EPJ B
Condensed Matter and Complex Systems
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Eur. Phys. J. B (2015) 88: 176 DOI: 10.1140/epjb/e2015-60211-6

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edp sciences
Società Italiana di Fisica
Springer

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THE EUROPEAN PHYSICAL JOURNAL B

Regular Article

Path integral approach to order by disorder selection in partially polarized quantum spin systems

Carlos Alberto Lamas[1,a], Daniel C. Cabra[1], Pierre Pujol[2], and Gerardo L. Rossini[1]

[1] IFLP-CONICET, Departamento de Física, Universidad Nacional de La Plata, C.C. 67, 1900 La Plata, Argentina
[2] Laboratoire de Physique Théorique, IRSAMC, CNRS and Université de Toulouse, UPS, 31062 Toulouse, France

Received 16 March 2015 / Received in final form 22 May 2015
Published online 8 July 2015 –

Abstract. In this paper we study the frustrated $J_1 - J_2$ quantum Heisenberg model on the square lattice for $J_2 > J_1/2$, in a magnetic field. In this regime the classical system is known to have a degenerate manifold of lowest energy configurations, where standard thermal order by disorder occurs. In order to study its quantum version we use a path integral formulation in terms of coherent states. We show that the classical degeneracy in the plane transverse to the magnetic field is lifted by quantum fluctuations. Collinear states are then selected, in a similar pattern to that set by thermal order by disorder, leaving a Z_2 degeneracy. A careful analysis reveals a purely quantum mechanical effect given by the tunneling between the two minima selected by fluctuations. The effective description contains two planar (XY-like) fields conjugate to the total magnetization and the difference of the two sublattice magnetizations. Disorder in either or both of these fields produces the locking of their conjugate observables.

1 Introduction

The properties of the two-dimensional Heisenberg model have received considerable interest in the last years [1–12], in part because of the possible connection between magnetism and high-temperature superconductivity. In this sense, one of the most typical examples of a two-dimensional frustrated spin system is given by the $J_1 - J_2$ antiferromagnetic Heisenberg model on the square-lattice.

The classical version of this model, for large enough J_2/J_1, has a continuous manifold of degenerate ground states related by the rotation of one sub-lattice with respect to the other. Thermal and quantum fluctuations can stabilize collinear spin configurations [1,3], a particular case of the phenomenon known as Order By Disorder (OBD) [2][1].

It is generally accepted that quantum and thermal fluctuation select the same ground state from the classical manifold. However, there exist some examples where the quantum fluctuations select a different ground state [13] than thermal ones. For this reason, it is interesting to study the quantum and thermal contributions to the free-energy in order to distinguish their selection features. Besides its theoretical interest, the present model is appropriate for describing compounds like Li_2VOSiO_4, Li_2VOGeO_4 and $VOMoO_4$ in which dominant magnetic interactions consist of first and second nearest neighbors exchange [6,7].

We focus here on the partially polarized system, in the presence of an external magnetic field. The quantum model at zero magnetic field (magnetization $m = 0$) has been subject of much recent controversy [14–16]. Notice that, as soon as we are considering the $m \neq 0$ situation, we are explicitly assuming that we have gone out of any putative non-trivial gapped phase related to the $m = 0$ case. The phenomenology at $m = 0$ is certainly very interesting and still the matter of many works, but it is not the concern of the present paper.

The path integral description of the magnetic degrees of freedom in partially polarized spin systems [9] represents a good alternative to study ordering due to disorder phenomena. In particular, semiclassical quantum fluctuations incorporated in such a path integral approach might go beyond the usual spin-wave fluctuation analysis.

For the strongly frustrated $J_1 - J_2$ antiferromagnetic Heisenberg model on the square lattice, in the presence of an external magnetic field, the arising effective description contains two planar XY-like fields. One of them (hereafter the symmetric field) is canonically conjugate to the magnetization degrees of freedom along the magnetic field, while the other (the antisymmetric field) is conjugate to the difference between sublattice magnetizations. The presence of topological terms (Berry phases) with coefficients that depend on the total magnetization may either allow or forbid the XY vortex proliferation that

[a] e-mail: lamas@fisica.unlp.edu.ar

[1] A related Heisenberg model consisting of interpenetrating triangular and dual hexagonal antiferromagnetic lattices, where coplanar spin configurations are selected, was recently proposed [12].

disorders the planar degrees of freedom. Delocalization of the spin components in the transverse plane is canonically related to the total magnetization. This approach thus allows for the study of plateaux formation in magnetization curves [17]. An effective potential is also found for the antisymmetric field, allowing for possible non-trivial instanton-like excitation processes. By tuning the couplings it may be possible to reduce the spin stiffness and favor the proliferation of such instanton configurations driving the system into a disordered phase where broken symmetries are restored.

From all the above, the magnetization dependent topological terms controlling the weight of field configurations with non-vanishing vorticity turn out to be relevant in the mechanism of quantum order by disorder selection. The aim of the the present work is the analysis of their consequences.

The paper is organized as follows: in Section 2 we present the model Hamiltonian and its classical ground state degeneracy. In Section 3 we derive a low energy effective action in the framework of path integration. Section 4 is devoted to computing the free energy of the system at finite temperature; vortex free and topological non trivial configurations are treated separately. In Section 5 we draw our main results arising from the presence of Berry phase terms in the low energy effective action. Section 6 presents the conclusions and open routes for further research.

