HARVESTING ENERGY FROM FAT-TAIL RANDOM VIBRATIONS

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Abstract: We illustrate how applied and computational mathematics tools in the field of probability, statistics and stochastic processes are applied in the preliminary design stage of an expectedly typical 21th century (nanotech) industrial process: the fabrication of carpets, paints and networks able to harvest energy from random vibrations.

Keywords: *first, second, third*
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1 INTRODUCTION

The term "industrial mathematics" still evokes to many of us huge plants with large-scale processes as forges, steel lamination or batch chemical processes, the realm of finite-element-like methods. Nonetheless, for already quite a few years, the true motor of industrial development have been the semiconductor and pharmaceutical industries, which benefit from a more probabilistic (not necessarily quantum) mathematical approach. The arrival of nanotechnology has definitely sided the scale towards this kind of techniques. Here we illustrate the use of formal and computational stochastic techniques in the problem of harvesting energy from random mechanical vibrations, for its use in powering micro- and nanodevices.

Several approaches are currently practiced, the most common being electromagnetic induction, and either electrostatic or piezoelectric transduction. The last one exploits the linear relationship between an input strain and an output voltage (and reciprocally) appearing in some crystals, but that strain must be produced by some "springs" attached to a moving mass submitted to the mechanical vibrations. Now if the "springs" had linear response, we could only exploit a narrow band of the vibration spectrum, by the known phenomenon of resonance. Hence the first design stage of these systems is to choose the nonlinearity that best suits the vibrations statistics and spectrum. For fat-tail (supra-Gaussian and Lévy-like) fluctuations, we have recently shown the convenience of square well-like oscillators [1].

2 A MODEL NONLINEAR PIEZOELECTRIC HARVESTER

We consider a 1D anharmonic oscillator mediating between a source of mechanical vibrations and a piezoelectric transducer. This produces a voltage $V(t) = K_c\, x(t)$, that is fed onto a load circuit with resistance R and capacitance τ_p/R. In turn, the transducer reacts back on the oscillator with a force $K_v V(t)^2$. The oscillator (of mass m and damping constant γ) obeys a square well-like potential $U(x)$ [1] and the source of mechanical vibrations (to which it is coupled with strength σ) is regarded as stochastic, producing a *strongly colored* instantaneous force $\eta(t)$. The system is thus described by

$$m\ddot{x} = -U'(x) - \gamma\dot{x} - K_v\, V + \sigma\,\eta(t), \tag{1}$$

$$\dot{V} = K_c\,\dot{x} - \frac{1}{\tau_p}V. \tag{2}$$

Being $V^2(t)/R$ the instantaneous power delivered to the load resistance, the measure of performance is V_{rms} during the observation interval.

The potential $U(x)$ is chosen to have the form proposed by Woods and Saxon in the mid fifties, within the shell model of nuclear structure [2]

$$U(x) := -\frac{V_0}{1+\exp\left(\frac{|x|-R}{a}\right)}. \tag{3}$$

It looks as in the left frame of Fig. 1 and takes the values $-V_0/(1+e^{-R/a})$ for $|x| \to 0$, $-V_0/2$ for $|x| = R$ and zero for $|x| \to \infty$, becoming a square well of depth V_0 and width $2R$ for $a \to 0$. As a grows the potential walls become smoother, maintaining the value $-V_0/2$ at $|x| = R$. For a large enough, $U(x)$ resembles a harmonic potential for $R \gg a$ and $|x| > a$. It thus allows to monitor the deviation from the linear case by just varying parameter a, whereas avoiding unreal infinite walls as it occurs with polynomial potentials.

