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Effects on light propagating in an electromagnetized vacuum, as predicted by a particular class of scalar–tensor theory of gravitation

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Abstract
The effect of static electromagnetic fields on the propagation of light is analyzed in the context of a particular class of scalar–tensor gravitational theories. It is found that for appropriate field configurations and light polarization, anomalous amplitude variations of the light as it propagates in either a magnetized or an electrified vacuum are strong enough to be detectable in relatively simple laboratory experiments.

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(Some figures may appear in colour only in the online journal)

1. Introduction

Scalar–tensor (ST) gravitational theories are the most firm candidates for extensions of General Relativity (GR). A great part of their interest comes from the fact that they are induced naturally in the reduction to four dimensions of string and Kaluza–Klein (KK) models [1, 2], resulting mostly in the form of a Brans–Dicke (BD) type of ST theory [3], often involving also non-minimal coupling to matter, leading to the so-called fifth force [4]. It is also interesting that ST theories are shown to be mathematically equivalent to theories with action depending non-linearly on the Ricci scalar, the so-called $f(R)$ theories [5]. Finally, ST theories are possibly the simplest extension of GR that could accommodate cosmological issues as inflation and universe-expansion acceleration, as well as possible space–time variation of fundamental constants [6]. On the other hand, observational and experimental evidence puts strong limits to the observable effects of a possible scalar field. For example, in the case of a massless scalar the BD parameter ω is constrained by precise Solar-system experiments to be a large number ($\omega > 4 \times 10^4$) [7]. In this way, ST gravity phenomenology appears to be very similar to that of GR, thus putting strong limits to possible experimental verifications. There is however a very interesting extension of ST gravity put forward by Mbelek and Lachièze-Rey (MLR) [8], which could allow electromagnetic (EM) fields to modify the space–time metric far more strongly than predicted by GR and standard ST theories. The theory was applied in cosmological [9] and galactic [10] contexts, and in [8] it was used to explain the discordancy in the measurements of Newton gravitational constant as due to the effect of the Earth's magnetic field. In [11] it was also shown that a ST theory of the MLR type can explain the unusual forces on asymmetric resonant cavities recently reported [12]. The key new element of the MRL theory is an additional, external scalar field ψ, minimally coupled to gravity, which is introduced in order to obtain bounded lower limits of the reduced action of a KK theory. The postulated reduced action for ψ includes a self-interaction potential $U(\psi)$ and a source J, the latter including contributions from the matter and EM fields, and from the internal scalar ϕ. It is the EM contribution to J which is responsible for a possible strong effect, relative to GR, of the EM fields on the metric. An additional interesting feature of the MLR theory, which was briefly discussed in [11], is that the matter contribution to J could allow a BD parameter ω close to one, and still fit the constraints determined by Solar-system experiments and observations.

2. The MLR Scalar–tensor theory

We will consider a MLR class of ST theory, whose action is given by (SI units are used)

$$\begin{aligned} S = &-\frac{c^3}{16\pi G_0}\int\sqrt{-g}\phi R\,\mathrm{d}\Omega + \frac{c^3}{16\pi G_0}\int\sqrt{-g}\frac{\omega(\phi)}{\phi}\nabla^\nu\phi\nabla_\nu\phi\,\mathrm{d}\Omega \\ &+\frac{c^3}{16\pi G_0}\int\sqrt{-g}\phi\left[\frac{1}{2}\nabla^\nu\psi\nabla_\nu\psi - U(\psi) - J\psi\right]\mathrm{d}\Omega \\ &-\frac{\varepsilon_0 c}{4}\int\sqrt{-g}\lambda(\phi)F_{\mu\nu}F^{\mu\nu}\,\mathrm{d}\Omega - \frac{1}{c}\int\sqrt{-g}j^\nu A_\nu\,\mathrm{d}\Omega + \frac{1}{c}\int\mathcal{L}_{\mathrm{mat}}\,\mathrm{d}\Omega. \end{aligned} \tag{1}$$

In (1) the internal, non-dimensional scalar field is ϕ, while the external scalar field is ψ. These fields have vacuum expectation values (VEV) $\phi_0 = 1$ and ψ_0, respectively. G_0 represents Newton gravitational constant, c is the velocity of light in vacuum, and ε_0 is the vacuum permittivity. $\mathcal{L}_{\mathrm{mat}}$ is the Lagrangian density of matter. The other symbols are also conventional, R is the Ricci scalar, and g the determinant of the metric tensor $g_{\mu\nu}$. The BD parameter $\omega(\phi)$ is considered a function of ϕ, as it usually results in the reduction to four dimensions of multidimensional theories [2]. The function $\lambda(\phi)$ in the term of the action of the EM field is of the type appearing in Bekenstein's theory and other effective theories [9]. The EM tensor is $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$, given in terms of the EM quadri-vector A_ν, with sources given by the quadri-current j^ν. U and J are, respectively, the potential and source of the field ψ. The source J contains contributions from the matter, EM field and the scalar ϕ. The model for J proposed in [8] is