2 The model and its classical degeneracy

We study a spin-S next-nearest-neighbor antiferromagnetic Heisenberg model on the square lattice in the presence of a homogeneous magnetic field. The Hamiltonian is given by:

$$H = J_1 \sum_{NN} \boldsymbol{S}(\boldsymbol{r}) \cdot \boldsymbol{S}(\boldsymbol{r}') + J_2 \sum_{NNN} \boldsymbol{S}(\boldsymbol{r}) \cdot \boldsymbol{S}(\boldsymbol{r}') - h \sum_{\boldsymbol{r}} S^z(\boldsymbol{r}) \qquad (1)$$

where J_1, J_2 are positive. The magnetic field h points in the z direction, vectors $\boldsymbol{r}$ and $\boldsymbol{r}'$ belong to the two dimensional square lattice $\boldsymbol{r} = n_x\hat{x} + n_y\hat{y}$, with $\hat{x} = a(1,0)$, $\hat{y} = a(0,1)$, a is the lattice spacing and the summations denoted as NN and NNN run on nearest-neighbor and next-nearest-neighbor sites, respectively. Despite its simplicity, this model is paradigmatic in quantum magnetism since it shows order from disorder selection [3] at $h = 0$, a magnetization plateau at saturation fraction $M = 1/2$, and field induced ordering [11]. In the following sections we study the degenerate classical ground state for $h > 0$ and the low energy theory describing thermal and quantum fluctuations around it.

At zero magnetic field and $J_2 < J_1/2$ the minimum energy configuration corresponds to a Néel order, whereas for $J_2 > J_1/2$ the ground state breaks up into two square $\sqrt{2} \times \sqrt{2}$ sub-lattices [1]. Each sub-lattice is ordered antiferromagnetically, leaving a classical ground state degeneracy associated to global rotations of all the spins belonging to one of the sub-lattices. In order to describe the strongly frustrated $J_2 > J_1/2$ regime we parameterize the ground state manifold as

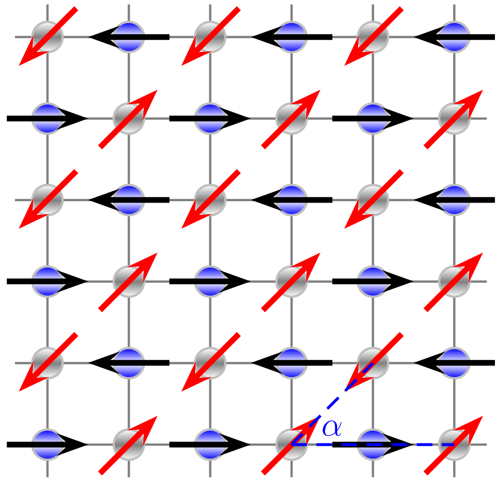

Fig. 1. Sketch of a given classical configuration on the plane. Each sub-lattice is ordered antiferromagnetically with a relative angle α between sub-lattices.

$$\boldsymbol{S}_l(\boldsymbol{r}) = \begin{pmatrix} \sqrt{S^2 - m^2}\cos(\boldsymbol{Q}\cdot\boldsymbol{r} + \alpha_l) \\ \sqrt{S^2 - m^2}\sin(\boldsymbol{Q}\cdot\boldsymbol{r} + \alpha_l) \\ m \end{pmatrix},$$

where $l = 1, 2$ is the sublattice index, m is the homogeneous magnetization, Q is a pitch angle and α_l is a sublattice dependent phase. To label the degenerate ground states we can choose one reference spin from each sublattice, and use the relative angle $\alpha = \alpha_2 - \alpha_1$ between these two spins to parameterize the non-trivial degeneracy, the ground-state energy being independent of α. In order to represent the ground state manifold, we take in the following $\boldsymbol{Q} = (\frac{\pi}{a}, 0)$, $\alpha_1 = 0$ and $\alpha_2 = \alpha$.

In the presence of a magnetic field the classical spins are canted towards the field direction ($m \neq 0$) and the classical energy can be written as:

$$\frac{E}{N_c} = 4m^2 J_1 + J_2(8m^2 - 4S^2) - 2mh,$$

where N_c is the number of unit cells. Minimizing with respect to m we obtain

$$m = \frac{h}{(4J_1 + 8J_2)}.$$

In Figure 1 we show the projection on the x-y plane of one of the many configurations that minimize the energy of the system.

3 Low energy effective action

In the absence of a magnetic field, the thermal and quantum fluctuations of the $J_1 - J_2$ Heisenberg model have been carefully studied within the spin-wave approach [3]. At non-zero magnetic field, since we are dealing with a magnetized classical state we can use a particular path integral approach in terms of coherent states [9,17] to estimate the free energy of the system.

