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## Stochastic Algorithms Applied to Vortex Detections in Turbulent Flow (Part I)

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**Abstract** The need to establish the downwind fluid dynamic field of aerodynamic bodies subjected to a given velocity field is well known, to verify their specific aerodynamic characteristics and therefore their efficiency. In this context, different techniques allow us to establish the characteristics of the field, on the one hand, visualizations, which define the qualitative characteristics of it, but it is almost always necessary to carry out quantitative determinations to describe the field correctly, particularly when the field is made up of turbulent wakes, characteristic of aerodynamic bodies under extreme operating conditions. In this sense, in the experimental field and for the definition of the turbulent flow field, it is common to use hot-wire anemometry techniques, which have great capabilities to quantify high-frequency events. Previous work has analysed the determination of changes in hot-wire anemometry signals for the detection of events in turbulent flows with different models, based on numerical algorithms, for the determination of change points (CPM - Change Point Model). The results obtained have shown agreement with conventional methodologies used for the determination of turbulent flow characteristics. The present work aims to compare the results obtained previously with the application of new CPM models developed in recent years. Previously applied and evaluated measurements are used, the implementation of the new models is carried out and the results are compared. All the algorithms used can detect changes in data that do not have a known distribution, i.e. non-parametric distributions, which are typical for turbulent flow field signals. The evaluation of measurement signals based on hot-wire anemometry is performed, considering measurements of the fluctuating components of the wind tunnel velocity at a specific point. The signals used correspond to periodic detachments downstream of a flow control device (Gurney mini-flap) at the trailing edge of an airfoil. In this way, the determination and characterization of vortices of different types is sought, to validate the different results obtained. The results show which are the best models to use for the experimental detection of such turbulent events in the flow field. These is the first approximation to the evaluation of the complete measurement with different arrange of the mini-flaps, in this case only for the static one.

**Keywords** Change point models, turbulent flow, event detection, vortex

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### Introduction

Turbulent flow in a fluid is that in which the variables: velocity, density, pressure, temperature, etc. behave randomly at each point of the fluid and at each instant of time [1]. The study of turbulent flows is of great



importance in many technological applications: Aeronautical, Naval, Mechanical and Structural Engineering, internal flow phenomena, transfer, combustion, etc. There are characteristics of the turbulent structure of a flow that change how the fluid moves in the environment of the objects, generating fluid-dynamic forces on them. Such changes can be losses in the amount of motion in the flow, generated by the appearance of eddies and viscous dissipation, and these effects are commonly observed in aeronautical and naval applications, internal flows, etc. [1]. If an engineering problem involving turbulence is to be improved or optimized, it will be necessary to understand and control the set of turbulent events or structures that govern it [2]. A meticulous analysis of a global turbulent flow would allow detecting the existence of turbulent structures in it, normally hidden. Since the mid-1960s the analysis of turbulence has been revolutionized thanks to the use of sophisticated methodologies for the analysis of experimental data. There is a vast literature on methodologies for the analysis of turbulent flow measurements, among the most recent ones, we can mention the so-called POD-Proper Orthogonal Decomposition [3], multiresolution analysis methods [4], the use of ARMA (Auto Regressive Moving Average) [5], etc. All these methodologies require significant computational power for their implementation, which makes them important for turbulence analysis in non-real time, but not in experimental test conditions. From the application of these methodologies, it was realized that in many cases the turbulence, which contained an important portion of the kinetic energy, was organized in different and particular structures. Such organization occurs, for example, in eddies of very different shape and size. The modern approach to turbulence concentrates on the meticulous study of the various turbulent structures, by analyzing how the flow behaves around the objects. The determination of such behaviour allows the effects on a moving object to be inferred. Of interest is the detection of fluid patterns by detecting, for example, the occurrence of specific turbulent events (eddies) that are generated as the flow detaches from the object. These eddies allow us to infer important effects related to the forces acting on the object, which are usually immersed in the motion of the turbulent flow [1]. When measurements, particularly in wind tunnel experiments, are performed with point velocity measurement equipment, Hot Wire Anemometry-HWA [6] or Laser Doppler Anemometry-LDA [7]), the possibility of processing the sensed random signal to detect this type of turbulent events and characterize their frequencies of occurrence and intermittency is of interest. From this, the aim is to analyze and understand how an object processes the fluid in which it is moving, and in this way to predict the behaviour of the moving object. Our aim, then, is to apply statistical techniques, for detecting changes in a sensed signal over time [8], for velocity fluctuations in a turbulent flow. Two main objectives are considered, on the one hand, the use of these methodologies to analyze measurements made with hot-wire anemometry, incorporating them as another tool that allows us to determine the occurrence of events in a fluid-dynamic field, to carry out their analysis and study. Secondly, it seems important to us that it can be applied as a tool in the data acquisition software with the final objective of optimizing the measurements made, taking advantage of the possibilities of detecting a particular change in the signal quickly.

