Optimization of an AC-DC Transfer Step-Up Scheme

Lucas Di Lillo, Vivian Grünhut, Fernando Kornblit, and Héctor Laiz, *Senior Member, IEEE*

***Abstract*—A set of statistical tools is proposed for the optimization of an alternating current–direct current voltage or current transfer step-up scheme. These tools are used to remove standards with level dependence and instabilities and to discard unusual measurements with the aim of minimizing the uncertainties. The method was applied to the new Instituto Nacional de Tecnologia Industrial current step-up.**

***Index Terms*—AC–direct current (dc) difference, alternating current (ac), thermal converter (TC).**

I. Introduction

The ALTERNATING current (ac)–direct current (dc) current transfer scale at the Instituto Nacional de Tecnologia Industrial (INTI) is realized using the well-known step-up and step-down procedures. At 10 mA, five Physikalisch-Technische Bundesanstalt thin-film multijunction thermal converters (PMJTCs), with one of them having a 20-mA shunt, form the basis of the system [1]. At other current levels, standards are calibrated against the standards of the neighboring range, and the assumption made was that the ac–dc transfer difference of each standard remains independent of the input current, from the reduced current at which it is calibrated against the neighboring standard to its rated current. The high sensitivity of the PMJTC allows large steps in the step-up calibration and many intermediate steps. In the proposed scheme, we used two standards to jump from one range to the other. At the highest range of the leap, one of these standards is at its rated power, and the other one is at a quarter of it. This redundancy is necessary for the statistical tools that will be introduced. Fig. 1 shows the 10-, 25-, and 50-mA range. At each current level, a system of equations is obtained, which is solved using the least square method [2]. For instance, at 50 mA, we have

$$\begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \delta_{\mathrm{PMJTC-4+SH-1}} \\ \delta_{\mathrm{PMJTC-1+SH-3}} \\ \delta_{\mathrm{PMJTC-3+SH-4}} \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ \delta'_{\mathrm{PMJTC-4+SH-1}} \\ \delta'_{\mathrm{PMJTC-1+SH-3}} \end{bmatrix} \tag{1}$$

where $\delta'_{\mathrm{PMJTC-4+SH-1}}$ and $\delta'_{\mathrm{PMJTC-1+SH-3}}$ are the values obtained for these standards in the previous step and a, b, and c are the measured values. This equation could be simply written

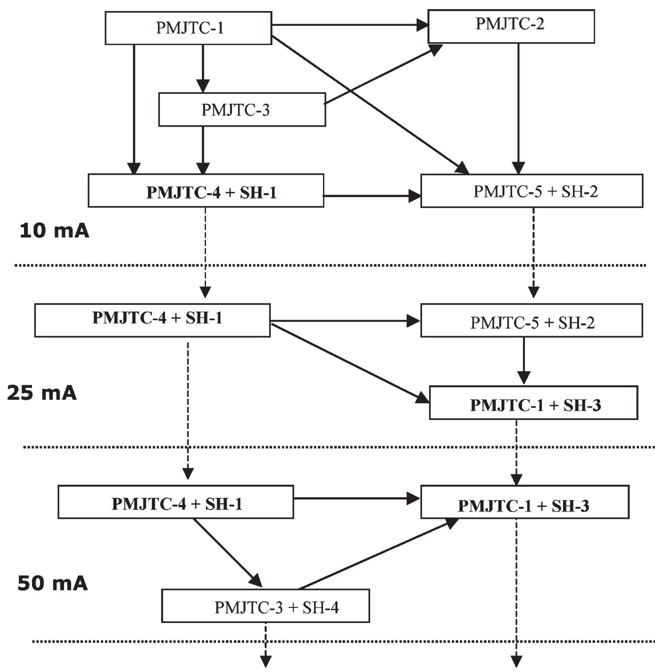

Fig. 1. — 10-, 25-, and 50-mA steps.

