Extended tachyon field, Chaplygin gas, and solvable *k*-essence cosmologies

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We investigate a flat Friedmann-Robertson-Walker spacetime filled with *k* essence and find the set of functions *F* that generate three different families of extended tachyon fields and Chaplygin gases. They lead to accelerated and superaccelerated expanding scenarios. For any function *F*, we find the first integral of the *k*-field equation when the *k* field is driven by an inverse square potential or by a constant one. In the former, we obtain the general solution of the coupled Einstein−*k*-field equations for a linear function *F*. This model shares the same kinematics of the background geometry with the ordinary scalar field one driven by an exponential potential. However, they are dynamically different. For a constant potential, we introduce a *k*-field model that exhibits a transition from a power-law phase to a de Sitter stage, inducing a modified Chaplygin gas.

I. INTRODUCTION

Cosmological inflation has become an integral part of the standard model of the Universe and perhaps the only known mechanism that can dynamically solve the flatness and the horizon problem of the universe [1]. It has gained certain support from the recent observations of the cosmic microwave background anisotropies [2–4]. Although particle physics, in particular M or string theory, provides several very weakly coupled scalar fields which are natural inflaton candidates, there exists no clearly preferred inflationary model. A link between string theory and inflation was investigated in Ref. [5] where the authors introduce *k* inflation and show that the *k* field may drive an inflationary evolution starting from rather generic initial conditions. Also, the process of a rolling tachyon field has been extensively used to implement power-law accelerated expansion scenarios [6–11]. In Ref. [12] it was pointed out that rolling tachyons can contribute a mass density to the universe that resembles classical dust. This has brought a new understanding of the role of the tachyon in string theory. The tachyonic matter, then, might provide an explanation for inflation at the early epochs and could contribute to some new form of cosmological dark energy [13] at late times [14]. These facts suggest enlarging the theoretical background and exploring some new possibilities; in this sense, we will introduce an extended tachyon field (ETF) which supplies accelerated and superaccelerated expanding scenarios.

In Ref. [15] it was argued that the coincidence problem may be solved by assuming a universe filled with a viscous fluid and dark energy modeled with a tachyonic field or a Chaplygin gas. The conjecture that dark matter and dark energy can be unified by using a generalized Chaplygin gas obeying an exotic equation of state has been investigated in several works in view of the cosmological observations [16–22]. The present situation is somewhat controversial, with some tests indicating good agreement between observational

data and the theoretical predictions of the model and others ruling out the model as an actual possibility for description for our Universe [23]. Some differences were found with the observed cosmic microwave background and mass power spectrum data. Due to these discrepancies, it may be useful to consider other candidates for dark energy, such as, for instance, the extended Chaplygin gas (ECG) generated from the ETF. Another interesting possibility appears to be selecting a *k*-field model whose equation of state is like that of two fluids, one obeying a baryotropic equation of state with constant baryotropic index γ, and the other an ECG. This "modified Chaplygin gas" interpolates between a power-law phase and a de Sitter phase.

From theoretical and experimental points of view it is important to find the exact shape of the potential, for instance, tachyonic inflation has been studied using phenomenological potentials that have not been derived from string theory and can be related to the so called "*k* inflation" [5,7,8]. Such *k* fields, described by a nonstandard kinetic term, are one of the recent suggested candidates to play the role of some unknown component of the universe. One of the purposes of introducing *k* essence is to provide a dynamical explanation of cosmological observations which does not require the fine-tuning of initial conditions [24–26]. In this sense, it may be considered as an alternative to quintessence, which requires a scalar field that slowly rolls down a potential to depict the observed acceleration of the present Universe. Also, it was argued that in certain dynamical regimes the *k* essence is equivalent to quintessence and it may prove difficult to distinguish between the two fields. In the light of these results, it seems clear that it is worth searching for the links between scalar fields and *k*-essence cosmologies, in particular, whether they are kinematically or dynamically equivalent.

In Sec. II, we introduce the ETF and find the power-law solution generated by an inverse square potential. At the end, we define the ECG and show the general behavior of the background geometry. In Sec. III, we find the first integral of the *k*-field equation for an inverse square potential and the general solution of the coupled Einstein−*k*-field equations

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for a linear function F. For power-law solutions we demonstrate that the linear k-field model, with a constant function F, is isomorphic to the model with divergent sound speed. Finally, we introduce the modified Chaplygin gas. In Sec. IV, we show the kinematical equivalence of the k field driven by a tachyonic potential and a linear function F, with the scalar field driven by an exponential potential. The conclusions are stated in Sec. V.

II. EXTEND TACHYON FIELD AND CHAPLYGIN GAS

The spatially flat homogeneous and isotropic spacetime is described by the Friedmann-Robertson-Walker (FRW) line element

$$ds^2 = -dt^2 + a^2(t)[dx_1^2 + dx_2^2 + dx_3^2], \qquad (1)$$

where $a(t)$ is the scale factor. This metric allows a particular calculational simplicity, on account of both the high degree of symmetry and the single metric degree of freedom. We assume that $8\pi G = 1$.

Let us present the Lagrangian of the k-essence field:

$$\mathcal{L} = -V(\phi)F(x), \quad x = g^{ik}\phi_i\phi_k, \qquad (2)$$

where $V(\phi)$ is a positive defined potential, $F(x)$ is an arbitrary function of x, ϕ is the k field, and $\phi_i = \partial V(\phi)/\partial x^i$. Associating the energy-momentum tensor of the k field,

$$T_{ik} = V(\phi)[2F_x\phi_i\phi_k - g_{ik}F], \quad F_x = \frac{dF}{dx}, \qquad (3)$$

with the energy-momentum tensor of a perfect fluid, we compute the energy density ρ, the pressure p, and the baryotropic index $\gamma = 1 + p/\rho$ of this equivalent fluid:

$$\rho = V(\phi)[F - 2xF_x], \quad p = \mathcal{L} = -V(\phi)F, \qquad (4)$$

$$\gamma = -\frac{2\dot{H}}{3H^2} = -\frac{2xF_x}{F - 2xF_x}. \qquad (5)$$

From Eq. (4), the Einstein field equations are

$$3H^2 = V[F - 2xF_x], \qquad (6)$$

$$\dot{H} = VxF_x, \qquad (7)$$

and the conservation equation reads

$$\dot{\rho} + 3H(\rho + p) = 0, \qquad (8)$$

where $H = \dot{a}/a$ is the expansion rate. Substituting Eq. (4) into the conservation equation (8), we find the field equation for the k field:

$$[F_x + 2xF_{xx}]\ddot{\phi} + 3HF_x\dot{\phi} + \frac{V'}{2V}[F - 2xF_x] = 0. \qquad (9)$$