## Methods

The problem of change detection has been a vast area of research since the 1950s [9-11]. Because the problem is very general, the literature is very diverse and takes place in very different fields. In particular, many of the methods have their origin in the quality control community, where the main objective is to monitor the results of an industrial manufacturing process, aiming to detect faults in the process as early as possible [8]. However, there are many other applications where change detection techniques are important, for example, in the study of genetic sequences, climatological studies, bioinformatics applications, intrusion in computer networks, financial market, etc. There is a lot of literature on all these topics, but their application to the analysis of fluid velocity signals is not well known. In our case, the sensed signal corresponds to the fluctuating values of the air velocity in a turbulent flow. In recent years [12], extensive work has begun on the subject of change detection in a process and certain basic criteria have been defined. Many statistical problems require the identification of change points in a data sequence. Statistical Process Control (SPC) refers to the monitoring of processes due to a change in their distribution. Traditional methods assume that the distribution of the process is fully known



before any change, including all its parameters, in which case the process is said to be "in control", and "out of control" if a change occurs that causes the process to correspond to a different distribution. The aim is to design control charts that can detect deviations from the baseline distribution. Usually, in control charts, the Average Run Length function (ARL) is employed, where  $ARL_0$  indicates the average number of observations between false positive detection assuming no change has occurred, and  $ARL_\delta$  indicates the average delay before a change in size  $\delta$  is detected. This is analogous to the classical idea applied in hypothesis test design of having a bounded Type I error and a controlled Type II error. Historically control charts were developed for monitoring changes in the mean value of a process, but variations have now been developed that also allow changes in standard deviation to be monitored in both Gaussian and non-Gaussian distributions, which prompted us to investigate the applicability of these new methodologies to the detection of changes in a turbulent random signal. Control charts traditionally require full knowledge of the process "in control", but this is not a problem if there is a large reference sample of observations that are known to generate the distribution "in control". In the case of fixed sample sizes, this is called Phase I analysis, while sequential monitoring of the process when observations are received over time is called Phase II analysis [12]. In some cases, the reference sample may be small or non-existent. In these cases, it would be impossible to accurately estimate the parameters "in control". This has important implications; it was found that even small deviations from the actual values can cause the charts to show a significantly different  $ARL_0$  from the desired value [13]. A worse situation can occur when the distribution "in control" is incorrectly specified, such as the use of a Gaussian distribution for processes exhibiting skewness. In these circumstances, non-parametric control charts are needed that assume no knowledge of the "in-control" distribution ("free distribution" charts), maintaining a desired value of  $ARL_0$  regardless of the true distribution of the process under study. In previous work [14,15], studies were initiated to analyse the application of Change Point Models (CPM), used to detect deviations in the sensed signal. In the present work, non-parametric tests for the implementation of CPM models are considered again, using applications of the detection algorithm, with routines coded in R language (<https://cran.r-project.org/web/packages/cpm/index.html> - <http://CRAN.R-project.org/package=ecp>) [16], employing the Mann-Whitney (CPM-MW), Cramer-von Mises (CPM-CvM), Kolmogorov-Smirnov (CPM-KS) [12], James-Matteson (CPM-JM) [21], and CUSUM [17,18] tests.

This work presents the results obtained from the analyses carried out for the device indicated in its fixed condition and comparing the different models used. From these results it is expected to continue with other configurations of the device, to evaluate its behaviour in all the established study cases.

