Influence Functional Approach to Decoherence During Inflation

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We show how the quantum to classical transition of the cosmological fluctuations produced during inflation can be described by means of the influence functional and the master equation. We split the inflaton field into the system-field (long-wavelength modes), and the environment, represented by its own short-wavelength modes. We compute the decoherence times for the system-field modes and compare them with the other time scales of the model.

1 Introduction

The emergence of classical physics from quantum behaviour is important for several physical phenomena in the early Universe. This is beyond the fundamental requirement that only after the Planck time can the metric of the Universe be assumed to be classical. For example, the inflationary era is assumed to have been induced by scalar inflaton fields, with simple potentials [1]. Such fields are typically assumed to have classical behaviour, although in principle a full quantum description should be used. The origin of large scale structure in the Universe can be traced back to quantum fluctuations that, after crossing the horizon, were frozen and became classical, stochastic, inhomogeneities [2].

It is generally assumed that several phase transitions have occurred during the expansion of the Universe [3]. As in the case for the inflaton fields, the (scalar) order parameter fields that describe these transitions are described classically. However, the description of early universe phase transitions from first principles is intrinsically quantum mechanical [4]. As a specific application [5] of the previous point, the very notion of topological defects (e.g. strings and monopoles) that characterize the domain structure after a finite-time transition, and whose presence has consequences for the early universe, is based on this assumption of classical behaviour for the order parameter [6], as it distributes itself between the several degenerate ground states of the ordered system.

In previous publications, we analysed the emergence of a classical order parameter during a second order phase transition and the role of decoherence in the process of topological defect formation [7, 8, 9, 10].

In the present paper our concern is directly related with the first point above, the quantum to classical transition of the inflaton. Any approach must take into account both the quantum nature of the scalar field and the non-equilibrium aspects of the process [11]. The problem of the quantum to classical transition in the context of inflationary models was first addressed by Guth and Pi [12]. In that work, the authors used an inverted harmonic oscillator as a toy model to describe the early time evolution of the inflaton, starting from a Gaussian quantum state centered on the maximum of the potential. They subsequently showed that, according to Schrödinger's equation, the initial wave packet maintains its Gaussian shape (due to the linearity of the model). Since the wave function is Gaussian, the Wigner function is positive for all times. Moreover, it peaks on the classical trajectories in phase space as the wave function spreads. The Wigner function can then be interpreted as a classical probability distribution for coordinates and momenta, showing sharp classical correlations at long times. In other words, the initial Gaussian state becomes highly squeezed and indistinguishable from a classical stochastic process. In this sense, one recovers a classical evolution of the inflaton rolling down the hill.

A similar approach has been used by many authors to describe the appearance of classical inhomogeneities from quantum fluctuations in the inflationary era [13]. Indeed, the Fourier modes of a massless free field in an expanding universe satisfy the linear equation

$$\phi_k'' + (k^2 - \frac{a''}{a})\phi_k = 0. \tag{1}$$

For sufficiently long-wavelengths ($k^2 \ll a''/a$), this equation describes an unstable oscillator. If one considers an initial Gaussian wave function, it will remain Gaussian for all times, and it will spread with time. As with the toy model of Guth and Pi, one can show that classical correlations do appear, and that the Wigner function can again be interpreted as a classical probability distribution in phase space. (It is interesting to note that a similar mechanism can be invoked to explain the origin of a classical, cosmological magnetic field from amplification of quantum fluctuations.)

However, classical correlations are only one aspect of classical behaviour. It was subsequently recognized that, in order to have a complete classical limit, the role of the environment is crucial, since its interaction with the system distinguishes the field basis as the pointer basis [14].

(We are reminded that, even for the fundamental problem of the space-time metric becoming classical, simple arguments based on minisuperspace models suggest that the classical treatment is only correct because of the interaction of the metric with other quantum degrees of freedom [15].)

While these linear instabilities cited above characterise *free* fields, the approach fails when interactions are taken into account. Indeed, as shown again in simple quantum mechanical models (e.g. the anharmonic inverted oscillator), an initially Gaussian wave function becomes non-Gaussian when evolved numerically with the Schrödinger equation. The Wigner function now develops negative parts, and its interpretation as a classical probability breaks down [9]. One can always force the Gaussianity of the wave function by using a Gaussian variational wave function as an approximate solution of the Schrödinger equation, but this approximation deviates significantly from the exact solution as the wave function probes the non-linearities of the potential [9, 16].