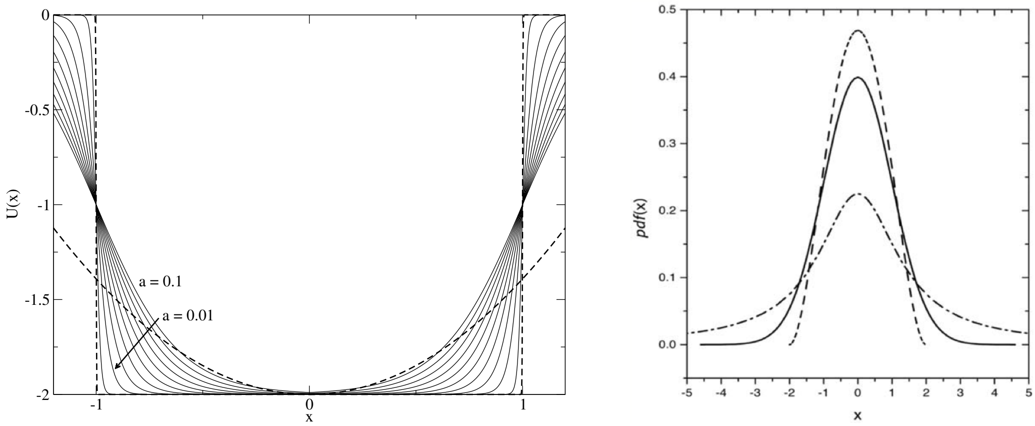

Figure 1: Left: Woods–Saxon potential; right: pdf of the Tsallis noise.

3 A MODEL OF FAT-TAIL RANDOM VIBRATIONS

In a former edition of this congress [3], we have introduced the Tsallis noise $\eta(t)$ and shown its usefulness for investigating non-Gaussian noise-induced effects. $\eta(t)$ is dynamically generated by

$$\dot{\eta} = -\frac{1}{\tau}\frac{d}{d\eta}V_q(\eta) + \frac{1}{\tau}\xi(t), \text{ with} \tag{4}$$

$$V_q(\eta) := \frac{1}{\tau(q-1)} \ln\left[1+\tau(q-1)\frac{\eta^2}{2}\right], \tag{5}$$

where $\xi(t)$ is Gaussian, centered, of variance 1 and white ($\langle\xi(t)\xi(t')\rangle = 2\,\delta(t-t')$).

For $q = 1$, $\eta(t)$ is an Ornstein–Uhlenbeck process with self-correlation time τ. For $q > 3$, its pdf is not normalizable. For $1 < q < 5/3$, $\eta(t)$ is a supra-Gaussian process, and for $q < 1$, its pdf has compact support (see right frame of Fig. 1). Our focus here are the supra-Gaussian ($1 < q < 5/3$) and Lévy-like ($5/3 < q < 3$) regimes; namely fat-tail distributions, which are commonly found in nature.

4 NUMERICAL RESULTS

Equations (1)–(5) have been integrated by Heun's method, and the efficiency of the energy harvesting process has been plotted in Fig. 2 as a function of q, up to its maximum value $q = 3$. These results spectacularly confirm our hypothesis that a square well-like oscillator better profits from the large and strongly self-correlated excursions of the environmental vibration process.

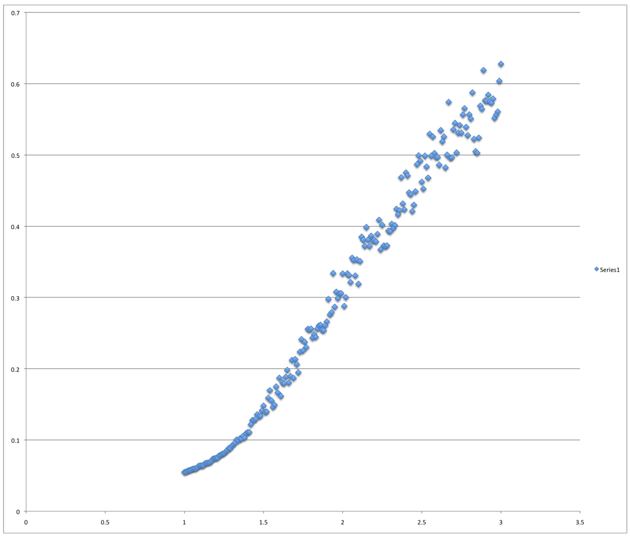

Figure 2: Efficiency of the energy harvesting process as a function of q, for $1 \geq q \geq 3$ (supra-Gaussian and Lévy-like regimes).