### Cumulative Sums (CUSUM)

For statistical analysis of the data, the concept of cumulative sums can be used to detect small deviations from the sample average. To obtain the cumulative sums plot, proceed as follows. First, the mean value of the fixed sample of observations is determined,

$$\bar{X} = \frac{X_1 + X_2 + \dots + X_{N-1} + X_N}{N} \quad (1)$$

where  $N$  is the total number of values in the sample,  $X_i$  are the corresponding values.

To obtain the sequence of values of the cumulative sums, the first value is considering  $S_0 = 0$ , then start to determine the successive sums by using the following formula the following recurrence formula,

$$S_i = S_{i-1} + (X_i - \bar{X}) \text{ for } i = 1, 2, \dots, N. \quad (2)$$

Then the graph of the cumulative sums of the  $S_i$  versus the corresponding period is made. This type of graph aims to see if there is a variation in the slope, which would indicate a possible change in the sample mean. The segment of the period in which the values tend to be above the mean can be detected in these graphs with an increase in the slope, and on the contrary, the segments of time in which the values are below the mean are visualised in the graph with a decrease in the slope. Employing such graphs, the possibility of the presence of changes in the value of the mean could be noticed.



Once a change in the slope of the cumulative sums graph is detected, the place where the change occurs is estimated. Let  $m$  be such that,

$$|S_m| = \max_{i=0, \dots, N} |S_i| \tag{3}$$

$S_m$  is the point most distant from zero on the cumulative sums graph. This point  $m$  estimates the last point before the change occurs and point  $m+1$  estimates the first point after the change. Once the first change is detected, the sample is divided into two parts, one before the change and one after, and the procedure described above is repeated to detect other possible changes.

**Tests CPM-CvM/ CPM-KS / CPM-MW**

We will consider the problem of detecting a change point in a fixed sequence of observations. By identifying the observations as  $\{X_1, \dots, X_t\}$ , the aim is to test whether they have been generated by the same probability distribution. We assume that the distribution is not known a priori. Using the language of statistical hypothesis testing, the null hypothesis is that there is no change point and all observations come from the same distribution, while the alternative hypothesis is that there is a change point  $\tau$  in the sequence that partitions it into two sets, with  $X_1, \dots, X_\tau$  coming from the pre-change  $F_0$  distribution, and  $X_{\tau+1}, \dots, X_t$  coming from a different  $F_1$  distribution after the change [11],

$$\begin{aligned} H_0: X_i \sim F_0 \quad \text{for } i = 1, \dots, t \\ H_1: X_1, \dots, X_\tau \sim F_0, \quad X_{\tau+1}, \dots, X_t \sim F_1 \end{aligned} \tag{4}$$

You can test the existence of a change point immediately after any observation,  $X_k$ , by partitioning the observations into two samples  $S_1 = \{X_1, \dots, X_k\}$  and  $S_2 = \{X_{k+1}, \dots, X_t\}$  of sizes  $n_1 = k$  and  $n_2 = t - k$ , respectively, and then applying a hypothesis test for two samples. We will use the CvM test for this, which is based on comparing the empirical distribution function of the two samples, as defined,

$$\hat{F}_{S_1}(x) = \frac{1}{k} \sum_{i=1}^k I(X_i \leq x) \tag{5}$$

$$\hat{F}_{S_2}(x) = \frac{1}{t-k} \sum_{i=k+1}^t I(X_i \leq x)$$

Where  $I(X_i < x)$  is the indicator function

$$I(X_i < x) = \begin{cases} 1 & \text{si } X_i < x \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

This test uses a statistic based on the square of the average distance between the empirical distributions, and it can be estimated as

$$W_{k,t} = \sum_{i=1}^t |\hat{F}_{S_1}(X_i) - \hat{F}_{S_2}(X_i)|^2 \tag{7}$$

We reject the null hypothesis  $H_0$  if  $W_{k,t} > h_{k,t}$  for some threshold  $h_{k,t}$