The stability of the k essence with respect to small wavelength perturbations requires that the effective sound speed

$$c_s^2 = \frac{p_x}{\rho_x} = \frac{F_x}{F_x + 2xF_{xx}} \qquad (10)$$

be positive. However, in Ref. [27] it was shown that a positive sound speed is not a sufficient condition for the theory to be stable. For the tachyon field, which is obtained from the k field by selecting $F = (1 - \dot{T}^2)^{1/2}$, with $\dot{T}^2 = -x = \dot{\phi}^2$, the sound speed is $c_s^2 = 1 - \gamma > 0$, where $\gamma = \dot{T}^2$. Let us look for the set of functions F, such that c_s^2 is proportional to the sound speed of the tachyon field. We express this proportionality as

$$c_s^2 = \frac{1 - \gamma}{2r - 1}, \qquad (11)$$

where r is a real constant, with $r < 1/2$ for $\gamma > 1$ and $r > 1/2$ for $\gamma < 1$. From Eqs. (5), (10), and (11), we obtain a differential equation for the function F

$$(1 - r)FF_x + (2r - 1)xF_x^2 + xFF_{xx} = 0. \qquad (12)$$

Integrating for $r = 0$, we obtain the set of functions $F_0(x) = x^{\gamma_0/2(\gamma_0 - 1)}$, with γ_0 constant, investigated in [28]; while for $r = 1$ we find three types of solution, $F_1 = (1 - \dot{T}_1^2)^{1/2}$, $F_1 = (1 + \dot{T}_1^2)^{1/2}$, and $F_1 = -(\dot{T}_1^2 - 1)^{1/2}$. The first function corresponds to the ordinary tachyon field $T = T_1$. Its energy density $\rho_1 = VF_1^{-1}$ and pressure $p_1 = -VF_1$ lead to the relation $p_1 = (\gamma_1 - 1)\rho_1$, where $\gamma_1 = \dot{T}_1^2$. The other two functions generate two "new tachyon fields" which will be investigated below. For the remaining values of r the general solution of Eq. (12) is given by $F_r^{2r} = c_1(-x)^r + c_2$, where c_1 and c_2 are arbitrary integration constants, the baryotropic index (5) is $\gamma_r = -c_1(-x)^r/c_2$, and we have $F_r^{2r} = c_2(1 - \gamma_r)$. From Eqs. (4) and (5), the energy density of the k field can be rewritten as $\rho = VF/(1 - \gamma)$. So, without loss of generality, we can split the solutions as follows: for $\gamma_r < 1$ and $r > 1/2$, it is necessary that $F_r > 0$ and $c_2 > 0$, with two options, $0 < \gamma_r < 1$ and $c_1 < 0$ or $\gamma_r < 0$ and $c_1 > 0$; for $\gamma_r > 1$ and $r < 1/2$, we have $F_r < 0$, c_2 and $c_1 > 0$. Finally,

$$F_r = (1 - \gamma_r)^{1/2r}, \quad \gamma_r = \pm\dot{T}_r^{2r} < 1, \qquad (13)$$

$$F_r = -(\gamma_r - 1)^{1/2r}, \quad \gamma_r = \dot{T}_r^{2r} > 1, \qquad (14)$$

where γ_r is the extended baryotropic index and the T_r field linked with F_r will be called the ETF. Inserting Eqs. (13) and (14) into Eq. (4), we get the energy density ρ_r and the pressure p_r of each ETF:

$$\rho_r = V(1 - \dot{T}_r^{2r})^{(1-2r)/2r}, \quad p_r = -V(1 - \dot{T}_r^{2r})^{1/2r},$$

$$0 < \gamma_r < 1, \qquad (15)$$

$$\rho_r = V(1 + \dot{T}_r^{2r})^{(1-2r)/2r}, \quad p_r = -V(1 + \dot{T}_r^{2r})^{1/2r}, \quad \gamma_r < 0, \qquad (16)$$

$$\rho_r = V(\dot{T}_r^{2r} - 1)^{(1-2r)/2r}, \quad p_r = V(\dot{T}_r^{2r} - 1)^{1/2r}, \quad 1 < \gamma_r, \qquad (17)$$

with

$$p_r = (\gamma_r - 1)\rho_r. \tag{18}$$

Equations (15) comprise perfect fluids that generalize the normal tachyon field, which is obtained for $r=1$. The fluids represented by Eqs. (16) with negative pressure and negative baryotropic index can describe phantom cosmologies, while those described by Eqs. (17) give rise to nonaccelerated expanding evolution. In the limit of large r, the fluids (15) and (16) satisfy the equation of state $p=-\rho=-V$ acting like a variable cosmological constant depending on the k field. In addition, the fluids (17) satisfy the equation of state $p=\rho = V$, behaving like stiff matter in the same limit. These exotic fluids satisfy the relations

$$p_r = -\frac{V^{2r/(2r-1)}}{\rho_r^{1/(2r-1)}}, \quad \gamma_r < 1, \tag{19}$$

$$p_r = \frac{V^{2r/(2r-1)}}{\rho_r^{1/(2r-1)}}, \quad \gamma_r > 1, \tag{20}$$

which, for a constant potential $V = V_0$, become exotic equations of state which extend that of the generalized Chaplygin gas.

A. Power-law expansion for the T_r field

We begin by investigating the atypical behavior of the $T_{1/2}$ field. Its sound speed diverges and the k-field equation (9) becomes a first order equation. Equation (5) can be integrated and its solution gives a link between the expansion rate and $T_{1/2}$:

$$H = \frac{2}{3T_{1/2}}, \tag{21}$$

where the integration constant has been chosen to set the singularity at $T_{1/2}=0$. For $r=1/2$, the three equations (15), (16), and (17) lead to $3H^2 = V$. So, combining them with Eq. (21), we obtain the inverse square potential

$$V_{1/2} = \frac{4}{3T_{1/2}^2}, \tag{22}$$

which is the uniquely allowed potential for $T_{1/2}$. As $T_{1/2}$ is not controlled by the k-field equation (9), it can be chosen freely, and this choice determines, after integrating Eq. (21), the form of the scale factor. Although the potential (22) diverges at $T_{1/2}=0$, it reasonably mimics the behavior of a typical potential in the condensate of bosonic string theory. One expects the potential to have a unique local maximum at the origin and a unique global minimum away from the origin at which V vanishes. In the most interesting case the global minimum is taken to lie at infinity. Obviously, more complicated potentials may be contemplated, but this is the simplest case to begin with.

