Game theory in models of pedestrian room evacuation

S. Bouzat* and M. N. Kuperman
Consejo Nacional de Investigaciones Científicas y Técnicas. FiEstIn, Centro Atómico Bariloche (CNEA), (8400) Bariloche, Río Negro, Argentina

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We analyze the pedestrian evacuation of a rectangular room with a single door considering a lattice gas scheme with the addition of behavioral aspects of the pedestrians. The movement of the individuals is based on random and rational choices and is affected by conflicts between two or more agents that want to advance to the same position. Such conflicts are solved according to certain rules closely related to the concept of strategies in game theory, cooperation and defection. We consider game rules analogous to those from the Prisoner's Dilemma and Stag Hunt games, with payoffs associated to the probabilities of the individuals to advance to the selected site. We find that, even when defecting is the rational choice for any agent, under certain conditions, cooperators can take advantage from mutual cooperation and leave the room more rapidly than defectors.

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I. INTRODUCTION

The fields of interdisciplinary physics and complex systems are nowadays producing important contributions to the problems of pedestrian dynamics and enclosures evacuation by introducing mathematical models which enable the prediction of the pedestrian flow in crowded areas [1,2]. Understanding the dynamics of pedestrians and anticipating the problems that may arise in emergency situations is critical in the design of large spaces that will be occupied by many people [2]. The efficient evacuation of the occupants of such places under a state of emergency is fundamental when trying to minimize the negative effects of panic and confusion, clogging, and avalanches. In addition to this important application, the mathematical modeling of pedestrian dynamics has a purely academic interest for interdisciplinary physics because of its similarity with makeshift linked to granular media [1].

Within this context, pedestrian dynamics has been extensively studied from both theoretical [3–8] and experimental point of views [9–13]. The associated pedestrian flow is usually modeled as a many-body system of interacting individuals. The literature on this subject is rather extensive [14–23], exposing several different approaches to the problem. Just to mention some examples, in Ref. [12], the authors introduce the active walker model to describe human trail formation, and they show that the pedestrian flow system exhibits various collective phenomena interpreted as self-organized effects. In Ref. [24] the author suggest that the behaviors of pedestrian crowds are similar to gases or fluids. Other authors prefer the formalism of cellular automata and lattice gases [25] to frame their models [25,26]. This is the approach we are going to adopt in this work.

One of the most difficult aspects of modeling pedestrian flow is to take into account the effects of subjective elements related to the characters of the agents that can affect the interactions among them and, consequently, the global motion of the crowd [13,27–30]. Most of the models have focused on describing the flow of individuals in pure mechanistic approaches, including the behavioral reaction of the evacuees during their movement through *social forces*. This is the most common case in many works based on molecular dynamics-like schemes [1,6]. The social force model considers that the motion of an agent is governed by the desire of reaching a certain destination and by the influence it suffers from the environment, which includes the other agents [1,12,23]. Meanwhile, the behavioral aspect has been hardly considered in lattice gas models.

By including a game dynamics considering cooperative and defective strategies for the resolution of conflicts between pedestrians, the present work introduces behavioral and characterological ingredients in a simple way. With the consideration of the different strategies we are pointing at describing internal states of the individuals, which are independent of what is happening outside. This intends to include psychological features of the individuals that were not considered in previous works.

In the present lattice gas model, N pedestrians are set on a rectangular room represented by an $L \times W$ lattice, with the restriction that each site cannot hold more than one walker. The pedestrians move to empty sites according to a preferential direction dictated by the need of escaping from the room [9,31]. Conflicts between agents that want to get to the same position are solved using specific game rules and taking into account previously defined strategies (of cooperation or defection) which represent the character of the agents. Hence, the considered pedestrian dynamics include games between agents which affect their possibilities of leaving the room. Our main results indicate that, under certain conditions, cooperators can take advantage from mutual cooperation and leave the room more rapidly than defectors, even in some situations in which defecting is the most favorable strategy *a priori*.

There are other works where game theory was introduced in the context of room evacuation [32,33]. In the first work the elements of game theory are actually not closely related to the behavioral characteristic of the players, and consider mainly the *rational player* concept, similarly to what happens by including social forces. In the second work, the authors propose a dynamics that can be associated to a Chicken game (see Ref. [34] and this work below), where defection is not the best choice. However, in that work the results do not reveal any emergent dynamics associated to the strategies.

*bouzat@cab.cnea.gov.ar

Summarizing, the aim of this work is twofold. First, we want to analyze the particular problem of room evacuation before mentioned. Second, we aim at showing a general way in which the characterological features of the pedestrians can be taken into account, together with the need of walking towards a desired direction.

In the following section we introduce some basic concepts of game theory and 2×2 symmetric games that are relevant for the pedestrian dynamics that we will consider.

II. 2 × 2 SYMMETRIC GAMES

A two players game can be characterized by the set of the strategies that the players can adopt and by the payoff received by each strategy when confronting any other. If the game is symmetric, i.e., both players have access to the same set of strategies and payoffs, the information can be loaded into an $n \times n$ matrix, with n the number of strategies. In 1966 Guyer and Rapoport [34] cataloged all the 2×2 games. There are 12 symmetric games; eight of them are trivial, in the sense that there is no conflict of interest as both players prefer the same outcome. The remaining four games represent four distinct social dilemmas.

In a 2×2 game there are only two different strategies that can be defined as cooperative (C) and defective (D).

Furthermore, we can consider only relative payoff values, and this left us with four generic payoffs represented in the following matrix:

	C	D
C	CC	CD
D	DC	DD

Here each matrix element indicates the payoff of a player adopting the strategy at the row when competing with a player adopting the strategy at the column. CC is the payoff to each of the two players, when both cooperate; DD is the payoff when both defect. When one player cooperates and the other defects, the payoffs of the cooperator and the defector are CD and DC, respectively. The relative values of the four payoffs characterize the whole family of symmetric 2×2 games.

In order to be consistent with the names assigned to the strategies, the payoff CC must be preferred to CD, meaning that a cooperation from the opponent is always preferable to defection. Also, DC must be better than DD. Still, there is something missing to define a dilemma. If defection is bad, it is natural to avoid it, unless there is a temptation to defect. There are three situations in which this can occur. Either there is an incentive to defect when the other player cooperates (DC > CC), or there is an incentive to defect when the other player defects (DD > CD), or both.