$$J = \beta_{\mathrm{mat}}(\psi,\phi)\frac{8\pi G_0}{c^4}T^{\mathrm{mat}} + \beta_{\mathrm{EM}}(\psi,\phi)\frac{4\pi G_0\varepsilon_0}{c^2}F_{\mu\nu}F^{\mu\nu}, \tag{2}$$

where T^{mat} is the trace of the energy–momentum tensor of matter,

$$T^{\mathrm{mat}}_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\mathcal{L}_{\mathrm{mat}}}{\delta g^{\mu\nu}}.$$

We have not included a term proportional to the energy tensor of the scalar ϕ, present in the original proposal of MLR, because in the weak-field (WF) limit to be considered it only amounts to a rescaling of the β coefficients in (2) [11]. These coefficients are in principle unknown functions of the scalars, but in the WF approximation they only contribute through the values of their first-order derivatives at the VEV ϕ_0 and ψ_0, thus appearing as adjustable constants.

Variation of (1) with respect to $g^{\mu\nu}$ results in ($T^{\mathrm{EM}}_{\mu\nu}$ is the usual EM energy tensor, and $T^{\phi}_{\mu\nu}$ the energy tensor associated to the scalar ϕ)

$$\phi\left(R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}\right)=\frac{8\pi G_0}{c^4}\left[\lambda\left(\phi\right)T^{\mathrm{EM}}_{\mu\nu}+T^{\mathrm{mat}}_{\mu\nu}\right]+T^{\phi}_{\mu\nu}$$
$$+\frac{\phi}{2}\left(\nabla_\mu\psi\nabla_\nu\psi-\frac{1}{2}\nabla^\gamma\psi\nabla_\gamma\psi g_{\mu\nu}\right)+\frac{\phi}{2}\left(U+J\psi\right)g_{\mu\nu}. \tag{3}$$

Variation with respect to ϕ gives

$$\phi R+2\omega\nabla^\nu\nabla_\nu\phi=\left(\frac{\omega}{\phi}-\frac{\mathrm{d}\omega}{\mathrm{d}\phi}\right)\nabla^\nu\phi\nabla_\nu\phi-\frac{4\pi G_0\varepsilon_0}{c^2}\phi\frac{\mathrm{d}\lambda}{\mathrm{d}\phi}F_{\mu\nu}F^{\mu\nu}$$
$$-\frac{\partial J}{\partial\phi}\psi\phi+\phi\left[\frac{1}{2}\nabla^\nu\psi\nabla_\nu\psi-U\left(\psi\right)-J\psi\right],$$

which can be rewritten, using the contraction of (3) with $g^{\mu\nu}$ to replace R, as

$$\left(2\omega+3\right)\nabla^\nu\nabla_\nu\phi=-\frac{\mathrm{d}\omega}{\mathrm{d}\phi}\nabla^\nu\phi\nabla_\nu\phi-\frac{4\pi G_0\varepsilon_0}{c^2}\phi\frac{\mathrm{d}\lambda}{\mathrm{d}\phi}F_{\mu\nu}F^{\mu\nu}+\frac{8\pi G_0}{c^4}T^{\mathrm{mat}}$$
$$+\phi\left[\frac{1}{2}\nabla^\nu\psi\nabla_\nu\psi-U\left(\psi\right)-J\psi\right]-\frac{\partial J}{\partial\phi}\psi\phi, \tag{4}$$

where it was used that $T^{\mathrm{EM}}=T^{\mathrm{EM}}_{\mu\nu}g^{\mu\nu}=0$.

The non-homogeneous Maxwell equations are obtained by varying (1) with respect to A_ν,

$$\nabla_\mu\left\{\lambda\left(\phi\right)F^{\mu\nu}\right\}=\mu_0 j^\nu, \tag{5}$$

with μ_0 the vacuum permeability.

Finally, the variation with respect to ψ results in

$$\nabla^\nu\nabla_\nu\psi+\frac{1}{\phi}\nabla^\nu\psi\nabla_\nu\phi=-\frac{\partial U}{\partial\psi}-J-\frac{\partial J}{\partial\psi}\psi+\frac{1}{\phi}\frac{8\pi G_0}{c^4}T^{\mathrm{mat}}. \tag{6}$$

Having included G_0, it is understood that ϕ takes values around its VEV $\phi_0=1$. The scalar ψ is also dimensionless and of VEV ψ_0.