It will be useful to investigate the existence of accelerated and superaccelerated expanding solutions when the T_r field

is driven by the potential $V_r = V_0/T_r^2$. To do that, let us consider an evolution of the form $a = t^n$ and a T_r field such that $T_r \propto t$. Power-law solutions are very important because they can always be obtained from any function F with an inverse square potential or from polynomial functions $F = (-x)^N$ with any potential. As these solutions are the usual ingredients in the quintessense and k-essence models, they allow us to recognize and compare the differences between the two cosmological models. The complete solution for the ETF is given by

$$T_r = \left(\frac{2}{3n}\right)^{1/2r} t, \quad n = \frac{1}{3}[1 + \sqrt{1 + 9\beta^2}], \quad 0 < \gamma_r < 1, \tag{23}$$

$$T_r = \left(\frac{2}{-3n}\right)^{1/2r} t, \quad n = \frac{1}{3}[1 - \sqrt{1 + 9\beta^2}], \quad \gamma_r < 0, \tag{24}$$

$$T_r = \left(\frac{2}{3n}\right)^{1/2r} t, \quad n = \frac{1}{3}[1 \pm \sqrt{1 - 9\beta^2}], \quad 1 < \gamma_r, \tag{25}$$

where

$$\beta^2 = \left[\frac{3}{V_0}\left(\frac{2}{3}\right)^{1/r}\right]^{2r/(1-2r)}. \tag{26}$$

The sound speed associated with the above solutions is given by

$$c_s^2 = \frac{1 - 2/3n}{2r - 1}. \tag{27}$$

Assuming a positive sound speed bounded by the light speed $0 \le c_s^2 \le 1$, we can see that $r > 1 - 1/3n$ is required for $n > 2/3$ or $n < 0$ and $r < 1 - 1/3n$ for $0 < n < 2/3$.

An accelerated power-law expanding universe $(n > 1)$ can be described by the set of ETFs having $r > 2/3$ and it is represented by the solutions (23). All these models are kinematically and dynamically different because the scale factor and the k field are strongly dependent on r, as can be seen from Eqs. (23) and (26). In particular, for $r=1$ Eq. (23) reduces to the solution of the tachyon field found in [7,8]. The solutions (24) with $n < 0$ can describe phantom cosmologies and we have to select ETFs with $r > 1$ to reach this scenario.

B. Extended Chaplygin gas

Recently a set of simple cosmological models was proposed based on the use of particular perfect fluids [29]. In this simple model the universe is filled with the so called Chaplygin gas, which is a perfect fluid characterized by the equation of state $p = -A/\rho$, where A is a positive constant. It describes the transition from a decelerated cosmological expansion to a cosmic accelerated de Sitter stage. Other possibilities are the inhomogeneous Chaplygin gas, which is able to combine the roles of dark energy and dark matter [30], and the generalized Chaplygin gas model discussed in Ref. [31], having two free parameters $p = -A/\rho^\alpha$ with $0 < \alpha \le 1$.

These cosmological models may give a unified macroscopic phenomenological description of dark energy and dark matter and generalize the usual cold dark matter models with a cosmological constant. On the other hand, the Chaplygin gas can be considered as the simplest tachyon cosmological model where the tachyon field is a purely kinetic *k*-essence model with a constant potential. In the same way, we will show that the generalized Chaplygin gas can be conceived as the simplest ETF model driven by a constant potential. This identification has the advantage of producing a variety of new Chaplygin gases, some of which lead to superaccelerated scenarios, as we will see in this subsection.

Coming back to our Eqs. (19) and (20), we see that in the case of a constant potential $V(T_r) = V_0$, with $r = 1$ and $0 < \gamma_r < 1$, the exotic equation of state (19) represents a Chaplygin gas [29]. This equation may be the consequence of a scalar field with a nonstandard kinetic term, e.g., the string theory motivated tachyon field [32,33]. For $1 < r$ and $0 < \gamma_r < 1$, the equation of state (19) represents a perfect fluid called the generalized Chaplygin gas [31]. Finally, an ECG will be characterized by the equation of state (19) with $1/2 < r < 1$, or by Eq. (20) with $r < 1/2$. Their properties will be investigated below. Using Eqs. (19), (20) and the relativistic energy conservation equation (8), we obtain the energy density of the ECG:

$$\rho = V_0 \left[1 + \left(\frac{a_0^3}{a^3} \right)^{2r/(2r-1)} \right]^{(2r-1)/2r} , \quad 0 < \gamma_r < 1, \quad (28)$$

$$\rho = V_0 \left[1 - \left(\frac{a_0^3}{a^3} \right)^{2r/(2r-1)} \right]^{(2r-1)/2r} , \quad \gamma_r < 0, \quad (29)$$

$$\rho = V_0 \left[-1 + \left(\frac{a_0^3}{a^3} \right)^{2r/(2r-1)} \right]^{(2r-1)/2r} , \quad 1 < \gamma_r. \quad (30)$$

Equation (28) with $r > 1$ gives the energy density of the generalized Chaplygin gas in terms of the scale factor interpolating between a dust dominated phase where the energy density is $\rho \approx V_0 (a_0/a)^3$, and a de Sitter phase where $\rho \approx V_0$, while during the intermediate stage it could be interpreted as a mixture of two fluids, one of which is the cosmological constant and the other is a perfect fluid with equation of state $p \propto \rho$. The additional free parameter r of the generalized Chaplygin gas can be used to compare it with observational data.

In any other case, Eqs. (28) with $1/2 < r < 1$ and Eqs. (29) and (30) represent new perfect fluids which are interesting from the cosmological point of view. The scale factor generated by the source (29) is nonsingular and has a bounce at the minimum $a = a_0$. The universe begins from a contracting era and ends in a superaccelerated stage. Such cosmologies may be interpreted as universes filled with baryotropic fluids having a negative constant baryotropic index and violating the weak energy condition $\rho > 0$, $\rho + p > 0$. The models will be dubbed phantom cosmologies following the standard terminology. Phantom matter can apparently be accommodated by current observations [34], and it can be based on the

motivation provided by string theory [35]. It looks interesting to admit that the origin of dark energy should be sought within a fundamental theory, such as string theory. However, at present there is no consensus as to whether a universe violating the weak energy condition should generically possess a future singularity or big rip [36].

