INSTITUTE OF PHYSICS PUBLISHING CLASSICAL AND QUANTUM GRAVITY

Class. Quantum Grav. **23** (2006) 3195–3203 doi:10.1088/0264-9381/23/9/027

# Duality gives rise to Chaplygin cosmologies with a big rip

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Received 2 December 2005
Published 7 April 2006
Online at stacks.iop.org/CQG/23/3195

**Abstract**
We consider modifications to the Friedmann equation motivated by recent proposals along these lines pursuing an explanation to the observed late time acceleration. Here we show that these approaches can be framed within a theory with modified gravity, and we discuss the construction of the duals of the cosmologies generated within that framework. We then investigate the modifications required to generate extended, generalized and modified Chaplygin cosmologies, and then show that their duals belong to a larger family of cosmologies we call enlarged Chaplygin cosmologies. Finally, by letting the parameters of these models take values not earlier considered in the literature we show that some representatives of that family of cosmologies display sudden future singularities. This fact indicates that the behaviour of these spacetimes is rather different from that of generalized or modified Chaplygin gas cosmologies. This reinforces the idea that modifications of gravity can be responsible for unexpected evolutionary features in the universe.

PACS number: 98.80.Cq

## 1. Introduction

All well-established cosmological models of the universe at large scales rest on the basic assumptions that matter is distributed homogeneously and isotropically, and that its structure is governed by an effective theory of gravity.

The first assumption implies the large-scale geometry of the universe is given by a Friedmann–Robertson–Walker metric in which the only non-trivial degree of freedom is the scale factor $a(t)$, where $t$ is the cosmological time. Hence, observations based on geometry can only tell us about $a$ and/or its derivatives. The second assumption of the two above determines how the geometry evolves. In usual practice, the different sources are treated as fluids with energy density $\rho$ and pressure $p$. Essentially, if one knows the total pressure as a function of the total energy density, then the total energy density $\rho$ can be determined by integrating the corresponding conservation equations. Finally, one can use the gravitational field equations within our effective theory of gravity to determine the metric.

General relativity is the standard effective theory of gravity, but possible corrections at the level of the Einstein equations have been investigated recently [1–5]. In a related fashion, other authors have considered low-curvature corrections to the Einstein–Hilbert action [11]. The modifications to Einstein's gravity investigated in all those references were designed so that they play an important role in the late asymptotic regimes, providing an explanation to the currently observed acceleration [6]. These proposals represent contenders to the less compelling suggestion that cosmic acceleration is due to dark energy associated with either a cosmological constant [8] or a new scalar field [9, 10].

Some of these modifications of gravity result from adding some new terms on the right-hand side (rhs) of the Einstein equations. In this paper, we show that these approaches can be framed within a theory with modified gravity, where we use the adjective 'modified' to indicate the presence of functions of $\rho$ and $p$ on the rhs of the Einstein equations. After that, and within that framework, we show how to apply duality transformations [12] in a similar fashion to previous works regarding conventional scalar field cosmologies or perfect fluid brane cosmologies. These transformations have received considerable attention because they lead to peculiar new cosmological models. For instance, the dual of a cosmological model with an initial singularity or big bang is another model with a final big rip singularity (the duality trades one singularity for the other [13]).

In the spirit of [5], we show later on that Chaplygin cosmologies, which are usually viewed as arising from a fluid with an exotic equation of state, can also arise in a modified gravity set-up from a perfect fluid with a linear relation between pressure and energy density. Specifically, we find a form of the modification to the Einstein equations which defines a class of cosmologies (first considered in [14]) which will call enlarged Chaplygin cosmologies. This class includes as particular cases generalized, extended and modified Chaplygin cosmologies (hereafter GC [15], EC [16] and MC [16] cosmologies) along with their duals. This larger class of Chaplygin cosmologies, which has been induced using duality transformations, extends the idea of modifications to pressure+energy density in the Einstein equations beyond the realm of accelerated cosmologies, because it includes both super-accelerated cosmologies and contracting (crunching) cosmologies.

Interestingly, this class of model displays a rich casuistry for the possible evolutions, for instance, scenarios with strong and weak sudden future singularities [17].

The outline of the paper is as follows. In section 2 we study the equations that govern flat FRW cosmologies in the set-up of modified gravity, and then we show how the duals of those models can be obtained. Then, we investigate the family of cosmologies that will be dubbed enlarged Chaplygin cosmologies which include GC, EC and MC cosmologies. In section 3, we concentrate on some of those enlarged Chaplygin cosmologies which we find particularly appealing, and highlight their properties. Finally, in section 4, the conclusions are reported.