As it is not known where the change point will be located, we do not know which value of  $k$  to use for partitioning the sample. That is why we specify a more general hypothesis  $H_0$ , i.e. there is no change in the sequence of values. The alternative hypothesis is then that there is a change point for some nonspecific value of  $k$ . Then we can make this test by calculating  $W_{k,t}$ , for each value  $1 < k < t$  and take the maximum value. However, the statistical variance  $W_{k,t}$  depends on the value of  $k$ . Because of this, we standardize the  $W_{k,t}$  statistics so that they have equal mean and variance for all values of  $k$ . For our case, standardization is simple, thus the mean and variance of  $W_{k,t}$  can be written as follows [11, 13]

$$\begin{aligned} \mu_{W_{k,t}} &= \frac{t+1}{6t} \\ \sigma_{W_{k,t}}^2 &= \frac{(t+1)[(1-3/4k)t^2 + (1-k)t - k]}{45t^2(t-k)} \end{aligned} \tag{8}$$

This leads to the maximization of the statistical test



$$W_t = \max_k \frac{W_{k,t} - \mu W_{k,t}}{\sigma W_{k,t}}, \quad 1 < k < t \quad (9)$$

If  $W_t > h_t$  for a chosen suitably threshold  $h_t$ , then the hypothesis  $H_0$  is rejected, and we conclude that a change has occurred at some point in the data. In this case, the best estimator  $\tau$  of the change point location is the  $k$  value that maximizes  $W_t$ . If  $W_t \leq h_t$ , then the hypothesis  $H_0$  is not rejected, and it is concluded that no change has occurred.

One of the most important issues in the implementation of this CPM is the number of pre-change observations; this has a great impact on the performance of the model. As the prior distribution change is unknown, it will be easy to detect changes when the number of previous observations is large, making it possible to obtain a better estimated distribution and a more accurate empirical function distribution.

For the KS test the statistic is defined as the maximum difference between the empirical distributions seen above (eq. 5 and 6) where,

$$D_{k,t} = \sup_x |\hat{F}_{S_1}(x) - \hat{F}_{S_2}(x)|, \quad (10)$$

As in the previous case, the procedure for hypothesis testing is similar. However, the standardisation of the KS statistic is more complex. This is due to the fact that there are no closed expressions for mean and variance of  $D_{k,t}$ , except asymptotically when  $t$  is large. Instead of considering the statistic  $D_{k,t}$  the  $p$ -value  $p_{k,t}$  is used, defined as the probability of observing a value more extreme than  $D_{k,t}$ . This value can be considered already standardised with respect to the sample size and easier to correct than the mean or variance for small samples.

We will have  $q_{k,t} = 1 - p_{k,t}$  and define by,

$$q_t = \max_k q_{k,t}, \quad (11)$$

In this case we will have  $q_t > h_t$ , where  $h_t$  is some possible chosen threshold, then the null hypothesis  $H_0$  is discarded and we conclude that a change has occurred at some point in the data sequence, just as for the CvM case.

Other popular nonparametric tests for changes in location and scale [16] use only the ranks of the observations, where the rank of the  $i$ th observation at time  $t$  is defined as

$$r(x_i) = \sum_{i \neq j}^t I(x_i \geq x_j) \quad (12)$$

where  $I$  is the indicator function. Detecting location changes using a change point model with a Mann–Whitney (MW) test statistic. The Mann-Whitney test ( $U$ ) is essentially an alternative form of the Wilcoxon Rank-Sum test for independent samples and is completely equivalent.

Let define the following test statistics for samples 1 and 2 where  $n_1$  is the size of sample 1 and  $n_2$  is the size of sample 2, and  $r_1$  is the adjusted rank-sum for sample 1 and  $r_2$  is the adjusted rank-sum of sample 2. It doesn't matter which sample is bigger.

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1 \quad U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2 \quad (13)$$

$$U = \min(U_1, U_2)$$

if the observed value of  $U < U_{crit}$  then the test is significant (at the  $\alpha$  level), i.e. we reject the null hypothesis. The values of  $U_{crit}$  for  $\alpha = .05$  (two-tailed) are given in the Mann-Whitney Tables. Then if a change is detected, Vostrikova segmentation [20] method let us to look for a new change point in each of the two samples.

The above formulations of the nonparametric CPMs rely on the calculation of ranks.

