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DARK MATTER TO DARK ENERGY TRANSITION IN *k*-ESSENCE COSMOLOGIES

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We implement the transition from dark matter to dark energy in k-essence cosmologies for a very large set of kinetic functions F, in a way alternative to recent proposals which use generalized Chaplygin gas and transient models. Here we require that the pressure admits a power-law expansion around some value of the kinetic energy where the pressure vanishes. In addition, for suitable values of the parameters of the model, the speed of sound of the dark matter will be low. We first present the discussion in fairly general terms, and later consider for illustration two examples.

Keywords: Dark matter; dark energy; k-essence.

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1. Introduction

As more and better astrophysical evidence is known, the evolution of the Universe is largely driven by dark energy with negative pressure together with pressureless cold dark matter (see Ref. 1 for the latest review). However, little is known about the origin of either component, which in the standard cosmological model would play very different roles: dark matter would be responsible for matter clustering, whereas dark energy would account for accelerated expansion. This lack of information leaves room to speculate with the idea that a single component acted in fact as both dark matter and dark energy.

On one hand the unification of those two components makes model building become considerable simpler, but on the other hand, and more importantly, it implies the existence of an era during which the energy densities of dark matter and dark energy are strikingly similar. One should therefore not be surprised that unifying dark matter energy has generated a considerable theoretical interest.

The first two unifying candidates to appear in the literature were the Chaplygin fluid and the tachyon field but here we will consider an emerging alternative: k-essence. In this paper we investigate scalar field models with non-canonical standard kinetic terms because they provide a unified description of dark matter and dark energy. Given that the equations of motion in classical theories seem to be of second order, the only kinetic terms considered will be functions of the square of the gradient of the scalar field (hereafter k-field). Moreover, since k-fields can be used for devising dark energy models, it is common place to interpret those fields as some kind of matter called k-essence.[3–5] Nevertheless, k-fields were not originally introduced for describing late time acceleration, but rather they were put forward as possible inflation driving agents.[6,7] Interestingly enough, as shown in Ref. 5, one can also construct tracking k-essence cosmologies.

In Ref. 3 it was shown that, under some assumptions on the form of the Lagrangian of k-essence models it is possible to obtain examples of universes which transit between a dark matter dominated era and a dark energy dominated one. In that paper, the author considered Lagrangians which functionally depend only on field derivatives and not on the field itself, and it was argued that if the pressure had an extremum at a given value of the field derivative, then the energy density of the model would scale like the sum of a nonrelativistic dust component and a cosmological constant-like component.

By sticking also with Lagrangians with a constant k-field potential, we will show that the transition between dark matter and dark energy can also be successfully modeled by requiring that there be a zero in the pressure. In this case, the parameters of the model can be chosen so that the sound speed is very low, thus letting the pressureless component behave as dark matter. We illustrate our results with two interesting examples: the first one is a quadratic toy model with necessarily large speed of sound, but the second one is a transient model in which the speed of sound may be as small as one wishes.

2. Basic Equations

Since we are interested in a large-scale description of the universe, we assume k-essence is the source of a spatially flat homogeneous and isotropic spacetime with scale factor $a(t)$ and Hubble factor $H = \dot{a}/a$ (as usual, here and throughout overdots denote differentiation with respect to t). Our models are derived from the factorizable Lagrangian $\mathcal{L} = -V(\phi)F(x)$, where $V(\phi)$ is a positive definite potential, $F(x)$ is an arbitrary function of x, ϕ is the k-field and $x = -\dot{\phi}^2$. That form of the Lagrangian is in turn suggested by the Born–Infeld Lagrangian $\mathcal{L} = -V(\phi)\sqrt{1+x}$ which was associated with the tachyon by computations in boundary string field theory.[10] Such Lagrangian also arises in open bosonic string theory[11] and is a key ingredient in the effective theory of D-branes.[12]

Associating the energy–momentum tensor of the k-field with that of a perfect fluid, we compute the energy density ρ and the pressure p, which read

$$\rho = V(\phi)[F - 2xF_x]\,, \tag{1}$$

$$p = -V(\phi)F\,. \tag{2}$$

The corresponding barotropic index γ is given by

$$\gamma \equiv 1 + p/\rho = -2xF_x(F - 2xF_x)^{-1}\,. \tag{3}$$

On the other hand, the Einstein field equations are

$$3H^2 = V[F - 2xF_x]\,, \tag{4}$$

$$\dot{H} = VxF_x\,, \tag{5}$$

whereas the conservation equation reads

$$\dot{\rho} + 3H(\rho + p) = 0\,. \tag{6}$$

Inserting Eqs. (1) and (3) into the conservation equation (6), we find the field equation for the k-field:

$$(\gamma/\dot{\phi})^{\cdot} + 3H(1-\gamma)(\gamma/\dot{\phi}) + V'(1-\gamma)/V = 0\,. \tag{7}$$

