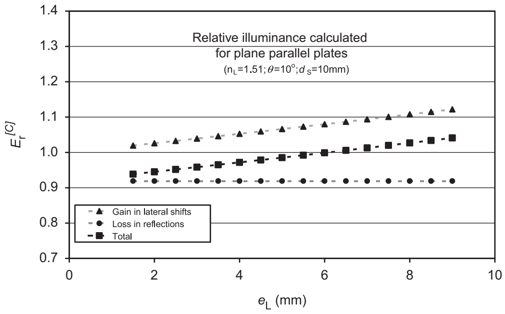

Fig. 10. Relative illuminance due to lateral shifts; to reflections and to both simultaneously in the presence of a plane parallel plate as a function of its thickness ($n_L = 1.51$, $\theta = 10^\circ$, $d_S = 10\,\text{mm}$).

4. Measured and calculated relative illuminance

In what follows we describe how we determine the parameters of the experimental setup and of the lenses employed and we show the relative illuminance experimentally and theoretically obtained.

4.1. Experimental setup and ophthalmic lenses parameters

We measure the relative illuminance, $\boldsymbol{E}_r^{[M]}$, using the setup of Fig. 3(c), which is mounted in 2 optical benches,

Table 1. Parameters of employed ophthalmic lenses ((*) indicates measured).

Type			Standard	Standard	Thin Cyl −0.12	Standard	Standard
Frontofocometer	$\Phi_G^{(F)}$ (*)	(dioptres)	−6	−4	0.12	4	6
Refraction index	n_G (*)		1.502	1.502	1.498	1.502	1.502
Lens diameter	H (*)	(mm)	65	65	65	65	65
Border thickness	e_{bor} (*)	(mm)	8.93	5.63	2	1.27	1.27
Measurements with spherometer	n_K (*)		1.53	1.53	1.53	1.53	1.53
	$\Phi_{1,K}$ (*)	(dioptres)	1.58	1.96	–	7.46	8.96
	R_1	(mm)	334.7	270.6	–	71.1	59.2
	$\Phi_1^{(K)}$	(dioptres)	1.50	1.85	–	7.06	8.49
	$\Phi_{2,K}$ (*)	(dioptres)	−8.08	−6.08	–	−3.29	−2.79
	R_2	(mm)	65.6	87.1	–	161.0	189.9
	$\Phi_2^{(K)}$	(dioptres)	−7.66	−5.76	–	−3.12	−2.64
Axial thickness	b_1		0.99	0.99	–	0.79	0.70
	a_1	(mm)	1.58	1.96	–	7.87	9.73
	b_2		0.75	0.86	–	0.96	0.97
	a_2	(mm)	8.62	6.29	–	3.31	2.80
	e_{ax}	(mm)	1.9	1.3	2.0	5.8	8.2
Lens power determined from measurements with spherometer	$\Phi_{1,K}+\Phi_{2,K}$	(dioptres)	−6.50	−4.13	–	4.17	6.17
	$\Phi_1^{(K)}+\Phi_2^{(K)}$	(dioptres)	−6.16	−3.91	–	3.95	5.84
	Φ_{Ad}	(dioptres)	−0.01	−0.01	–	−0.09	−0.12
	$\Phi_G^{(K)}$	(dioptres)	−6.14	−3.90	–	4.03	5.96
Frontofocometer and spherometer	$\Phi_G^{(F)}-\Phi_G^{(K)}$	(dioptres)	0.14	−0.10	–	−0.03	0.04
	$\delta_{\%}\Phi_G$		−2.4	2.6	–	−0.8	0.6

one for the glare source and another for the photometer and the ophthalmic lenses support. The source consists in a reflecting incandescent lamp Osram Concentra Spot R63 220V-E27/ES-30° of 60 W and a stop of adjustable diameter adjacent to it. To avoid the influence on illuminance measurements of voltage fluctuations, we employ a voltage stabiliser. We use 2 photometers, a POCKET LUX LMT with sensor diameters $d_S = 10$ mm and $d_{SE} = 30$ mm and a LMT B360 with $d_S = 30$ mm and $d_{SE} = 80$ mm. We analyze four monofocal organic standard ophthalmic lenses Orma 15 of Essilor of powers ± 6 and ± 4 dioptres and also one lens of standard material specially fabricated for this work with border thickness 2 mm and power 0.12 dioptres though, because of fabrication errors, it has cylinder −0.12 dioptres so we approximate its power to that of a 0.06 dioptres spherical lens. Besides the ophthalmic lenses, we study 2 plane parallel plates of Crown glass and zero power, a thin one of thickness $e_L = 1.95$ mm and quadrangular contour of $50\,mm \times 50\,mm$ and a thick one with $e_L = 6.6$ mm and rectangular contour of $30\,mm \times 105\,mm$. The lenses and plates are clean and without scratches and their powers are measured with a digital frontofocometer Nidek Autolensmeter LM 820.