as $\mathbf{A} \cdot \delta = \mathbf{B}$ and, using the least square method, δ can be estimated by

$$\hat{\delta} = (\mathbf{A}^T \cdot \mathbf{A})^{-1} \cdot \mathbf{A}^T \cdot \mathbf{B} = \mathbf{C} \cdot \mathbf{B}. \tag{2}$$

The residual vector $\mathbf{B} - \mathbf{A} \cdot \hat{\delta}$ represents the lack of fit of the model (1), which can be quantified from the residual variance as

$$\hat{\sigma}^2_{\mathrm{step}} = SS/df \tag{3}$$

where the residual sum of squares SS is calculated as

$$SS = \|\mathbf{B} - \mathbf{A} \cdot \hat{\delta}\|^2 \tag{4}$$

and df, the number of degrees of freedom used to estimate the residual variance, is calculated as the number of rows minus the number of columns of $\mathbf{A}$.

The associated uncertainties of $\hat{\delta}$ are calculated as the square root of the diagonal terms of the covariance matrix

$$\mathrm{cov}(\hat{\delta}) = \mathbf{C} \cdot \mathrm{cov}(\mathbf{B}) \cdot \mathbf{C}^{-1} \tag{5}$$

where $\mathrm{cov}(\mathbf{B})$ is the covariance matrix of $\mathbf{B}$. The diagonal terms of $\mathrm{cov}(\mathbf{B})$ are

$$\mathrm{var}(i) = u_A^2(i) + u_C^2(i) + u_M^2(i), \quad i = a, b, c \tag{6}$$

$$\mathrm{var}(i) = u^2(\hat{\delta}_{\mathrm{pi}}), \quad i = d, e \tag{7}$$

Manuscript received July 11, 2006; revised November 1, 2006.
The authors are with Instituto Nacional de Tecnologia Industrial, San Martin B1650WAB, Argentina (e-mail: laiz@inti.gov.ar).
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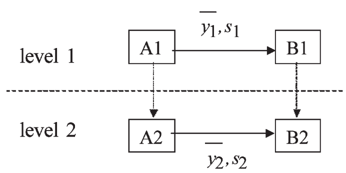

Fig. 2. Step with two transfers as reference.

where $u_A(i)$ is the Type-A standard uncertainty associated to the repeatability of each bilateral comparison, $u_C(i)$ is the Type-B standard uncertainty associated to the comparison system, $u_M(i)$ is the standard uncertainty of the scheme of the comparison, which is calculated as the residual standard deviation of the least squares fit, i.e.,

$$u_M(i) = \sqrt{\hat{\sigma}^2_{step}} \tag{8}$$

and $u(\hat{\delta}_{\text{pi}})$ is the standard uncertainty of the ac–dc transfer difference of the standards arising from the previous measurement step [2]. The standard uncertainty of $\hat{\delta}_i$ is

$$u(\hat{\delta}_i) = \sqrt{\text{cov}(\hat{\delta})_{ii}}. \tag{9}$$

To use this method, we require thermal converters (TCs) with stable level-independent ac–dc differences. The level independence and stability are measured using different strategies. However, the fulfillment of both requirements is usually checked, taking advantage of the operator experience. Three statistical tests are proposed to check these requirements quantitatively and objectively.

II. Statistical Tests

A. *Testing the Level Dependence*

We propose a method to test the current level independence of the ac–dc difference of a standard. This test should be applied to all the pairs of standards used to step up (or down) from one range to another. Let us suppose that two of these standards, A and B, are used to step from current level 1 to current level 2; let us call their values at level 1 A1 and B1, and the corresponding values at level 2 A2 and B2 (Fig. 2). Both standards have been compared n times at both levels, and the averages $\overline{y}_1$ and $\overline{y}_2$ and standard deviations s_1 and s_2 of the measured differences have been calculated.