### Test CPM-JM

It assumes that at most only one change point exists [19]. A natural way to proceed is to choose  $\tau$  as the most likely location for a change point, based on some criterion. Here,  $\tau$  is chosen from some subset of  $\{1, 2, \dots, T-1\}$ , then a test for homogeneity is performed. This should necessarily incorporate the fact that  $\tau$  is unknown.

Now, suppose there is a known number of change points  $k$  in the series, but with unknown locations. Thus, there



exist change points  $0 < \tau_l < \dots < \tau_k < T$ , that partition the sequence into  $k+1$  clusters, such that observations within clusters are identically distributed, and observations between adjacent clusters are not. Then it maximizes the objective function using dynamic programming.

This is a nonparametric technique, which they call *E-Divisive* [21], for performing multiple change point analysis of a sequence of multivariate observations. The *E-Divisive* method combines bisection [20] with a multivariate divergence measure from Székely and Rizzo [22].

For random variables  $X; Y \in R^d$ ; let  $\phi_x$  and  $\phi_y$  denote the characteristic functions of  $X$  and  $Y$ , respectively. A divergence measure between multivariate distributions may be defined as

$$\int_{\mathbb{R}^d} |\phi_x(t) - \phi_y(t)|^2 w(t) dt \tag{14}$$

in which  $w(t)$  denotes an arbitrary positive weight function, for which the above integral exists. Using the following weight function [22]

$$w(t; \alpha) = \left( \frac{2\pi^{d/2}\Gamma(1-\alpha/2)}{\alpha^{2\alpha}\Gamma((d+\alpha)/2)} |t|^{d+\alpha} \right)^{-1} \tag{15}$$

For some fixed constant  $\alpha \in (0, 2)$ . Then, if  $E|X|^\alpha, E|Y|^\alpha < \infty$ , a characteristic function-based divergence measure may be defined as

$$D(X, Y; \alpha) = \int_{\mathbb{R}^d} |\phi_x(t) - \phi_y(t)|^2 \left( \frac{2\pi^{d/2}\Gamma(1-\alpha/2)}{\alpha^{2\alpha}\Gamma((d+\alpha)/2)} |t|^{d+\alpha} \right)^{-1} dt \tag{16}$$

then we may employ an alternative divergence measure based on Euclidean distances, defined in [22]

$$\mathcal{E}(X, Y; \alpha) = 2E|X - Y|^\alpha - E|X - X'|^\alpha - E|Y - Y'|^\alpha \tag{17}$$

An empirical divergence measure analogous to previous equation (17) may be defined as

$$\hat{\mathcal{E}}(X_n, Y_m; \alpha) = \frac{2}{mn} \sum_{i=1}^n \sum_{j=1}^m |X_i - Y_j|^\alpha - \binom{n}{2}^{-1} \sum_{1 \leq i < k \leq n} |X_i - X_k|^\alpha - \binom{m}{2}^{-1} \sum_{1 \leq j < k \leq m} |X_j - Y_k|^\alpha \tag{18}$$

Let

$$\hat{Q}(X_n, Y_m; \alpha) = \frac{mn}{m+n} \hat{\mathcal{E}}(X_n, Y_m; \alpha) \tag{19}$$

denote the scaled sample measure of divergence discussed above. This statistic leads to a consistent approach for estimating change point locations. Let  $Z_1, \dots, Z_T \in R^d$  be an independent sequence of observations and let  $l \leq \tau < k \leq T$  be constants. Now define the following sets,  $X_\tau = \{Z_1, Z_2, \dots, Z_\tau\}$  and  $Y_\tau = \{Z_{\tau+1}, Z_{\tau+2}, \dots, Z_k\}$ . A change point location  $\hat{\tau}$  is then estimated as

$$(\hat{\tau}, \hat{\kappa}) = \underset{(\tau, \kappa)}{\operatorname{argmax}} \hat{Q}(X_\tau, Y_\tau(\kappa); \alpha) \tag{20}$$

