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Non-Markovian model for the study of pitting corrosion in a water pipe system

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The main studies on pitting consist in proposing Markovian stochastic models, based on the statistics of extreme values and focused on growing the depth of wells, especially the deepest one. We show that a non-Markovian model, described by a nonlinear Fokker–Planck (nFP) equation, properly depicts the time evolution of a distribution of depth values of pits that were experimentally obtained. The solution of this equation in a steady-state regime is a q-Gaussian distribution, i.e. a long-tail probability distribution that is the main characteristic of a non-extensive statistical mechanics. The proposed model, that is applied to data from four inspections conducted on a section of a line of regular water service in power water reactor (PWR) nuclear power plants, is in agreement with experimental results.

Keywords: Pitting; non-Markovian; nonlinear Fokker–Planck equation; Tsallis statistics; long-tail distributions.

1. Introduction

Corrosive processes are the main sources of failures in equipment and structures that make up an industrial plant and, for these reasons, the study of such processes is a central research issue for reliability engineering.[1] Specifically, it has been of great interest to analyze corrosion in pipelines since, in any industrial plant, they form a network to transport different types of fluids that meet the most varied purposes: transportation of fuels, raw materials and chemical waste, cooling or heating systems, among others. Over time, corrosive processes of different types were detected both on its inner surface by the influence of the fluid transported, as well as on the external surface due to the action of the chemical environment in which the pipeline is immersed. Thus, there is a significant reduction in the thickness increasing the risk of collapse due to possible cracks. Therefore, knowing the dynamics of corrosion allows the adoption of security policies and more effective maintenance, minimizing the risk of accidents and avoiding production downtime.

According to Burstein *et al.*,[2] the main source of ruptures in pipelines is the pitting. Besides, according to Valor *et al.*,[3] a stochastic phenomenon characterized by an initial metastable state was associated with the initial formation of the well, followed by a stable state due to the growth of the depth of the well over time. In order to explain these phenomena[3,4] based on the statistics of extreme values,[5] Markovian models are commonly proposed. These models are focused on growing deeper wells and the depth values obey the Gumbel distribution of type.[6] Recently, Camacho *et al.*[7] proposes a Markovian model based on the Fokker–Planck equation that studies the degradation of corroded pipes at nuclear plants. These Markovian models do not fully explain the experimental data due to presence of long-tail in the correspondent distributions.

In this paper, we propose a nonlinear Fokker–Planck (nFP) equation as an alternative procedure to the analysis of pitting corrosion. The nFP provides a stochastic distribution compatible with the experimental data of Ref. 7. In steady-state regime, nFP has as a solution a q-Gaussian distribution of probability that is a watermark of the nonextensive statistical mechanics proposed by Tsallis.[8,9] The modeling that we propose in this paper, in addition to characterizing a non-Markovian stochastic process, also provides a best-fit to the experimental data.

In the next section, the data set to be analyzed is presented and the experimental conditions are described. The main results obtained from the statistical analysis are discussed in Sec. 3. A description and interpretation of the proposed model is done in Sec. 4. The final conclusions are discussed in Sec. 5.

2. Experimental Data

The studied setup corresponds to water flowing in laminar regime through a stainless steel pipe of thickness of 6 mm. We recall that the measured data set is the same as used in Ref. 7 and it corresponds to four sets of measurements of depth for 246 corrosion points distributed along a section of 150 m. Each series was obtained by a Pipeline Inspection Gauge (PIG) inspection tool applying the technique of Magnetic Flux Leakage (MFL). The inspected section corresponds to a range of typical water service in nuclear power plants Power Water Reactor (PWR) and the inspections occurred at regular intervals of three years. The distribution of the depths along the length of the section, for each series is illustrated in Fig. 1.