The social dilemmas can be represented by four different games that fulfill at least one of the aforementioned conditions:

(1) Prisoner's Dilemma: DC > CC > DD > CD
(2) Stag Hunt: CC > DC > DD > CD
(3) Deadlock: DC > DD > CC > CD
(4) Chicken: DC > CC > CD > DD

We want to focus on the two first cases. In particular, the Prisoner's Dilemma (PD) represents situations in which obtaining cooperation is difficult because of the increased individual incentives to defect. Despite that players can realize that they would be better if they both cooperate than if they both defect, defecting is individually the best choice. It is a dominant strategy. Under this scenario, organizational cooperation involves convincing the individuals to work towards a common goal even if they have to give up personal incentives to defect.

In turn, Stag Hunt (SH) represents a situation in which coordination is difficult because of the uncertainty about what the opponent will do. Unlike in PD, both players realize that they are best off when they coordinate on (C, C), but do not want to select C if they think that the other player will not do the same. Therefore, organizational coordination involves being convinced that others will work towards the common goal as well, in which case it is individually rational for everyone to do so.

III. MODEL

In this work we analyze the evacuation of a rectangular room of size $L \times W$ with a single door of length L_d located at the center of one of the walls. We consider a discrete time and space dynamics in which the pedestrians (or agents) can occupy the sites of a square lattice and can perform jumps to first neighbor sites according to certain rules. When more than one agent attempts to jump to the same site at the same time a conflict occurs. To solve the conflict the involved agents (which in a square lattice can be between two and four) play a game using previously defined strategies. As a result of the game, either there can be a winner which finally gets to the desired position, or there can be no winner and all the involved agents lose their opportunity to move at that time step.

The discrete sites are labeled as (x,y) with $1 \leqslant x \leqslant L$ and $1 \leqslant y \leqslant W$. The sites with $x = 1, x = L$, and $y = W$ belong to the walls and cannot be occupied by agents. The same occurs for the sites with $y = 1$ excepting for those with $x_l \leqslant x \leqslant x_r$ that correspond to the door exit. We consider a door symmetrically located setting $x_l = L/2 - L_d/2$ and $x_r = L/2 + L_d/2$. For the sake of simplicity, throughout the paper we consider $L_d = L/10$.

We define an initial density of agents $0 \leqslant \rho \leqslant 1$, which implies an initial number of agents equal to $\mathrm{Int}(\rho \times \mathrm{L} \times \mathrm{W})$. The agents are placed at random positions $1 < x < L$, $1 < y < W$ at $t = 0$. At each time step, the dynamics involves three stages. First, every agent chooses a neighboring site where to attempt to jump. Second, all the conflicts are identified and solved according to the game rules. Finally, the winners of the conflicts (as well as the agents that can move without conflict) jump to their desired positions. The agents that reach the door are taken out of the system. The simulation finishes when all the agents have abandoned the room. In the following subsections we explain each of the three instances in detail.

A. First stage: Choosing the site where to attempt to jump

Each agent can attempt to jump to any of the four neighboring sites, which we simply label as up, down, left, and right [see Fig. 1(a)]. With probability R $(0 \leqslant R \leqslant 1)$, the direction of the jump attempt is chosen at random. Conversely, with probability $1 - R$ the direction of the jump attempt is chosen according to a desired (or rational) direction, which is defined for each agent pointing essentially toward the door in a way that

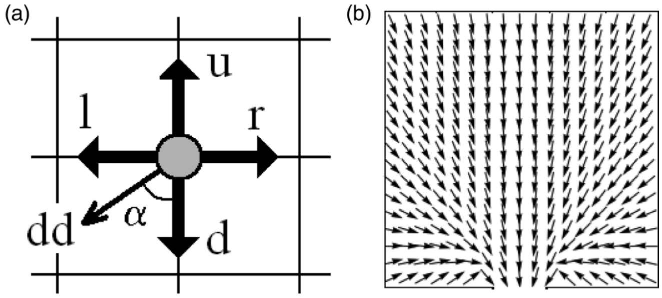

FIG. 1. Model for agents motion. (a) Allowed directions (up, down, left, right) for the motion of an agent (central circle) and desired direction (dd). (b) Field of desired direction in a square room.

we explain in detail below. Importantly, the angle α defining such a desired direction is continuous between 0 and 2π. Then the desired direction is projected on the discrete allowed directions in order to define the probabilities for attempt to jump. This is done as follows. For instance, consider the case depicted in Fig. 1(a). In such a case, we define the probabilities for an attempt to jump to the up, down, right, and left directions as $p_u = R/4, p_d = R/4 + (1-R)\cos(\alpha)/Z, p_r = R/4$, and $p_l = \frac{1}{4}R + (1-R)\sin(\alpha)/Z$, respectively. Here $Z = |\cos(\alpha)| + |\sin(\alpha)|$ is a normalization constant required to have $p_u + p_d + p_r + p_l = 1$. Note that in this case p_u and p_r contain only the term proportional to R, associated to the random choice, while p_d and p_l include also a term which is proportional to $(1-R)$ and to the projections of the desired direction. Now, considering an arbitrary desired direction defined by an angle $0 \leqslant \alpha \leqslant 2\pi$ increasing clockwise from $\alpha = 0$ (corresponding to the down direction), we have the general definition

$$\begin{aligned}
p_u &= \frac{1}{4}R + (1-R)\frac{-\cos(\alpha)\{1-\mathrm{sgn}[\cos(\alpha)]\}}{2Z},\\
p_d &= \frac{1}{4}R + (1-R)\frac{\cos(\alpha)\{1+\mathrm{sgn}[\cos(\alpha)]\}}{2Z},\\
p_r &= \frac{1}{4}R + (1-R)\frac{-\sin(\alpha)\{1-\mathrm{sgn}[\sin(\alpha)]\}}{2Z},\\
p_l &= \frac{1}{4}R + (1-R)\frac{\sin(\alpha)\{1+\mathrm{sgn}[\sin(\alpha)]\}}{2Z}.
\end{aligned} \tag{1}$$

Note that the desired or rational direction contributes only to the probabilities of jumping to the discrete directions on which it has a positive projection. Moreover, the probability R measures the randomness of the motion in such a way that for $R = 0$ only the two discrete directions defined by the desired direction are allowed, while for $R = 1$ all four directions have equal probabilities.