To estimate multiple change points, we iteratively apply the above technique as follows. Suppose that  $k - 1$  change points have been estimated at locations  $0 < \hat{\tau}_1 < \dots < \hat{\tau}_{k-1} < T$ . This partitions the observations into  $k$  clusters  $\hat{C}_1, \hat{C}_2, \dots, \hat{C}_k$ , such that  $\hat{C}_i = \{Z_{\hat{\tau}_{i-1}+1}, \dots, Z_{\hat{\tau}_i}\}$ , in which  $\hat{\tau}_0 = 0$  and  $\hat{\tau}_k = T$ . Given these clusters, we then apply the procedure for finding a single change point to the observations within each of the  $k$  clusters. Specifically, for the  $i$ th cluster  $\hat{C}_i$  denote a proposed change point location as  $\hat{\tau}(i)$  and the associated constant  $\hat{\kappa}(i)$ , as defined by equation (20). Now, let

$$i^* = \underset{i \in \{1, \dots, k\}}{\operatorname{argmax}} \hat{Q}(X_{\hat{\tau}(i)}, Y_{\hat{\tau}(i)}(\hat{\kappa}(i)); \alpha) \tag{21}$$

in which  $\hat{\tau}_k = \hat{\tau}(i^*)$  denotes the  $k$ th estimated change point, located within the cluster  $\hat{C}_{i^*}$ , and  $\hat{\kappa}_k = \hat{\kappa}(i^*)$  the corresponding constant. This iterative procedure has running time  $\mathcal{O}(kT^2)$ , in which  $k$  is the unknown number of change points.

### Experimental Setup

The measurements were performed in one of the close circuit boundary layer wind tunnels of our laboratory (UIDET-LaCLyFA) at the Aeronautics Department Engineering College, at the National University of La Plata, which has a test section 1 m high and 1.4 m wide. The model was a small wing with a chord length of 45 cm (C) and a wingspan of 80 cm (main wing length), built with a NACA 4412 airfoil. A flow control device (Gurney



mini-flap) with a length  $h = 2\% C$  was added, located at the trailing edge (TE) of the airfoil at an angle of  $90^\circ$  to the chord axis, a second arrangement located at a distance of  $8\%C$  from the TE, and a third version rotating up to  $30^\circ$ . The airfoil was submitted to a flow with an angle of attack of  $0^\circ$  (incidence chord angle relative to the free stream direction) to obtain a Reynolds number of 300,000 for the tests.

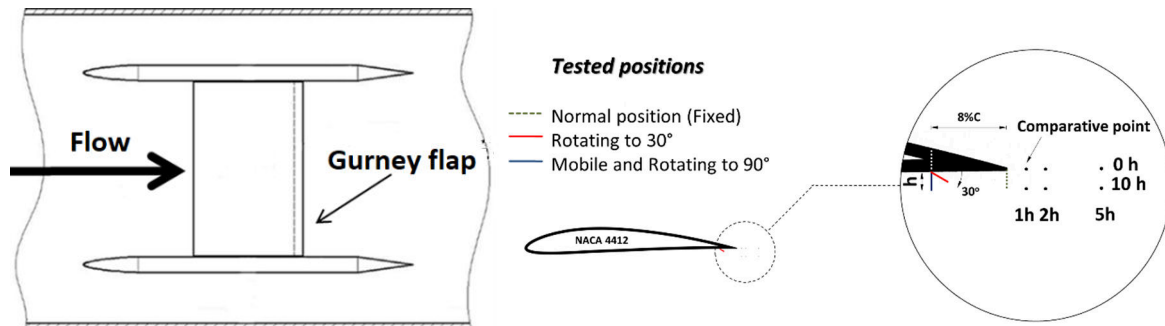


Figure 1: Wind tunnel setup and trailing edge measurement points.



Figure 2: Wind tunnel setup image.

The flow velocities components were measured using a constant temperature hot-wire anemometer and a double wire sensor. Signal acquisition was at 4,000 Hz, with a low-pass filter of 2,000 Hz and 8,192 samples. The measurements presented correspond to a point in the wake generated by the profile at a distance of 1h downstream from the trailing edge at the height of the chord, with the passive flow control device at the TE (Gurney mini-flap).