To calculate the relative illuminance $E_r^{[C]}$ by means of Eq. (15), the parameters n_G; e_{ax}; Φ_G; D_G; g; B and d_S appearing in Eqs. (4), (5), (10), (11) and (14) must be determined. The ophthalmic lenses parameters are shown in Table 1 (symbol (*) indicating parameters which are directly measured or fabrication data). The lenses refraction indices (n_G) and diameters (H) are fabrication data and we measure the border thickness (e_{bor}). We determine the lens power not only with the frontofocometer but also with a spherometer since this enables us to evaluate R_1; R_2 and e_{ax} except for the 0.12 dioptres lens whose surface powers are so small that cannot be suitably measured. In the spherometer we read the values of the powers $\Phi_{1,K}$ and $\Phi_{2,K}$ corresponding to a lens of refraction index $n_K = 1.53$ (index K indicating magnitudes measured with spherometer). Using these values, we compute the radii, $R_1 = 1000(n_K-1)/\Phi_{1,K}$ and $R_2 = 1000(1-n_K)/\Phi_{2,K}$ and, considering index n_G, we determine the real powers, $\Phi_1^{(K)} = 1000(n_G-1)/R_1$ and $\Phi_2^{(K)} = 1000(1-n_G)/R_2$. Replacing the values of $H = 2h$; e_{bor}; R_1 and R_2 of Table 1 in Eq. (3), we calculate the axial thickness, $e_{ax} = e_{bor} + a_1 - a_2$, and, according to Eq. (2), the lens power measured with spherometer is $\Phi_G^{(K)} = \Phi_1^{(K)} + \Phi_2^{(K)} - \Phi_{Ad}$. Concerning precision, the frontofocometer (0.12 dioptres resolution for the lens power) is more precise than the spherometer (0.25 dioptres resolution for each surface power) but is not always available in research laboratories whereas the spherometer can be cheaply

Table 2. Measured and calculated relative illuminances in lenses ((*) indicates measured).