The source (30) leads to two types of solutions according to the value of the parameter r. For $r < 0$ the universe has a finite time span, interpolating between two dust dominated phases, and has a maximum at $a = a_0$. For $0 \leq r \leq 1/2$ the universe begins at a singularity with a finite scale factor, $a = a_0$, and ends in a dust dominated phase. From the cosmological point of view these solutions are not relevant because they do not describe the present observed accelerated expansion stage.

III. SOLVABLE *k*-ESSENCE COSMOLOGIES

In this section, we will show some cases where the coupled Einstein–*k*-field equations (6), (9) admit a first integral or can be solved exactly. Expressing the energy density of the *k* field as $\rho = VF/(1 - \gamma)$ and using the conservation equation (8), we get the *k*-field equation in terms of the baryotropic index γ,

$$\left(\frac{\gamma}{\dot{\phi}} \right)^{\cdot} + 3H \left(\frac{\gamma}{\dot{\phi}} \right)(1 - \gamma) + \frac{V'}{V}(1 - \gamma) = 0, \quad (31)$$

where $V' = dV/d\phi$. This form of writing the field equation allows us to show that the first integral of the *k*-field equation (9) for any function F is given by

$$\frac{\gamma}{\dot{\phi}} = \phi^{-1} \left(\frac{2}{3H} + \frac{c}{a^3 H^2} \right), \quad (32)$$

where we have assumed an inverse square potential,

$$V = \frac{V_0}{\phi^2}, \quad (33)$$

and c is an arbitrary integration constant. The usual linear field solution $\phi = \phi_0 t$ along with the power-law scale factor $a = t^{2/3\gamma}$ (with constant γ) is obtained from the last equation for a vanishing integration constant. Combining Eqs. (5), (7), and (33) with Eq. (32), it can be rewritten as

$$\dot{\phi} F_x - \left[H + \frac{3c}{2a^3} \right] \frac{\phi}{V_0} = 0, \quad (34)$$

or as

$$-V_0 F_x \dot{H} = \left[H + \frac{3c}{2a^3} \right]^2 . \quad (35)$$

Then, for the tachyonic potential (33), Eqs. (32), (34), and (35) are three different forms of writing the first integral of the *k*-field equation (9) or (31).

At this point, using Eqs. (5), (33), and (32), it is interesting to see the coupled Einstein–k-field equations (6), (9) as a system of differential equations for the function F. Hence, by integrating we obtain

$$F = \frac{3h_0^2}{V_0} + \sqrt{-x}\left[b + \frac{3c}{2V_0}\int \frac{(2h_0+ch)h}{(-x)^{3/2}}dx \right], \quad (36)$$

$$H\phi = -\frac{3c}{2}\int \frac{d\phi}{a^3} = h_0 + ch, \quad (37)$$

where h_0 and b are arbitrary constants and $\dot{h} = -3\dot{\phi}/2a^3$. In the particular case $c=0$, Eq. (36) becomes

$$F^\infty = \frac{3h_0^2}{V_0} + b\sqrt{-x}, \quad (38)$$

which, after a redefinition of the constants, turns into the function $F_{1/2}$ that generates the ETF, $T_{1/2}$ [see Eqs. (13), (14) for $r=1/2$]. For this "divergent" k-essence theory the sound speed (10) diverges and the k-field equation (9) becomes a first order equation that is consistent only with an inverse square potential. This divergent model is related to the linear k-field model, $\phi=\phi_0 t$, obtained by evaluating F,F_x at $x=x_0=-\phi_0^2$ in the Einstein–k-field equations (6), (9). The linear k-field model driven by the potential (33) leads to the power-law solutions $a=t^\lambda$ with

$$\lambda = \frac{1}{3}\frac{f+2\phi_0^2 f'}{\phi_0^2 f'}, \quad V_0 = \frac{\lambda}{f'}, \quad (39)$$

where we have defined $f=F(-\phi_0^2)$, $f'=F_x(-\phi_0^2)$.

From Eqs. (38), (39) it is easy to show that the linear model is isomorphic to the divergent one. This can be done by constructing a one-to-one mapping between these two models. In fact, choosing in the divergent model the constants $h_0^2=\lambda^2\phi_0^2$ and $b=-2\lambda\phi_0/V_0$, we find the same power-law solutions obtained from the linear model. In Ref. [28] it was suggested that this might be the reason why a model with a diverging sound speed leads to serious problems as discussed in a recent paper [37]. In addition, assuming a series expansion of the function $F(x)$ around $x=x_0$, the background cosmology is completely determined by the first two coefficients (f,f') of the expansion of F and the value of ϕ_0. Hence, the model is insensitive to the remaining coefficients in the expansion of the function F and both the linear and divergent models should be considered as equivalent. This means that the power-law solutions generated by an inverse square potential possess a degeneracy. Possibly this degeneracy may be removed by perturbing the solutions.

A. Inverse square potential and linear function F

From the k-field equation (35), it can be seen that a linear function

$$F = 1 + mx, \quad (40)$$

where m is a constant, decouples the dynamics of the background geometry from the dynamics of the k field. Therefore, this choice clearly introduces a rather small degree of nonlinearity into the dynamical equations, allowing us to obtain the general solution of the coupled Einstein–k-field equations. On the other hand, the function (40) mimics the behavior of other models. For instance, when $x \ll 1$ the tachyonic function $F=(1+x)^{1/2}$ can be approximated by $F \approx 1+x/2$ and it has the form (40) [8]. In Ref. [25] a set of models was introduced where F admits a power series expansion similar to Eq. (40). This form is reminiscent of a Born-Infeld action with higher order corrections in x, and particular cases were investigated in [26,37]. Hence, the knowledge of the general solution for the linear function (40) should be of interest, at least for understanding the asymptotic behavior of many other models generated by an analytical function $F(x)=F(0)+F_x(0)x+\cdots$. In fact, keeping the first order term in the expansion of F, it adopts the linear form (40), after redefining the potential V to set the constant $F(0)=1$.