These equations can be approximated in the WF limit keeping only the lowest significant order in the perturbations $h_{\mu\nu}$ of the metric $g_{\mu\nu}$ about the Minkowski metric $\eta_{\mu\nu}$, with signature $(1,-1,-1,-1)$, and of the scalar fields about their VEV ϕ_0 and ψ_0

$$-\eta^{\gamma\delta}\partial_{\gamma\delta}\overline{h}_{\mu\nu}=2(\partial_{\mu\nu}\phi-\eta^{\gamma\delta}\partial_{\gamma\delta}\phi\eta_{\mu\nu}), \tag{7}$$

$$\partial_\gamma\overline{h}^\gamma_\nu=0, \tag{8}$$

$$\left(2\omega_0+3\right)\eta^{\gamma\delta}\partial_{\gamma\delta}\phi=-\left.\frac{\partial J}{\partial\phi}\right|_{\phi_0,\psi_0}\psi_0, \tag{9}$$

$$\partial_\nu F^{\mu\nu}=-\mu_0[1-\lambda_0'(\phi-\phi_0)]j^\mu-F^{\mu\nu}\partial_\nu(\lambda_0'\phi-\overline{h}/2) \tag{10}$$

$$\eta^{\gamma\delta}\partial_{\gamma\delta}\psi=-\left.\frac{\partial J}{\partial\psi}\right|_{\phi_0,\psi_0}\psi_0, \tag{11}$$

where $\omega_0=\omega\left(\phi_0\right)$, $\lambda_0'\equiv\mathrm{d}\lambda/\mathrm{d}\phi|_{\phi_0}$, and

$$\overline{h}_{\mu\nu}\equiv h_{\mu\nu}-\tfrac{1}{2}h\eta_{\mu\nu},$$

with $\overline{h}=\eta^{\mu\nu}\overline{h}_{\mu\nu}$. In these equations only the EM sources were included, as we assume that the effect of the near matter on the metric is also weak, and can thus be included, do to the linearity of the WF equations for $\overline{h}_{\mu\nu}$, as an additional term in $\overline{h}_{\mu\nu}$, determined from its own independent equations, and considered also as a perturbation to a local Minkowski metric.

As done in [9] and [10], the condition of recovering GR–Maxwell equations when the scalar fields are not excited requires that the β coefficients in (2) and the potential $U(\psi)$ be all zero when evaluated at the VEV ϕ_0 and ψ_0, and also that $\lambda(\phi_0) = 1$.

With these considerations, using the expression of the source J given in (2), with only the EM terms, we finally obtain the complete set of equations for the EM field (making explicit the electric and magnetic field vectors **E** and **B**, respectively)

$$\Box\Theta = \varkappa(B^2 - E^2/c^2), \tag{12a}$$

$$\nabla\cdot\mathbf{E} = \frac{\widetilde{\rho}}{\varepsilon_0} - \nabla\Theta\cdot\mathbf{E}, \tag{12b}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}, \quad \nabla\cdot\mathbf{B} = 0, \tag{12c}$$

$$\nabla\times\mathbf{B} = \mu_0\widetilde{\mathbf{j}} + \frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t} + \frac{1}{c^2}\frac{\partial\Theta}{\partial t}\mathbf{E} - \nabla\Theta\times\mathbf{B}, \tag{12d}$$

where the auxiliary field Θ is defined as $\Theta \equiv \lambda_0'\phi - \overline{h}/2$, and the EM sources were redefined as $\widetilde{j}^\mu \equiv [1 - \lambda_0'(\phi - \phi_0)]j^\mu$. The constant $\varkappa$ is given by

$$\varkappa = -\frac{8\pi G_0\varepsilon_0\left(\lambda_0' - 3\right)}{(2\omega_0 + 3)\,c^2}\psi_0\left.\frac{\partial\beta_{\mathrm{EM}}}{\partial\phi}\right|_{\phi_0,\psi_0}. \tag{13}$$

According to [8], except for the factor $\left(\lambda_0' - 3\right)$, the constant (13) can be determined in order to fit the values of G measured at different places on Earth, and has thus a value

$$\varkappa \simeq 5\left(\lambda_0' - 3\right)\times 10^{-8}\frac{A^2}{N^2}. \tag{14}$$

Incidentally, for the particular KK theory proposed in [8] $\lambda_0' = 3$, so that $\varkappa = 0$. We anyway will consider a non-zero $\varkappa$ since different KK theories can have $\lambda_0' \neq 3$, and also to study some of the effects predicted by the system (12) with $\varkappa \neq 0$ in order to compare them with those predicted by an alternative reformulation of the theory, to be presented in the following section, in which the first equation of the system (12) is modified, and which predicts observable effects even in the case $\varkappa = 0$.