When interactions are taken into account, classical behaviour is recovered only for "open systems", in which the unobservable degrees of freedom interact with their environment. When this interaction produces *both* a diagonalization of the reduced density matrix and a positive Wigner function, the quantum to classical transition is completed [17].

In Ref.[9] we have considered an anharmonic inverted oscillator coupled to a high temperature environment. We showed that it becomes classical very quickly, even before the wave function probes the non-linearities of the potential. Being an early time event, the quantum to classical transition can now be studied perturbatively. In general, recoherence effects are not expected [18]. Taking these facts into account, we have extended the approach to field theory models [7, 8]. In field theory, one is usually interested in the long-wavelengths of the order parameter. Even the early universe is replete with fields of all sorts which comprise a rich environment, in the inflationary example, we considered a model in which the system-field interacts with the environment-field, including only its own short-wavelengths. This is enough during inflation. Assuming weak self-coupling constant (inflaton potential is flat) we have shown that decoherence is a short time event, shorter than the time $t_{\rm end}$, which is essentially the time by which different modes in the system sector cross the horizon. As a result, perturbative calculations are justified[8]. Subsequent dynamics can be described by a stochastic Langevin equation, the details of which are only known for early times [19].

In our approach, the quantum to classical transition is defined by the diagonalization of the reduced density matrix. In phase transitions the separation between long and short-wavelengths is determined by their stability, which depends on the parameters of the potential. During Inflation, this separation is set by the existence of the Hubble radius. Modes cross the aparent horizon during their evolution, and they are usually treated as classical. The main mativation of this talk is to present a formal way to understand this statement within the open quatum sytem approach. In the last sense, decoherence is the critical ingredient if we are to dynamically demonstrate the quantum-to-classical transition of the open system.

The paper is organized as follows. In Section 2 we introduce our model. This is a theory containing a real system field ϕ, massless and minimally coupled to de Sitter background. We compute the influence functional by integrating out the environmental sector of the field, composed by the short-wavelength modes. Section 3 is dedicated to reviewing the evaluation of the master equation and the diffusion coefficients which are relevant in order to study decoherence. In Section 4 we evaluate upper bounds on the decoherence times. As we will see, decoherence takes place before the end of the inflationary period. Section 5 contains our final remarks.

2 The Influence Functional and the Density Matrix

Let us consider a massless quantum scalar field, minimally coupled to a de Sitter spacetime $ds^2 = a(\eta)[d\eta^2 - d\vec{x}^2]$ (where η is the conformal time $d\eta = dt/a(t)$), with a quartic self-interaction. The classical action is given by

$$S[\phi] = \int d^4x\, a^4(\eta) \left[\frac{\phi'^2}{2a^2(\eta)} - \frac{\nabla\phi^2}{2a^2(\eta)} - \lambda\phi^4\right], \quad (2)$$

where $a(\eta) = -1/(H\eta)$ and $\phi' = d\phi/d\eta$ ($a(\eta_i) = 1$ [$\eta_i = -H^{-1}$], and H is the Hubble radius). Let us make a system-environment field splitting

$$\phi = \phi_< + \phi_>, \quad (3)$$

where the system field contains the modes with wavelengths longer than the critical value $\Lambda^{-1} = 2\pi/\lambda_c$, while the bath field contains wavelengths shorter than Λ^{-1}. As we set $a(\eta_i) = 1$, a physical length $\lambda_{\rm phys} = a(\eta)\lambda$ coincides with the corresponding comoving length at the initial time. Therefore, the splitting between system and environment gives a system sector constituted by all the modes with physical wavelengths shorter than the critical length λ_c at the initial time η_i.

After splitting, the total action (2) can be written as

$$S[\phi] = S_0[\phi_<] + S_0[\phi_>] + S_{\rm int}[\phi_<, \phi_>], \quad (4)$$

where S_0 denotes the free field action and the interaction term is given by

$$S_{\rm int}[\phi_<, \phi_>] = -\lambda \int d^4x\, a^4(\eta)\, \left\{\phi_<^4(x) + \phi_>^4(x) + 6\phi_<^2(x)\phi_>^2(x) + 4\phi_<^3(x)\phi_>(x) + 4\phi_<(x)\phi_>^3(x)\right\}. \quad (5)$$