Combining Eqs. (35) and (40), we obtain the following nonlinear second-order differential equation for the scale factor:

$$\frac{d^2 s}{d\tau^2} + s^\sigma \frac{ds}{d\tau} + \frac{1}{4}s^{2\sigma+1} = 0, \quad \sigma = -3mV_0, \quad (41)$$

where we have used the new variables s and τ, defined by

$$s = a^{-3/\sigma}, \quad \tau = \frac{3c}{mV_0}t. \quad (42)$$

The general solution of Eq. (41) can be found by changing to nonlocal variables z and η, defined by [38]

$$z = \frac{s^{\sigma+1}}{\sigma+1}, \quad \eta = \int s^\sigma d\tau, \quad \sigma \neq 1, \quad (43)$$

$$z = \ln s, \quad \eta = \int \frac{d\tau}{s}, \quad \sigma = -1. \quad (44)$$

Then, in these new variables, Eq. (41) becomes a linear homogeneous differential equation with constant coefficients,

$$\frac{d^2 z}{d\eta^2} + \frac{dz}{d\eta} + \frac{\sigma+1}{4}z = 0, \quad \sigma \neq -1, \quad (45)$$

equivalent to a damped harmonic oscillator equation, and

$$\frac{d^2 z}{d\eta^2} + \frac{dz}{d\eta} + \frac{1}{4} = 0, \quad \sigma = -1. \quad (46)$$

On the other hand, expressing Eq. (34) for the k field in terms of the independent variable η, we get

$$\frac{d\phi}{\phi d\eta} = -\frac{3da}{\sigma a d\eta} + \frac{1}{2}. \quad (47)$$

Once $z(\eta)$ is known from Eq. (45), one can compute $s(\tau)$ from Eq. (43), the scale factor $a(\tau)$ from Eq. (42), and the k field $\phi(\tau)$ from Eq. (34). Following this procedure and inserting the general solutions of Eqs. (45) and (47) into the Einstein equation (6), we find the scale factor, the k field, and the relationship among the integration constants:

$$a=\left[\sqrt{-B}e^{-\eta/2}\sinh\left(\frac{\sqrt{-\sigma}}{2}\eta+\eta_0\right)\right]^{-\sigma/3(\sigma+1)}, \quad \sigma<-1,$$
(48)

$$a=a_0\exp\left[-\frac{\eta}{12}+V_0e^{-\eta/81c^2\phi_0^2}\right], \quad \sigma=-1,$$
(49)

$$a=\left[\sqrt{B}e^{-\eta/2}\cosh\left(\frac{\sqrt{-\sigma}}{2}\eta+\eta_0\right)\right]^{-\sigma/3(\sigma+1)}, \quad -1<\sigma<0,$$
(50)

$$a=\left[\sqrt{B}e^{-\eta/2}\sin\left(\frac{\sqrt{\sigma}}{2}\eta+\eta_0\right)\right]^{-\sigma/3(\sigma+1)}, \quad 0<\sigma.$$
(51)

The k field is given by

$$\phi=\phi_0 a^{-3/\sigma}e^{\eta/2},$$
(52)

where

$$B=\frac{4(\sigma+1)V_0}{27c^2\phi_0^2},$$
(53)

and η_0 and ϕ_0 are arbitrary integration constant.

For $\sigma<-1$, the solution expands from a singularity as $t^{1/3}$ and ends as $t^{-\sigma/3}$. When $\sigma>-3$ the scale factor displays a power-law inflationary scenario. For $-1<\sigma<0$, the universe expands from a singularity as $t^{1/3}$ and its final behavior is given by $t^{1/3}$. For $\sigma>0$ the solution represents a contracting universe which begins at a finite time and reaches a minimum where it bounces, exhibiting a final superaccelerated expansion $a(t)\propto(t_0-t)^{-\sigma/3}$. The universe has a finite time span and bounces when the k field satisfies the condition $\dot{\phi}^2=-1/m$.

As this model displays an accelerated expanding stage at late times it may be an interesting alternative to describe the epoch where dark energy dominates.

B. The explicit solution

For $\sigma=-4$ or $mV_0=4/3$, we can solve Eq. (41) by means of the substitution

$$s^{-4}=\frac{1}{2}\frac{v^{-4}}{\int v^{-4}d\tau},$$
(54)

so that Eq. (41) reads as

$$\ddot{v}=0.$$
(55)

Inserting its solution into Eq. (54), using Eq. (42), and integrating Eq. (47), we get the general solution for the evolution and the k field satisfying Eq. (6):

$$a(\tau)=\left(\frac{3}{2}\right)^{1/3}\left[\frac{9c^2V_0}{16\phi_0^2}t^4-ct\right]^{1/3},$$
(56)

$$\phi(\tau)=\frac{2}{3t^{1/2}}\left[-\frac{\phi_0^2}{c}+\frac{9V_0}{16}t^3\right]^{1/2},$$
(57)

where we have chosen $c<0$ to set the singularity at $t=0$. The scale factor (56) exhibits a transition from $a\propto t^{1/3}$ to an accelerated expansion where $a\propto t^{4/3}$. Curiously, it coincides with the solutions found in [39] where a FRW spacetime filled with a scalar field driven by an exponential potential was considered. In the next section we will investigate this relation between the scalar field and the k-essence field. The k field diverges at the singularity as $\phi\propto t^{-1/2}$ and behaves as $\phi\propto t$ for large time. It has a minimum and a turning point where the kinetic energy vanishes.

Another set of solutions can be found when $c=0$. Here, Eq. (35) reduces to $\dot{H}=-H^2/mV_0$ and we get the power-law expansion $a=t^{mV_0}$ and a linear k field $\phi=\phi_0t$.

C. The polynomial function $F_\gamma(x)=(-x)^{\gamma/2(\gamma-1)}$

This polynomial function yields a constant baryotropic index γ and the power-law expansion $a=t^{2/3\gamma}$ [28]. For F_γ the general solution of Eq. (34) is

$$\phi=[b+4\phi_0t^{(2-\gamma)/\gamma}]^{\gamma/(2-\gamma)}, \quad \gamma\neq2,$$
(58)

$$\phi=\phi_0t^b, \quad \gamma=2,$$
(59)

where b and ϕ_0 are integration constants. Consistency between these solutions and the Eq. (6) gives the following relation between the integration constants:

$$V_0=\frac{4(1-\gamma)}{3\gamma^2}(4\phi_0)^{\gamma/(1-\gamma)}, \quad \gamma\neq2,$$
(60)

$$b=\pm\frac{1}{\sqrt{3V_0}}, \quad \gamma=2.$$
(61)

For large cosmological time the k field behaves as $\phi\approx t$. More details about the cosmological model generated by the set F_γ can be found in Ref. [28].