The stability of k-essence with respect to small wavelength perturbations requires that the effective sound speed[7]

$$c_s^2 = p_x/\rho_x = F_x(F_x + 2xF_{xx})^{-1}\,, \tag{8}$$

be positive. However, in Ref. 8 it was shown that a positive sound speed is not a sufficient condition for the theory to be stable. In the next section we turn our attention to the description of unifying dark matter energy using purely kinetic k-essence.

3. *k*-Essence with a Constant Potential

The first ever studied k-essence models had a constant potential; the inflationary behaviour they described had, then, a purely kinetic origin and it was dubbed k-inflation.[3,6,9] Specifically, in such cosmologies inflation is pole-like, i.e. the scale factor evolves like a negative power of time. An earlier theoretical framework in which (pole-like) k acceleration arises naturally is the pre-big bang model of string cosmology.[13] In this setup, acceleration is just due to a scalar field called the dilaton, and it will only manifest itself in the string conformal frame. Finally, for other ideas on kinetic inflation one may take a look at Ref. 14, where acceleration was put down to a dynamical Planck mass.

Coming back to k-essence, for a constant potential $V = V_0$, the k-field equation (7) admits the first integral

$$a^3F_x\dot{\phi} = 6c/V_0\,, \tag{9}$$

where c is an arbitrary integration constant. Alternatively, Eq. (9) can be written as

$$\dot{H} = -6c\dot{\phi}/a^3 \,, \tag{10}$$

after using Eq. (5). Combining Eq. (9) with the Friedmann equation (4), the barotropic index associated with this kind of k-essence can be written in a more convenient form

$$\gamma = (1 + V_0^2 a^6 F F_x / 72c^2)^{-1} \,. \tag{11}$$

We anticipate that there is a large set of models which describe universes that are dust-dominated in their early stages, that is, $\gamma \approx 1$ or equivalently $p \approx 0$, which according to the conservation equation (6) implies that $\rho \approx a^{-3}$. These models are precisely generated by the set of functions F which at early times satisfy the condition $a^6 F F_x \ll 1$. By construction, such models will behave at intermediate times as if they were filled with a perfect fluid with equation of state $p \propto \rho$. Finally, such universes end in a stable de Sitter accelerated expansion scenario. In a way, these models play the same role that the generalized Chaplygin gas, i.e. they interpolate between dark matter at early times and dark energy at late times. However, they do not share some of the unwanted features of Chaplygin cosmologies. We dwell now on the details of the construction of those cosmologies.

In this paper we take the class of functions which admit an expansion in powers in the form

$$F(x) = F_0 + F_1(x - x_0) + F_2(x - x_0)^2 + \cdots \,, \tag{12}$$

and then look at the definition of the barotropic index (11), we see that near x_0 the condition $a^6 F F_x \approx 0$ leads to $\gamma \approx 1$, obtaining a matter filled universe. Thus, there are two options for getting a dust cosmology: either making $F(x_0) = 0$, i.e. taking $F_0 = 0$, where $x_0 = x(t_0)$, or making $F_x(x_0) = 0$ as suggested in Ref. 3, i.e. taking $F_1 = 0$. To investigate the first option we define the small quantity ε as

$$x = x_0(1 + \varepsilon) \,. \tag{13}$$

Then, using Eqs. (1), (9) and (13), and working up to the first order in ε, we obtain

$$\rho \approx 12c\sqrt{-x_0}/a^3 \,. \tag{14}$$

Remarkably, we get a dust-like evolution, without making any assumption on F_1 and F_2. Nevertheless, at zeroth order in ε the sound speed reads $c_s^2 \approx F_1/(F_1 + 4x_0F_2)$ and its value does indeed depend on F_1 and F_2. If we now impose $4x_0F_2 \gg F_1$ we get $c_s^2 \approx 0$, i.e. we can describe matter which is easily concentrated by gravity. In addition, from Eqs. (4) and (5) we get