With the knowledge of the flow field generated by the presence of this device and the knowledge that it generates periodic vortex structures, periodic counter-rotating vortices (see Figure 3), previous work [23] evaluated the possibility of using this methodology to detect the expected wake events. These events were identified by applying the wavelet transform to the signal. Wavelets are localized in both space and frequency; therefore, the wavelet transform analyzes a signal locally in the frequency domain and in space or time [3]. The characteristic frequency localization in time of the wavelet transform provides a great opportunity to discover the positions of singularities and discontinuities in a signal, which is not possible with ordinary Fourier analysis [4]. The results of this methodology and the change point models were compared.



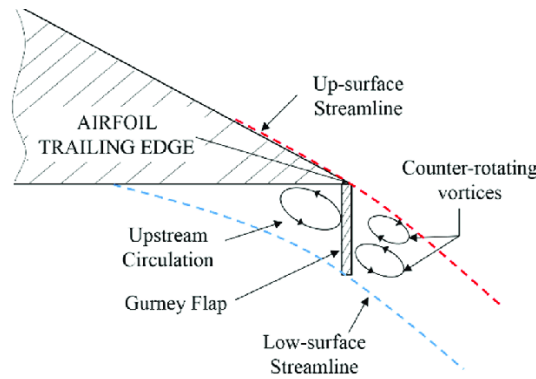


Figure 3: Counterrotating vortices scheme downstream the Gurney mini-flap and numeric simulations [24]

To compare, we present the result analysis found in the calculations for the vertical component of the velocity ( $v$ ) of the analyzed signal. Figure 4 presents the wavelet map applying wavelet transform to the signal using a Mexican Hat wave type (Ricker wavelet), in which a maximum can be traced in the signal [4]. Hence, the occurrence of a turbulent periodic event associated with one of the counter-rotating vortices that are released downstream of the device is observed.

In Figure 4, the value corresponding to the scales ordinate is defined with the following expression:

$$Scale = \frac{\ln(\Delta t)}{\ln(10)} \tag{22}$$

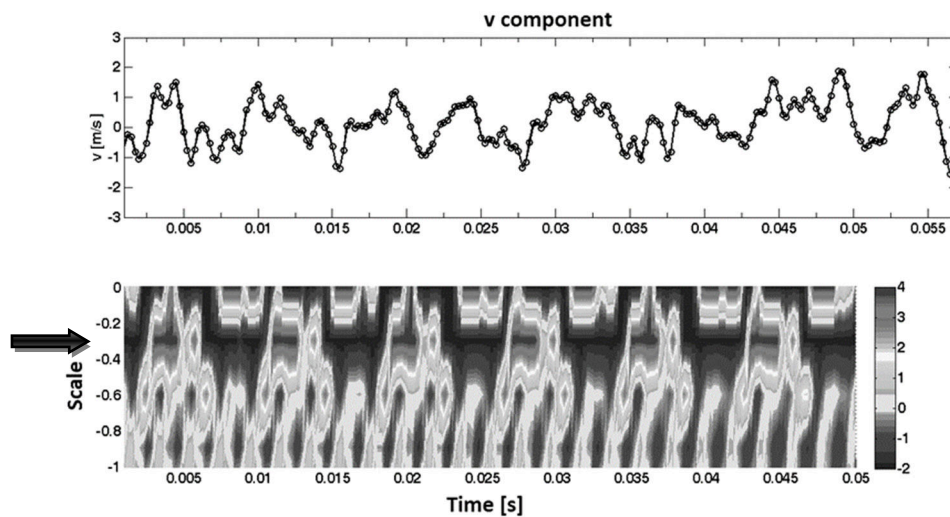


Figure 4: Wavelets map and time series of  $v$  component velocity fluctuations for the first 0,05 seconds (Arrow indicates vortex scale: -0.3) [23]

**Results & Discussion**

Based on the measurements made and the implementation of the different CPM models, a comparison of the results between the models was made. It was found that, in general, the different models used are very accurate in detecting the expected events obtained by the methods usually used. As mentioned earlier, these models allow the determination of changes in a signal for which no probability distribution is known, so they can be applied to any flow case to evaluate the possible events contained therein. As can be seen in Table 1, the comparison of the results does not show a preference for any of the models. However, in this table, as in previous works, the model CPM-KS is the one with the best comparison results. This may be due to the fact that this first analysed case, corresponding to the mini-flap in TE and fixed, and as shown in Figure 3, is the case in which the predominance



of counter-rotating vortices that periodically and alternately detach from the device with a characteristic frequency is clear.