D. Constant potential case

We have seen that the generalized Chaplygin gas model was proposed as unified dark matter. It is derived from a Lagrangian containing nonstandard kinetic energy terms (i.e., nonquadratic) and can be considered as driven by a constant potential. Below we show that, even in the absence of any potential energy term, a general class of models exists, including the ECG, connecting a dust dominated era at early times with an accelerated expansion stage at late times.

For a constant potential $V = V_0$, the k-field equation (31) has the first integral

$$\frac{\gamma}{\dot{\phi}} = \frac{c}{a^3 H^2}, \qquad (62)$$

or, after using Eqs. (5) and (7), it turns into

$$a^3 F_x \dot{\phi} = \frac{3c}{2V_0}, \qquad (63)$$

where c is an arbitrary integration constant. Also, combining them with the Friedmann equation (6), the baryotropic index associated with this kind of k essence can be written in a more convenient form,

$$\gamma = \frac{1}{1 + 2V_0^2 a^6 F F_x / 9c^2}. \qquad (64)$$

From the last equation we see that for a large set of models, i.e., the class of models generated by the set of functions F satisfying the condition $a^6 F F_x \ll 1$ at early time, the universe is dust dominated in the beginning. At intermediate times it behaves as if it were filled with a perfect fluid with equation of state $p \propto \rho$. Finally, the universe ends in an accelerated expansion scenario. So these alternative models play the same role as the generalized Chaplygin gas, i.e., interpolating between dark matter at early time and dark energy at late time.

Now, we investigate a simple kinetic k-essence model generated by a function F satisfying the more general condition $a^6 F F_x \approx \text{const}$ at early time. This model is generated by the following function:

$$F = \frac{1}{(2\alpha - 1)V_0} [2\alpha\alpha_0 \sqrt{-x} - (-x)^\alpha], \qquad (65)$$

where α and α_0 are two real constants. The energy density and pressure of the k field are calculated from Eqs. (4) and (65):

$$\rho = (-x)^\alpha, \quad p = -\frac{1}{(2\alpha - 1)} [2\alpha\alpha_0 \sqrt{-x} - (-x)^\alpha], \qquad (66)$$

the equation of state is

$$p = \frac{1}{2\alpha - 1} \left[\rho - \frac{2\alpha\alpha_0}{\rho^{-1/2\alpha}} \right], \qquad (67)$$

and the sound speed becomes

$$c_s^2 = \frac{1}{2\alpha - 1} \left[1 - \frac{\alpha_0}{\rho^{(2\alpha - 1)/2\alpha}} \right]. \qquad (68)$$

Solving the conservation equation (8), we obtain the energy density

$$\rho = \left[\alpha_0 + \frac{c_0}{a^3} \right]^{2\alpha/(2\alpha - 1)}, \qquad (69)$$

where c_0 is a redefinition of the integration constant c. Restricting consideration to positive sound speed, the source (69) and the ECG induce a similar evolution of the scale factor. However, there is a significant change when both constants α_0, c_0 are positive and $\alpha > 1/2$, because in this case the baryotropic index becomes

$$\gamma = \frac{2\alpha}{2\alpha - 1} \left[1 - \frac{\alpha_0}{\rho^{(2\alpha - 1)/2\alpha}} \right]. \qquad (70)$$

So, near the singularity, the energy density diverges and the baryotropic index behaves as $\gamma \approx 2\alpha/(2\alpha - 1)$, indicating that the model begins to evolve with a power-law dominated phase, where the scale factor is $a \propto t^{(2\alpha - 1)/3\alpha}$. In this limit the sound speed (68) behaves as $c_s^2 \approx (2\alpha - 1)^{-1}$. For large α, the model is initially dust dominated, with approximately vanishing sound speed, approaching the ECG generated by the source (28) in the limit of large r. At late times the model ends in a de Sitter stage. Such a "modified Chaplygin gas" may be considered as an alternative model to the generalized Chaplygin gas investigated in [31]. It allows the evolution of the initial perturbations in the energy density into a nonlinear regime to form a gravitational condensate of particles that could play the role of cold dark matter. The cool dark matter condensates gravitationally into regions where the pressure $p \approx 0$ and the k field is close to $\phi_c = (2\alpha\alpha_0)^{1/(2\alpha - 1)}$. In this case, the model yields an energy density which scales like the sum of a nonrelativistic dust component at $\phi = \phi_c$ with equation of state $p = 0$ and a cosmological-constant-like component $p = -\rho$.

IV. LINKING SCALAR AND k-ESSENCE COSMOLOGIES

The observed acceleration of the present Universe has been investigated by assuming that the dark energy can be described by quintessence and more recently by k essence. The last one involves an effective scalar field theory generated by a Lagrangian with a noncanonical kinetic term. Particular cases of k-essence are the generalized Chaplygin gas and tachyon dark energy models. Quintessence and k-essence frameworks are usually based in a homogeneous scalar field driven by an exponential potential in the case of quintessence or an inverse square potential in the case of k essence. Both encounter the so-called coincidence problem, namely, why are the energy densities of dark energy and dark matter of the same order today? The standard quintessence model appears promising at this point, as it can solve this problem for flat FRW universes provided the dark matter component is assumed to be dissipative [40]. The system is attracted to a stationary and stable solution characterized by the constancy of both density parameters at late times. In addition, a class of k-essence models has been claimed to solve the coincidence problem by linking the onset of dark energy domination to the epoch of matter domination [24,25]. From these satisfactory results and taking into ac-

count that both models involve evolving scalar fields, we believe it is reasonable to explore whether quintessence and k-essence frameworks have similarities. We will deal with this question by investigating in which cases quintessence resembles k essence.

Another interesting aspect to be considered is related to current observations that would indicate that the universe is superaccelerated and it could be filled with a nonstandard fluid that violates the weak energy condition. In this sense, a kind of matter was recently proposed described by a homogeneous scalar field with negative kinetic energy term. The fluid with negative pressure obeys an equation of state of the form $p = (\gamma - 1)\rho$, where γ is taken negative, and the models are known as phantom or ghost cosmologies [41–45]. The phantom and quintessence cosmologies can be investigated simultaneously by taking a scalar field with both signs of the kinetic term driven by an exponential potential. The dynamical equations of these cosmological models are

$$3H^2 = \frac{1}{2}q\dot{\varphi}^2 + \mathcal{V}, \tag{71}$$

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{1}{q}\frac{d\mathcal{V}}{d\varphi} = 0, \tag{72}$$

$$\mathcal{V}(\varphi) = \mathcal{V}_0 e^{-qA\varphi}, \tag{73}$$

where q, A are real numbers and $\mathcal{V}_0$ is a positive constant. The exponential potential is interesting because it may be considered just as a limit of a more complex potential [44]. For negative values of q the above equations describe a phantom cosmology [45].