$$F_x(x_0) = -3H^2(x_0)/2x_0V_0 \,, \tag{15}$$

$$\dot{H}(x_0) = -3/2H^2(x_0) \,. \tag{16}$$

If we compare now with an evolution $a = t^{2/3}$, for which $H = 2/3t$ and $\dot{H} = -3H^2/2$, at $x = x_0$ it follows that $t_0^2 = -2/3x_0V_0F_1$. For our choice $F(x_0) = 0$ the function F leads to an energy density $\rho \propto a^{-3}$ and a scale factor that matches $a = t^{2/3}$ up to the second derivative at t_0, which may be chosen arbitrarily by choosing the value of F_1. However, for the option $F_x(x_0) = 0$, we find $F_0 = 3H^2/V_0$, $\dot{H}(x_0) = 0$ and $t_0^2 = 4/3V_0F_0$ showing that the main difference with respect to our option is that in this case the scale factor only matches $a = t^2/3$ at x_0 up to the first derivative. Therefore, it is the first option that best describes matter.

In the case of a constant potential and for an expanding universe $H > 0$, the variable $\Gamma = \gamma/\dot{\phi}$ of Eq. (7) is a Liapunov function for any function F provided that $\dot{\phi} > 0$ is a positive function (if $\dot{\phi} < 0$, then we must take the variable $-\Gamma$). This means that γ is a decreasing positive function, so the particular solution $\Gamma = 0$, which corresponds to the de Sitter evolution is stable whenever the barotropic index is restricted to $0 \leq \gamma < 1$.

Our model behaves as a sum of dark matter Eq. (14) with equation of state $p = 0$ (see Eq. (14) and a cosmological-constant-like component $p = -\rho = V_0(F(x_s) - 2x_sF_x(x_s))$, where $x_s = \lim_{t\to\infty} x$. From $t = t_0$ onwards (where $x(t_0) = x_0$), the matter content of the model will mimic successively all possible $p = (\gamma - 1)\rho$ fluids between $p = 0$ (dust) and $p = -\rho$ (dark energy). So, the model interpolates between these two phases. For $t < t_0$, the energy density of the k-essence fluid behaves as $\rho \approx a^{-3\gamma}$ with $\gamma > 1$.

4. An Exactly Solvable Toy Model

The main results we just obtained can be illustrated with an exactly solvable quadratic model. To that end we put forward the quadratic function $F(x)$

$$F = \frac{b}{6} + x - \frac{x^2}{2b}\,, \tag{17}$$

which becomes null at $x = b(1 \pm 2/\sqrt{3})$ and has an extremum at $x = b$, where b is a free parameter of the model. Inserting Eq. (17) into Eq. (4) we arrive at

$$\dot{\phi}^2 = -b/3 \pm \sqrt{2b/V_0}\,H\,. \tag{18}$$

We then substitute the latter into Eq. (10) and by integration we get the relative expansion:

$$H = \pm\left[\sqrt{bV_0/18} + 9c^2\sqrt{2b/V_0}\,\eta^2\right]\,, \tag{19}$$

where a new parameter $\eta = \int dt/a^3$ has been introduced (in addition, an arbitrary integration constant in the definition of η has been adjusted so that the expression scale factor coincides asymptotically at large time t with the one corresponding to a de Sitter solution). Now, combining the last two equations we get

$$\dot{\phi}^2 = 18c^2b\eta^2/V_0 \tag{20}$$

and the scale factor can be obtained by integrating Eq. (19) to yield

$$a^3 = -\left(\sqrt{bV_0/2}\,\eta + 9c^2\sqrt{2b/V_0}\,\eta^3\right)^{-1}, \qquad \eta \le 0\,, \tag{21}$$

where the singularity has been set at $\eta = -\infty$. The equation of state and the sound speed of this toy model are respectively

$$p = -4bV_0/9 + \rho/3 + \sqrt{8b/27V_0}\,\rho^{1/2}\,, \tag{22}$$

$$c_s^2 = dp/d\rho = 1/3 + \sqrt{2bV_0/27}\,\rho^{-1/2}\,. \tag{23}$$

Now, by making a time shift in η so that $\eta = \delta - \delta_0$ with $\delta > 0$, it will be possible to cast (21) in the form