## 2. Dual Chaplygin cosmologies within modified gravity

We consider now the dynamics of flat FRW cosmologies governed by the equations of motion of a gravitational theory with modified gravity in the aforementioned sense:

$$3H^2 = \rho_{\text{mod}}(\rho), \tag{1}$$

$$-2\dot{H} = \rho_{\text{mod}}(\rho) + p_{\text{mod}}(p), \tag{2}$$

where $H = \dot{a}/a$ is the Hubble factor, $a$ is the scale factor, and the overdot denotes differentiation with respect to cosmic time. Here $\rho$ and $p$ are, respectively, the energy density and the pressure of the fluid that fills the universe, and they satisfy the conservation equation

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{3}$$

In addition, we assume it is a conventional perfect fluid with equation of state $p = (\gamma - 1)\rho$, where $\gamma$ is the barotropic index.

Now, from equations (1) and (2) one gets

$$\dot{\rho}_{\rm mod} + 3H(\rho_{\rm mod} + p_{\rm mod}) = 0, \tag{4}$$

and combining the latter with equation (3) we arrive at

$$\rho'_{\rm mod} = \frac{\rho_{\rm mod} + p_{\rm mod}}{\rho + p} \tag{5}$$

where the prime denotes differentiation with respect to the energy density $\rho$.

A duality transformation links a cosmology with scale factor $a$ with another one with scale factor $a^{-1}$. In consequence, this leads to the transformation $H \to -H$, which leaves equation (1) unchanged but reverses the sign of the sum $(\rho_{\rm mod} + p_{\rm mod})$ in equation (2). Hence $p_{\rm mod}$ must transform according to

$$p_{\rm mod} \to -(2\rho_{\rm mod} + p_{\rm mod}) \tag{6}$$

to leave equation (2) unmodified. Since $\rho_{\rm mod}(\rho)$ is invariant under the transformation, it is its derivative with respect to $\rho$. Combining that with (5) one deduces that $(\rho + p) \to -(\rho + p)$, so that if the seed fluid satisfies the weak energy condition the source of the dual cosmology will automatically violate it.

Let us assume our fluid has an EC gas equation of state:

$$p_{\rm mod} = -\frac{A}{\rho^{\alpha}_{\rm mod}}, \tag{7}$$

with $\alpha$ and $A$ being constants. In the particular case of $A > 0$ and $0 < \alpha \leqslant 1$ the latter reduces to the GC gas. Chaplygin gas cosmologies can alternatively be considered as the simplest tachyon cosmological models where the tachyon field is a purely kinetic k-essence model with a constant potential. In the same way, the EC gas can be conceived as the simplest extended tachyon field model driven by a constant potential. This identification has the advantage of producing a variety of new Chaplygin gas cosmologies which includes some super-accelerated scenarios.

From equation (6) the dual equation of state for an EC cosmology is given by

$$p^{d}_{\rm mod} = -2\rho_{\rm mod} + \frac{A}{\rho^{\alpha}_{\rm mod}}. \tag{8}$$

From the latter we see the application of the duality transformation has generated a nonlinear term in the energy density, and therefore its equation of state does not have the original form. This suggests broadening the equation of state defining EC cosmologies so that the duality transformation keeps the form of the equation of state mentioned. This forces us to introduce the alternative equation of state

$$p_{\rm mod} = (\gamma_0 - 1)\rho_{\rm mod} - \gamma_0 \frac{A}{\rho^{\alpha}_{\rm mod}}, \tag{9}$$

with $A, \gamma_0$ and $\alpha$ being arbitrary constants. Keeping the assumption $p = (\gamma - 1)\rho$ and integrating equation (5), combined with the requirement $\alpha \neq -1$, one arrives at

$$\rho_{\rm mod} = [A \pm \rho^{\gamma_0(1+\alpha)/\gamma}]^{1/1+\alpha}. \tag{10}$$

The cosmologies derived from the latter were presented in [14], where some aspects of them were discussed. We will dub them as enlarged Chaplygin gas cosmologies. Since they have one additional parameter with respect to EC and MC cosmologies, they are richer in possible evolutions. The EC and MC cases are obtained by respectively taking $\gamma_0 = 1$ and $\gamma_0 = 1/(1+\alpha)$ in equation (9). The first case includes the result obtained by the modification of gravity proposed in [5], where only the Friedmann equation was modified. In the second case the MC equation of state mimics the mixture of a barotropic perfect fluid and a GC gas. This is actually a purely kinetic k-essence model [10] which describes the unification of dark matter and dark energy [14, 18–21]. MC cosmologies have the merit that, as recently shown in [22], they are observationally favoured over GC cosmologies. In general, for positive $\alpha$ and $A$ one has $\rho_{\text{mod}} \approx A^{1/1+\alpha}$ for $\rho \approx 0$, and $\rho_{\text{mod}} \approx \rho^{\gamma_0}$ for very large $\rho$ and $\gamma_0 > 0$. In contrast, if $A < 0$ consistency requires $\rho$ be bounded from below so that $\rho^{\gamma_0(1+\alpha)/\gamma} \leqslant |A|$.