The desired direction is defined through a target position (x_T, y_T) toward which the agents point. For an agent located at x, y we set $x_T = (L+1)/2$ (independently of x and y), $y_T = -L/10 + 2L/5\{-y/\sqrt{(y-L/2+5)^2+y^2} + \sin[\tan^{-1}(a)]\}$ for $y < a(x-x_r)$ or $y < -a(x-x_l)$ and $y_T = -L/10$ otherwise. Here x_r and x_l are the right and left limits of the door and $a = 3$. Figure 1(b) shows the field of desired directions for a square room. The dependence of y_T on (x,y) is chosen in order to produce a recirculation pattern that prevents the agents from remaining "trapped" close to the bottom wall. This can be considered as equivalent to an effective repulsion exerted by the bottom wall. Such recirculation pattern is important to get a realistic escape dynamics like the one shown in Fig. 2. The consideration of a fixed target position (independent of the agent position) produces nonrealistic dynamics close to the $y = 1$ wall. For instance, for a fixed target position at the center of the door, an agent located at the exit would walk along the door until it reaches its center instead of getting out of the room immediately.

Now we can finally explain the detailed procedure for determining the attempt to jump for each agent at each time step. First, the desired direction and the probabilities given in Eq. (1) are computed. Second, using the probabilities (1), the agent performs two attempts of finding an empty neighbor site for the jump. This means that one of the four allowed directions is selected according to the probabilities given by Eq. (1). Then, in the case that the corresponding neighbor site is empty, it is marked as chosen for an attempt to jump by the considered agent. Meanwhile, if the neighboring site is not empty, the procedure of selecting one of the four allowed directions according to the Eq. (1) is repeated once. In case that a nonempty neighbor site is selected again, the agent will not move at that time step.

The consideration of two attempts of finding an empty site models the existence of certain degree of information that the agent has concerning the situation of the surrounding sites. The agent is allowed to select a second site if the first choice corresponds to a nonempty site. Note that the consideration of a single attempt would represent a situation in which the agent selects the site to try to move without taking into account whether the site is empty or not. Thus, the agent would act like a blind person that only knows where he wants to go, but he cannot see if there is someone there. Conversely, if a relatively large number of attempts is considered, say, four or more, the original probabilities p_u, p_d, p_r, and p_l would begin to lose sense, since the agent would always find an empty site (if there is any). From another point of view, each time step of the dynamics models the action of the agents during a short time. Thus, it is reasonable to assume that, during such a small time, the agent can change its decision of where to try to perform its next step only once. Nevertheless, we have verified that the results of the model show only small quantitative changes when the number of attempts is changed in the range from two to ten.

B. Second stage: Solving the conflicts

When a given site is chosen by more than one agent on their attempt to jump, we say that there is a conflict at that site. Thus, once all the pedestrians have chosen their sites where to attempt to jump (excepting those that have lost their opportunity of jumping due to having chosen a nonempty site), all the conflicts have to be identified and solved in order to determine what agents will be able to move at this time step. The conflicts and their solutions imply an effective interaction between the agents which affects the dynamics.

A conflict can involve two, three, or four agents, due to the existence of four allowed directions for arriving to a given site. Each conflict is solved through a game. As a result of the game, a *winner* may be selected to jump to the desired site, while the rest of the players (the *losers*) will lose their opportunity to move at that time step. It is important to stress