It will be demonstrated that the scale factor obtained from Eqs. (71)–(73) or from the k-essence model generated by the linear function $F = 1 + mx$ and driven by an inverse square potential [see Eqs. (6), (33), (35) and (40)] is the same. To this end, we sketch the procedure followed in Ref. [46]. From Eqs. (71)–(73), we find

$$\dot{H} = -\frac{1}{2}q\dot{\varphi}^2, \tag{74}$$

and the first integral of the Klein-Gordon equation (72)

$$\dot{\varphi} = AH + \frac{c_1}{a^3}, \tag{75}$$

where c_1 is an arbitrary integration constant. Now, inserting Eq. (75) into Eq. (74), we obtain the second order differential equation for the scale factor,

$$\frac{d^2 S}{d\zeta^2} + S^\nu \frac{dS}{d\zeta} + \frac{1}{4}S^{2\nu+1} = 0, \quad \nu = -6/qA^2, \tag{76}$$

where we have used the new variables S and ζ, defined by

$$S = a^{-3/\nu}, \quad \zeta = c_1 qAt. \tag{77}$$

With the following identification of the parameters:

$$mV_0 = \frac{2}{qA^2}, \quad \frac{3c}{2} = \frac{c_1}{A}, \tag{78}$$

Eqs. (41) and (76) coincide. Therefore, both models are described by the same scale factor and they are geometrically equivalent. Also, from Eqs. (6), (7), (40) and Eqs. (71), (72), we get a relationship between the two potentials:

$$3H^2 + \dot{H} = V(t) = \mathcal{V}(t), \tag{79}$$

showing they are the same function of the cosmological time t, so that

$$\frac{V_0}{\phi^2} = \mathcal{V}_0 e^{-qA\varphi}. \tag{80}$$

After inserting the k field (52) into the last equation we find the scalar field

$$\varphi = \frac{1}{qA}\ln\varphi_0 + \frac{\eta}{qA} + A\ln a, \tag{81}$$

where $\varphi_0 = \mathcal{V}_0\phi_0^2/V_0$. Equation (80) supplies the link between the scalar field and the k-essence field,

$$\phi = \frac{\phi_0}{\varphi_0^{1/2}}e^{qA\varphi/2}, \tag{82}$$

displaying that these models are dynamically not equivalent. Thus, the homogeneous quintessence and phantom fields are different from the k-essence field.

Summarizing, from the kinematical point of view, which is the background geometry, we have obtained exact equivalences, and it is impossible to differentiate between quintessence, k essence, and phantom cosmologies because they share the same scale factor. To distinguish between them, it appears necessary to focus our attention on the scalar field.

V. CONCLUSIONS

We have investigated the set of T_r fields whose effective sound speed is proportional to the sound speed of the tachyon field. They can be grouped into three types according to whether they yield phantom expansion or accelerated expansion with or without inflation. We have shown these behaviors by finding exact power-law solutions for an inverse square potential and proved that the $T_{1/2}$ field is compatible only with this potential. Each T_r field produces an ECG, and the set of all these gases can be divided into three kinds, one of which contains the generalized Chaplygin gas and the others give rise to "perfect fluids," leading to new evolutions. There exist basically nonsingular bouncing solutions with a final superaccelerated stages or singular ones with a finite time span, as well as particular singular solutions that begin with a finite scale factor. In this manner, both the ETF and the ECG may be considered fair candidates to implement phantom cosmologies.

For an inverse square potential, we have found the first

integral of the k-field equation for any function F and shown that the coupled Einstein–k-field equations can be solved in some cases. In particular, the divergent k-essence theory generated by the $T_{1/2}$ field becomes an intrinsic component of all k-essence models. Therefore, for power-law expansions the linear k-field model driven by an inverse square potential and the divergent model are isomorphic. We have obtained the general solution of the Einstein–k-field equations for a linear function F. From the kinematical point of view this model and the quintessence scalar field one driven by an exponential potential are the same. However, they are dynamically nonequivalent, because the k field and the scalar field are linked by the Einstein equation, e.g., both potentials are the same function of the cosmological time.

For a constant potential, we have studied a k-essence field associated with a perfect fluid whose equation of state contains one term proportional to the energy density and another with the form of an ECG. This model, essentially different from the ECG model, smoothly interpolates between a power-law dominated phase and a de Sitter phase. In this "modified Chaplygin gas" scenario, the value of the k field where initial perturbations condense gravitationally to cold dark matter can be chosen.

[1] A.R. Liddle and D.H. Lyth, *Cosmological Inflation and Large Scale Structure* (Cambridge University Press, Cambridge, England, 2000).

[2] C.L. Bennett *et al.*, Astrophys. J., Suppl. Ser. **148**, 1 (2003).

[3] C.B. Netterfield *et al.*, Astrophys. J. **571**, 604 (2002).

[4] N.W. Halverson *et al.*, Astrophys. J. **568**, 38 (2002).

[5] C. Armendariz-Picon, T. Damour, and V. Mukhanov, Phys. Lett. B **458**, 209 (1999).

[6] G.W. Gibbons, Phys. Lett. B **537**, 1 (2002).

[7] A. Feinstein, Phys. Rev. D **66**, 063511 (2002).

[8] T. Padmanabhan, Phys. Rev. D **66**, 021301(R) (2002).

[9] M. Sami, D. Chingangham, and T. Qureshi, Phys. Rev. D **66**, 043530 (2002).

[10] A. Frolov, L. Kofman, and A. Linde, Phys. Lett. B **545**, 8 (2002).

[11] S. Sugimoto and S. Terashima, J. High Energy Phys. **07**, 025 (2002).

[12] A. Sen, J. High Energy Phys. **04**, 048 (2002); **07**, 065 (2002); Mod. Phys. Lett. A **17**, 1797 (2002).

[13] A.G. Riess *et al.*, Astron. J. **116**, 1009 (1998); S. Perlmutter *et al.*, Astrophys. J. **517**, 565 (1998); G. Efstathiou *et al.*, Mon. Not. R. Astron. Soc. **330**, L29 (2002).