According to Ref. 7, the used data refer to one of the examples described in the report produced by the Electric Power Research Institute, EPRI Final Report

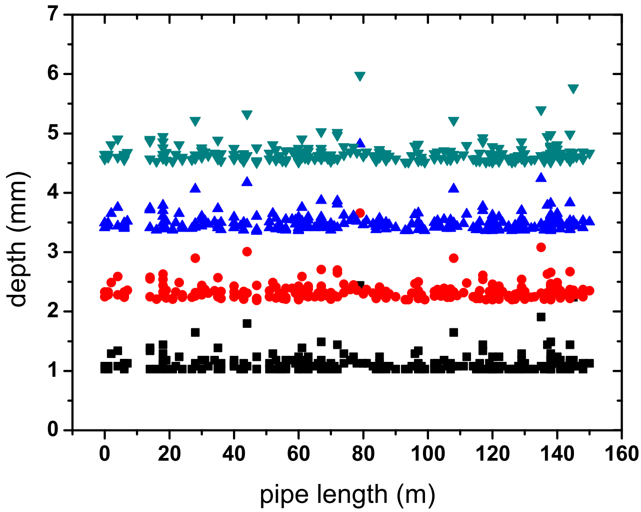

Fig. 1. (Color online) Distribution of depths of corrosion points along a 150 m section of a water service line. Each level observed in the graph corresponds to an inspection by MFL PIG. The four inspections were conducted over 12 years.

2005,[10] whose content presents a series of methods and applications for managing the life cycle of water service lines in the U.S. nuclear power systems.

3. Pitting Model Based on Tsallis Generalized Statistical Mechanics

3.1. *Foundations of generalized statistical mechanics*

The generalized statistical mechanics arose in 1988 from the formalism proposed by Tsallis[8] that is based on a nonadditive entropic term as a central postulate, i.e.

$$S_q = \frac{1 - \sum_i^W P_i^q}{1 - q}, \tag{1}$$

where W is the number of microstates accessible to the system, P is the probability associated with the i-th microstate and q is the so-called entropic index. Equation (1) defines a physical nonextensive entropy for $q \neq 1$, paving the way for a generalization of the Boltzmann–Gibbs statistical mechanics, since for $q \to 1$ the Shannon entropy is recovered. A probability distribution that maximizes the Tsallis's entropy corresponds to a generalized Gaussian one, commonly called q-Gaussian distribution,[9] and is given by

$$P(x) = A[1 - B(1 - q)(x - x_0)^2]^{\frac{1}{1-q}}, \tag{2}$$

where A is a normalization constant, B is a scaling factor associated with the standard deviation of the distribution, and x_0 is the average value of the variable x that characterizes the studied system. The limit $q \to 1$ in Eq. (2) recovers the conventional Gaussian distribution. Currently, the statistical mechanics of Tsallis consists in a wide field of research in increasing development, being applied to the modeling of astrophysical phenomena,[11–13] sunspots,[14] proteins,[15] among others complex systems.[16–20] In particular, we highlight the modeling of anomalous diffusion processes from nonlinear equations which are generalizations of the standard Fokker–Planck equation.[21] Basically, a Fokker–Planck-like equation governs the dynamical evolution of the probability density that is associated with the characteristic phase space of a stochastic process.

3.2. *Proposed pitting model*

As illustrated in Fig. 2, the average depths of corrosion points, measured for each of the four series, describe a linear increase in the time for that the rate was $0.392 \pm 0.003\,\text{mm/year}$ (mm per year). The Pearson's correlation coefficient was $R = 0.999$ indicating a robust adjustment.

The next step was to characterize the distribution of the depth values as a function of time, from the analysis of the histograms corresponding to the series. Figure 3 shows the distribution of depths to the first inspection carried out after three years of service. The linear fit suggests that the distribution of depths corresponds to a power law. This result implies that shallow wells are quite common at this stage but deep wells are rare events. For other inspections carried out at 6, 9 and 12 years of service, the distributions of the measured depths are not in agreement with power laws. As illustrated in Fig. 4, the Gaussian distribution (grey line) does not fit well the experimental data. It is important to emphasize that the

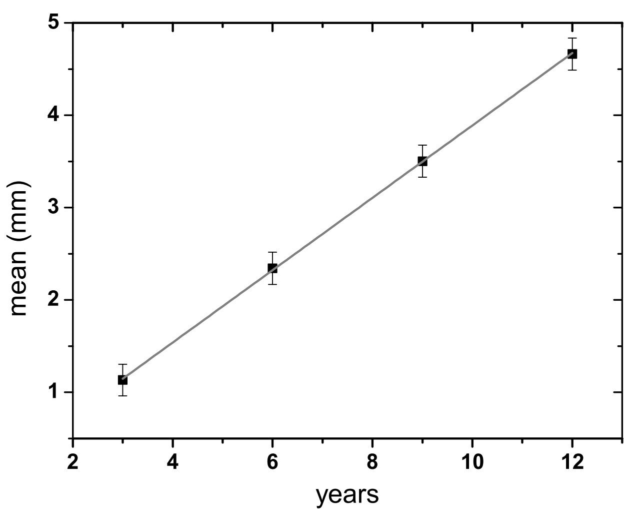

Fig. 2. Temporal evolution of the average depth of the wells. The symbols represent the average (squared) calculated for each of the inspections. The continuous curve is a linear fit whose Pearson's correlation coefficient is $R = 0.999$.