The total density matrix (for the system and bath fields) is defined by

$$\rho[\phi_<^+, \phi_>^+, \phi_<^-, \phi_>^-, t] = \langle\phi_<^+\phi_>^+|\hat{\rho}|\phi_<^-\phi_>^-\rangle, \quad (6)$$

where $|\phi_<^\pm\rangle$ and $|\phi_>^\pm\rangle$ are the eigenstates of the field operators $\hat{\phi}_<$ and $\hat{\phi}_>$, respectively. For simplicity, we will assume that the interaction is turned on at the initial time η_i and that, at this time, the system and the environment are not correlated (we ignore, for the moment, the physical consequences of such a choice, it has been disussed in [8]). Therefore, the total density matrix can be written as the product of the density matrix operator for the system and for the bath

$$\hat{\rho}[\eta_i] = \hat{\rho}_<[\eta_i]\hat{\rho}_>[\eta_i]. \quad (7)$$

We will further assume that the initial state of the environment is the vacuum.

We are interested in the influence of the environment on the evolution of the system. Therefore the reduced density matrix is the object of relevance. It is defined by

$$\rho_{\rm r}[\phi^+_<, \phi^-_<, \eta] = \int \mathcal{D}\phi_> \rho[\phi^+_<, \phi_>, \phi^-_<, \phi_>, \eta]. \quad (8)$$

The reduced density matrix evolves in time by means of

$$\rho_{\rm r}[\eta] = \int d\phi^+_{<i} \int d\phi^-_{<i}\; J_{\rm r}[\eta, \eta_i]\; \rho_{\rm r}[\eta_i], \quad (9)$$

where $J_{\rm r}[\eta, \eta_i]$ is the reduced evolution operator

$$J_{\rm r}[\phi^+_{<f}, \phi_{<f}, \eta|\phi^+_{<i}, \phi^-_{<i}, \eta_i] = \int_{\phi^+_{<i}}^{\phi_{<f}} \mathcal{D}\phi_< \int_{\phi_{<i}}^{\phi_{<f}} \mathcal{D}\phi_< \times \exp\frac{i}{\hbar}\{S[\phi^+_<] - S[\phi^-_<]\}F[\phi^+_<, \phi^-_<]. \quad (10)$$

The influence functional (or Feynman-Vernon functional) $F[\phi^+_<, \phi^-_<]$ is defined as

$$\begin{aligned} F[\phi^+_<, \phi^-_<] &= \int d\phi^+_{>i} \int d\phi^-_{>i}\; \rho_{\phi_>}[\phi^+_{>i}, \phi^-_{>i}, \eta_i] \int d\phi_{>f} \\ &\times \int_{\phi^+_{>i}}^{\phi_{>f}} \mathcal{D}\phi^+_> \int_{\phi^-_{>i}}^{\phi_{>f}} \mathcal{D}\phi^-_> \exp\left(i\{S[\phi^+_>] + S_{\rm int}[\phi^+_<, \phi^+_>]\}\right) \\ &\times \exp\left(-i\{S[\phi^-_>] + S_{\rm int}[\phi^-_<, \phi^-_>]\}\right). \end{aligned}$$

This functional takes into account the effect of the environment on the system. The influence functional describes the averaged effect of the environmental degrees of freedom on the system degrees of freedom to which they are coupled. With this functional, one can indetify a noise and dissipation kernel related by some kind of fluctuation-dissipation relation. This relation is important when one is interested in possible stationary states where a balance in eventually reached. During inflation we have a very flat potential well away from its minimum, and we are, in general, only interested in the dynamics over some relatively small time. For example, we would neglect dissipation during the slow roll period; but it is not correct during the eventual reheating phase.

We define the influence action $\delta A[\phi^+_<, \phi^-_<]$ and the coarse grained effective action (CGEA) $A[\phi^+_<, \phi^-_<]$ as

$$F[\phi^+_<, \phi^-_<] = \exp\frac{i}{\hbar}\delta A[\phi^+_<, \phi^-_<], \quad (11)$$

$$A[\phi^+_<, \phi^-_<] = S[\phi^+_<] - S[\phi^-_<] + \delta A[\phi^+_<, \phi^-_<]. \quad (12)$$