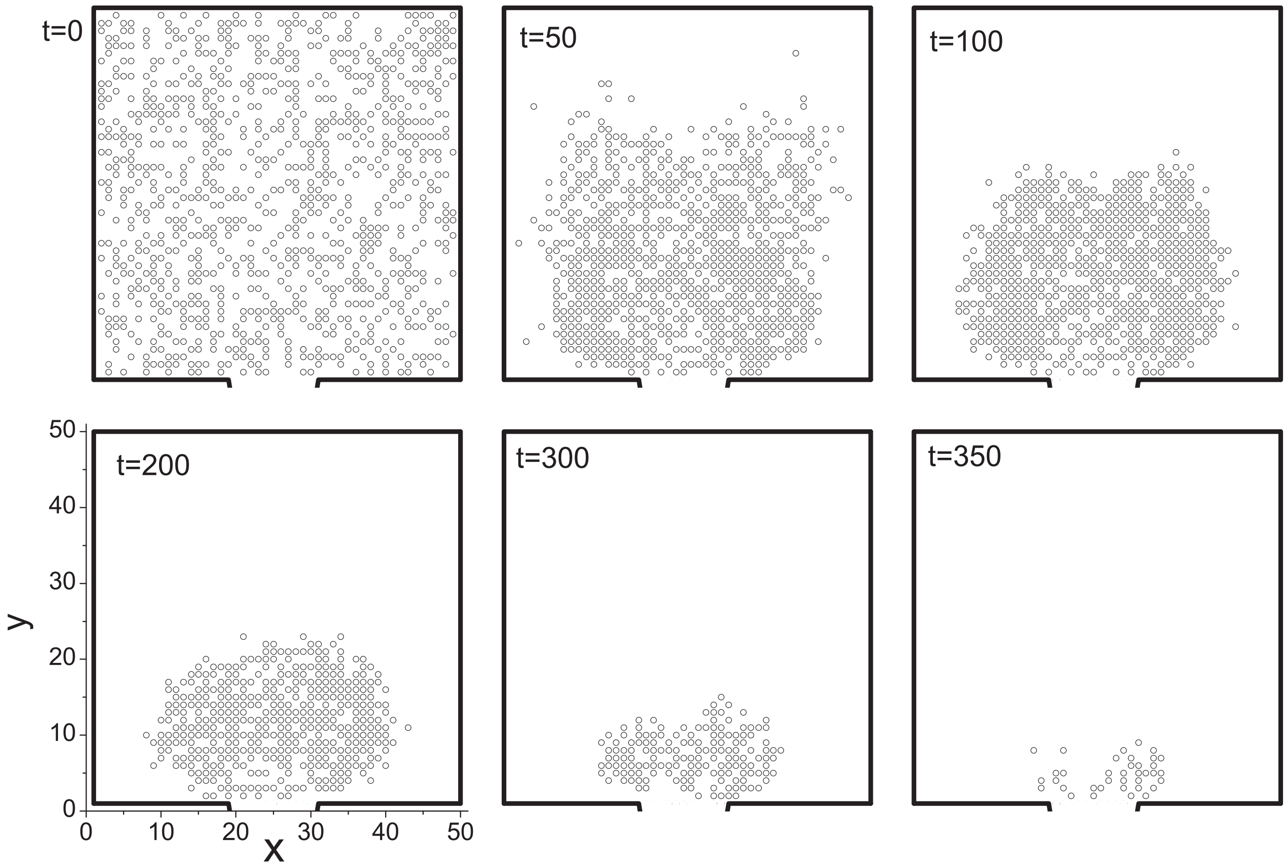

FIG. 2. Room evacuation for a population with only cooperators. Parameters $L = W = 50, \rho = 0.4, R = 0.3$.

that, with a certain probability that we indicate later, a game can have no winner. In such a case, all the players will lose their opportunity to move.

The definition of the game demands the consideration of strategies for the players. We consider that at the beginning of the dynamics each pedestrian adopts an attitude (strategy) that can be either cooperative (C) or a defective (D). In this work we are not including evolutionary aspects so that the strategies of the agents are conserved throughout the evacuation process. At the moment of the conflict, and according to the strategies chosen by each of the individuals involved in the conflict, the players will be assigned a probability to win, i.e., to jump to the desired site.

For a game which involves only two agents the game rules considered are the following. First, in a two cooperators game there is always a winner (that jumps to the desired position), which is chosen at random between the two players. Second, in a game between a cooperator and a defector, the cooperator has no chance to win while the defector can win with probability $1/P$. Here $P \geqslant 1$ is a parameter that measures the conflictive attitude of the defectors, which makes them focus more on the competition than on the movement itself. Third, in a game between two defectors, one of them is randomly selected, and then, with probability $1/P$, it becomes a winner. Thus, each of the agents has probability $1/(2P)$ of wining, while the probability of having a winner is $1/P$. Note that the larger the value of P, the lower the probabilities of stepping for the defectors, both when interacting with defectors and with cooperators. In particular, in a game with at least one defector, the existence of a winner is ensured only for $P = 1$. Thus, values $P > 1$ are used to model situations in which defectors sometimes lose their opportunity to move due to their conflictive attitude. In this sense, P is a punishment to defection.

The rules for solving conflicts on n agents are the following. In a game with n cooperators and no defectors, there is always a single winner, which is randomly selected between all the players. In a game with a single defector and $m = n - 1$ cooperators, all the cooperators lose, and the defector has probability $1/P$ of wining (i.e., the probability of having a winner is $1/P$). In a game with more than one defector and a number of cooperators equal to $m \leqslant (n - 2)$, one of the defectors is randomly selected, and then, with probability $[(n - m - 1)P]^{-1}$, it becomes the winner. Hence, each defector has probability $[(n - m)(n - m - 1)P]^{-1}$ of winning while the cooperators always loose. The probability of having a winner is $[(n - m - 1)P]^{-1}$.

By considering the probabilities of winning of the agents as payoffs, it is possible to give an interpretation of the rules for solving conflicts in terms of game theory. This can be done by following the ideas in Ref. [35] for iterative normal games (see also Ref. [36]), where the payoff matrices are defined in terms of temporal averages of the payoffs of individual games. For instance, note that in our system, any cooperator will get probability of moving equal to 1 on half of the two-players games in which it confronts another cooperator, while it will get 0 on the other half. Thus, the average probability 1/2 can be considered as the (average) payoff for the CC interaction of the two players game. Now, by considering the average probabilities of moving for the cases of CD, DC and DD interactions, we get to the following payoff matrix for the two players games:

	C	D
C	$\frac{1}{2}$	0
D	$\frac{1}{P}$	$\frac{1}{2P}$

Note that in the first row we have the payoffs for C agents, while in the second one those for D agents. It is important to stress the fact that the matrix contains mean values that make sense only when considering several matches. For instance, the CC element should not be interpreted as if in a single encounter between two cooperators each one obtains an independent probability $1/2$ of moving, which will lead to a probability $1/4$ of both moving at the same time.

Using the above indicated matrix, the two players interaction considered is straightforwardly analyzable in terms of game theory. We observe that when $1 \leqslant P < 2$ the matrix is analogous to the payoff matrix of the Prisoner's Dilemma, while for values of P higher than 2, the analogous situation is the Stag Hunt. As mentioned before, in both cases there is an incentive to defect. While in the first case D is clearly a dominant strategy, this is not true for the Stag Hunt. Still, in both cases, cooperators can take advantages only from mutual cooperation, that is more attractive when $P > 2$. The lower bound for P is imposed to fulfill the requirement $2\text{CC} > \text{DC} + \text{CD}$, which prevents alternating cooperation and defection in an evolutionary game.

For encounters of n players we can construct the following payoff matrix containing the probabilities of winning of a C player (first row) and a D player (second row) in the cases in which all the rest players are cooperators (first column) and in the cases in which the player (C or D) competes against $m < n - 1$ cooperators and $n - 1 - m$ defectors (second column):

	$(n-1)$C, 0D	mC, $(n-1-m)$D
C	$\frac{1}{n}$	0
D	$\frac{1}{P}$	$\frac{1}{(n-m)(n-m-1)P}$

Note that the matrix elements are calculated according to the above indicated rules for solving conflicts of n players.

C. Third stage: Movement of agents

As stated before, once all the conflicts have been solved, all the winners as well as all the agents that can move without conflict are finally moved to their selected sites. In case that one agent gets to a position with $y = 1$ (i.e., it reaches the exit), it is taken out of the system, and the number of escaped agents is increased by one.