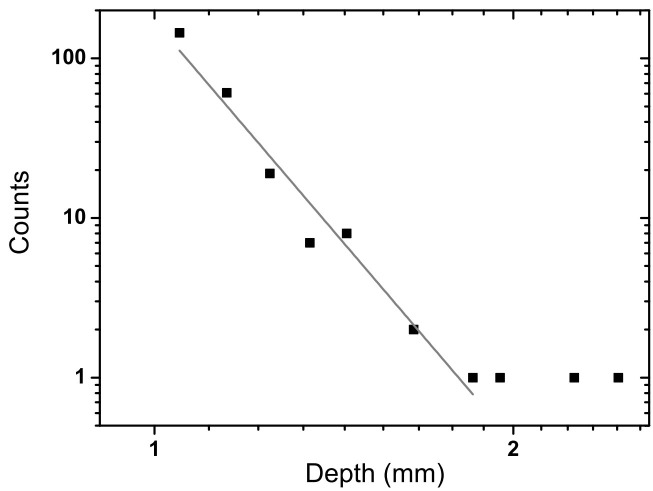

Fig. 3. Depth distribution of corrosion pits in the pipeline after three years of service. The dots represent the experimental data, with the continuum line being a linear fit. The fitting suggests that the distribution is characterized by a power law with Pearson's correlation coefficient $R = 0.98$, $F_{\text{value}} = 155.12$ and Prob $> F = 5.9210^{-5}$.

proposed model in Ref. 7 consists of Gaussian distributions of depths, i.e. a Markovian process. This proposal does not fully agree with the experimental results. Markovian processes are not well represented by long-tail distributions. It is not possible to obtain long-tail distributions by only using random processes. The results shown in Fig. 4 imply that a depth distribution is not characterized as a random event. In fact, there are long-range correlations in time for the pitting depths so that the depth-distribution shape behaves as a long-tail distribution. Therefore, the adoption of a stochastic Markovian model for the study of pitting could not provide an accurate result. In this context, we propose a formalism from the Tsallis statistics[9] based on a non-Markovian model that is able to describe the experimental data. The black curves in Fig. 4 show adjustments of Eq. (2) applied to the inspections.

We briefly describe the fitting procedure used in Fig. 4. We optimized the χ^2-value and we use analysis of variance (ANOVA) to validate the hypothesis testing. Despite the fact that ANOVA test favors the average values (central ones), the obtained results show that q-Gaussian distributions, i.e. long-tail ones, exhibit higher F_{values} when compared to Gaussian distributions. Besides, the fitting using a q-Gaussian distribution presents better χ^2-value and the Pearson correlation coefficient. These results indicate a high reliability in the fittings using q-Gaussian distribution. Clearly, in this case, these are much more proper than the fittings using the current Gaussian distribution.

From Fig. 4, it is also observed that the time evolution of the distribution of depths elapses slowly over time. In this case, it is possible to characterize the pitting