Type of ophthalmic lens			Standard	Standard	Thin Cyl −0.12	Standard	Standard
Both cases							
Lenses parameters (copied from Table 1)	n_G		1.502	1.502	1.498	1.502	1.502
	R_1	(mm)	334.7	270.6		71.1	59.2
	R_2	(mm)	65.6	87.1		161.0	189.9
	e_{ax}	(mm)	1.9	1.3	2.0	5.8	8.2
	$\Phi_G^{(K)}$	(dioptres)	−6.14	−3.90	0.06	4.03	5.96
Source parameters	B (*)	(mm)	739.5	739.5	739.5	739.5	739.5
	g (*)	(mm)	125	125	125	125	125
	θ	(degree)	9.6	9.6	9.6	9.6	9.6
	d_{gl} (*)	(mm)	21.0	21.0	21.0	21.0	21.0
Case $D_S = 12$ mm							
Sensor parameters	d_S (*)	(mm)	10.0	10.0	10.0	10.0	10.0
	d_{SE} (*)	(mm)	30	30	30	30	30
	s'_I	(mm)	3.04	3.04	3.04	3.04	3.04
	D_S (*)	(mm)	12	12	12	12	12
	D'_G	(mm)	9.0	9.0	9.0	9.0	9.0
Calculated relative illuminance	m_G		0.931	0.955	1.001	1.051	1.077
	$\boldsymbol{E}_{r,m}$		0.867	0.913	1.001	1.104	1.160
	k		0.0009	0.0006	0.0009	0.0026	0.0037
	$\boldsymbol{E}_{r,g}$		1.023	1.016	1.025	1.073	1.105
	$(1-n_G)/(1+n_G)$		−0.201	−0.201	−0.199	−0.201	−0.201
	$\boldsymbol{E}_{r,p}$		0.921	0.921	0.922	0.921	0.921
	$\boldsymbol{E}_r^{[C]}$		0.812	0.850	0.948	1.099	1.186
Measured relative illuminance	E (*)	(lx)	60.06	60.06	60.06	60.06	60.06
	E_G (*)	(lx)	48.20	49.88	56.08	65.83	72.32
	$\boldsymbol{E}_r^{[M]}$		0.803	0.831	0.934	1.096	1.204
	$\delta_{\%}\boldsymbol{E}_r$		−1.2	−2.3	−1.5	−0.3	1.5
	$\varepsilon_{\%}$		−19.7	−16.9	−6.6	9.6	20.4
Case minimum D_S							
Sensor parameters	d_S (*)	(mm)	10.0	10.0	10.0	10.0	10.0
	d_{SE} (*)	(mm)	30	30	30	30	30
	s'_I	(mm)	3.04	3.04	3.04	3.04	3.04
	D_S (*)	(mm)	1.74	1.30	0.00	0.70	0.59
	D'_G	(mm)	−1.3	−1.7	−3.0	−2.3	−2.4
Calculated relative illuminance	m_G		0.989	0.995	1.000	1.003	1.004
	$\boldsymbol{E}_{r,m}$		0.979	0.990	1.000	1.006	1.007
	k		0.0009	0.0006	0.0009	0.0026	0.0037
	$\boldsymbol{E}_{r,g}$		1.023	1.016	1.025	1.073	1.105
	$(1-n_G)/(1+n_G)$		−0.201	−0.201	−0.199	−0.201	−0.201
	$\boldsymbol{E}_{r,p}$		0.921	0.921	0.922	0.921	0.921
	$\boldsymbol{E}_r^{[C]}$		0.923	0.927	0.947	1.000	1.033
Measured relative illuminance	E (*)	(lx)	60.00	60.00	60.00	60.00	60.00
	E_G (*)	(lx)	54.36	55.50	57.22	60.60	63.30
	$\boldsymbol{E}_r^{[M]}$		0.906	0.925	0.954	1.010	1.055
	$\delta_{\%}\boldsymbol{E}_r$		−1.9	−0.2	0.7	1.0	2.1
	$\varepsilon_{\%}$		−9.4	−7.5	−4.6	1.0	5.5

acquired and yields different precisions depending on which parameters are known and on the degree of difficulty of the calculation. We compare the lens power measured with frontofocometer, $\Phi_G^{(F)}$, to that obtained with the spherometer when different approximations are considered and to do so, besides $\Phi_G^{(K)}$, we evaluate $\Phi_1^{(K)}+\Phi_2^{(K)}$ and $\Phi_{1,K}+\Phi_{2,K}$. The difference between $\Phi_G^{(F)}$ and $\Phi_G^{(K)}$ results to be at most 0.14 dioptres so the perceptual difference $\delta_{\%}\Phi_G = 100\ (\Phi_G^{(F)}-\Phi_G^{(K)})/\Phi_G^{(F)}$ is less than 2.6%. If the axial thickness is neglected and the power is approximated by $\Phi_1^{(K)}+\Phi_2^{(K)}$, the difference with $\Phi_G^{(K)}$ is Φ_{Ad}, which is lesser than 0.12 dioptres whereas, if neither the lens index nor its thickness are known and power is assumed to be $\Phi_{1,K}+\Phi_{2,K}$, the difference with $\Phi_G^{(K)}$ results to be 0.36 dioptres for the −6 dioptres lens and much smaller for the other lenses though this difference increases in cases in which n_G and n_K differ more from each other.

Regarding the distance lens-sensor, we measure $\boldsymbol{E}_r^{[M]}$ setting $D_S = 12$ mm for every lens and also reducing D_S to the minimum possible value, $D_{S,min}$. Usually $D_{S,min} \neq 0$ because the sensor contacts the lens 2nd surface (which is concave) in points other than its vertex and, replacing $a_2 = D_{S,min}$, $h_2 = d_{SE}/2$ and the measured value of R_2 in Eq. (3), we get $D_{S,min} = R_2\,[1-(1-(d_{SE}/(2R_2))^2)^{1/2}]$. With these values of D_S, we find the experimental vertex distance, $D'_G = D_S - |s'_I|$ to be considered in Eq. (4) instead of D_G.