To study the propagation of EM waves we consider the case of a vacuum with uniform and static electric and magnetic fields $\mathbf{E}_0$ and $\mathbf{B}_0$, so that one can linearize the system (12) in the perturbations as

$$\nabla\cdot\delta\mathbf{E} = -\nabla\delta\Theta\cdot\mathbf{E}_0 - \nabla\Theta_0\cdot\delta\mathbf{E}, \tag{15a}$$

$$\nabla\times\delta\mathbf{E} = -\frac{\partial\delta\mathbf{B}}{\partial t}, \quad \nabla\cdot\delta\mathbf{B} = 0, \tag{15b}$$

$$\nabla\times\delta\mathbf{B} = \frac{1}{c^2}\frac{\partial\delta\mathbf{E}}{\partial t} + \frac{1}{c^2}\frac{\partial\delta\Theta}{\partial t}\mathbf{E}_0 - \nabla\delta\Theta\times\mathbf{B}_0 - \nabla\Theta_0\times\delta\mathbf{B}, \tag{15c}$$

$$\nabla^2\Theta_0 = -\varkappa\left(B_0^2 - E_0^2/c^2\right), \tag{15d}$$

$$\Box\delta\Theta = 2\varkappa(\mathbf{B}_0\cdot\delta\mathbf{B} - \mathbf{E}_0\cdot\delta\mathbf{E}/c^2). \tag{15e}$$

Starting with this system we consider now different simple configurations.

2.1. Case $\mathbf{E}_0 = 0$

For the case without zero-order electric field, $\mathbf{E}_0 = 0$, and perturbations $\delta\mathbf{E}$, $\delta\mathbf{B}$, $\delta\Theta \sim \exp \mathrm{i}\,(\mathbf{k}\cdot\mathbf{x} - \varpi t)$ one has from (15)

$$\mathbf{k}\cdot\delta\mathbf{E} = \mathrm{i}\nabla\Theta_0\cdot\delta\mathbf{E}, \tag{16a}$$

$$\mathbf{k}\times\delta\mathbf{E} = \varpi\delta\mathbf{B}, \tag{16b}$$

$$\mathbf{k}\times\delta\mathbf{B} = -\frac{\varpi}{c^2}\delta\mathbf{E} - \mathbf{k}\times\mathbf{B}_0\delta\Theta + \mathrm{i}\nabla\Theta_0\times\delta\mathbf{B}, \tag{16c}$$

$$(k^2 - \varpi^2/c^2)\delta\Theta = 2\varkappa\mathbf{B}_0\cdot\delta\mathbf{B}. \tag{16d}$$

For propagation along the magnetic field, $\mathbf{B}_0 \parallel \mathbf{k}$, with $\nabla\Theta_0 \perp \mathbf{k}$, one has

$$\delta E_{\shortparallel} = \mathrm{i}\nabla\Theta_0\cdot\delta\mathbf{E}_\perp/k, \tag{17a}$$

$$\delta\mathbf{E}_\perp = -\frac{\varpi}{k^2}\mathbf{k}\times\delta\mathbf{B}, \tag{17b}$$

$$\mathbf{k}\times\delta\mathbf{B} = -\frac{\varpi}{c^2}\left(\delta\mathbf{E}_\perp + \delta E_{\shortparallel}\frac{\mathbf{k}}{k}\right) + \mathrm{i}\nabla\Theta_0\times\delta\mathbf{B}, \tag{17c}$$

replacement of the first two equations in the last results in

$$\left(1 - \frac{\varpi^2}{k^2c^2}\right)\mathbf{k}\times\delta\mathbf{B} + \mathrm{i}\left[\nabla\Theta_0\times\delta\mathbf{B} + \frac{\varpi^2}{k^4c^2}\nabla\Theta_0\cdot(\mathbf{k}\times\delta\mathbf{B})\,\mathbf{k}\right] = 0. \tag{18}$$

Projection of equation (18) in vector components parallel and perpendicular to $\mathbf{k}$, results in a homogeneous, linear system for the components of $\delta\mathbf{B}$, which has non-zero solutions only if

$$\varpi^2 = k^2c^2,$$

the usual dispersion relation for EM waves in vacuum. For the kind of propagation considered we thus have the standard plane EM wave for $\delta\mathbf{E}_\perp$ and $\delta\mathbf{B}$, with only the addition of a longitudinal component of amplitude

$$\delta E_{\shortparallel} = \mathrm{i}\nabla\Theta_0\cdot\delta\mathbf{E}_\perp/k.$$