If both standards are equally affected by the level change, the averages will be similar at both levels. If not, we conclude that one of them has a larger level coefficient than the other. As both PMJTCs are of similar design and technology but are being used at quite different powers, i.e., at different heater temperatures, if a change is measured between $\overline{y}_1$ and $\overline{y}_2$, it can be assigned to the higher powered PMJTC. To test if the difference between $\overline{y}_1$ and $\overline{y}_2$ is statistically significant, a simple two-sample *t*-test for mean differences [3] is applied, based on the statistic

$$T = (\overline{y_1} - \overline{y_2}) \Big/ \sqrt{\frac{s_1^2 + s_2^2}{n}}. \tag{10}$$

A value of $|T|$ greater than a critical value $t_{2n-2;\alpha/2}$ (which depends on a previously stated type of risk α) leads to the conclusion that the difference between the averages is statistically significant, and therefore, the level dependence must be considered an uncertainty component assigned to the highest powered standard. Otherwise, it can be assumed that the difference is negligible or attributed to random errors, which are included in the least square calculation. For example, with the data from the 10- to the 25-mA step at 100 kHz, with $n = 12$, $\overline{y_1} = 10.91$, $\overline{y_2} = 13.32$, $s_1 = 0.79$, and $s_2 = 0.22$, we obtain $|T| = 2.939$. For a commonly used value of $\alpha = 0.05$, $t_{2n-2\alpha/2} = 2.074$ [3]. Thus, $|T| > t_{2n-2\alpha/2}$, and we conclude that the difference between the averages is statistically significant. Therefore, we correct the value of the higher powered TC by $(\overline{y}_1 - \overline{y}_2)$. The standard uncertainty associated to this correction is estimated by

$$u_{\text{ld}} = \sqrt{\frac{s_1^2 + s_2^2}{n}} \tag{11}$$

and incorporated to the uncertainty of the highest powered standard.

B. *Testing the Consistency Between Pairs of Standards*

To verify that the values assigned to both reference standards A and B at the same step are consistent, we propose to compare the values given to each standard by solving the equations of each step twice, according to the following procedure.

- First, the step is solved, considering both standards providing a link condition to the previous step [i.e., the last two rows in (1)]. Let us call δ_{AB} the output vector of the step.
- Then, one of the link conditions is eliminated from the model (deleting one of the two last lines in the design matrix A). Therefore, other values will be obtained for all the transfers, i.e., δ_B.
- Finally, both estimations are compared by means of parameter E_n [4], which is given by

$$E_n(i) = \frac{\delta_{AB}(i) - \delta_B(i)}{u\left(\delta_{AB}(i) - \delta_B(i)\right)}. \tag{12}$$

The standard uncertainty in the denominator must be calculated, suppressing all the correlations between $\delta_{AB}(i)$ and $\delta_B(i)$. Values of $E_n(i)$ greater than 2 for any i express lack of consistency.

For example, at 50 mA and 100 kHz, we obtain $\delta_{AB} = \{14.07; 19.15; 11.18\}$ for the transfers $\text{PMJTC} - 4 + \text{SH} - 1$, $\text{PMJTC} - 1 + \text{SH} - 3$, and $\text{PMJTC} - 3 + \text{SH} - 4$, respectively, while $\delta_B = \{15.65; 21.73; 13.26\}$. Therefore, $|\delta_{AB} - \delta_B| = \{1.57; 2.58; 2.09\}$, $u(\delta_{AB} - \delta_B) = \{10.15; 10.14; 10.14\}$, and $E_n = \{0.15; 0.25; 0.21\}$. As E_n is always smaller that 2, we conclude that there is consistency between the two reference standards at this step.