4.2. Comparison between measured and calculated relative illuminance

The distances g; B' and d_{gl} are measured while B and θ are calculated and, as in previous papers [4,5], θ is approximately 10° (Table 2). For every ophthalmic lens, we determine $\boldsymbol{E}_r^{[C]}$ replacing in Eq. (15) the measured parameters; $\boldsymbol{E}_r^{[M]}$ measuring E_G and E with the photometer whose sensor diameter is $d_S = 10$ mm and the perceptual difference $\delta_{\%}\boldsymbol{E}_r = 100\ (\boldsymbol{E}_r^{[M]}-\boldsymbol{E}_r^{[C]})/\boldsymbol{E}_r^{[M]}$. In spite of the approximations involved, our formulas predict the relative illuminance in standard ophthalmic

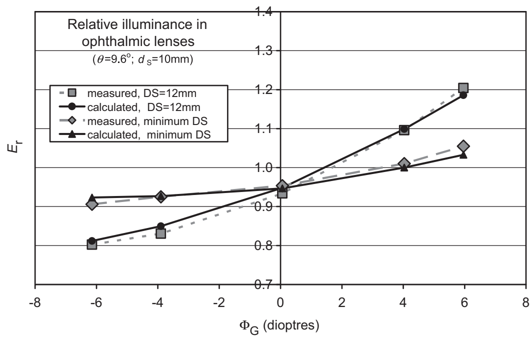

Fig. 11. Measured and calculated relative illuminance in the presence of an ophthalmic lens as a function of its power in the cases $D_S = 12$ mm and D_S minimum ($\theta = 9.6°$, $d_S = 10$ mm).

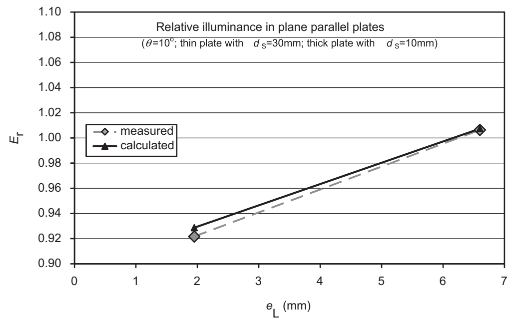

Fig. 12. Measured and calculated relative illuminance in the presence of a thin plane parallel plate (with $d_S = 30$ mm) and a thick plate (with $d_S = 10$ mm) as a function of its thickness ($\theta = 10°$; the scale is different from that of Fig. 11).

Table 3. Relative illuminances in plates ((*) indicates measured).

Plane parallel plate thickness			Thin	Thick
Plate parameters	n_L (*)		1.51	1.51
	e_L (*)	(mm)	1.95	6.60
Source parameters	B (*)	(mm)	719.0	718.9
	g (*)	(mm)	126	126.8
	θ	(degree)	9.9	10.0
	d_{gl} (*)	(mm)	20.0	20.0
Sensor parameters	d_S (*)	(mm)	30.0	10.0
	D_S (*)	(mm)	12	12
Calculated relative illuminance	k		0.0009	0.0031
	$\boldsymbol{E}_{r,g}$		1.010	1.088
	$(1-n_L)/(1+n_L)$		−0.203	−0.203
	$\boldsymbol{E}_{r,p}$		0.919	0.919
	$\boldsymbol{E}_r^{[C]}$		0.929	1.007
Measured relative illuminance	E (*)	(lx)	60.00	60.04
	E_L (*)	(lx)	55.30	60.42
	$\boldsymbol{E}_r^{[M]}$		0.922	1.006
	$\delta_{\%}\boldsymbol{E}_r$		−0.8	−0.1
	$\varepsilon_{\%}$		−7.8	0.6

lenses of up to ± 6 dioptres with an error which is at most 3%. Additionally, we obtain that, if $D_S = 12$ mm, a diverging ophthalmic lens introduces an illuminance loss that increases with power absolute value and we have $\varepsilon_{\%} = -19.7\%$ for -6 dioptres whereas a converging lens of power larger than 2 dioptres introduces a gain that increases with power and $\varepsilon_{\%} = 20.4\%$ for 6 dioptres. If the distance lens-sensor is $D_{S,min}$, the effect of pupil size variation is small and we get $\varepsilon_{\%} = -9.4\%$ for the -6 dioptres lens and $\varepsilon_{\%} = 5.5\%$ for the 6 dioptres one. In Fig. 11, we plot $\boldsymbol{E}_r^{[C]}$ and $\boldsymbol{E}_r^{[M]}$ as a function of lens power for both values of D_S.