First, we introduce two angular fields by sub-lattice to represent the spins as:

$$\boldsymbol{S}_l(\boldsymbol{r}) = \begin{pmatrix} S\sin(\theta_l(\boldsymbol{r})\cos(\phi_l(\boldsymbol{r})) \\ S\sin(\theta_l(\boldsymbol{r}))\sin(\phi_l(\boldsymbol{r})) \\ S\cos(\theta_l(\boldsymbol{r})) \end{pmatrix}$$

and parameterize their fluctuations around the classical solution in terms of fields φ and ϑ,

$$\phi_l(\boldsymbol{r}) = \boldsymbol{Q}\cdot\boldsymbol{r} + \alpha_l + \varphi_l(\boldsymbol{r})$$
$$\theta_l(\boldsymbol{r}) = \theta_0 + \vartheta_l(\boldsymbol{r}),$$

where θ_0 is the classical solution ($m = S\cos\theta_0$), which in the present case is given by $\theta_0 = \arccos\left(\frac{h}{S(4J_1+8J_2)}\right)$. Expanding up to second order in the fluctuating fields we have:

$$S\cos\left(\theta_0 + \vartheta_l(\boldsymbol{r})\right) \simeq S\cos\theta_0 - S\sin\theta_0\,\vartheta_l(\boldsymbol{r}) - \frac{S}{2}\cos\theta_0(\vartheta_l(\boldsymbol{r}))^2$$
$$S\sin\left(\theta_0 + \vartheta_l(\boldsymbol{r})\right) \simeq S\sin\theta_0 + S\cos\theta_0\,\vartheta_l(\boldsymbol{r}) - \frac{S}{2}\sin\theta_0(\vartheta_l(\boldsymbol{r}))^2.$$

Should we take φ and ϑ as canonical conjugates and calculate the Poisson brackets $\{S^z, S^\pm\}_{\varphi,\vartheta}$, we would obtain $i\hbar\{S^z, S^\pm\}_{\varphi,\vartheta} = -S(\sin\theta_0 - \vartheta\cos\theta_0)\,(\pm\hbar S^\pm)$. In order to generate the correct $SU(2)$ algebra, one instead defines the canonical conjugate for φ_l as

$$\frac{a^2}{S}\Pi_l(\boldsymbol{r}) = -\sin\theta_0\vartheta_l(\boldsymbol{r}) - \frac{1}{2}\cos\theta_0(\vartheta_l(\boldsymbol{r}))^2. \tag{2}$$

The spin operators in terms of the canonical conjugate pairs read

$$S_l^\pm(\boldsymbol{r}) = e^{\pm i\boldsymbol{Q}\cdot\boldsymbol{r}}e^{\pm i\varphi_l(\boldsymbol{r})}\left[S\sin\theta_0 - a^2\frac{m}{S\sin\theta_0}\Pi_l(\boldsymbol{r}) - a^4\frac{S^2}{S^2-m^2}\frac{1}{S\sin\theta_0}\Pi_l^2(\boldsymbol{r})\right], \tag{3}$$

$$S_l^z(\boldsymbol{r}) = m + a^2\Pi_l(\boldsymbol{r}). \tag{4}$$

Notice from the last equation that the Π_l fields describe fluctuations in the spin components along the magnetic field direction.

Using expressions (3) and (4) in Hamiltonian (1), taking the continuum limit and retaining terms up to second order in the fields, we can write $H = H_0 + H_\varphi + H_\pi$. Here H_0 is a contribution independent of the fields while H_φ and H_π are given by:

$$\begin{aligned} H_\varphi = \int d^2r\Big\{ & \left(S^2-m^2\right)\left(J_2 + \frac{J_1}{2}\cos(\alpha)\right) \\ & \times\left[\left(\partial_x\varphi_1\right)^2 + \left(\partial_x\varphi_2\right)^2\right] \\ & + \left(S^2-m^2\right)\left(J_2 - \frac{J_1}{2}\cos(\alpha)\right) \\ & \times\left[\left(\partial_y\varphi_1\right)^2 + \left(\partial_y\varphi_2\right)^2\right]\Big\} \\ H_\pi = \int d^2r & \left\{4J_1a^2\Pi_1\Pi_2 + 4J_2a^2\left(\Pi_1^2+\Pi_2^2\right)\right\}. \end{aligned}$$

The $\boldsymbol{r}$ dependence in the fields has been omitted for simplicity. As a further step towards a spin coherent states path-integral formulation, we write an effective action as

$$\mathcal{S} = \mathcal{S}_\varphi + \mathcal{S}_\pi,$$

where

$$\begin{aligned} \mathcal{S}_\varphi = \int d^2r\int d\tau\Big\{ & \left(S^2-m^2\right)\left(J_2 + \frac{J_1}{2}\cos(\alpha)\right) \\ & \times\left[\left(\partial_x\varphi_1\right)^2 + \left(\partial_x\varphi_2\right)^2\right] \\ & + \left(S^2-m^2\right)\left(J_2 - \frac{J_1}{2}\cos(\alpha)\right) \\ & \times\left[\left(\partial_y\varphi_1\right)^2 + \left(\partial_y\varphi_2\right)^2\right] \\ & + i\left(\frac{S-m}{a^2}\right)\left(\partial_\tau\varphi_1 + \partial_\tau\varphi_2\right)\Big\} \\ \mathcal{S}_\pi = \int d^2r\int d\tau & \left\{4J_1a^2\Pi_1\Pi_2 + 4J_2a^2\left(\Pi_1^2+\Pi_2^2\right)\right. \\ & \left. - i\Pi_1\partial_\tau\varphi_1 - i\Pi_2\partial_\tau\varphi_2\right\}. \end{aligned} \tag{5}$$

The first order derivatives of φ_1, φ_2 with respect to imaginary time appear in the path integral approach as the Berry connection associated to the continuous set of spin coherent states (parameterized on the sphere by the same angular variables as the classical spins); in the present Euclidean effective action Berry terms are easily recognized for they are purely imaginary. The integral of the Berry connection along different closed trajectories gives rise to complex Berry phases [18]. These will lead to important interference effects when computing the free energy of the system.