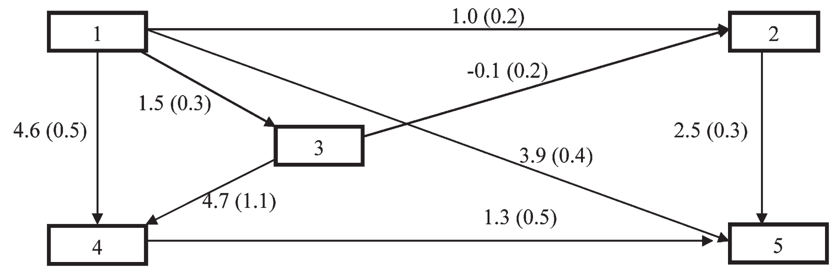

Fig. 3. Comparison of five standards at 10 mA and 10 kHz. The values near the arrows are the measured values with the standard deviation of the measurements (in brackets).

C. Testing the Stability of a Standard

A statistical method to test the inner consistency of each step is proposed. At each step, the ac–dc differences δ are estimated by means of the least squares method, using (2). If a transfer is not stable enough during the time when the measurements are performed, the least squares fit will be poor, and some components of the residual vector $\mathbf{B} - \mathbf{A} \cdot \hat{\delta}$ will be too far from their expected value, which is 0. Thus, the residual sum of squares (4) and the estimated residual variance (3) will be too high.

To test the step consistency, the residual sum of squares of the complete model must be compared with the residual sum of the squares obtained from a reduced model. If one of the transfers is suspected of being unstable, it is discarded from the scheme. A reduced matrix $\mathbf{A_r}$ is obtained from $\mathbf{A}$, by eliminating the column corresponding to the discarded standard, and all the rows related to the measurements in which this standard was involved. In addition, a reduced vector of observations $\mathbf{B_r}$ is obtained from $\mathbf{B}$, and new estimations for the nondiscarded transfers can be calculated.

Following the same procedure as for the full model, the reduced sum of squares SS_r, the reduced degrees of freedom df_r, and the reduced residual variance $\hat{\sigma}_r^2$ are obtained. Then, an F statistic can be calculated as

$$F = \frac{(SS - SS_r)/(df - df_r)}{SS_r/df_r}. \tag{13}$$

It can be shown that under the stability hypothesis (that is, all the residuals in the step come from the same Gaussian distribution), F is distributed according to a Fisher–Snedecor distribution with $df - df_r$ degrees of freedom in the numerator and df_r degrees of freedom in the denominator [5]. The statistical properties of F depend only on df and df_r, not on the typical standard deviations of the step.

A type-one risk α (the risk of detecting a nonexisting instability) is previously stated. So, if the calculated value of F is greater than the tabulated critical value $f_{df-dfr,dfr,\alpha}$, we could conclude that the model consistency is significantly weaker for the full model than for the reduced one. Then, the lack of stability of the discarded transfer can be considered significant.

The power of the F-test, that is, the probability of detecting an actual lack of consistency, was evaluated by Monte Carlo simulations. For example, the F-test was applied for the step in Fig. 3 where transfer 4 was evaluated as possibly unstable. In order to estimate the F-test power in this case, $M = 5000$ simulated copies of the step values were obtained by assigning random numbers to each of the pair-comparisons. Such random numbers were generated from Gaussian distributions, where the common mean value is 0 and the common standard deviation is σ_0 (i.e., combination of the actual Type-A sources of uncertainty associated to the lack of fit and repeatability of the step). First, one of the comparisons in which the suspected transfer participates was contaminated in all the simulations by adding a constant value c to the simulated measurements. Next, the F-test was applied for each simulation, and the F-test power was estimated as the relation between the cases in which the contamination was detected and the total number of simulations. Fig. 4 shows the estimated power for a test level $\alpha = 0.1$ and for different values of c between 0 and $5 \cdot \sigma_0$.