$$a^3 = -(a_0 + a_1\delta + a_2\delta^2 + a_3\delta^3)^{-1}\,, \tag{24}$$

where a_0, a_1, a_2 and a_3 are constants. If we get closer to the singularity (i.e. $\delta \to \infty$) then $a^3 \propto \delta^{-3}$, and taking into account that $d\eta = d\delta$ it turns out that $t \propto \delta^{-2}$ and the universe starts off like $a \propto t^{1/2}$. If we now move away from the singularity and approach x_0, then $a^3 \propto \delta^{-2}$, which implies $t \propto \delta^{-1}$, and the scale factor satisfies $a \propto t^{2/3}$, as corresponds to a matter field universe. Finally, if we keep on moving away from the singularity to reach the last epoch of the universe, then $a^3 \propto \delta^{-1}$ and $t \propto -\ln\delta$, and the scale factor satisfies $a \propto \exp\sqrt{bV_0/18}\,t$, as corresponds to a de Sitter model.

The equation of state tells us that initially we have a radiative fluid $p \approx \rho/3$ (at high energies), but as the model proceeds to the asymptotic regime the pressure tends to a constant value and $p = -\rho = -bV_0/6$, so the fluid acts like a cosmological constant. The fluid interpolates between these two limits and passes through a dust-dominated epoch when p becomes null; by the definition of p as a function of F this will happen at $x = x_0 = b(1 - 2/\sqrt{3})$ for $b > 0$ (recall that by definition $x < 0$). In what concerns the speed of sound, we know it initially takes the value corresponding to radiation, i.e. $c_s^2 = 1/3$. From there on it grows monotonically till it reaches the upper limit $c_s^2 = 1$ at the last stage in which the evolution is de Sitter-like ($H = \text{const.}$) and the energy density takes the limiting value $\rho = bV_0/6$. Another way to deduce this result is to note from combining Eqs. (20) and (21) that on the $\eta \to 0$ limit we have $x \propto a^{-6}$, and the Einstein equation (4) comes down to the one corresponding to a free scalar field, i.e. our k-essence behaves like a stiff fluid with $c_s^2 \approx 1$.

Now, since the speed of sound is large at the stage where the evolution is dust-like, there would either be a blow-up of the perturbations or excessively damped oscillations.[15] Nevertheless, even though this model has not much physical merit, it is mathematically interesting because it shows the possibility of having k-essence which behaves like dust when the pressure is not extremal.

5. A Transient Model

Now, we investigate a simple transient kinetic k-essence model introduced in Ref. 9. It is generated by the following function F

$$F = \beta V_0^{-1}\left[2\alpha\alpha_0\sqrt{-x} - (-x)^\alpha\right] , \tag{25}$$

where α and α_0 are two real constants and $\beta = 1/(2\alpha - 1)$. The energy density and the equation of state of the k-field are calculated from Eqs.‘(1) and (25)

$$\rho = (-x)^\alpha , \qquad p = \beta[\rho - 2\alpha\alpha_0\rho^{1/2\alpha}] , \tag{26}$$

and the sound speed becomes

$$c_s^2 = \beta[1 - \alpha_0\rho^{-1/2\alpha\beta}] . \tag{27}$$

Solving the conservation equation (6), we obtain the energy density in terms of a

$$\rho = [\alpha_0 + c_0/a^3]^{2\alpha\beta} , \tag{28}$$

where c_0 is a redefinition of the integration constant c. Now we will apply the results obtained in the last section and we will expand the function F in powers around $x_0 = (2\alpha\alpha_0)^{2\beta}$ where F and p become null. In this case we can evaluate $\rho(x_0)$ using Eq. (26) and then calculate the speed of sound given by Eq. (27). The result is $c_s^2 = 1/2\alpha$ whereas for the barotropic index we get $\gamma(x_0) = 1$. Thus, for large α, the model is cold dark matter dominated at $x = x_0$, with approximated vanishing sound speed, $c_s^2 \approx 0$, while from Eq. (28) the energy density of the transient k-essence fluid becomes $\rho \approx \alpha_0 + c_0/a^3$. In addition, at late times the model ends in a de Sitter stage. Such "transient model" may be considered as an alternative model to the generalized Chaplygin gas. It allows the evolution of the initial perturbations in the energy density into a nonlinear regime and near x_0 it could play the role of cold dark matter (i.e. dark matter unable to resist gravitational clumping). Finally, the model yields an energy density which scales like the sum of a nonrelativistic dust component at x_0 with equation of state $p = 0$ and a cosmological-constant-like component $p = -\rho$.