After writing the path-integral one can readily integrate out the Gaussian Π-fields to get the effective action:

$$\begin{aligned}\mathcal{S}_{eff} = \int d^2r \int d\tau \Big\{ &(S^2-m^2)\left(J_2 + \frac{J_1}{2}\cos(\alpha)\right) \\ &\times \left[(\partial_x\varphi_1)^2 + (\partial_x\varphi_2)^2\right] \\ &+ (S^2-m^2)\left(J_2 - \frac{J_1}{2}\cos(\alpha)\right) \\ &\times \left[(\partial_y\varphi_1)^2 + (\partial_y\varphi_2)^2\right] \\ &+ \frac{1}{4a^2(2J_2+J_1)}(\partial_\tau\varphi_1 + \partial_\tau\varphi_2)^2 \\ &+ \frac{1}{4a^2(2J_2-J_1)}(\partial_\tau\varphi_1 - \partial_\tau\varphi_2)^2 \\ &+ i\left(\frac{S-m}{a^2}\right)(\partial_\tau\varphi_1 + \partial_\tau\varphi_2)\Big\}. \end{aligned} \tag{6}$$

Notice that one can write this action in a decoupled form in terms of symmetrical and anti-symmetrical fields

$$\varphi_s = \frac{1}{2}(\varphi_1 + \varphi_2), \tag{7}$$

$$\varphi_a = \frac{1}{2}(\varphi_1 - \varphi_2), \tag{8}$$

which yields

$$\mathcal{S}_{eff} = \mathcal{S}_s + \mathcal{S}_a,$$

with

$$\begin{aligned}\mathcal{S}_s = \int d^2r \int d\tau \Big\{ &2(S^2-m^2)\left(J_2 + \frac{J_1}{2}\cos(\alpha)\right)(\partial_x\varphi_s)^2 \\ &+ 2(S^2-m^2)\left(J_2 - \frac{J_1}{2}\cos(\alpha)\right)(\partial_y\varphi_s)^2 \\ &+ \frac{1}{a^2(2J_2+J_1)}(\partial_\tau\varphi_s)^2 + i2\left(\frac{S-m}{a^2}\right)(\partial_\tau\varphi_s)\Big\}, \end{aligned} \tag{9}$$

$$\begin{aligned}\mathcal{S}_a = \int d^2r \int d\tau \Big\{ &2(S^2-m^2)\left(J_2 + \frac{J_1}{2}\cos(\alpha)\right)(\partial_x\varphi_a)^2 \\ &+ 2(S^2-m^2)\left(J_2 - \frac{J_1}{2}\cos(\alpha)\right)(\partial_y\varphi_a)^2 \\ &+ \frac{1}{a^2(2J_2-J_1)}(\partial_\tau\varphi_a)^2\Big\}. \end{aligned} \tag{10}$$

Both the symmetrical and the anti-symmetrical fields are governed by XY-like actions. We briefly recall below the role of topologically distinct field configurations in the two-dimensional XY model [19].

In order to compute the finite temperature free energy of the system one performs a path integral over periodic trajectories in imaginary time. Here vorticity comes into play, as planar angular variables can wind an integer number of times before reaching their initial value to satisfy periodic boundary conditions. The winding number of a configuration encodes topological information, in the sense that configurations with different winding cannot be continuously deformed into each other. A continuous parameterization of trajectories cannot then sweep all periodic configurations, and the path integral must (at least conceptually) be carried out separately on each winding sector.

When applying the coherent states path integral to magnetized systems (as originally discussed in Ref. [9]) one gets for the azimuthal angles not only an XY-like action but also a Berry phase contribution, which takes the form of a winding number (integral of a first order derivative of an angular field). Thus the Berry term measures vorticity; this is the reason why it is dubbed topological. Before decoupling, such contribution is shown in the last line in equation (6). However, after decoupling such a term is present only in the symmetric field effective action, in equation (9), and not in the corresponding equation (10) for the anti-symmetric field. Most importantly, the coefficient $S-m$ in front of the Berry term dictates the weight of topologically non trivial configurations (vortices) in the quantum free energy. On generic grounds, the issue about the presence of magnetization plateaux is closely related to the coefficient of the Berry phase term in equation (9) (see for instance [9]).