IV. RESULTS

We characterize the evacuation dynamics of the system by computing the *mean exit time*, which is defined as the average over realizations of the number of time steps required to evacuate the room completely. We also study the time dependence of the *mean number of escaped agents*, i.e., the average over realizations of the number of agents that have reached the exit at a given time.

Before considering the general case of a population with both cooperators and defectors, we study the extreme cases in which all the agents have the same strategy.

A. Only cooperators: The random game case

First, we consider a population in which all the agents are cooperators. In this case, any conflict leads to a random game in which there is always a winner. Namely, for a game between n cooperators, each of them has a probability $1/n$ of becoming the winner and, thus, moving to the desired site. Note that the parameter P results irrelevant. In Fig. 2 we show snapshots of the positions of all the agents at six different stages of the evacuation process for a single realization and for a system consisting in a room with $L = W = 50$.

Figure 3 shows the results for the mean exit times and the number of escaped agents as a function of time for the evacuation dynamics from a square room considering different system parameters ρ, R, and L. In Fig. 3(a) we see that the mean exit time increases with R more rapidly than exponentially. This means that the randomness of the dynamics strongly slows down the evacuation process. Note that the randomness may be associated to an uncertainty of the agents in their knowledge of the position of the door. We can also see that the mean exit time increases with the initial density, as could be expected. Figure 3(b) shows that such a growth with ρ is linear at fixed R.

In Fig. 3(c) we study the dependence of the mean exit time on the system size. We see that the growth is slower than exponential and faster than linear.

Finally, Fig. 3(d) shows the evolution of the mean number of escaped agents for different values of ρ and R. Here we see that the growth is linear along most of the evolution. Small deviations from the linear regime occur for very short and very long times. This is due to the fact that the flux of pedestrians out of the room is controlled mainly by the local density at the exit (and by the parameter R, which is constant). The local density at the exit at small times coincides with ρ, and then it increases until it reaches a quasistationary value. Such quasistationary density determines the slope of the linear growth of the evacuation profile. Finally, at large times, when the evacuation process is about to finish, the density at the exit decreases and the flux at the door is reduced. Figure 3(d) also shows that the slope of the linear growth is essentially independent of the initial density. This indicates that the quasistationary value of the density at the door depends only on R, as could be expected.

B. Only defectors

Now we consider a population in which all the agents are defectors. This is an extreme case opposite to the one considered in the previous subsection. We focus on the dependence of the results on the parameter P, which now becomes relevant as it rules the probabilities of motion resulting from all the conflicts.

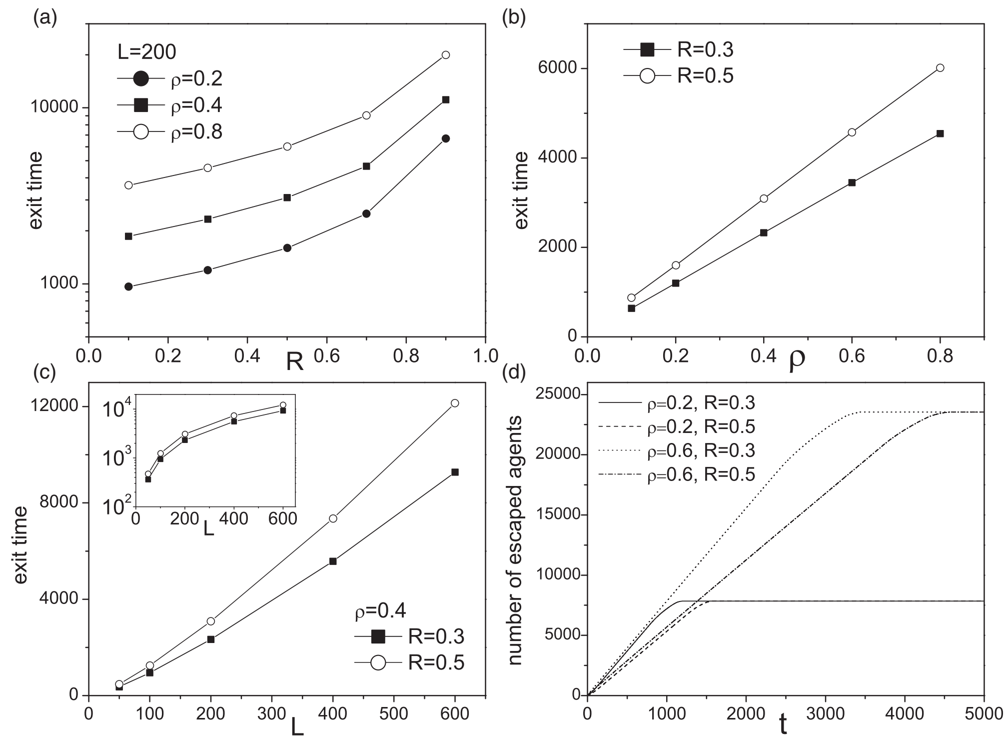

FIG. 3. Results for a population of only cooperators in a square room ($L = W$). (a) Exit time as a function of R for different values of ρ and $L = 200$. (b) Exit time as a function of ρ for different values of R and $L = 200$. (c) Exit time as a function of L for different values of ρ. The small inset shows the same curves in logarithmic scale. (d) Evolution of the number of escaped agents for different values of ρ and R for $L = 200$.

Figure 4(a) shows the mean exit time as a function of the initial density ρ for different values of P, while Fig. 4(b) shows the mean exit time as a function of P for different values of ρ. For the sake of comparison, in both insets we also include the results for the only-cooperators case studied in the previous subsection. It can be seen that the mean exit time grows linearly both with ρ [Fig. 4(a)] and with P [Fig. 4(b)]. Moreover the mean exit time for the only-cooperators case is always smaller than that for the only-defector case at any value of P. This last fact can be understood as a consequence of the delays for stepping after the conflicts, which are present only for games with defectors.