For plane parallel plates, in Table 3 we show the calculated and measured results obtained for $\theta = 10^{\circ}$ using the photometer with $d_S = 30$ mm for the thin plate and the one with $d_S = 10$ mm for the thick plate. The thin plate (with thickness similar to that of a diverging lens) introduces a small illuminance loss ($\varepsilon_{\%} = -7.8\%$) while the thick one (with thickness a little larger than that of the converging 4 dioptres lens) causes a slight gain ($\varepsilon_{\%} = 0.6\%$). In Fig. 12 we plot the relative illuminance and we get $\delta_{\%}\boldsymbol{E}_r < 1\%$.

5. Conclusion

When a subject wearing ophthalmic lenses is under the presence of a peripheral glare source, it is useful to evaluate how these lenses modify the illuminance reaching the eye without measuring it. In this paper, we develop a methodology to theoretically estimate the relative illuminance $\boldsymbol{E}_r$ (defined as the quotient between the illuminance at the corneal vertex with and without a lens) as a function of parameters that can be simply determined or are fabrication data. We consider the light originated at the central source point that is incident upon a standard monofocal ophthalmic lens of spherical surfaces. We assume that certain restrictions (distant small source, eccentricity angle verifying the paraxial approximation, eye fixating a forward object, system lens-eye free from vignetting, circular pupils and uniform flux at the cornea) are valid. We take into account 3 effects that modify the illuminance: change of pupil size of the optical system with and without ophthalmic lens; lateral shifts of rays transmitted through the lens and reflections in both lens surfaces. The second and third effects modify the relative illuminance, which would be obtained if only the pupil size variation effect were present in up to 15% for a 4 mm pupil diameter and a lens power of 6 dioptres. Considering the 3 effects, we obtain that diverging ophthalmic lenses introduce an illuminance loss whereas converging lenses of power greater than a critical one (usually smaller than 2 dioptres), introduce a gain that increases as power increases. A diverging ophthalmic lens is thin in the axial zone so the gain due to lateral shifts is small and cannot compensate the loss due to reflections and decrease of pupil size whereas a thick converging lens corresponding to a relatively high power is such that the gain due to lateral shifts and to increase of pupil size can be much greater than the loss due to reflections. We experimentally verify our theoretical formulas considering 5 organic ophthalmic lenses of refraction index 1.5 and power ± 6; ± 4 and 0.12 dioptres and, to avoid the effect of change of pupil size, we also consider 2 glass plane parallel plates of thicknesses 1.95 and 6.6 mm. Using a glare source of diameter 21 mm forming an eccentricity angle of 9.6° and at 740 mm from a photometer sensor of diameter 10 mm, there is gain or loss of about 20% if power is, respectively 6 or -6 dioptres though these percentages vary if the sensor is the eye since pupil diameter is usually much smaller than 10 mm. For the plates, using a source of diameter 20 mm at 10° and 719 mm apart from the sensor, there is a loss of 8% for the thin plate and a sensor of diameter 30 mm and a gain of 1% for the thick plate and a sensor of diameter 10 mm. Experimental results differ in less than 3% from the theoretically predicted ones.

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Appendix A. Reflection coefficients in ophthalmic lenses

We estimate reflection coefficients when light originated at source point J is incident upon an ophthalmic lens (Fig. 13(a)) regarding its 1st and 2nd spherical surfaces as constituted by small plane surfaces tangent to it at points such as T_1 and T_2 in Fig. 13(b). Since an appreciable amount of light is lost in each refraction, we restrict our study to the 1st reflection at the 1st and 2nd lens surfaces. We analyze whether the paraxial approximation is an adequate assumption for the evaluation of coefficients tracing rays that correspond to an array of points at the 1st surface and computing, for each