We will calculate the influence action perturbatively in λ and we will consider only terms up to order λ^2 and one loop in the $\hbar$ expansion. The influence action has the following form

$$\begin{aligned} &\delta A[\phi^+_<, \phi^-_<] = \left\{\langle S_{\rm int}[\phi^+_<, \phi^+_>]\rangle_0 - \langle S_{\rm int}[\phi^-_<, \phi^-_>]\rangle_0\right\} \\ &+\frac{i}{2}\left\{\langle S^2_{\rm int}[\phi^+_<, \phi^+_>]\rangle_0 - \left[\langle S_{\rm int}[\phi^+_<, \phi^+_>]\rangle_0\right]^2\right\} \\ &-i\left\{\langle S_{\rm int}[\phi^+_<, \phi^+_>] S_{\rm int}[\phi^-_<, \phi^-_>]\rangle_0\right. \\ &\left. -\ \langle S_{\rm int}[\phi^+_<, \phi^+_>]\rangle_0 \langle S_{\rm int}[\phi_<, \phi_>]\rangle_0\right\} \\ &+\frac{i}{2}\left\{S^2_{\rm int}[\phi^-_<, \phi^-_>]\rangle_0 - \left[\langle S_{\rm int}[\phi^-_<, \phi^-_>]\rangle_0\right]^2\right\}, \end{aligned} \quad (13)$$

where $\langle\ \rangle_0$ is the quantum average, assuming the environment is initially in its vacuum state.

The influence functional can be computed, and the result is

$$\begin{aligned} {\rm Re}\delta A = &\ -\lambda\int d^4x\; a^4(\eta)\; P(x) \\ &+\lambda^2\int d^4x \int d^4y\; a^4(\eta)\; a^4(\eta')\; \theta(\eta-\eta') \\ &\times\left\{64\Delta_3(x){\rm Re}G^{\Lambda}_{++}(x-y)\Sigma_3(y)\right. \\ &\left.+288\Delta_2(x){\rm Im}G^{\Lambda 2}_{++}(x-y)\Sigma_2(y)\right\}, \quad (14) \\ {\rm Im}\delta A = &\ -\lambda^2\int d^4x \int d^4y\; a^4(\eta)\; a^4(\eta') \\ &\left\{-32\Delta_3(x){\rm Im}G^{\Lambda}_{++}(x-y)\Delta_3(y)\right. \\ &\left.+144\Delta_2(x){\rm Re}G^{\Lambda 2}_{++}(x-y)\Delta_2(y)\right\}, \quad (15) \end{aligned}$$

where $G^{\Lambda}_{++}(x-y)$ is the Feynmann propagator of the environment field, where the integration over momenta is restricted by the presence of the infrared cutoff Λ. We have also defined,

$$\begin{aligned} P &= \frac{1}{2}(\phi^{+4}_< - \phi^{-4}_<) \quad ; \quad \Delta_3 = \frac{1}{2}(\phi^{+3}_< - \phi^{-3}_<) \\ \Delta_2 &= \frac{1}{2}(\phi^{+2}_< - \phi^{-2}_<) \quad ; \quad \Sigma_3 = \frac{1}{2}(\phi^{+3}_< + \phi^{-3}_<) \\ \Sigma_2 &= \frac{1}{2}(\phi^{+2}_< + \phi^{-2}_<) \quad . \end{aligned}$$

3 Master Equation and Diffusion Coefficients

In this Section we will obtain the evolution equation for the reduced density matrix (master equation), paying particular attention to the diffusion terms, which are responsible for decoherence. We will closely follow the quantum Brownian motion (QBM) example [21, 22], translated into quantum field theory [7, 24].

The first step in the evaluation of the master equation is the calculation of the density matrix propagator $J_{\rm r}$ from Eq.(10). In order to solve the functional integration which defines the reduced propagator, we perform a saddle point approximation

$$J_{\rm r}[\phi^+_{<f},\phi^-_{<f},\eta|\phi^+_{<i},\phi^-_{<i},\eta_i]\approx \exp iA[\phi^+_{<{\rm cl}},\phi^-_{<{\rm cl}}], \quad (16)$$