An important comment is due here. In related systems where order by disorder does not take place, the antisymmetric field is found to be gapped via a mass-like term [9,17,20,21]. This forces φ_a to very weakly fluctuate around zero, so that the fields φ_1 and φ_2 remain tightly bounded in every region of imaginary time. Thus, if one of the fields has some vorticity, the other will have the same vorticity. The presence of a such a mass term then implies that the antisymmetric field is essentially vortex free and the contribution from the Berry phase term (which arises precisely from the vorticity) concerns only the symmetric field. By contrast, in the system at hand the antisymmetric field in equation (10) remains massless. The fields φ_1 and φ_2 are not tightly bounded and can thus produce different contributions for the vorticity. This plays an important role in the Ising transition discussed in the following sections.

4 Analysis of the free energy

Once the low energy fields are recognized as XY-like fields, the consideration of vortex proliferation and the associated Kosterlitz-Thouless (KT) disorder transition is due. In fact, studying the entire XY-like model from scratch is technically very difficult. We proceed first describing the vortex free contributions (non winding configurations) to the free energy, essentially treating the angular fields as real bosonic fields. Then we discuss the weight of topologically non-trivial (vortex) contributions.

4.1 Vortex free regime

Let us consider the preceding action first assuming that the fields φ_l ($l = 1,\ 2$) have only vortex free configurations.

These regular fields can be treated as standard periodic scalar fields: one can Fourier transform and express the fields in terms of the momentum and Matsubara frequency modes

$$\varphi_l(\boldsymbol{r},\tau) = \sum_{n=-\infty}^{\infty} \frac{1}{(2\pi)\beta} \int d^2k e^{i\boldsymbol{k}\cdot\boldsymbol{r}} e^{-i\omega_n\tau} \varphi_l(\boldsymbol{k},\omega_n)$$

where $\omega_n = \frac{2\pi n}{\beta}$. We obtain for the action

$$\mathcal{S} = \frac{1}{(2\pi)^2} \sum_{n=-\infty}^{\infty} \int d^2k \Bigg\{ \left(\epsilon^2 + \frac{J_2}{a^2\,(4J_2^2 - J_1^2)}\omega_n^2\right) \times \left(|\varphi_1(\boldsymbol{k},\omega_n)|^2 + |\varphi_2(\boldsymbol{k},\omega_n)|^2\right) - \left(\frac{J_1}{2a^2\,(4J_2^2 - J_1^2)}\omega_n^2\right) \times \left(\varphi_1(\boldsymbol{k},\omega_n)\varphi_2^*(\boldsymbol{k},\omega_n) + \varphi_1^*(\boldsymbol{k},\omega_n)\varphi_2(\boldsymbol{k},\omega_n)\right) \Bigg\}$$

where

$$\epsilon^2 = (S^2 - m^2)\left[\left(J_2 + \frac{J_1}{2}\cos(\alpha)\right)k_x^2 + \left(J_2 - \frac{J_1}{2}\cos(\alpha)\right)k_y^2\right]. \tag{11}$$

We can evaluate $\mathcal{Z} = \int \mathcal{D}[\phi] e^{-\mathcal{S}}$ by integrating the fields,

$$\log(\mathcal{Z}) = N(\beta) - \frac{1}{2}\sum_{n=-\infty}^{\infty} \int \frac{d^2k}{(2\pi)^2} \log\left[\left(\epsilon^2 + \frac{J_2}{a^2\,(4J_2^2 - J_1^2)}\omega_n^2\right)^2 - \left(\frac{J_1}{2a^2\,(4J_2^2 - J_1^2)}\omega_n^2\right)^2\right]. \tag{12}$$

Notice that all dependence in $\boldsymbol{k}$ and the parameter α is contained in ϵ. After some algebra we can write

$$\log(\mathcal{Z}) = N'(\beta) - \frac{1}{2}\sum_{n=-\infty}^{\infty} \int \frac{d^2k}{(2\pi)^2} \left\{ \log\left[\omega_n^2 + \omega_+^2\right] + \log\left[\omega_n^2 + \omega_-^2\right] \right\} \tag{13}$$

where $\omega_\pm = 2\epsilon a\sqrt{J_2 \pm J_1/2}$. Now we use that

$$\log\left(\omega_n^2 + \omega_\pm^2\right) = \log\left(\omega_n^2 + \frac{1}{\beta^2}\right) + \int_{\frac{1}{\beta^2}}^{\omega_\pm^2} \frac{d(t^2)}{\omega_n^2 + t^2} \tag{14}$$

and $\log(\omega_n^2 + \frac{1}{\beta^2}) = -2\log(\beta) + \log(2\pi^2 n^2 + 1)$. Then we have

$$\log(\mathcal{Z}) = \tilde{N}(\beta) - \frac{1}{2}\sum_{n=-\infty}^{\infty} \int \frac{d^2k}{(2\pi)^2} \left\{ \int_{\frac{1}{\beta^2}}^{\omega_+^2} \frac{d(t^2)}{\omega_n^2 + t^2} + \int_{\frac{1}{\beta^2}}^{\omega_-^2} \frac{d(u^2)}{\omega_n^2 + u^2} \right\}, \tag{15}$$