We consider now propagation perpendicular to the zero-order field. Taking $\mathbf{E}_0 = 0$, $\mathbf{B}_0 = B_0\mathbf{e}_z$, $\nabla\Theta_0 = a\mathbf{e}_x$, $\mathbf{k} = k_x\mathbf{e}_x + k_y\mathbf{e}_y$, the general system (15) can be reduced to

$$(\varpi^2 - k^2c^2)\delta B_z + \mathrm{i}ak_xc^2\delta B_z = k^2c^2B_0\delta\Theta, \tag{19a}$$

$$(\varpi^2 - k^2c^2)\delta B_x - \mathrm{i}ak_yc^2\delta B_y = 0, \tag{19b}$$

$$(\varpi^2 - k^2c^2)\delta B_y + \mathrm{i}ak_xc^2\delta B_y = 0, \tag{19c}$$

$$(k^2c^2 - \varpi^2)\delta\Theta = 2\varkappa c^2B_0\delta B_z. \tag{19d}$$

In the case $\delta B_z = 0$, this system has non-trivial solution only for the dispersion relation

$$\varpi^2 - k^2c^2 + \mathrm{i}ak_xc^2 = 0,$$

from which, $\varpi = kc + \mathrm{i}\gamma$, with $(\cos\theta = k_x/k)$

$$\gamma = -\frac{ac\cos\theta}{2}. \tag{20}$$

In the case $\delta B_z \neq 0$, one also has $\varpi = kc + i\gamma$, with

$$\gamma = -\frac{1}{2}\left[\pm\sqrt{2\varkappa c^2 B_0^2 + \left(\frac{ac\cos\theta}{2}\right)^2} + \frac{ac\cos\theta}{2}\right]. \tag{21}$$

Using the equation for Θ_0 in (15) one can estimate that

$$a = |\nabla\Theta_0| \sim \varkappa B_0^2 L,$$

with L a characteristic length of the zero-order magnetic field distribution, so that

$$\frac{a}{\sqrt{\varkappa}B_0} \sim \sqrt{\varkappa}B_0 L, \tag{22}$$

which, using the value (14) is seen to be much smaller than one for reasonable field distributions ($B_0 \sim 1T$, $L \sim 1m$), so that the expression (21) can be approximated by

$$\gamma = \pm\sqrt{\frac{\varkappa}{2}}B_0 c, \tag{23}$$

a value much larger, according also to (22), than that corresponding to the case $\delta B_z = 0$, equation (20).

2.2. *Case* $\mathbf{B}_0 = 0$

For the case without zero-order magnetic field, $\mathbf{B}_0 = 0$, and perturbations $\delta\mathbf{E}$, $\delta\mathbf{B}$, $\delta\Theta \sim \exp i\,(\mathbf{k}\cdot\mathbf{x} - \varpi t)$ one has

$$\mathbf{k}\cdot\delta\mathbf{E} = -\mathbf{k}\cdot\mathbf{E}_0\delta\Theta + i\nabla\Theta_0\cdot\delta\mathbf{E}, \tag{24a}$$

$$\mathbf{k}\times\delta\mathbf{E} = \varpi\delta\mathbf{B}, \tag{24b}$$

$$\mathbf{k}\times\delta\mathbf{B} = -\frac{\varpi}{c^2}\delta\mathbf{E} - \frac{\varpi}{c^2}\mathbf{E}_0\delta\Theta + i\nabla\Theta_0\times\delta\mathbf{B}, \tag{24c}$$

$$(k^2c^2 - \varpi^2)\delta\Theta = -2\varkappa\mathbf{E}_0\cdot\delta\mathbf{E}. \tag{24d}$$

For propagation along the electric field, $\mathbf{E}_0 \parallel \mathbf{k}$, with $\nabla\Theta_0 \perp \mathbf{k}$, one then has

$$\delta\Theta = \frac{(i\nabla\Theta_0 - \mathbf{k})}{\mathbf{k}\cdot\mathbf{E}_0}\cdot\delta\mathbf{E}, \tag{25a}$$

$$(\mathbf{k} - i\nabla\Theta_0)\times(\mathbf{k}\times\delta\mathbf{E}) = -\frac{\varpi^2}{c^2}\delta\mathbf{E} + \frac{\varpi^2}{c^2}\mathbf{E}_0\frac{(\mathbf{k} - i\nabla\Theta_0)}{\mathbf{k}\cdot\mathbf{E}_0}\cdot\delta\mathbf{E}. \tag{25b}$$

The scalar product of equation (25*b*) with $\mathbf{k}$ leads to an identity satisfied by any arbitrary longitudinal component $\delta E_{\shortparallel}$, while the homogeneous, linear system for the components $\delta\mathbf{E}_\perp$ obtained from equation (25*b*) has non-zero solutions only if $\varpi^2 = k^2c^2$, thus resulting the standard plane EM wave relations for $\delta\mathbf{E}_\perp$ and $\delta\mathbf{B}$. On the other hand, the longitudinal component $\delta E_{\shortparallel}$ must be zero in order to satisfy equation (24*d*). In this way, the standard plane EM wave is the only solution in this case.