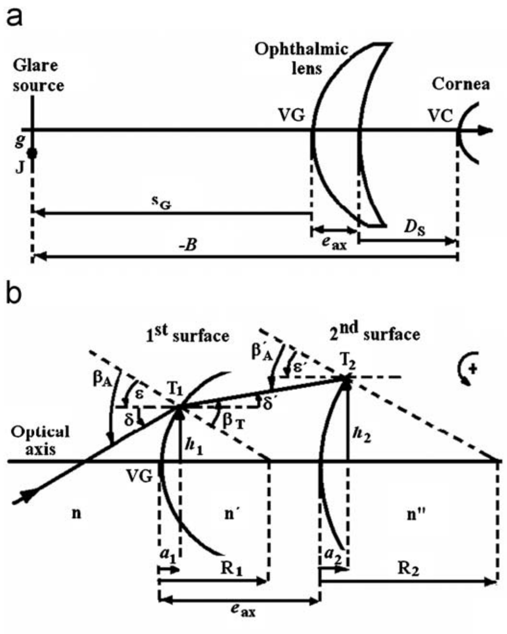

Fig. 13. Reflections in an ophthalmic lens. (a) Distances between components. (b) Incidence on the first and second surface of the lens ($a_1>0$, $R_1>0$, $h_1>0$, $a_2>0$, $R_2>0$ and $h_2>0$ in the figure).

ray, the incidence angle; the coefficients without this assumption (C_r and C_l); the approximated coefficients (C_0) and $\boldsymbol{\delta_{\%}C} = 100(C-C_0)/C$ (where C stands for either C_r or C_l).

For the lens 1st surface, we find the incidence angle (β_A) in terms of the ray height at the 1st surface (h_1); the distance from the optical axis to J (g); the 1st surface radius (R_1) and the distance from the lens vertex VG to the object plane (s_G) calculated in terms of the axial thickness (e_{ax}); the distance lens-sensor (D_S) and the distance source-sensor (B), this is, $s_G = -B+D_S+e_{ax}$. If ε and δ are the angles which the 1st surface normal and the ray, respectively form with the optical axis, using Eq. (3), we have

$$\sin(\varepsilon) = \frac{h_1}{R_1} \quad a_1 = R_1\left(1 - \sqrt{1 - \left(\frac{h_1}{R_1}\right)^2}\right)$$
$$\tan(\delta) = \frac{h_1 - g}{a_1 - s_G} \quad \beta_A = \varepsilon + \delta \tag{16}$$

For the lens 2nd surface we calculate the incidence angle β'_A in terms of the ray height at the 1st surface (h_1); the sagitta at the 1st surface (a_1); the axial thickness (e_{ax}); the 2nd surface radius (R_2) and the angle between the optical axis and the ray emerging from the 1st surface (δ') computed in terms of the refraction angle at the 1st surface (β_T) obtained using Eq. (1) and the angle between the normal to the 1st surface and the optical axis (ε), this is, $\delta' = \beta_T - \varepsilon$. After a little algebra, we get

$$a_2 = \left(\frac{1}{1+\tan^2(\delta')}\right)\left(\left(R_2\left(1 - \tan(\delta')\left(\frac{V}{R_2}\right)\right)\right) - \sqrt{U}\right)$$
$$h_2 = h_1 + (e_{ax} - a_1 + a_2)\tan(\delta') \quad \sin(\varepsilon') = \frac{h_2}{R_2} \quad \beta'_A = \varepsilon' + \delta' \tag{17}$$

where for simplicity, we define

$$V = h_1 + (e_{ax} - a_1)\tan(\delta')$$
$$U = R_2^2\left(1 - 2\tan(\delta')\left(\frac{V}{R_2}\right) - \left(\frac{V}{R_2}\right)^2\right) \tag{18}$$

To calculate C_r and C_l, for the 1st surface, we use Eq. (12) for the values of β_A evaluated in Eq. (16) while, for the 2nd surface, we replace n by n'; n' by n''; $\beta_A$by β'_A and β_T by $\beta_T{}'$ in Eq. (12) anduse the values of β'_A evaluated in Eq. (17). For the ± 6 dioptres lenses of Section 4 ($n_G = 1.502$, $B = 738.6$ mm and $\theta = 10°$), we obtain that both for the 1st and 2nd surfaces, the largest values of $\boldsymbol{\delta_{\%}C}$ occur for the upper marginal ray. If the aperture is 10 mm, the largest value for the 1st surface is $\boldsymbol{\delta_{\%}C} = -10.7\%$ and corresponds to the ray incident with $\beta_A = 15.4°$ upon the 6 dioptres lens and the largest value for the 2nd surface is $\boldsymbol{\delta_{\%}C} = -13.6\%$ and corresponds to the ray incident with $\beta_A = 11.3°$ upon the -6 dioptres lens, whereas if the aperture is 4 mm these percentages are reduced to -6.5% and -7.5%, respectively.

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