[14] G.W. Gibbons, Phys. Lett. B **537**, 1 (2002); Padmanabhan [8]; G. Shiu and Ira Wasserman, Phys. Lett. B **541**, 6 (2002); T. Padmanabhan and T. Roy Choudhury, Phys. Rev. D **66**, 081301 (2002); D. Choudhury, D. Ghoshal, D.P. Jatkar, and S. Panda, Phys. Lett. B **544**, 231 (2002); K. Hashimoto, J. High Energy Phys. **07**, 035 (2002); H.S. Kim, *ibid.* **01**, 080 (2003); Sugimoto and Terashima [11]; J.A. Minahan, J. High Energy Phys. **07**, 030 (2002); L. Cornalba, M.S. Costa, and C. Kounnas, Nucl. Phys. **B637**, 378 (2002); H.B. Benaoum, hep-th/0205140; Xin-zhou Li, Jian-gang Hao, and Dao-jun Liu, Chin. Phys. Lett. **19**, 1584 (2002); Yun-Song Piao, Phys. Rev. D **66**, 121301 (2002); Jian-gang Hao and Xin-zhou Li, *ibid.* **68**, 043501 (2003); S. Nojiri and S.D. Odintsov, Phys. Lett. B **571**, 1 (2003); L. Raul, W. Abramo, and Fabio Finelli, *ibid.* **575**, 165 (2003).

[15] L.P. Chimento, A.S. Jakubi, and D. Pavón, Phys. Rev. D **67**, 087302 (2003).

[16] M.C. Bento, O. Bertolani, and A.A. Sen, Phys. Rev. D **67**, 063003 (2003).

[17] L.M.G. Beca, P.P. Avelino, J.P.M. de Carvalho, and C.J.A.P. Martins, Phys. Rev. D **67**, 101301 (2003).

[18] N. Bilic, R.J. Lindebaum, G.B. Tupper, and R.D. Viollier, astro-ph/0307214.

[19] T. Multamaki, M. Manera, and E. Gaztanaga, Phys. Rev. D **69**, 023004 (2004).

[20] P.P. Avelino, L.M.G. Beca, J.P.M. de Carvalho, C.J.A.P. Martins, and E.J. Copeland, Phys. Rev. D **69**, 041301 (2004).

[21] A. Dev, D. Jain, and D.D. Upadhyaya, Astron. Astrophys. **417**, 847 (2004).

[22] D. Carturan and F. Finelli, Phys. Rev. D **68**, 103501 (2003); L. Amendola, F. Finelli, C. Burigana, and D. Carturan, J. Cosmol. Astropart. Phys. **07**, 005 (2003).

[23] H.B. Sandvik, M. Tegmark, M. Zaldarriaga, and I. Waga, Phys. Rev. D (to be published), astro-ph/0212114.

[24] C. Armendariz-Picon, V. Mukhanov, and Paul J. Steinhardt, Phys. Rev. Lett. **85**, 4438 (2000).

[25] C. Armendariz-Picon, V. Mukhanov, and Paul J. Steinhardt, Phys. Rev. D **63**, 103510 (2001).

[26] M. Malquarti, E.J. Copeland, A.R. Liddle, and M. Trodden, Phys. Rev. D **67**, 123503 (2003).

[27] S.M. Carroll, M. Hoffman, and M. Trodden, Phys. Rev. D **68**, 023509 (2003).

[28] L.P. Chimento and A. Feinstein, Mod. Phys. Lett. A **19**, 761 (2004).

[29] A. Kamenshchik, U. Moschella, and V. Pasquier, Phys. Lett. B **511**, 265 (2001).

[30] N. Bilic, G.B. Tupper, and R.D. Viollier, Phys. Lett. B **535**, 17 (2002); N. Bilic, R.J. Lindebaum, G.B. Tupper, and R.D. Viollier, astro-ph/0310181.

[31] M.C. Bento, O. Bertolani, and A.A. Sen, Phys. Rev. D **66**, 043507 (2002).

[32] A. Sen, J. High Energy Phys. **04**, 048 (2002).

[33] Gibbons [14].

[34] S. Hannestad and E. Mörstell, Phys. Rev. D **66**, 063508 (2002); P.H. Frampton and T. Takahashi, Phys. Lett. B **557**, 135 (2003); S.M. Carroll, M. Hoffman, and M. Trodden, Phys.

Rev. D **68**, 023509 (2003); A. Melchiorri, L. Mersini, C.J. Odman, and M. Trodden, *ibid.* **68**, 043509 (2003); J.S. Alcañiz, *ibid.* **69**, 083521 (2004); J.A.S. Lima, J.V. Cunha, and J.S. Alcañiz, *ibid.* **68**, 023510 (2003).

[35] L. Mersini, M. Bastero-Gil, and P. Kanti, Phys. Rev. D **64**, 043508 (2001); M. Bastero-Gil, P.H. Frampton, and L. Mersini, *ibid.* **65**, 106002 (2002); P.H. Frampton, Phys. Lett. B **555**, 139 (2003).

[36] R.R. Caldwell, M. Kamionkowski, and N.N. Weinberg, Phys. Rev. Lett. **91**, 071301 (2003); A. Yurov, astro-ph/0305019; P.F. González-Díaz, Phys. Rev. D **68**, 021303 (2003); M. Sami and A. Toporensky, gr-qc/0312009; J.G. Hao and X.Z. Li, astro-ph/0309746; V. Sahni and Y. Shtanov, J. Cosmol. Astropart. Phys. **11**, 014 (2003).

[37] M. Malquarti, E.J. Copeland, and A.R. Liddle, Phys. Rev. D **68**, 023512 (2003).

[38] L.P. Chimento, J. Math. Phys. **38**, 2565 (1997).

[39] A.B. Burd and J.D. Barrow, Nucl. Phys. **B308**, 929 (1988).

[40] L.P. Chimento, A.S. Jakubi, and D. Pavón, Phys. Rev. D **62**, 063508 (2000); **67**, 087302 (2003).

[41] R.R. Caldwell, Phys. Lett. B **545**, 23 (2002).

[42] A. Melchiorri, L. Mersini, C.J. Odman, and M. Trodden, Phys. Rev. D **68**, 043509 (2003).

[43] S.M. Carroll, M. Hoffman, and M. Trodden, Phys. Rev. D **68**, 023509 (2003).

[44] R. Easther, Class. Quantum Grav. **10**, 2203 (1993).

[45] L.P. Chimento and R. Lazkoz, Phys. Rev. Lett. **91**, 211301 (2003).

[46] L.P. Chimento, Class. Quantum Grav. **15**, 965 (1998).