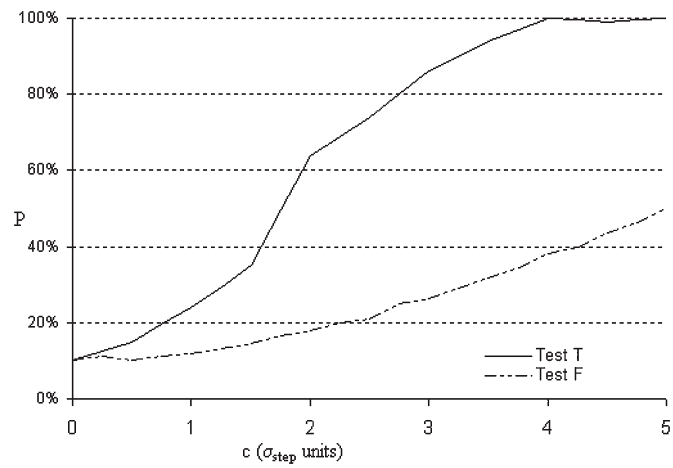

Fig. 4. Power of the F and T tests. P is the percentage of detection, and c is the contamination in σ units. In both cases, the type of risk α is 0.1.

It is clear from Fig. 4 that the power of the F-test is not good. For instance, the test for $\alpha = 0.10$ detects a $3 \cdot \sigma_0$ contamination, with a probability of 0.26. Therefore, we proposed a modification of the test to increase its power, based on the Monte Carlo simulations of the measurements.

Each one of the comparisons presented in the step is repeated by the generation of N random numbers with Gaussian distributions centered around the average of an actual measurement. Those generations are performed with a common standard deviation $\sigma_\sim$. Once the simulations are done, simulated versions of the F statistics $\{F_1, \ldots, F_N\}$ are computed by means of the same procedure for the F test. These copies of F could be used for statistical calculations. The mathematical properties of F are quite hard to work with. For instance, it has no

TABLE I
UNCERTAINTIES AT 25 mA, 0.6 A, AND 5 A (IN $\mu A \cdot A^{-1}$)

Standards	*f* = 1 kHz	*f* = 20 kHz	*f* = 100 kHz
0.25	1.2	2.1	10.1
0.6 A	1.5	2.6	10.3
5 A	1.6	2.9	11.0

TABLE II
CORRELATION COEFFICIENTS FROM THE 0.6- AND 5-A STANDARDS AND THE 10-mA BASIC STANDARDS

Standards	*f* = 1 kHz	*f* = 20 kHz	*f* = 100 kHz
0.6 A	0.70	0.80	0.97
5 A	0.64	0.68	0.96

finite expected value for $df_r \leq 3$, and it has no finite standard deviation for $df_r \leq 4$ [3]. Simulations show that $\log(F_i)$ has a probability distribution that is close to a Gaussian distribution to allow the proper use of a t-test based on the following statistic:

$$T = \frac{\overline{\log(F)} - \mu_{\log(F)}}{s\left(\log(F)\right)/\sqrt{N}} \tag{14}$$

where $s(\log(F))$ is the sample standard deviation of $\log(F_1), \ldots, \log(F_N)$, and $\mu_{log}(F)$ is the theoretical expected value of $\log(F_i)$, which can be calculated as

$$\mu_{\log(F)} = \int \log(x) \cdot f_{\nu 1, \nu 2}(x) \cdot dx \tag{15}$$

where $f_{\nu 1}, \nu_2(x)$ is the density function of the F distribution [3].

The distribution of T can be approximated by a t distribution, with $N-1$ degrees of freedom. Thus, the condition to conclude instability or lack of consistency in the step is

$$T > t_{N-1,\alpha}. \tag{16}$$

The power of the T-test was evaluated for the same case and in a similar way as for the F test. The results are shown in Fig. 4. Note that for each contamination, $M \cdot N$ simulations were needed: M values simulating the measurement results must be generated, and for each one of these, N simulated F_is must be obtained.