For propagation perpendicular to the zero-order field, one has now $\mathbf{B}_0 = 0$, $\mathbf{E}_0 = E_0\mathbf{e}_z$, $\nabla\Theta_0 = a\mathbf{e}_x$, $\mathbf{k} = k_x\mathbf{e}_x + k_y\mathbf{e}_y$, and the general system (15) can be reduced to

$$(\varpi^2 - k^2c^2)\delta B_z + iak_xc^2\delta B_z = 0, \tag{26a}$$

$$(\varpi^2 - k^2c^2)\delta B_x - iak_yc^2\delta B_y = \frac{2\varkappa\varpi^2E_0^2k_y}{k^2(\varpi^2 - k^2c^2)}(k_x\delta B_y - k_y\delta B_x), \tag{26b}$$

$$(\varpi^2 - k^2c^2)\delta B_y + iak_xc^2\delta B_y = -\frac{2\varkappa\varpi^2E_0^2k_x}{k^2(\varpi^2 - k^2c^2)}(k_x\delta B_y - k_y\delta B_x). \tag{26c}$$

If $\delta B_z \neq 0$, one has $\varpi = kc + \mathrm{i}\gamma$, with the same value (20) as in the case of $\mathbf{E}_0 = 0$. For the polarization $\delta B_z = 0$, one obtains, analogously to the case $\delta B_z \neq 0$ of the previous subsection, $\varpi = kc + \mathrm{i}\gamma$ with

$$\gamma = -\frac{1}{2}\left[\pm\sqrt{2\varkappa E_0^2 + \left(\frac{ac\cos\theta}{2}\right)^2} + \frac{ac\cos\theta}{2}\right], \tag{27}$$

which, proceeding as after equation (21), can be approximated by

$$\gamma = \pm\sqrt{\frac{\varkappa}{2}}E_0. \tag{28}$$

3. Experimental possibilities

From the previous section it is seen that the more noticeable effects are obtained for propagation perpendicular to the zero-order fields, with the appropriate polarization of the wave in each case. Moreover, comparing (28) with (23) it is clear that, from a practical point of view, magnetic fields are preferable. In any case, when the beam traverses a length ΔL, the relative variation of the amplitude A of any field is given by

$$\frac{\Delta A}{A} = \exp\left(\frac{\gamma\Delta L}{c}\right). \tag{29}$$

For the case of a magnetic field of 1 T, with $\delta B_z \neq 0$, one can thus estimate from (14) and (23)

$$\frac{\gamma}{c} \sim 10^{-4}\,\mathrm{m}^{-1}.$$

Although this effect is relatively large, there is the problem that both, growing and decreasing modes are always present, so that a wave entering the region with a static magnetic field results in a superposition of both modes, and so the variation of amplitude of the growing mode cancels with that of the decreasing mode at first order in $\gamma\Delta L/c$, and the effect is only observable at second order, much hindering the experiment.

There is however a further possibility. It was argued in [11] that, in order for the MLR theory to be consistent with the lack of strong gravitational effects due to the magnetic field of the Earth, the nonlinear terms in equations (4) and (6) should come into play. This does not mean that the WF approximation breaks down (second-order terms are still much smaller than first-order terms), but rather that the Laplacian terms in equations (9) and (11) are zero for this particular type of source, so that the equalities are satisfied by the higher order terms.

More explicitly, for the case of a static magnetic field outside its sources one can write $\mathbf{B} = \nabla\Psi$, with $\nabla^2\Psi = 0$, so that, from equations (4) and (6) for the static case, one has instead of equations (9) and (11),

$$(2\omega_0 + 3)\nabla^2\phi + \omega_0'\nabla\phi\cdot\nabla\phi - \tfrac{1}{2}\nabla\psi\cdot\nabla\psi = \chi_\phi\nabla\Psi\cdot\nabla\Psi, \tag{30a}$$

$$\nabla^2\psi + \nabla\phi\cdot\nabla\psi = \chi_\psi\nabla\Psi\cdot\nabla\Psi, \tag{30b}$$

where $\omega_0' \equiv (\mathrm{d}\omega/\mathrm{d}\phi)_{\phi_0}$, and

$$\chi_\phi \equiv \frac{8\pi G_0\varepsilon_0}{c^2}\psi_0\left.\frac{\partial\beta_{\mathrm{EM}}}{\partial\phi}\right|_{\phi_0,\psi_0},$$

$$\chi_\psi \equiv \frac{8\pi G_0\varepsilon_0}{c^2}\psi_0\left.\frac{\partial\beta_{\mathrm{EM}}}{\partial\psi}\right|_{\phi_0,\psi_0}.$$