6. Conclusions

The Universe at present seems to be expanding because of the joint action of dark energy with negative pressure and pressureless cold dark matter. However, it is uncertain where this dark energy could come from, and it is therefore fair to speculate the possibility of modeling those two components by a single one: k-essence. Following that line we present a new way of using k-essence for modeling a transition form dark matter to dark energy.

Specifically, it suffices that the pressure admits a power-law expansion around x_0. After that we prove that for a large subclass of models the speed of sound of the dark matter component can be as low as required for structure formation.

We then move on and discuss an exactly solvable quadratic toy model, which is undoubtedly of formal interest because it shows explicitly that the dust epoch is not necessarily associated with an extremum in the pressure.

Finally, we revisit an exact transient model (a sort of modified Chaplygin gas), which describes successfully universes dominated by clustering dark matter at early times.

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References

1. V. Sahni, astro-ph/0403324, in *Proc. of the Second Aegean Summer School on the Early Universe*, Syros, Greece, September 2003.
2. A. Yu. Kamenshchik, U. Moschella and V. Pasquier, *Phys. Lett.* **B511**, 265 (2001); N. Bilic, R. J. Lindebaum, G. B. Tupper and R. D. Viollier, *ibid.* **B511**, 265 (2001); T. Padmanabhan and T. R. Choudhury, *Phys. Rev.* **D66**, 08131 (2002); V. Gorini, A. Kamenshchik and U. Moschella, *ibid.* **D67**, 063509 (2003); V. Gorini, A. Kamenshchik, U. Moschella and V. Pasquier, gr-qc/0403062. A. Dev, J. S. Alcañiz and D. Jain, *Phys. Rev.* **D67**, 023515 (2003); M. C. Bento, O. Bertolami and A. A. Sen, *ibid.* **D66**, 043507 (2002); *Gen. Rel. Grav.* **35**, 2063 (2003); M. Makler, S. Q. de Oliveira, and I. Waga, *Phys. Rev.* **D68**, 123521 (2003).
3. R. J. Scherrer, *Phys. Rev. Lett.* **93**, 011301 (2004).
4. T. Chiba, T. Okabe and M. Yamaguchi, *Phys. Rev.* **D62**, 023511 (2000); M. Malquarti, E. J. Copeland, A. R. Liddle and M. Trodden, *ibid.* **D67**, 123503 (2003).
5. C. Armendáriz-Picón, V. Mukhanov and P. J. Steinhardt, *Phys. Rev. Lett.* **85**, 4438 (2000); C. Armendáriz-Picón, V. Mukhanov, and P. J. Steinhardt, *Phys. Rev.* **D63**, 103510 (2001); T. Chiba, *ibid.* **D66**, 063514 (2002); L. P. Chimento and A. Feinstein, *Mod. Phys. Lett.* **A19**, 761 (2004).
6. C. Armendáriz-Picón, T. Damour and V. Mukhanov, *Phys. Lett.* **B458**, 209 (1999).
7. J. Garriga and V. F. Mukhanov, *Phys. Lett.* **B458**, 219 (1999).
8. S. M. Carroll, M. Hoffman and M. Trodden, *Phys. Rev.* **D68**, 023509 (2003).
9. L. P. Chimento, *Phys. Rev.* **D69**, 123517 (2004).
10. A. A. Gerasimov and S. L. Shatashvili, *JHEP* **0010**, 034 (2000); D. Kutasov, M. Marino and G. Moore, *ibid.* **0010**, 045 (2000); S. Sugimoto and S. Terashima, *ibid.* **0207**, 025 (2002).
11. E. S. Fradkin and A. A. Tseytlin, *Phys. Lett.* **B163**, 123 (1985).
12. R. G. Leigh, *Mod. Phys. Lett.* **A4**, 2767 (1989).
13. G. Veneziano, *Phys. Lett.* **B265**, 287 (1991); K. A. Meissner and G. Veneziano, *ibid.* **B267**, 33 (1991); *Mod. Phys. Lett.* **A6**, 3397 (1991); M. Gasperini and G. Veneziano, *Astropart. Phys.* **1**, 317 (1993); J. E. Lidsey, D. Wands and E. Copeland, *Phys. Rep.* **337**, 343 (2000).
14. J. Levin, *Phys. Rev.* **D51**, 462 (1995).
15. H. Sandvik, M. Tegmark, M. Zaldarriaga and I. Waga, *Phys. Rev.* **D69**, 123524 (2004).