where $\phi^{\pm}_{<{\rm cl}}$ is the solution of the equation of motion $\delta ReA/\delta\phi^+_<|_{\phi^+_<=\phi^-_<} = 0$ with boundary conditions $\phi^{\pm}_{<{\rm cl}}(\eta_i)=\phi^{\pm}_{<i}$ and $\phi^{\pm}_{<{\rm cl}}(\eta)=\phi^{\pm}_{<f}$. Since we are working up to λ^2 order, we can evaluate the influence functional using the solutions of the free field equations. This classical equation is $\phi^{''}_< + 2\mathcal{H}\phi^{'}_< - \nabla^2\phi_< = 0$, ($\mathcal{H}=a'(\eta)/a(\eta)$). A Fourier mode $\phi^{\rm cl}_<(x)=\int_{|\vec k|<\Lambda}\phi^{\rm cl}_{\vec k}\exp\{i\vec k.\vec x\}$, satisfies

$$\psi^{''}_{\vec k} + \left(k^2 - \frac{a''(\eta)}{a(\eta)}\right)\psi_{\vec k} = 0, \quad (17)$$

where we have used $\psi_{\vec k}=a(\eta)\phi_{\vec k}$ and the fact that $\frac{a''(\eta)}{a(\eta)} = \frac{2}{\eta^2}$. It is important to note that for longwavelength modes, $k \ll 2/\eta^2$, Eq. (17) describes an unstable (upside-down) harmonic oscillator [12].

As a classical solution can be written as

$$\phi^{\pm{\rm cl}}_{\vec k}(\eta) = \phi^{\pm}_i(\vec k)u_1(\eta,\eta_f) + \phi^{\pm}_f(\vec k)u_2(\eta,\eta_f), \quad (18)$$

where

$$u_1 = \frac{\sin[k(\eta-\eta_f)](\frac{1}{k}+k\eta\eta_f)+\cos[k(\eta-\eta_f)](\eta_f-\eta)}{\text{Idem num. } \eta\rightarrow\eta_{\rm i}},$$
$$u_2 = \frac{\sin[k(\eta_i-\eta)](\frac{1}{k}+k\eta\eta_i)+\cos[k(\eta-\eta_i)](\eta-\eta_i)}{\text{Idem num. } \eta\rightarrow\eta_{\rm f}}.$$

We will assume that the system-field contains only one Fourier mode with $\vec k = \vec k_0$. This is a sort of "minisuperspace" approximation for the system-field that will greatly simplify the calculations, therefore we assume

$$\phi^{\pm{\rm cl}}_<(\vec x,\eta) = \phi^{\pm{\rm cl}}_{\vec k_0}(\eta)\cos(\vec k_0.\vec x), \quad (19)$$

where $\phi^{\pm{\rm cl}}_{\vec k_0}$ is given by (18).

In order to obtain the master equation we must compute the final time derivative of the propagator $J_{\rm r}$. After that, all the dependence on the initial field configurations $\phi^{\pm}_{<i}$ (coming from the classical solutions $\phi^{\pm{\rm cl}}_<$) must be eliminated. In previous publicatiuons, we have shown that the free propagator satisfies [8]

$$\phi^{\pm{\rm cl}}_<(\eta)J_0 = \Big[\phi^{\pm}_{<f}[u_2(\eta,\eta_f) - \frac{u'_2(\eta_f,\eta_f)}{u'_1(\eta_f,\eta_f)}u_1(\eta,\eta_f)] \mp i\frac{u_1(\eta,\eta_f)}{u'_1(\eta_f,\eta_f)}\partial_{\phi^{\pm}_{<f}}\Big]J_0. \quad (20)$$

These identities allow us to remove the initial field configurations $\phi^{\pm}_{\rm i}$, by expressing them in terms of the final fields $\phi^{\pm}_{\rm f}$ and the derivatives $\partial_{\phi^{\pm}_{\rm f}}$, and obtain the master equation.

The full equation is very complicated and, as for quantum Brownian motion, it depends on the system-environment coupling. In what follows we will compute the diffusion coefficients for the different couplings described in the previous section. As we are solely interested in decoherence, it is sufficient to calculate the correction to the usual unitary evolution coming from the noise kernels (imaginary part of the influence action). The result reads

$$\begin{aligned} i\hbar\partial_\eta\rho_{\rm r}[\phi^+_{<f},\phi^-_{<f},\eta] &= \langle\phi^+_{<f}|\Big[\hat H_{\rm ren},\hat\rho_{\rm r}\Big]|\phi^-_{<f}\rangle \\ &- i\Big[\Gamma_1\, D_1(\vec k_0;\eta) + \Gamma_2\, D_2(\vec k_0;\eta)\Big]\rho_r[\phi^+_{<f},\phi^-_{<f},t] \\ &+ \ \dots, \end{aligned} \quad (21)$$