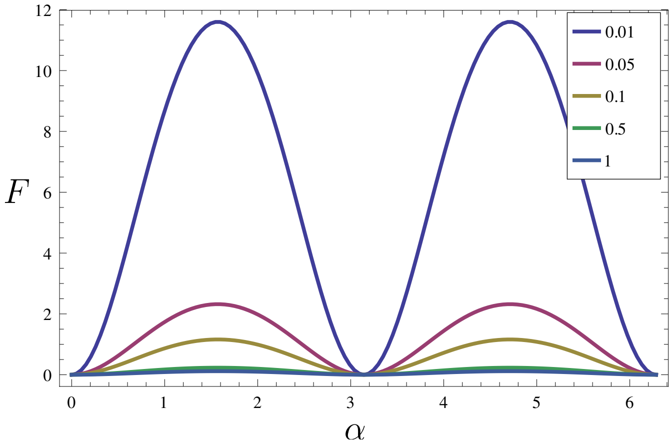

Fig. 2. Free energy in terms of the sub-lattices relative angle α. The plot corresponds to $J_2 = 0.6J_1$ and low magnetization, for different values of β denoted in the plot legend. Minimum values correspond to collinear configurations ($\alpha = 0$ and $\alpha = \pi$).

where we have included all the vacuum contributions in the first term. Now we can perform the summation to obtain

$$\log(\mathcal{Z}) = \int \frac{d^2k}{(2\pi)^2} \left\{ \log\left[\operatorname{csch}\left(\frac{\beta\omega_+}{2}\right)\right] + \log\left[\operatorname{csch}\left(\frac{\beta\omega_-}{2}\right)\right] \right\}.$$

Finally, after some rearrangements, we obtain an expression for the free energy,

$$F = \frac{1}{2}\int \frac{d^2k}{(2\pi)^2}(\omega_+ + \omega_-) + \frac{1}{\beta}\int \frac{d^2k}{(2\pi)^2}\left[\log\left(1 - e^{-\beta\omega_+}\right) + \log\left(1 - e^{-\beta\omega_-}\right)\right]. \tag{16}$$

Equation (16) makes apparent the role of quantum and thermal fluctuations: the first term represents the quantum zero point contribution to the free energy whereas the second term is the thermal contribution. From equation (16) it is easy to extract the zero temperature (quantum) contribution F_Q, which is simply the first term,

$$F_Q = \frac{1}{2}\int \frac{d^2k}{(2\pi)^2}(\omega_+ + \omega_-)$$

and the thermal contribution of the purely classical model F_{Cl}. The latter is obtained from the second term in the limit $\beta \to 0$ whose dominant contribution is given, up to α independent terms, by:

$$F_{Cl} = \frac{1}{\beta}\int \frac{d^2k}{(2\pi)^2}\log[\omega_+\omega_-].$$

We show in Figure 2 plots of the free energy in equation (16) in terms of the relative angle between sub-lattices, for low magnetization and several temperatures.

The free energy shows (as expected) two minima. The minima of both quantum and thermal contributions are located in $\alpha = 0$ and $\alpha = \pi$, corresponding to the collinear configurations $(0, \pi)$ and $(\pi, 0)$. This translates into an emergent Z_2 symmetry of the system at large scales. The selection of an angle, at either the values 0 or π implies the spontaneous breaking of the Z_2 symmetry. One expects this symmetry to be restored at higher temperatures.

Both F_{Cl} and F_Q coincide with the results obtained by Henley [1] for the planar model using classical and quantum spin wave theory. Our approach provides nevertheless a more complete analysis, allowing us to study the crossover from the quantum to the purely classical regime and more importantly, allowing the study of non-trivial topological contributions given by the Berry phase terms. We discuss these contributions in the following section.

4.2 The Z_2 and KT transition temperatures

An emergent Z_2 chiral symmetry in a continuous frustrated magnet can be found in many examples as the $J_1 - J_2$ XY model [1], the $J_1 - J_2$ Heisenberg model as well as the fully frustrated XY model [2]. At low temperatures one expects a quasi-long-range order for the spin variables and an ordered pattern for the chiral degree of freedom (broken Z_2 symmetry) while at high temperatures both degrees of freedom should be disordered with short range correlations. One then expects a Kosterlitz-Thouless (KT) transition at finite temperature T_{KT} for the spin degrees of freedom and an Ising-like transition for the chiral degrees of freedom at a temperature $T_{Ising} \geq T_{KT}$ (for restoring Z_2 symmetry implies disorder). Numerical Monte Carlo results [4] suggest that these critical temperatures, although very close, are different. Moreover, the model seems to have a rather large cross-over scale at the vicinity of the critical points making quite difficult the confirmation of the nature of the transitions as traditional Ising and KT type [22,23].

Notice that in the presence of a magnetic field the length of the planar component of the spin decreases with the magnetization or magnetic field h. One then expects both KT and Ising critical temperatures to decrease with h.

The situation becomes more interesting when we consider the topological term in equation (9). When a magnetization plateau is present (either $S - m$ integer or rational, as discussed in detail in Sect. 5), even at zero temperature the spin degrees of freedom have short range correlations implying $T_{KT} \to 0$. The question whether T_{Ising} also tends to zero for this value of the magnetization or remains non-zero is governed by the structure of the ground state of the quantum system and is discussed qualitatively below.