Finally, we conclude that within the modified gravity picture GC, EC and MC cosmologies can be constructed in terms of the energy density of the fluid filling the universe.

The way to obtain the duals of these enlarged Chaplygin cosmologies is to apply (6) to (9), which is equivalent to the transformations

$$\gamma_0 \to -\gamma_0. \tag{11}$$

If we now denote with the superindex $d$ the quantities obtained under the duality transformation we will have

$$p^d_{\text{mod}} = -(\gamma_0 + 1)\rho_{\text{mod}} + \gamma_0 \frac{A}{\rho^\alpha_{\text{mod}}} \tag{12}$$

for the cosmologies which are dual to those obtained from (9), thus giving other cosmologies of the same kind.

The duals of EC and MC cosmologies belong to the larger family of dual cosmologies (12), and since those have an initial singularity due to the vanishing of the scale factor (generically in the GC case, and just for $\gamma_0 > 0$ in the MC case), their duals will have a sudden future singularity[3] which the scale factor will blow up in a finite time (see [13] for details).

## 3. General behaviour

Let us now analyse the behaviour of some of the new cosmologies we have presented here. Chaplygin cosmologies were advocated to explain the late-time acceleration suggested by supernovae data [6] and, in the following, we will mainly address the enlarged Chaplygin cosmologies which can serve that purpose. Interestingly, the expansion of the universe seems not to be just accelerated, but super-accelerated ($H > 0$ and $\dot{H} > 0$), i.e., the universe can be described as filled with a phantom fluid. Keeping that in mind, we will mainly care for the enlarged Chaplygin cosmologies with late-time super-acceleration, because such asymptotic behaviour (which is driven by super-negative pressure) seems to have some motivation in the framework of string theory [23]. GC and MC cosmologies are typically super-accelerated at late times but as time grows $\dot{H}$ becomes smaller and smaller, i.e., they end up being de Sitter cosmologies. In order to highlight the novel aspects of enlarged Chaplygin cosmologies we will concentrate on those super-accelerated models which do not have a de Sitter-like late-time limit[4]. Our discussion will remain qualitative, and it will be based on the assumption $p = (\gamma - 1)\rho$, which will give the expression of $\rho_{\text{mod}}$ in terms of the scale factor at the end of the day.

[3] In the recent literature two main classes of sudden cosmologies have been considered. One is the class typically arising in phantom cosmologies, which is characterized by a blow up of $H$, $\dot{H}$ and $a$ at a finite time. The other class has been considered in [25] and it corresponds to a situation in which $\dot{H}$ explodes in a finite time, whereas $H$ and $a$ stay finite.

[4] Generalized Chaplygin cosmologies which avoid the big rip by having a de Sitter-like late-time limit have been shown to exist in [14, 24].

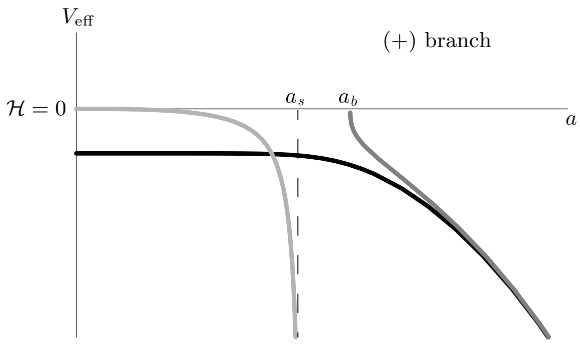


**Figure 1.** Effective potential energy curves for some enlarged Chaplygin cosmologies in the (+) branch with $\gamma_0 < 0$, and different choices for the rest of parameters which lead to particularly interesting universes. For $1+\alpha > 0$, we have considered the cases $A > 0$ and $A < 0$, which are represented by the black and dark grey curve respectively, whereas the case with $1+\alpha < 0$ and $A < 0$ is represented by the light grey curve.