where we have defined $\Gamma_1 = \frac{V\lambda^2}{\Lambda^{-3}}\frac{(\phi^{+3}_{<f}-\phi^{-3}_{<f})^2}{H^5}$ and $\Gamma_2 = \frac{V\lambda^2}{\Lambda^{-3}}\frac{(\phi^{+2}_{<f}-\phi^{-2}_{<f})^2}{H^3}$. V is the spatial volume inside which there are no coherent superpositions of macroscopically distinguishable states for the system field. The ellipsis denotes other terms coming from the time derivative that not contribute to the diffusive effects. This equation contains time-dependent diffusion coefficients $D_i(t)$. Up to one loop, only D_1 and D_2 survive. Coefficient D_1 is related to the interaction term $\phi^3_<\phi_>$, while D_2 to $\phi^2_<\phi^2_>$. These coefficients can be (formally) written as

$$\begin{aligned} D_1(k_0,\eta) &= 2\frac{H^5}{\Lambda^3}\int_{\eta_i}^{\eta} d\eta' a^4(\eta)a^4(\eta')F^3_{\rm cl}(k_0,\eta,\eta') \\ &\times\ {\rm Im}G^{\Lambda}_{++}(3k_0,\eta,\eta')\,\theta(3k_0-\Lambda), \end{aligned} \quad (22)$$

and

$$\begin{aligned} D_2(k_0,\eta) = 36\frac{H^3}{\Lambda^3}\int_{\eta_i}^{\eta} d\eta' a^4(\eta)a^4(\eta')F^2_{\rm cl}(k_0,\eta,\eta') \\ \times\ \left[{\rm Re}G^{\Lambda 2}_{++}(2k_0,\eta,\eta') + 2{\rm Re}G^{\Lambda 2}_{++}(0,\eta,\eta')\right], \end{aligned} \quad (23)$$

where the function $F_{\rm cl}$ is

$$F_{\rm cl}(k_0,\eta,\eta') = \frac{\sin[k_0(\eta-\eta')]}{k_0\eta} + \frac{\eta'}{\eta}\cos[k_0(\eta-\eta')]. \quad (24)$$

The explicit expression of these coefficient are complicated functions of conformal time, the particular mode k_0, and the cutoff Λ, and we will show them in a separate publication [23]. For the porpuse of this talk, we will use some analytical approximations, which will allow to obtain an estimation of the scale of decoherence in a particular case.

4 Decoherence

The effect of the diffusion coefficient on the decoherence process can be seen considering the following approximate solution to the master equation

$$\rho_{\rm r}[\phi^+_<, \phi^-_<; \eta] \approx \rho^{\rm u}_{\rm r}[\phi^+_<, \phi^-_<; \eta] \times \exp\left[-\sum_j \Gamma_{\rm j}\int_{\eta_i}^{\eta_f} d\eta\, D_{\rm j}(k_0, \Lambda, \eta)\right], \quad (25)$$

where $\rho^{\rm u}_{\rm r}$ is the solution of the unitary part of the master equation (i.e. without environment), and Γ_j includes the coefficients in front each diffusion term in Eq.(21). The system will decohere when the non-diagonal elements of the reduced density matrix are much smaller than the diagonal ones.

The decoherence time-scale sets the time after which we have a classical field configuration, and it can be defined as the solution to

$$1 \approx \sum_j \Gamma_j \int_{\eta_i}^{\eta_D} d\eta D_j(k_0, \Lambda, \eta). \quad (26)$$

We will solve this equation only in a very approximated way in order to find upper bounds for the decoherence times coming from each diffusion coefficients. A more refined evaluation of decoherence times will be shown in Ref. [23]. For example, when we consider a mode $k_0 \leq H$, we can probe that a good approximation to D_1 is given by

$$D_1^{\rm approx}(k_0 \sim H, \eta) \sim -\frac{(1 + H\eta)}{H\eta^7 k_0^3 \Lambda^3}. \quad (27)$$

This is valid for modes shorter or the same order than H; but it is an overestimation for $k_0 \ll H$. As D_1 is defined under the constraint $\Lambda/3 < k_0 < \Lambda$, and considering we will set the critical length $\Lambda \leq H$, ours is a good estimation. Analysing the coefficient D_2, it is possible to find an approximated expression for small values of k_0 (respect to H). For $k_0/H > 1$, that corresponds to those modes out of the aparent horizon at η_i, the diffusion coefficient D_2 is an oscilatory function and it has a maximun when $k_0 \sim \Lambda$, this is the behavior noted for conformally coupled fields [24]. Finally, for long-wavelength modes, we can write

$$D_2^{\rm approx}(k_0 < H, \eta) \sim \frac{1}{H\eta^4\Lambda^3}. \quad (28)$$

Both approximations above, are close to the exact coefficients when $\Lambda \leq H$.