These equations have the solutions $\nabla\phi \propto \nabla\psi \propto \nabla\Psi$, so that $\nabla^2\phi = \nabla^2\psi = 0$, thus nullifying the contribution of the vacuum magnetic field as a source of $\bar{h}$ (from which the gravitational force is derived in the WF limit). This solution for the case of the Earth's magnetic field is compatible with the proposal in [8], where the solution with $\nabla^2\phi \neq 0$ was used, on the condition that

$$\omega_0' \sim 2\omega_0 + 3. \tag{31}$$

In this way, from the contraction of equation (7) with $\eta^{\mu\nu}$, one sees that to a static magnetic field corresponds $\bar{h} = 0$, so that equation (15*d*) in the system (15) is replaced by

$$\nabla\Theta_0 = \lambda_0'\nabla\phi \equiv \Gamma\nabla\Psi = \Gamma\mathbf{B}_0,$$

where the constant Γ depends, other than on λ_0' , on χ_ϕ, χ_ψ and ω_0', and is directly determined from the system (30) with the condition $\nabla^2\phi = \nabla^2\psi = 0$. From the data in [8], together with the *assumption* $\chi_\psi \approx \chi_\phi$, and condition (31), one has

$$\Gamma \simeq \lambda_0'\sqrt{-\frac{8\pi G_0\varepsilon_0}{(2\omega_0+3)\,c^2}\psi_0\left.\frac{\partial\beta_{\rm EM}}{\partial\phi}\right|_{\phi_0,\psi_0}} \approx \lambda_0' \times 10^{-4}\frac{A}{N}. \tag{32}$$

In this case the system (15) can be written as (with $\mathbf{E}_0 = 0$)

$$\nabla\cdot\delta\mathbf{E} = -\Gamma\mathbf{B}_0\cdot\delta\mathbf{E}, \tag{33a}$$

$$\nabla\times\delta\mathbf{E} = -\frac{\partial\delta\mathbf{B}}{\partial t}, \quad \nabla\cdot\delta\mathbf{B} = 0, \tag{33b}$$

$$\nabla\times\delta\mathbf{B} = \frac{1}{c^2}\frac{\partial\delta\mathbf{E}}{\partial t} - \Gamma\mathbf{B}_0\times\delta\mathbf{B}, \tag{33c}$$

where the term with $\delta\Theta$ can be neglected as it is small, since from (15) one can estimate that

$$\left|\frac{\nabla\delta\Theta\times\mathbf{B}_0}{\nabla\Theta_0\times\delta\mathbf{B}}\right| \sim \Gamma B_0 L, \tag{34}$$

with L a characteristic length of spatial variation of the zero-order magnetic field. Note that the magnitude of the right-hand side of equation (34) is similar to that of equation (22).

Proceeding as before one has

$$\mathbf{k}\cdot\delta\mathbf{E} = \mathrm{i}\Gamma\mathbf{B}_0\cdot\delta\mathbf{E}, \tag{35a}$$

$$\mathbf{k}\times(\mathbf{k}\times\delta\mathbf{E}) = -\frac{\varpi}{c^2}\delta\mathbf{E} + \mathrm{i}\Gamma\mathbf{B}_0\times(\mathbf{k}\times\delta\mathbf{E})\,. \tag{35b}$$

For propagation perpendicular to $\mathbf{B}_0$ one thus obtains

$$\left(\frac{\varpi^2}{c^2} - k^2\right)\delta\mathbf{E}_\perp + \frac{\varpi^2}{kc^2}\delta E_\parallel\mathbf{k} = \mathrm{i}\Gamma B_0\delta E_\perp\cos\alpha\,\mathbf{k}, \tag{36a}$$

$$k\delta E_\parallel = \mathrm{i}\Gamma B_0\delta E_\perp\cos\alpha, \tag{36b}$$

where α is the angle between $\mathbf{B}_0$ and $\delta\mathbf{E}_\perp$. As a result the dispersion relation is that of a normal EM wave, $\varpi = kc$, and the only anomalous effect is the presence of a small longitudinal component of the electric field.

In the case of propagation along $\mathbf{B}_0$ one has instead

$$\left(\frac{\varpi^2}{c^2} - k^2\right)\delta\mathbf{E}_\perp = -\mathrm{i}\Gamma kB_0\delta\mathbf{E}_\perp, \tag{37a}$$

$$k\delta E_\parallel = \mathrm{i}\Gamma B_0\delta E_\parallel, \tag{37b}$$

so that $E_{\parallel} = 0$, and $\varpi = kc + \mathrm{i}\gamma$, with

$$\gamma = -\frac{\Gamma B_0 c}{2}. \tag{38}$$

This effect is similar in magnitude to that in (23) (for $\lambda_0' \sim 1$), but with the advantage that only one sign is possible, so that there are not coexisting growing and decaying modes, and the growth (or decay) could be observed at first order.