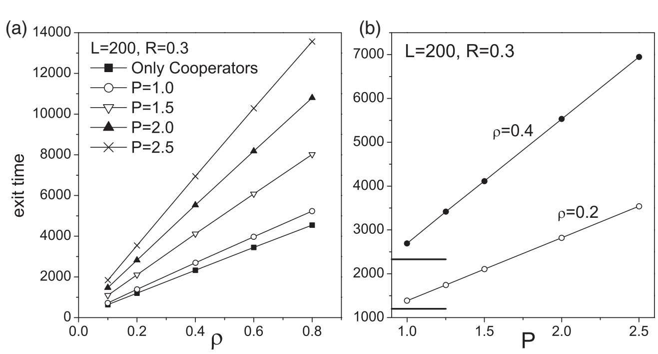

FIG. 4. Evacuation dynamics for systems with only defectors. (a) Mean exit time as a function of the initial density of agents ρ for different values of P. For the sake of comparison we also indicate the results for a system with only cooperators (squares). (b) Mean exit time as a function of P for different values of ρ. The segments on the left indicate the values for systems with only cooperators.

C. Heterogeneous populations

Finally we study the evacuation problem for heterogeneous populations with both cooperators and defectors. In this case there is an additional relevant parameter of the system. Namely, the initial fraction of defectors ρ_D ($0 \leqslant \rho_D \leqslant 1$) defined in such a way that the number of defectors is $\mathrm{Int}(\rho_D \times \rho \times L \times W)$ with ρ, the initial density of agents.

In Fig. 5(a) we show the mean exit time as a function of the initial fraction of defectors ρ_D for fixed values of ρ, R, and L considering different values of P. It can be seen that for $P = 1$ the exit time is almost independently of ρ_D. This is so because in this case the penalization for defectors is null excepting for games with more than two defectors, which are likely only at large ρ_D. At larger values of P the exit time grows approximately linearly with ρ_D. In Fig. 5(b) we show the normalized exit time, defined as $[t(\rho_D) - t(\rho_D = 0)]/t(\rho_D = 1)$, where $t(\rho_D)$ is the mean exit time obtained for an initial fraction of defectors ρ_D. The plot shows us that the curves for large enough P are close to collapse, indicating a quasilinear behavior with ρ_D. However, a close look to the figure reveals that the dependence on ρ_D may be exactly linear only for a value of P close to $P = 2$ (i.e., the limit between PD and SH games), while it is superlinear for smaller values of P (PD game) and slightly sublinear for larger values of P (SH game).

One important question we pose here is whether cooperators can or cannot take advantages from mutual cooperation

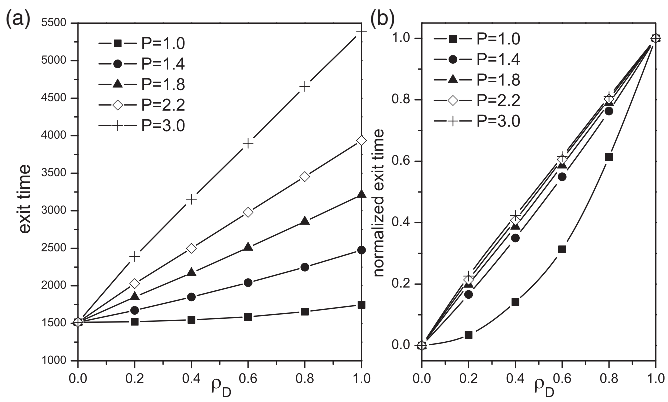

FIG. 5. Evacuation dynamics for heterogeneous populations. (a) Exit time as a function of the initial fraction of defectors (ρ_D) for different values of P. (b) Normalized exit time (defined as $[t(\rho_D) - t(\rho_D = 0)]/t(\rho_D = 1)$, where $t(\rho_D)$ is the mean exit time obtained for an initial fraction of defectors ρ_D) as a function of the fraction of defectors for the same systems as in (a). All the calculations are for $L = 200, \rho = 0.4$, and $R = 0.3$.

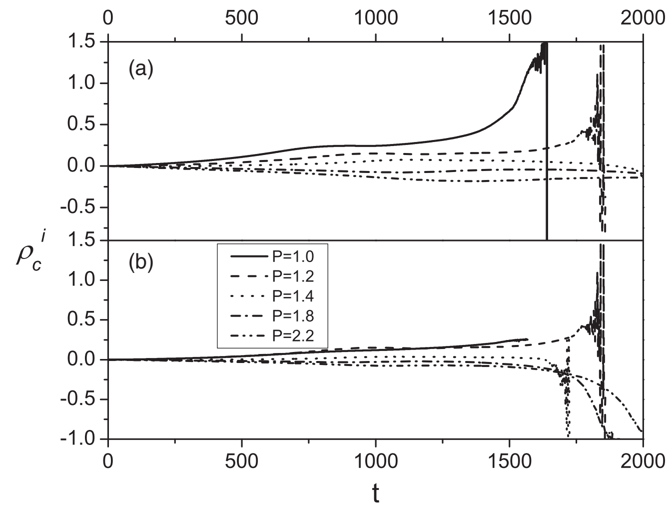

FIG. 6. Normalized instantaneous fraction of cooperators within the room [defined in Eq. (2)] for different values of P. Results for $L = W = 200$, $R = 0.3$, and $\rho = 0.4$ considering an initial fraction of cooperators equal to $\rho_c(0) = 0.4$ (a) and $\rho_c(0) = 0.8$ (b).

as it was verified in other systems where cooperation arise as an emerging phenomena [37–46]. With this goal in mind we characterize the dynamics of the system by three quantities, all of them aiming at revealing the relative success of the cooperators in finding the exit. First, we sample the composition of the population inside the room, comparing the instantaneous fraction of cooperators with the initial one. We define a normalized instantaneous fraction of cooperators in the room as

$$\rho_c^i(t) = \frac{\rho_c(t) - \rho_c(0)}{\rho_c(0)}, \tag{2}$$

where $\rho_c(t)$ is the fraction of cooperators in the room at time t [so that $\rho_c(0) = 1 - \rho_D$]. A positive (negative) value of $\rho_c^i(t)$ indicates that the population in the room has a greater (lower) fraction of cooperators than the initial one. Meanwhile, a positive (negative) derivative of $\rho_c^i(t)$ indicates that defectors (cooperators) are being more successful in finding the exit. Figure 6 shows the time behavior of this quantity for two values of the initial density of defectors and several values of P for two different values of ρ. We observe that for small values of P we have $\rho_c^i > 0$ throughout the evolution, meaning that the defectors clearly surpass the cooperators in finding the exit. Note that not only $\rho_c^i(t)$ is positive, but its derivative increases with time, indicating the continuous increment of the fraction of cooperators in the room. But as P increases and even still in the PD regime ($P = 1.8$ in the figure), the values of ρ_c^i become negative, meaning that the cooperators start to profit from mutual cooperation. While defectors lose time in futile arguments, the cooperators leave the room. $\rho_c^i(t)$ and its derivative are negative, reflecting the ability of cooperators to reach the exit.