In order to quantify decoherence time we have to fix the values of $\Gamma_{1,2}$. For this, we have to assume values to λ, V, Δ, and Σ. We will use the more conservative choice in order to have a lower bound to the decoherence time.

Assuming slow roll condition $1/2(d\phi/dt) \ll U = \lambda\phi^4$, the classical equations (using $\ddot{\phi} \ll U'$) are

$$H^2 = \frac{8\pi U}{3m_{\rm pl}^2} \;\; ; \;\; \dot{\phi} = -\frac{U'}{3H}, \quad (29)$$

these equations are obtained under the following conditions

$$\epsilon_U = \frac{m_{\rm pl}^2}{2}\frac{U'}{U} \ll 1 \;\; ; \;\; \sigma_U = m_{\rm pl}^2 \frac{U''}{U} \ll 1, \quad (30)$$

where $m_{\rm pl}^2 = 1/G$ is the Plank mass.

Definig the end of the inflationary period setting $\epsilon_U \sim 1$, one can set $\phi(N) \approx \sqrt{8N} m_{\rm pl}$, where $N = \ln a(\eta_f)/a(\eta)$ is the e-fold number. Thus, we assume the mean value of the system field at time of decoherence is ϕ, and we set $(\phi^+_< - \phi^-_<) \sim 10^{-5}\phi$, and $V \sim H^{-3}$.

From previous considerations, we can show

$$t_{D_1} \leq \frac{1}{6H}\ln\left\{\frac{5k_0^3H^3}{\lambda^2 10^{-15}\phi^6}\right\} \sim \frac{1}{6H}\ln\left\{\frac{\lambda\phi^6 10^{15}}{m_{\rm pl}^6}\right\}, \quad (31)$$

and

$$t_{D_2} \leq \frac{1}{3H}\ln\left\{\frac{3H^4}{\lambda^2 10^{-10}\phi^4}\right\} \sim \frac{1}{3H}\ln\left\{\frac{10^{10}\phi^4}{m_{\rm pl}^4}\right\}. \quad (32)$$

Using $N = Ht_{\rm end} \geq 60$ as an estimative scale to the end of inflationary period,

$$\frac{t_{D_1}}{t_{\rm end}} \leq \ln\{\lambda 10^{15}\} \leq 1, \quad (33)$$

corresponding to the diffusion term D_1 and values of $\lambda \sim 10^{-8}$. The scale coming from D_2 is

$$\frac{t_{D_2}}{t_{\rm end}} \leq \frac{1}{10}, \quad (34)$$

.

From scales t_{D_1}, and t_{D_2}, we can see that for a given mode $k_0 < \Lambda \leq H$, decoherence is effective by the time in which inflation is ending.

5 Final Remarks

Let us summarize the results contained in this paper. After the integration of the high frequency modes in Section 2, we obtained the CGEA for the low energy modes. From the imaginary part of the CGEA we obtained, in Section 3, the diffusion coefficients of the master equation. System and environment are two sectors of a single scalar field, and the results depend on the "size" of these sectors, which is fixed by the critical wavelength Λ^{-1}.

In Section 4 we analysed the decoherence times for those modes in the system whose wavelength is shorter than the critical value.

We have shown some analitycal approximations that allow to conclude that if we consider a critical length $\Lambda \sim H$, those modes with wavelength $k_0 \ll \Lambda$ are the more affected by diffusion throught coefficient D_2. For these modes, we can show that the effect is not dependent of the critical Λ [23].

If one consider a cutoff $\Lambda \geq H$, and modes $H < k_0 < \Lambda$, diffusive effects are larger for those modes in the system whose wavelength is close to the critical Λ^{-1} [24].

In Ref. [23] we will present the complete expression of the diffusive terms, an also an extensive analysis of the evaluation of the timescale for decoherence in several cases of interest.

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