The evolution equations of these universes can be derived from a given Hamiltonian $\mathcal{H}$ satisfying

$$\mathcal{H} = a^3\left[3\frac{\dot{a}^2}{a^2} - \rho_{\rm mod}(a)\right]. \tag{13}$$

The explicit expression of $\rho_{\rm mod}(a)$ follows from combining equations (5) and (7). One would then get

$$\rho_{\rm mod} = \left[A \pm \left(\frac{a_0}{a}\right)^{3\gamma_0(1+\alpha)}\right]^{1/1+\alpha} \tag{14}$$

with $a_0 > 0$ being an arbitrary constant. We see there are two alternative expressions for $\rho_{\rm mod}$ which we will respectively call the (+) branch and the (−) branch. The Friedmann equation implies in turn the Hamiltonian constraint $\mathcal{H} = 0$ must hold. These universes have qualitatively the same evolution as a massive particle moving in one dimension under the effective potential $V_{\rm eff} = -\rho_{\rm mod}(a)$ and with a velocity proportional to $\log a$. One can thus take advantage of the typical study of the potential energy curves to get a very pictorial insight. Even though we will mainly be concerned with just some enlarged Chaplygin cosmologies which depict novel cosmological evolutions, the analysis applies to any of them all the same.

Enlarged Chaplygin cosmologies with late-time super-acceleration and without a de Sitter-like late-time limit are obtained assuming $\gamma_0 < 0$ (not considered in the literature so far) together with $1+\alpha > 0$, and different possibilities arise depending on which branch is being considered and depending also on the sign of $A$.

Let us first concentrate on the (+) branch. For $A < 0$, the scale factor never vanishes and has a lower limiting value $a_b$ where the energy density, as given by (14), becomes null. The initial behaviour of the universe corresponds to a contracting (crunching) phase, and contraction lasts till the scale factor reaches the minimum value $a_b$. Then a bounce occurs, and the universe commences to expand and has a final expanding phase. The scale factor evolves between two infinite values. For large values of the scale factor, (14) gives $\rho_{\rm mod} \approx a^{3|\gamma_0|}$, which implies $a \approx |t - t_{bu}|^{-2/3|\gamma_0|}$ with $t_{bu}$ representing any of the two time instants where the blow up occurs. The birth and end of this universe are signalled by sudden singularities and the final one has the typical characteristics of a big rip. All these features can be better understood by looking at the dark grey curve in figure 1.

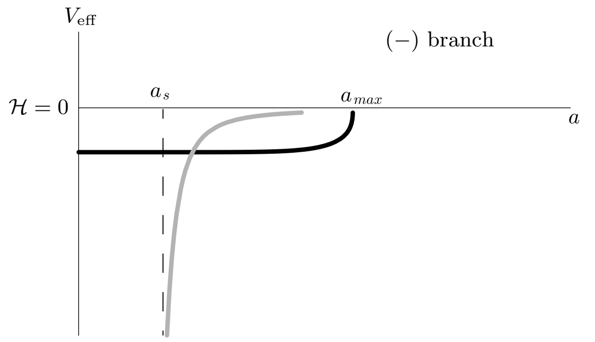


**Figure 2.** Effective potential energy curves for some enlarged Chaplygin cosmologies in the $(-)$ branch with $\gamma_0 < 0$, $A > 0$, and different choices for the rest of the $\alpha$ which lead to particularly interesting universes. The models with $1 + \alpha > 0$ are represented by the black curve, whereas the case with $1 + \alpha < 0$ is represented by the grey curve.

In contrast, when $A > 0$, the scale factor diverges only once in the whole history of the universe, and this occurs where the energy density $\rho_{\rm mod}$ diverges. This singularity, which we fix at $t = 0$, divides the time axis in two disconnected regions, $t > 0$ and $t < 0$. Thus, when solving the Einstein equations for this case, one gets a scale factor with two disconnected branches, one for each region. In the $t > 0$ region, the solution represents a contracting universe, which contracts forever, with an initial infinite scale factor and a vanishing final limit in the far future. In the $t < 0$ region, the solution represents an expanding universe which begins to evolve from a vanishing scale factor and ends in a final big rip at $t = 0$. A typical solution with such a dramatic end is depicted by the black curve in figure 1.