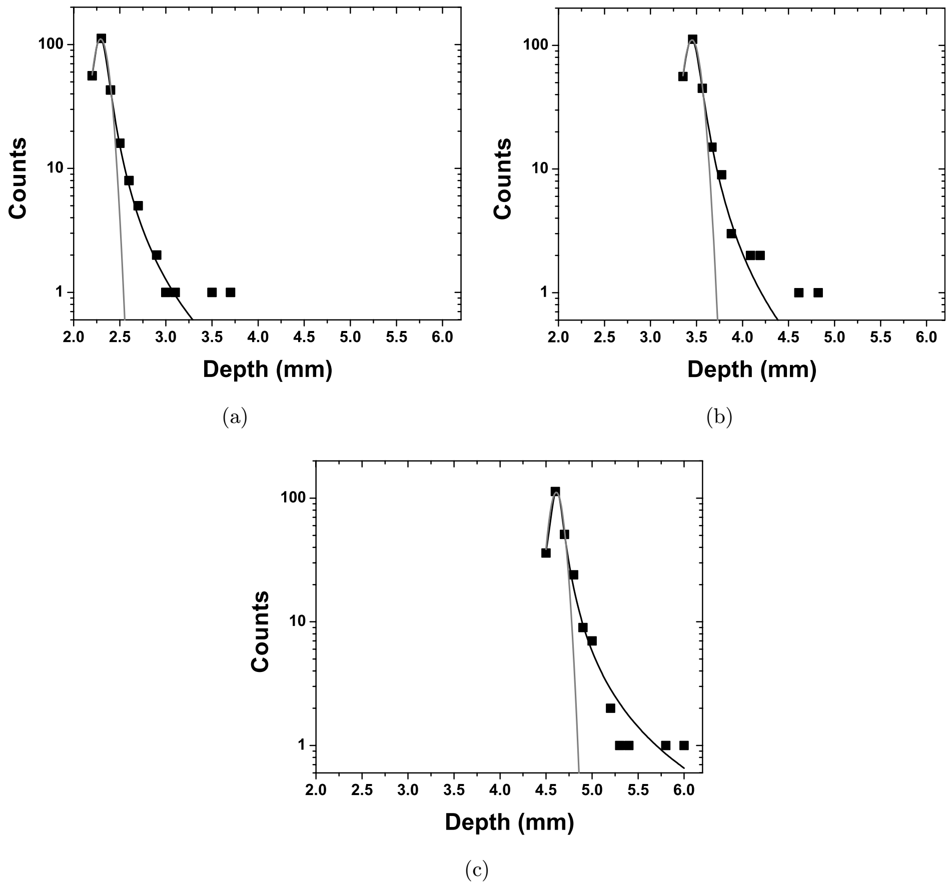

Fig. 4. Distribution of depths of pits after (a) 6, (b) 9 and (c) 12 years of service. The dots represent the experimental data, the grey line accounts the fit of the Gaussian distribution curve and, in black, the q-Gaussian distribution fit. We recall that the entropic indexes (q), Pearson correlation coefficient and ANOVA parameters to q-Gaussian distributions are (a) $q = 1.90 \pm 0.05$, $R = 0.999$, $F_{\text{value}} = 5348.12$ and Prob $> F = 0$, (b) $q = 1.83 \pm 0.06$, $R = 0.999$, $F_{\text{value}} = 4780.17$ and Prob $> F = 5.97 \times 10^{-12}$, (c) $q = 2.16 \pm 0.11$, $R = 0.995$, $F_{\text{value}} = 2584.03$ and 5.13×10^{-11}. On the other hand, the Gaussian best fitting presents $R = 0.982$ and $F_{\text{value}} = 355.64$.

as a quasi-stationary process. On the other hand, the results obtained from the fitting indicate that a long-tail distribution of depths is the proper distribution. We remark that a q-Gaussian distribution is the long-tail one proposed by our model.

3.3. *nFP equation*

The Fokker–Planck equation is a continuity equation consisting in a drift term and a diffusive term. The drift term statistically weighs the action of inhibitory mechanisms of the stochastic process and it is coupled up to the emergence of systematic trends. On the other hand, the diffusive term measures the evolution of the dispersion around the mean value. Plastino and Plastino[22] introduced the following one-dimensional equation:

$$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial x}[K(x)P] + \frac{Q}{2}\frac{\partial^2}{\partial x^2}[P^{2-q}], \tag{3}$$

where $P(x,t)$ is the probability density. The first term is the drift term that accounts for systematic tendencies present in the process. The parameter $K(x)$ is called "drift coefficient". The Q parameter is called "diffusion coefficient" that is constant and defined positive with the exponent q being the entropic index. The choice of a nonlinear diffusive term aims to produce $P(x,t)$ solutions whose time dependence is conditioned to this term in a clear reference to anomalous diffusion. The steady-state regime corresponds to a system of dynamic equilibrium reached for long times, and induces a probability distribution independent on time. The condition $P(x) \to 0$ for $x \to \pm\infty$, imposed on Eq. (3), provides a generalized exponential as a natural solution for the steady-state regime. Even more, if we consider $K(x) = -\gamma x$ that is the well-known Ornstein–Uhlenbeck process,[23] the solution obtained is the q-Gaussian distribution. According to Schwämmle *et al.*,[24] Eq. (3) is a particular case of a large class of nonlinear equations of the Fokker–Planck type whose steady-state solutions maximize the Tsallis entropy. Such a connection is established from the Boltzmann H theorem.[25,26] The nFP can be reformulated to contain a nonlinear drift term. In this context, the q-Gaussian distribution also is a stationary solution for such an equation:

$$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial x}[K(x)P^q] + \frac{Q}{2}\frac{\partial^2 P}{\partial x^2}. \tag{4}$$

This equation is not associated with anomalous diffusion since the nonlinearity is not related to the diffusion term.