As shown in Table 1, only for the CUSUM and CPM-JM models are there cases in which differences of more than 9% are found compared to the wavelet determination. Smaller differences are found for the other models. However, lower difference values are found than for the other models, so it is not considered that the CUSUM and CPM-JM models should be discarded to proceed with the evaluation of the other configurations and application conditions of the device.

It was also found that the CPM-MW, CPM -CvM and CPM-JM models do not detect some of the changes predicted by the application of the wavelet transform. Nevertheless, all the models shown in this work are considered useful for evaluating turbulent events under different conditions.

The evaluation of the results of the model for mobile, rotating and more distant from the TE cases will be processed and analysed in the second part of this work. However, we believe that this review of the most suitable models for evaluation will primarily allow us to determine and analyse the behaviour of the downstream fluid dynamic field of the different variants of the flow control device studied.

**Table 1:** Change point detection with different models for the static case in the TE.

Wavelet	CUSUM		Non parametric distributions CPM models								
			CPM MW		CPM-KS		CPM-CvM		CPM-JM		
Change [s]	Change [s]	Relative Difference [%]	Change [s]	Relative Difference [%]	Change [s]	Relative Difference [%]	Change [s]	Relative Difference [%]	Change [s]	Relative Difference [%]	p-value
0.00225	0.00250	11.11	0.00225	0	0.00225	0	0.00225	0	0.00250	11.11	0.002
0.00425	0.00475	11.76	0.00450	5.88	0.0045	5.88	0.0045	5.88	0.00500	17.65	0.002
0.00825	0.00900	9.09	0.00875	6.06	0.00875	6.06	0.00875	6.06	0.00900	9.09	0.002
0.01225	0.01225	0	0.01175	4.08	0.01175	4.08	0.01175	4.08	0.01200	2.04	0.004
0.01575	0.01575	0	0.01550	1.59	0.01550	1.59	0.01550	1.59	0.01725	9.52	0.002
0.02025	0.02000	1.23	0.01975	2.47	0.02000	1.23	0.01975	2.47	0.02000	1.23	0.002
0.02375	0.02200	7.37	0.02175	8.42	0.02150	9.47	0.02175	8.42	----	----	----
0.02550	0.02475	2.94	0.02450	3.92	0.02450	3.92	0.02450	3.92	0.02475	2.94	0.012
0.02825	0.02675	5.31	0.02800	0.88	0.02800	0.88	0.02800	0.88	0.02950	4.42	0.006
0.03350	0.02925	12.69	0.03350	0	0.03350	0	0.03350	0	0.03375	0.75	0.002
0.03625	0.03375	6.90	0.03775	4.14	0.03775	4.14	0.03775	4.14	0.03775	4.14	0.002
0.04025	0.03775	6.21	0.04050	0.62	0.04050	0.62	0.04050	0.62	0.04050	0.62	0.002
0.04425	0.04075	7.91	0.04300	2.82	0.04300	2.82	0.04300	2.82	0.04375	1.13	0.006
0.04825	0.04400	8.81	---	---	0.04800	0.52	----	----	0.05025	4.14	0.686

**Conclusions**

The studies performed allowed us to compare the results obtained by applying different stochastic models based on change-point models for non-parametric distributions and detecting changes in a temporal signal. In our case, a fluctuating velocity signal of the turbulent flow field at a point. Detection of periodic events in the turbulent flow generated by a flow control device installed near TE is achieved. Table 1 shows a summary of the results of the detection of events in the signal using the different models compared to the detections by applying the wavelet transform. It can be seen that the implemented CUSUM, Mann-Whitney, Kolmogorov-Smirnov, Cramer von Mises and James-Matteson models show a good correlation of the results with those of the wavelet transform.

It is believed that the method is acceptable for the analysis of fluctuating velocity data and allows the detection of events in a turbulent flow signal. It is therefore intended to continue the implementation of these methods to analyse the application of the instrument in the various remaining test configurations.

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