Leaving aside super-accelerated cosmologies, there are other enlarged Chaplygin cosmologies with interesting unusual futures. The name big rip [26] refers usually to sudden future singularities like those described before in which the scale factor diverges in a finite time. Nevertheless, lately, other sudden future singularities have attracted some attention. They display the property that $\dot{H}$ blows up in a finite time but $a$ stays finite [25][5]. Enlarged Chaplygin gases with $\gamma_0 < 0$, $A < 0$ and $1 + \alpha < 0$ are examples of that kind of evolution, but they do not display late-time super-acceleration. Interestingly, one will only be allowed to say a sudden future singularity is strong only if $a$ blows up [17]. The light grey curve in figure 1 represents this kind of universe, where we have let $a_s$ denote the finite value of the scale factor associated with the blow up.

So far, all this has concerned the (+) branch. Now, we have to repeat the analysis for the $(-)$ branch. Let us consider first the cases with $\gamma_0 < 0$ and $1 + \alpha > 0$, i.e. super-accelerated cases. The first difference we face is that the case $A < 0$ is not admissible unless $\alpha = 0$, so we will not consider this case any further. When $A > 0$, the universe begins its history with a null value of the scale factor, at which there is no blow up of the energy density, then it expands till the scale factor reaches a maximum value $a_{\rm max}$, and then enters an everlasting contracting phase without final singularities. The scale factor evolves between two finite values (see the black curve in figure 2).

In the $(-)$ branch there are another two cases worth mentioning which do not belong, however, to the realm of super-accelerated cosmologies. First, we have the models stemming from the choice $1 + \alpha < 0$, $\gamma_0 < 0$ and $A > 0$. These have sudden singularities which would

[5] However, unlike in the models in [25], the universes considered in this paper do not have an initial big bang.

represent the final stage of a contracting phase in a universe with an initial infinite scale factor (see the grey curve in figure 2). The second interesting case is the one corresponding to $\gamma_0 = 1$, $A > 0$ and $1 + \alpha < 0$. These are models with a late-time de Sitter-like phase; thus they avoid the big rip, as was first shown in [14] and then discussed in full detail in [24]. Their duals are depicted by the black curve in figure 2. Given that our dualities move the $H \equiv \text{const}$ regime from the $a \simeq 0$ region to the infinite scale factor region, the avoidance of the big rip proved in [14] and [24] follows immediately.

Before closing it is also worth mentioning the alternative equation of state

$$p_{\text{mod}} = -\rho_{\text{mod}} + \frac{A}{\rho_{\text{mod}}^{\alpha}}, \tag{15}$$

with

$$\rho_{\text{mod}} = [\log \rho^{A(\alpha+1)/\gamma}]^{1/(1+\alpha)}. \tag{16}$$

It generates a peculiar class of Chaplygin cosmologies [20] which do not, however, belong to the class defined by the equation of state (9). Comparing the expression (15) with its dual equation of state

$$p_{\text{mod}}^{d} = -\rho_{\text{mod}} - \frac{A}{\rho_{\text{mod}}^{\alpha}}, \tag{17}$$

we see that the duality transformations map both cosmologies into others within the same class. The evolution of these cosmologies has been the subject of a detailed study in [21], so we submit the interested reader to this reference.

## 4. Conclusions

In this paper, we have investigated possibilities offered by modifications to Einstein's theory of gravity, and we have discussed some consequences of them. Our study rests on the customary large-scale description of the universe under the assumption of isotropy and homogeneity. Specifically, we address some modifications to the Friedmann equations which are motivated by recent attempts in this spirit to explain the observed late-time acceleration.

We have proposed framing those modified gravity approaches within a theory of modified gravity with the presence of functions of $\rho$ and $p$ on the rhs of the Einstein equations. We have anticipated formally how, given a cosmology derived from these assumptions, one can obtain its dual. This can have interesting applications to the construction of phantom cosmologies with a big rip taking as seeds more conventional cosmologies with a big bang.

We then applied our proposal to GC, EC and MC cosmologies, and afterwards looked at the duals of those cosmologies. Finally we have found that they belong to a larger family of cosmologies which we call enlarged Chaplygin cosmologies. These have one additional parameter compared with the cosmologies above and, in consequence, the collection of evolutions they can depict is richer. In particular, we concentrated on values of the parameters not considered previously in other references, and we have found that these enlarged Chaplygin cosmologies can have future sudden singularities with typical big rip (blow up in the scale factor) or rather as those described in [25] (regular scale factor).

Finally, it would be interesting to check the solar and astrophysical constraints on this model in the manner of related works [27–32], but we have deferred this for future work.

## Acknowledgments

We are grateful to J M Aguirregabiria for valuable suggestions. RL is supported by the Spanish Ministry of Science and Education through research grant FIS2004-01626. LPC is partially funded by the University of Buenos Aires under project X224, and the Consejo Nacional de Investigaciones Científicas y Técnicas under proyect 02205.

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