We have studied several initial configurations with different initial conditions including varying values for the initial density of individuals, the initial fraction of defectors and the room size. With some small variations, we have found qualitatively the same results indicating the success of defectors at small P and the success of collaborators at large P.

To complement this measure, we analyze the strategy of the individuals exiting the room at each time step and calculate the fraction of cooperators among them. We compare this fraction with the corresponding to the cooperators remaining in the room and define the normalized fraction of exiting cooperators as

$$\rho_c^e(t) = \frac{\eta_c(t) - \rho_c(t)}{\rho_c(t)}. \tag{3}$$

Here $\eta_c(t)$ is the ensemble averaged fraction of cooperators among exiting individuals at time t. Figure 7 shows the time behavior of $\rho_c^e(t)$ under the same conditions analyzed in Fig. 6. Again, the fraction of exiting defectors is higher than

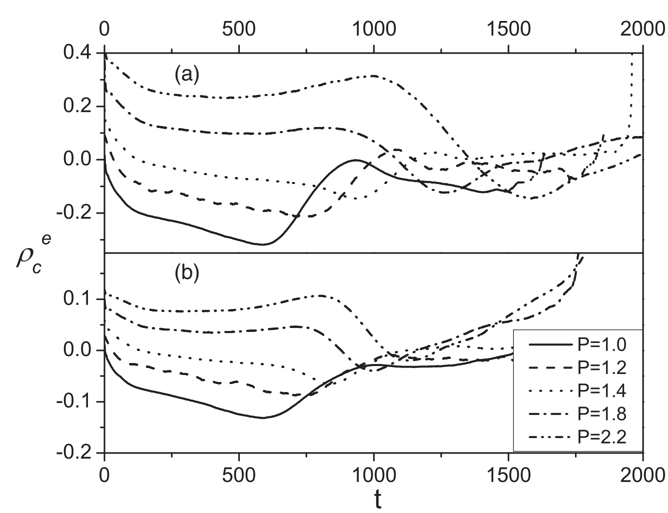

FIG. 7. Normalized fraction of cooperators exiting the room [defined in Eq. (3) for different values of P]. Results for $L = W = 200$, $R = 0.3$, and $\rho = 0.4$ considering an initial fraction of cooperators equal to $\rho_c(0) = 0.4$ (a) and $\rho_c(0) = 0.8$ (b).

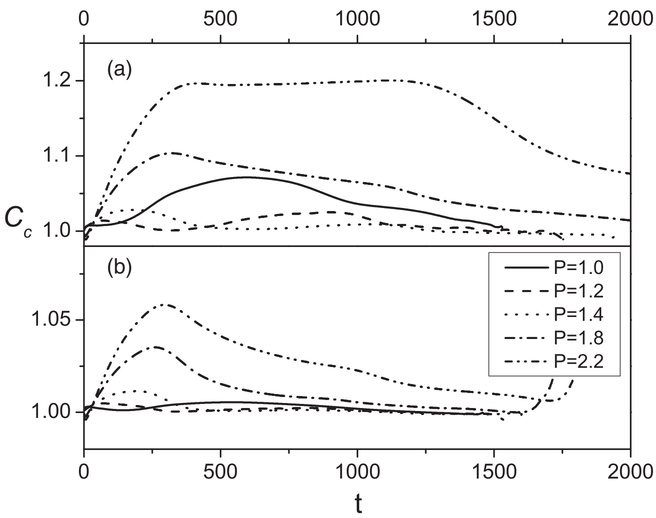

FIG. 8. Clustering of cooperators in the room [defined in Eq. (4)] for different values of P. Results for $L = W = 200$, $R = 0.3$, and $\rho = 0.4$ considering an initial fraction of cooperators equal to $\rho_c(0) = 0.4$ (a) and $\rho_c(0) = 0.8$ (b).

the expected one for small values of P, but the situation is reversed as P increases.

As cooperators can advantage defectors only when interacting with other cooperators, any evidence of cooperators performing better than defectors at leaving the room must be reflected in the emergence of some degree of clustering of cooperators that enhance the mutual cooperation. If cooperators are not clustered, the defectors will outstrip them. Although, as we show later, the effect of clustering of cooperators can be visually appreciated, it is better to provide a quantitative characterization of this phenomenon. For these, we count the instantaneous fraction of cooperating neighbors of each cooperating individual i in the room $\omega_c(i,t)$, and then we define

$$C_c(t) = \frac{1}{N_c(t)} \sum_i \omega_c(i,t)/\rho_c(t), \qquad (4)$$

where $N_c(t)$ and $\rho_c(t)$ are the number and the fraction of cooperators in the room at time t. We exclude isolated individuals from this count. Note that, with this definition, a value $C_c(t) = 1$ indicates that the mean fraction of C neighbors of a C individual is the same as the fraction of C individuals in the room, indicating no clustering effects. Meanwhile, a value $C_c(t) > 1$ reveals aggregation of cooperators.