In a non-Markovian stochastic processes, the proper use of nFP equations for modeling is discussed by Frank.[23] The proposed generalized form, that is not unique, is given by,

$$\begin{aligned}\frac{\partial}{\partial t}P(\mathbf{x},t) = &-\sum_i \frac{\partial}{\partial x_i}[D_i(\mathbf{x},t,P)P(\mathbf{x},t)] \\ &+ \sum_{i,k}\frac{\partial^2}{\partial x_i \partial x_k}[Q_{i,k}(\mathbf{x},t,P)P(\mathbf{x},t)],\end{aligned} \tag{5}$$

where $x(t)$ is a multidimensional random variable $P(x,t)$ is the probability density function and the generalized coefficients D_i and $Q_{i,k}$ have into account the drift and the diffusion coefficients, respectively. Equation (5) is linear and corresponds to drift and diffusion processes. In this context, the corresponding coefficients do not depend on the probability density distribution, P. When the generalized coefficients D_i and $Q_{i,k}$ explicitly depend on $P(x,t)$, the solution of nFP equation will be the proper solution for modeling stochastic non-Markovian processes. However, a licit way to establish the dependence of those coefficients with $P(x,t)$ is the main task of this approach. In this context, Eqs. (3) and (4) are in correspondence to the conditions:

$$\begin{cases} D(x,t,P) = K(x), \\ Q(x,t,P) = (Q/2)P^{1-q}, \end{cases} \tag{6}$$

$$\begin{cases} D(x,t,P) = -\gamma x P^{q-1}, \\ Q(x,t,P) = Q/2. \end{cases} \tag{7}$$

By considering an Ornstein–Uhlenbeck-type process in the Fokker–Planck equation, Plastino and Plastino proposed the following time-dependent solution:

$$P(x,t) = D(t)[1 - \beta(t)(1-q)(x - x_0(t))^2]^{\frac{1}{1-q}}, \tag{8}$$

that emerges from Eq. (3) when Eq. (8) is applied. The detailed development was derived in Ref. 22. However, the analogy with an anomalous diffusion process is not consistent since the standard deviation does not change significantly for the four series. This restriction does not apply to Eq. (4). From numerical integration, Schwämmle *et al.*[24] calculated the probability distribution for Eq. (4) analyzing their second-order moments and varying the entropic index for values in the range $0.7 \leq q \leq 1.5$.

3.4. *Non-Markovian model and corrosion by pites*

A non-Markovian process is one for which future probabilities are not determined by the most recently known value, and depend on the previous history. The proposed model describes this type of process. Thus, let us consider the current probability density $j(x,t)$ given by:

$$j(x,t) = -\left[\gamma(x - x_0)[P(x,t)]^q + \frac{Q}{2}\frac{\partial}{\partial x}P(x,t)\right]. \tag{9}$$

Replacing Eq. (9) in Eq. (4), we recover the one-dimensional continuity equation in its most usual form, i.e.:

$$\frac{\partial}{\partial t}P(x,t) + \frac{\partial}{\partial x}j(x,t) = 0. \tag{10}$$

The current density must vanish for the stationary regime, which ensures the condition $P(x) \to 0$ as $x \to \pm\infty$. Thus, we have the equation:

$$\frac{dP}{dx} = -\frac{2\gamma}{Q}(x - x_0)P^q. \tag{11}$$

And for $q \neq 1$, the proper solution is:

$$P(x) = A[1 - B(1-q)(x - x_0)^2]^{\frac{1}{1-q}}, \tag{12}$$

where $B = (\gamma/Q)[A^{1-q}]$. From the numerical calculations we obtained that the three analyzed series have values for the entropic index within the range $1 < q < 3$. Under this condition it is easy to verify that the mean value is equal to x_0 and the normalization constant A is given by:

$$A = \left\{ \left[\frac{(q-1)\gamma}{\pi Q} \right]^{\frac{1}{2}} \frac{\Gamma(\alpha)}{\Gamma(\alpha - 1)} \right\}^{\frac{2}{3-q}}, \tag{13}$$

where $\alpha = \frac{1}{(q-1)}$ and $\Gamma(x)$ is the gamma function. The result given by Eq. (12) supports a non-Markovian model related to the long-tail distribution obtained for the pitting corrosion.