4. Table-top experiments with optical fibers

Due to the relevance of first-order effects in the propagation of EM waves, it seems plausible that the use of simple and readily available nowadays fiber optics would allow the verification of the theoretical results of the previous section. Especially, the last result of section 3 shows a direct method for measuring the additional amplitude change caused by the propagation of a single mode inside an ordinary polymer fiber. Given the significance of separating between alternative extensions of general relativistic theories for modern cosmology we propose that such an experiment is of great importance due to its simplicity.

Specifically, our proposal is to get a sufficiently large fiber appropriately coiled which, with existing materials, can be made to easily reach a kilometer of total distance for the propagating mode. By taking the logarithm of equation (29) as $10\log_{10}(\frac{\Delta A}{A})$, for the amplitude variation to be in dB units, we see that a magnetic field of 2 T would result in an amplitude variation of 1 dB in 1 km distance. It is possible to reduce such a distance by an order of magnitude only through a large magnetic field of about 10 T or more which can be produced in current NMR devices while use of superconducting elements could reach even higher values [13, 14]. Actually, recent reports from the NHMFL at Los Alamos claim a 100 T machine is already operational [15]. At the moment we will only assume the strongest existing rare earth magnets like Boron–Neodymium for a tabletop experiment where sufficiently high accuracy power meters are available. The central idea is to detect the difference between measurement on the fiber coil, with and without the B field. Present day power meters have a power resolution of about 0.1 dB and lower level near −95 dBm [16]. Fortunately, existing manufacturers may be able to provide bobbins totaling 25 km of fiber or more so that a measurement of 10–20 dB of additional amplitude variations is in principle possible.

With respect to the fiber coiling process, one has to take into account that any angles introduced to the fiber material introduce additional attenuation to any propagating mode. Technical data for existing fiber materials suggest that there should be a certain curvature with angles small enough not to cause severe damping during normal propagation. This can be achieved with a flattened coil frame like the one shown in figure 1.

As our scope is not in the engineering details of an actual experiment we only emphasize the main points where care must be taken using some simplified configurations. In the flattened fiber coil of figure 1 care must be taken so that on the upper path the fiber is parallel to the direction of the external magnetic field. The return path, though, must be outside the region of influence or else the amplitude variation effect will be cancelled and no difference will be measured. For this reason we also made the flattened electromagnets shown in such a way that the homogenized flux of the applied B field will only affect the upper part of the fiber's path.

It is also possible to make up an homogeneous magnetic field using Neodymium magnets in a special configuration known as a cylindrical 'Halbach array' [17]. Such arrays have been in use for a long time in magnetic trains, very fast brushless motors and similar electrical engineering applications. In such a case, the upper- or lower-part of the flattened fiber coil should be put inside the region of homogeneous B flux of a Halbach cylinder.

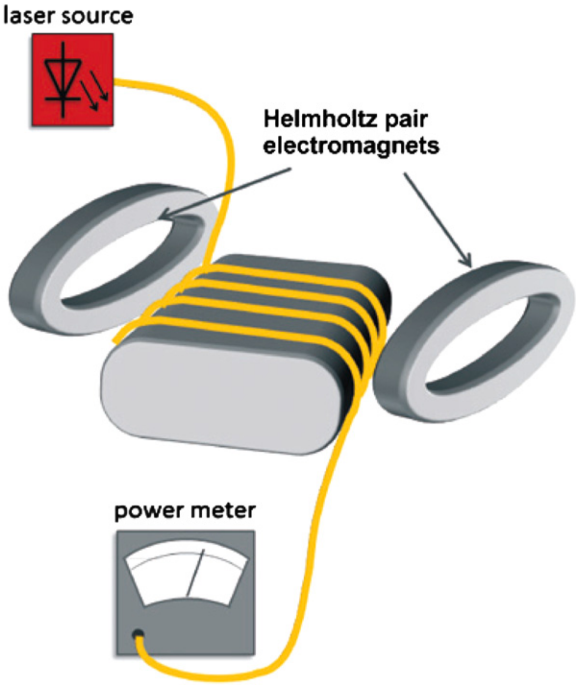

Figure 1. Proposed configuration of the optical fiber and Helmholtz coils.

5. Conclusions

We have here reported for the first time some new results on the possible gravitational influence on classical EM fields in a particular class of scalar–tensor extentions of General Relativity. We also used the linearized version of the perturbed Maxwell equations to analyze the propagation of ordinary modes. The analysis led us to conclude the possibility of easy, low cost experiments with fiber optics that would allow the verification of the said theories. We believe that the present state of cosmology with the recurring acute problems of inflation and initial conditions, the CMB anisotropy as well as the dark matter and dark energy, fully justifies the continuation of the present research in more areas where evidence can be accumulated experimentally, especially considering that the MLR theory considered has special relevance in cosmological and galactic contexts [9, 10].

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