The results in Fig. 8 confirm the occurrence of the expected clustering phenomenon. We observe that for $P > 1.5$, $C_c(t)$ is considerably larger than 1 at the same time intervals at which we have $\rho_c^e(t) > 0$ (see Fig. 6) and $d\rho_c^i(t)/dt < 0$ (see Fig. 7). This means the clustering emerges when cooperators are performing better than defectors in reaching the exit. The temporal behavior of $C_c(t)$ shows that the clustering of cooperators starts from the very beginning of the run and decreases once the majority of the cooperators have left the room. This confirms that the dynamics of the system leads to a partial segregation into cluster of cooperators, which leaves them in a situation of taking advantage from the benefits of mutual cooperation. For values of $1 < P < 1.5$, $C_c(t)$ fluctuates very close to 1, indicating a negligible clustering effect. The high values of $C_c(t)$ found in the case $P = 1$ seems to contradict our previous arguments because this indicates a clustering effect in a situation in which defectors are performing better at finding the exit, since we have $[\rho_c^e(t) < 0]$ and $[d\rho_c^i(t)/dt > 0]$. However, the effect at $P = 1$ is different in nature from the one observed for high values of P. Actually, the origin of this clustering of collaborators is associated to the fact that defectors, free to move due to low values of P manage to approach the exit, displacing the cooperators and producing a segregation that isolates the later from the door.

The results thus indicate that cooperators can take advantage from mutual cooperation for large enough values of P, typically $P > 1.5$, well within the PD regime where still the DC payoff is larger than the CC one. The benefits from mutual cooperation are further enhanced in the SH regime ($P > 2$).

The snapshots shown in Fig. 9 help us to appreciate the clustering effects better. In Fig. 9(a) corresponding to the case $P = 1$ we see how the cooperators remain clustered far from

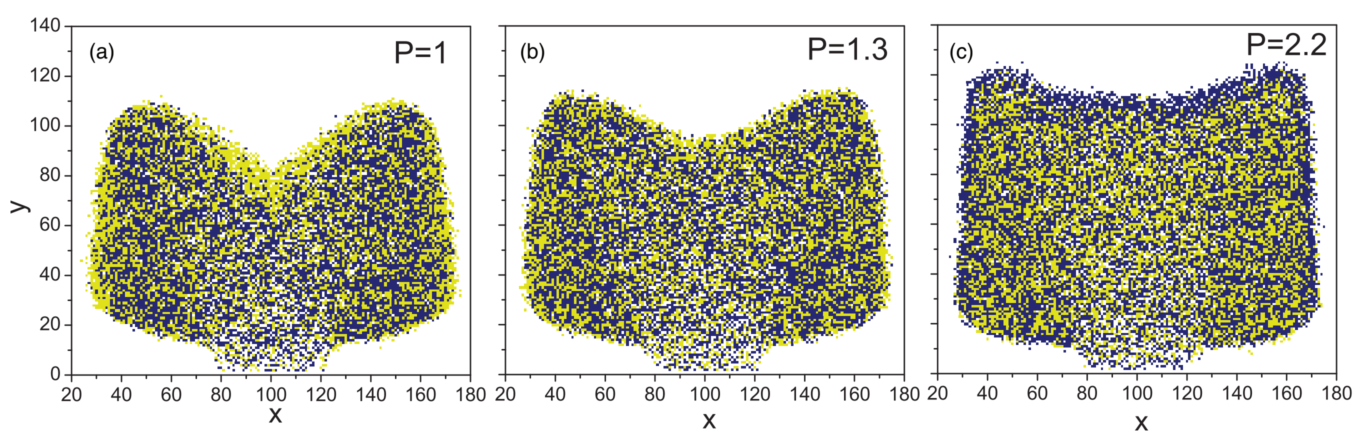

FIG. 9. (Color online) Instantaneous state of the C and D populations within a square room of size $L = 200$. Results for selected times of three different runs for (a) $P = 1$, (b) $P = 1.3$, and (c) $P = 2.2$. Each of the snapshots corresponds to a time at which $C_c(t)$ is close to attaining its maximum value, namely, $t = 700, t = 500$, and $t = 600$ for panels a, b, and c, respectively. In all the panels the gray squares (yellow online) indicate C agents whereas dark squares (dark blue online) indicate D agents. The remaining parameters are $R = 0.3, \rho = 0.4$, and $\rho_c = 0.4$ in all the cases.

the door while the defectors leave the room more easily, as just explained. In Fig. 9(b) ($P = 1.3$) the clustering effects are negligible. Finally, Fig. 9(c) ($P = 2.2$) shows a situation in which collaborators are taking advantage of clustering to leave the room before the defectors.

V. CONCLUSIONS

One of the shortcomings of the lattice gas models used in pedestrian dynamics is the lack of inclusion of behavioral aspects. Considering this situation, the present work has a clear goal: To develop a simple model able to consider some individual attitudes that may affect the movement of interacting pedestrians. Motivated by this objective, we have amalgamated models for pedestrian evacuation with game theory concepts and analyzed the emergence of nontrivial effects that may manifest. The results show that the emerging phenomena observed in an evolutionary Prisoner's Dilemma are also present here. In the works [37–46] the systems are spatially extended and allow for the cooperators to form clusters. In the present work the evolutionary aspects have not been so far included, but the growth of clusters is promoted by the mobility of the individuals. As a result of this clustering, the cooperators can profit from the only advantage they have over the defectors: the mutual cooperation. When this happens, cooperators have more success in reaching the exit than defectors, as reflected in the plots.

It is important to understand the limitation and scope of the present model. We are not intending to reproduce a real situation in detail but to point out the effects that behavioral aspects may have on the denouement of an emergency evacuation scenario. When comparing our results with previous works, we observe that if we strip our model away from the game theory elements, nothing new can be said. The results are in agreement with previous works based on gas lattice scheme. The temporal behavior of the escape time as well as the dependence with the parameters involved in the model reproduce the already known results [3,4,11,18]. But when different strategies or behaviors among the pedestrians are considered, some unexpected results arise. We have not found any literature on mathematical modeling of room evacuation where the importance of cooperative behavior is discussed. In Ref. [47], however, the authors present a related work on car dynamics. They analyze the cooperation and defection at a nonsignalized crossroad in a car traffic model. They find that cooperation maximizes the flow of vehicles and minimizes the number of accidents. Nevertheless, in such work, cooperation is *a priori* the best choice, and thus the comparison with the present work is limited. Searching the literature, we have not found direct empirical evidence about what was pointed out in this work.

Future work envisions the inclusion of evolutionary strategies, differentiated social roles, off-lattice dynamics, and different geometries and obstacles.

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