5 Analytical treatment

Also in a former edition of this congress [4], we have shown how to extract a Fokker–Planck description for a process like $x(t)$—driven in turn by the self-correlated and non-Gaussian process $\eta(t)$—that keeps however information on its spectral and statistical features. Here, we have the additional complication that the oscillator's inertia is not negligible, as usually assumed in Fokker-Planck treatments. The calculation is still under way, as well as another one involving a multiplicative quadratic noise.

6 An electronic analogy

In order to vividly illustrate the mechanism taking place, we have fed (through the audio output of a computer and using `Matlab`'s audio functions) a Zener diode, as a metaphor of a square well. The circuit is shown in the left frame of Fig. 3, whereas the succession of scope captures in the right frame of Fig. 3 and Fig. 4 illustrates how as q increases, surpassing the dashed line becomes more probable.

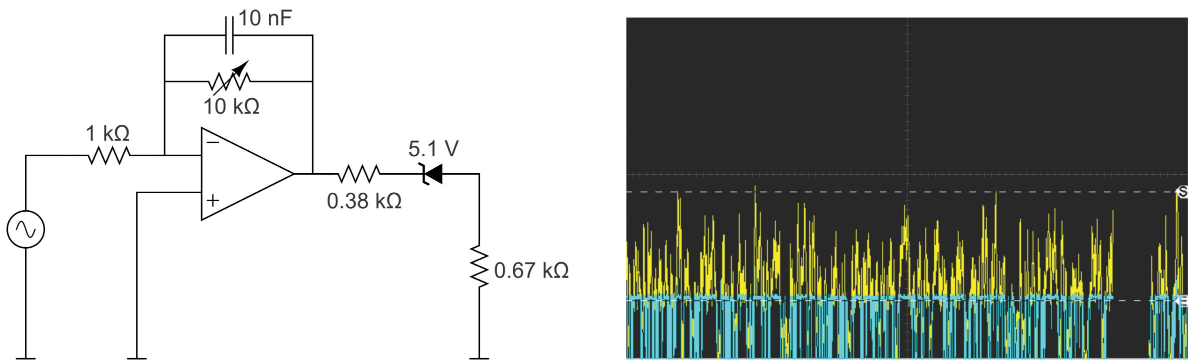

Figure 3: Left: circuit of the electronic analogy; right: scope capture for $q = 1.0$.

7 Conclusions

The typical industrial processes of the 21th century will heavily use stochastic methods. We have illustrated the use of analytical and numerical stochastic techniques in the preliminary design stage of the

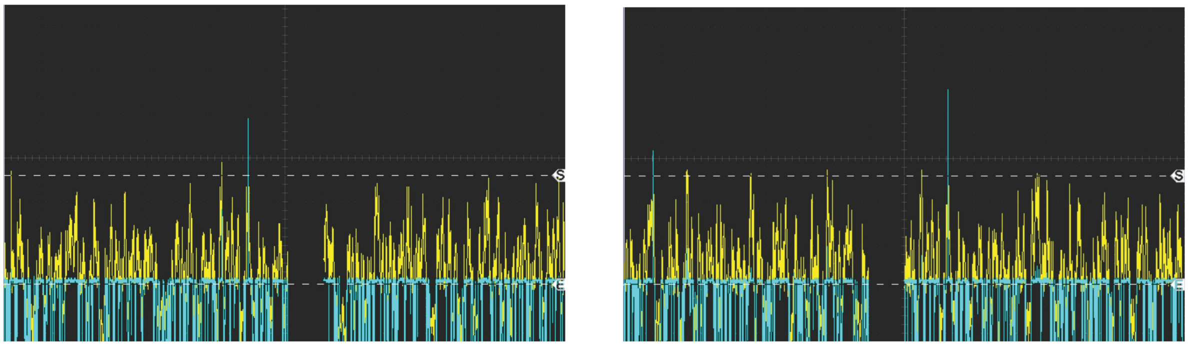

Figure 4: Left: scope capture for $q = 1.3$; right: scope capture for $q = 1.6$.

fabrication of carpets, paints and networks able to harvest energy from random vibrations. If oscillators with square well-like response can be fabricated at the nanoscale, they will optimally exploit the large self-correlated excursions of the environmental vibration process.

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