4. Conclusion

In this paper, we have shown that a non-Markovian stochastic model can describe the temporal evolution of the pit-depths distribution based on nonextensive statistical mechanics. This model describes the dynamic development of pitting corrosion with a nonlinear shape of the Fokker–Planck equation, whose q-Gaussian distribution is the solution for the steady-state regime. This class of distribution fits better the experimental data than Markovian models. Besides, the proposed model has better accuracy to map the experimental values that are in the tail of the distribution curve. That said, the proposed model is able to model pitting corrosion in pipelines. Finally, the Markovian model does not provide a good agreement with the experimental data for the temporal evolution for quasi-stationary processes.

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References

1. P. D. T. OConnor and A. Klyner, *Pratical Reliability Engineering* (John Wiley and Sons, New York, 2012).
2. T. Burstein, C. Liu, R. M. Souto and S. P. Vines, *Corros. Eng. Sci. Technol.* **39**, 25 (2004).
3. A. Valor, F. Caleyo, L. Alfonso, D. Rivas and J. M. Hallen, *Corros. Sci.* **49**, 559 (2007).
4. F. Caleyo, J. C. Velázquez, A. Valorand and J. M. Hallen, *Corros. Sci.* **51**, 1925 (2009).
5. E. Castillo, *Extreme Value Theory in Engineering* (Academic Press, San Diego, 1988).
6. E. J. Gumbel, *Ann. Inst. Henri Poincare* **5**, 115 (1935).
7. E. N. Camacho, P. F. Frutuoso e Melo, P. L. C. Saldanha and E. P. da Silva, A Fokker–Planck model of pitting corrosion in underground pipelines to support risk-informed decision making in Advances, *in Safety, Reliability and Risk Management Bérenguer*, eds. Grall and Guedes Soares (Taylor and Francis Group, London, 2012).
8. C. Tsallis, *J. Stat. Phys.* **52**, 479 (1988).
9. C. Tsallis, *Introduction to Nonextensive Statistical Mechanics: Approaching a Complex World* (Springer, New York, 2009).

10. EPRI Final Report, Life cycle management sourcebook for nuclear plant service water systems, Technical report, Eletric Power Research Institute, Palo Alto, California (2005).
11. M. A. Moret, V. Senna, G. F. Zebende and P. Vaveliuk, *Physica A* **389**, 854 (2010).
12. C. Vignat, A. Plastino and A. R. Plastino, *Physica A* **390**, 2491 (2011).
13. A. C. P. Rosa Jr., J. C. O. de Jesus and M. A. Moret, *Physica A* **392**, 6079 (2013).
14. M. A. Moret, *Phys. Lett. A* **378**, 494 (2014).
15. M. A. Moret, *Physica A* **390**, 3055 (2011).
16. P. Douglas, S. Bergamini and F. Renzoni, *Phys. Rev. Lett.* **96**, 110601 (2006).
17. B. Liu and J. Goree, *Phys. Rev. Lett.* **100**, 055003 (2008).
18. R. G. DeVoe, *Phys. Rev. Lett.* **102**, 063001 (2009).
19. R. M. Pickup, R. Cywinski, C. Pappas, B. Farago and P. Fouquet, *Phys. Rev. Lett.* **102**, 097202 (2009).
20. L. Telesca, *Physica A* **389**, 1911 (2010).
21. H. Risken, *The Fokker–Planck Equation: Methods of Solutions and Applications*, 2nd edn. (Springer, Berlin, 1996).
22. A. R. Plastino and A. Plastino, *Physica A* **222**, 347 (1995).
23. T. D. Frank, *Nonlinear Fokker–Planck Equations: Fundamentals and Applications* (Springer, Berlin, 2005).
24. V. Schwämmle, E. M. F. Curado and F. D. Nobre, *Eur. Phys. J. B* **70**, 107 (2009).
25. C. F. Delale, *J. Stat. Phys.* **37**, 451 (1984).
26. M. Shiino, *Phys. Rev. A* **36**, 2393 (1987).