In Figure 3 we suggest two different scenarios for the behavior of both critical temperatures as a function of magnetization, one for which the chiral symmetry Z_2 is broken and another where it is not. It is our purpose in the next section to argue that both scenarios are possible, and that in principle by tunning the microscopic parameters one could pass from one case to the other. Numerical

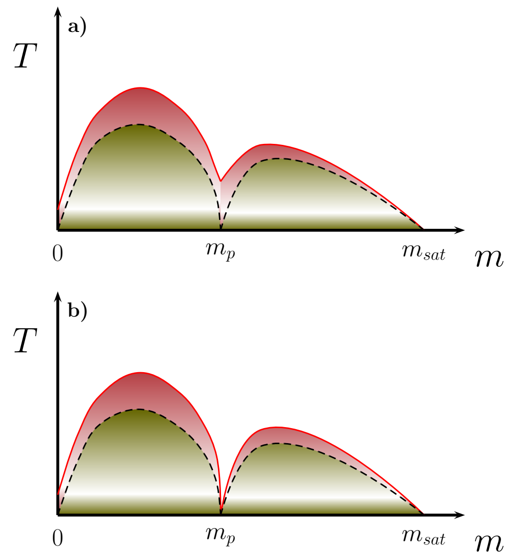

Fig. 3. Possible scenarios for the KT and Ising transitions. Dashed black lines correspond to KT transitions whereas solid red lines correspond to Ising transitions. In case (b) the Z_2 symmetry is restored at zero temperature by tunneling processes for values of the magnetization where the Berry phase term disappears.

techniques would reveal more useful to predict a precise value for the transition point, though this challenging task is beyond the scope of the present work. Notice that in the $m = 0$ limit the model is $SU(2)$ symmetric, having only a Z_2 transition [5,24].

5 Vorticity and Berry phase contributions

In the calculation of the free-energy of Section 4.1 the vortex contribution to the action was not taken into account. Indeed, the Matsubara decomposition of the fields φ_1 and φ_2 assumes periodicity in imaginary time and is only well defined for smooth configurations, namely if the field configurations with vortices are excluded. In general, the vortex free description of the preceding section remains valid as long as the stiffness of the fields is large enough to penalize vortex configurations (that is, below the KT transition temperature).

On general grounds a topological Berry term provides in the path integral a weight for each configuration, measured by its vorticity (2π times the integer winding number): when appearing with a non integer coefficient the vortex contributions tend to cancel out by destructive interference, thus penalizing vortex proliferation. On the other hand, when the coefficient is integer vortex configurations interfere constructively, just as in the XY model. This rationale leads to the distinction of integer or non-integer values of $S - m$ when evaluating the influence of the topological term in equation (9).

In the particular case of integer $S - m$, the Berry term provides a trivial phase $e^{i(S-m)2\pi} = 1$ in the free energy computation, then vortices can arise as in a standard XY model. The consequence of vortex proliferation is the presence of a magnetization plateau [9,17]: this is monitored by the behavior of the symmetric field

$$\varphi_s = \frac{1}{2}(\varphi_1 + \varphi_2) \tag{17}$$

which governs the physics of the total magnetization of the system. It is invariant under global rotations, thus a Goldstone mode of the system, and is conjugate to the fluctuation of the total magnetization $\Pi_1 + \Pi_2$. Its delocalization due to vortex proliferation translates into a locking of its conjugate field at a fixed (quantized) value. This is nothing else than the presence of a magnetization plateau.

When $S - m = \frac{p}{q}$ is rational, only vortices with vorticity q can proliferate (larger q weakens the effect). The consequence is again a magnetization plateau, characterized by a degeneracy of the ground state given by q [9].

For generic values of $S - m$ (neither integer nor simple fractions), due to the Berry phase term in the symmetric action, destructive interference takes place between different vortex configurations [9]. Vortex effects are indeed averaged out of the partition function.

The behavior of the anti-symmetric field

$$\varphi_a = \frac{1}{2}(\varphi_1 - \varphi_2) \tag{18}$$

is also affected by the Berry term in the symmetric action, in a way particular to the present model. Usually, such a field gets a mass term in the effective action. In the present case, the flatness of the potential obtained in equation (10) for the anti-symmetric field is the result of the continuous degeneracy of the classical ground states, not protected by any symmetry. Hence, it is not a Goldstone mode and a potential term could appear in addition. Such a potential term can be taken from the free energy of fluctuations around the configurations selected by the order from disorder mechanism [12]. In this sense, the free energy shown in Figure 2 plays the role of a pseudo potential for φ_a. This field is conjugate to $\Pi_1 - \Pi_2$, directly linked to the relative spin angle α between sub-lattices.

The presence of two minima in the pseudo potential is important, since it allows for non-trivial tunneling processes, if allowed by the Berry phase term in the symmetric action. In weakly frustrated systems, the antisymmetric field is gapped due to the presence of a mass term in the effective action. This term fixes the value of the field φ_a preventing vortex formation, and hence tunneling events. Then, in the weakly frustrated case, only the symmetric field φ_s may present a non-zero vorticity.

In the present case, due to the strong frustration, the antisymmetric field is not necessarily locked. The double minima potential then allows for tunneling processes where the vortices corresponding to φ_a can still proliferate, disordering it and restoring the Z_2 symmetry.

Such processes involve the anti-symmetric combination of φ_1 and φ_2, the original sublattice fields. Nevertheless, the proliferation (or not) of vortex configurations for φ_s affects the number of available vortex configurations for each sublattice field, and then for their anti-symmetric combination φ_a. In this way the Berry phase for φ_s, in particular the value of the coefficient $S - m$, influences the tunneling process.

We discuss this mechanism below.

Tunneling effects and Ising transition

In the preceding sections we have assumed that, at low enough temperatures, the emergent Z_2 symmetry is broken. This is certainly the case for generic values of $S - m$, because the tunneling between the two minima of the effective potential for φ_a implies topologically non trivial processes, generically suppressed by the Berry phase term.

Let us consider in contrast the case where $S - m$ is an integer. The Berry phase term can just be dropped off from the action and the computation of the partition function now allows for the presence of such topologically non trivial processes where, in a localized region of the space, the relative angle between the two sub-lattices goes from 0 to π and then goes back to 0 again when evolving in imaginary time. two minima of the effective potential.

The inclusion of these kind of processes in the partition function is analogous to the well known low temperature expansion of the Ising model. In that case, the "instanton-like" excitations correspond to a small domain of "−" spins in a sea of otherwise fully ordered "+" spins in an effective three dimensional classical Ising model. In our case, the energetic cost of a domain wall is proportional to the stiffness of the antisymmetric field φ_a which is in turn controlled by the couplings J_1 and J_2.

Once the condition on $S - m$ for coherent tunneling is satisfied, it is the value of the microscopic parameters which makes favorable breaking or not the Z_2 symmetry. Reducing the stiffness of φ_a can be easily achieved by for instance approaching the limit $J_2 \to \frac{J_1}{2}$. This favors the proliferation of those instantons towards a point at which the system could enter the disordered phase, restoring the Z_2 symmetry. Whether simply approaching this limit may be enough to restore the Z_2 symmetry is a question that goes beyond the scope of the present article but the possibility of a zero temperature Ising transition is certainly an interesting issue that deserves further analysis. Related results in quasi one-dimensional systems can be found in [17,20,21].

6 Conclusions and further discussions

In this paper we have studied the order-by-disorder selection in the $J_1 - J_2$ Heisenberg model on the square lattice, in the presence of a magnetic field, by using a path integral approach appropriate for partially polarized spin systems. Quantum and thermal fluctuations select the collinear states from the largely degenerate manifold of classical ground states. The low energy effective theory of quantum fluctuations is written in terms of a symmetric field φ_s, related to global magnetization, and an anti-symmetric field φ_a related to the spin imbalance between sublattices. The path integral approach provides a topological Berry phase, with consequences going beyond the usual spin-wave analysis.

While the global magnetization is described by the symmetric field, we mainly focus on the Z_2 symmetry still present after order by disorder selection, and the influence of the obtained Berry terms in Z_2 symmetry restoration. Let us recall that restoring the Z_2 symmetry implies disordering the field φ_a. The locking of its conjugate variable Π_a to a (quantized) value which is not necessarily zero measures the difference of magnetization between sublattices, also known as spin imbalance. In this case not only the total magnetization would be locked to a special value [9] (which reveals the disordering of φ_s and the presence of a plateau in the magnetization curve) but also the difference of magnetization between the two sub-lattices would be locked to quantized values.

In view of the results in this work, one could further discuss some perspectives for future investigation:

The restoration of the rotational symmetry may also indicative of the formation of singlets in the system, triggering a transition from a ground state pattern with a clear semiclassical interpretation of the spins (with no singlet formation) to a more quantum-mechanical and less degenerate ground-state. Such kind of plateau phases were dubbed classical and quantum plateaux by Hida and Affleck in the study of one-dimensional systems [25].

Another feature which is presumably related to the formation of singlets mentioned above is the possible factorization of the wave function into separable states. The most known examples of factorized systems may be the Majumdar-Gosh chain [26], as well as the 2-dimensional Shastry-Sutherland spin 1/2 system [27], but such kind of phenomena have been shown to occur in a large variety of quantum magnets which are known to have OBD mechanism. In the presence of a magnetic field, and close to saturation, it has been shown that the wave function can be written as the tensor product of localized magnons in a sea of polarized spins [28]. In fact, the kagome model at the $\frac{1}{3}$ plateau has a wave function which has a large overlap with a test wave function consisting, again, in a tensor product of resonating plaquettes [13]. Such wave function factorization also occurs in highly frustrated one dimensional systems [21]. In particular, it has been rigorously shown that a fully dimerized wave function is the ground state of a family of zig-zag ladder Hamiltonians [29]. In the case at hand, the factorization would separate the two sublattices, with very little entanglement between sites belonging to each of them.

The quantum nature of the ground state of the highly frustrated spin system discussed in this work and the possible mechanisms involving the tunneling effect described above deserve future investigation.

P.P. acknowledges Xavier Plat for enlightening discussions. C.A.L., D.C.C. and G.L.R. are partially supported by PIP 0747 CONICET, PICT 1724 and PICT 2013-0009 ANPCyT. P.P. is partially supported by the "Agence Nationale de la Recherche" under Grant No. ANR 2010 BLANC 0406-0. The authors hereby state that they have contributed equally to the present paper.

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