Regarding the simulation, the following caution must be taken into account: The simulated results come from a probability distribution centered on the actual measured value. So, there are positive correlations among them and, therefore, among the $\log(F_i)$. Thus, the denominator in (14) underestimates the standard deviation of the numerator, and the values of T will be inappropriate. This problem can be avoided by considering a value of $\sigma_\sim$ that is quite larger than σ_0. In that case, the correlations among $\log(F_i)$ are negligible in practice. The simulation shown in Fig. 4 was carried out, with $\sigma_\sim = 10 \cdot \sigma_0$. It can be seen that the percentage of false detection (contamination $= 0$) was a bit higher than its expected value α. This bias could be minimized by increasing $\sigma_\sim$ even more, but then the number of needed simulations N would also have to be larger to obtain good results.

Finally, it must be noted that both tests could be applied to only one transfer suspected of being unstable or to all the transfers in the step, discarding them one by one. If any of the ratios F are greater than the critical value, the corresponding standard should be replaced. In this case, the power of both tests will increase, because if some comparison between the two transfers A and B is affected by errors, we have two chances of detecting it: when the "suspected" transfer is A and again when it is B. One application of the test with $N = 50\,000$ simulations takes approximately 1 min in a regular personal computer.

Fig. 3 shows the results of the measurements at 10 mA and 1 kHz. If we apply (14) to each standard suspected of being unstable, we get $T_1 = 0.55$, $T_2 = 0.25$, $T_3 = 0.29$, $T_4 = 2.2$, and $T_5 = 0.88$. If we choose a type-I risk of $\alpha = 0.1$, we get critical $t_{4,0.1} = 1.64$ [3]. As $T_4 > 1.64$, we conclude that transfer 4 is unstable and should be replaced.

III. RESULT

The uncertainty assigned to all the transfers in the step-up process as well as the correlations among them can be easily estimated by the Monte Carlo simulation. The results of the analytical (GUM) and Monte Carlo approaches are quite similar. Table I depicts the values for 25 mA, 0.6 A, and 2 A. Table II shows the correlation coefficients between the standards at 10 mA, 600 mA, and 5 A at different frequencies.

IV. CONCLUSION

The systematic monitoring of a step-up procedure is a complex operation and difficult to manage rationally. The proposed statistical method provides an objective tool to assess the quality of a step-up scheme. Unstable or level-dependent standards can be discarded with a base on objective numbers. Besides, the uncertainty components can be calculated directly from the measurements.

The method has proved useful for the reduction of the uncertainties due to the step-up procedure. From 25 mA to 5 A, only 1 $\mu A \cdot A^{-1}$ is added at 1 and 100 kHz.

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Lucas Di Lillo was born in Argentina in 1970. He received the Licenciatura degree in physics from the University of Buenos Aires, Buenos Aires, Argentina, in 2001.

In 1994, he joined the Instituto Nacional de Tecnología Industrial, San Martin, Argentina, where he was engaged in dielectric measurements. In 2000, he joined the AC–DC Transfer Laboratory, and now, he is in charge of the National Standards of AC–DC Transfer and Power.

Vivian Grünhut, photograph and biography not available at the time of publication.

Fernando Kornblit received the Licenciatura degree in mathematics from the University of Buenos Aires, Buenos Aires, Argentina, in 1983 and the "Expert in Industrial Quality" degree from the Universidad de San Martin, Argentina, in 2001.

He joined the Instituto Nacional de Tecnología Industrial (INTI), San Martin, in 1988, working in fields related to metrology, uncertainty evaluations, and applied statistics. He is also with the Universidad de Buenos Aires and the Universidad de San Martín on a part-time basis.

Héctor Laiz (M'87–SM'03) was born in Argentina in 1964. He received the electrical engineer degree from the University of Buenos Aires, Buenos Aires, Argentina, in 1988 and the Doctorate degree from the Technische Universität Braunschweig, Braunschweig, Germany, in 1999.

In 1989, he joined Instituto Nacional de Tecnología Industrial (INTI), San Martin, Argentina, where he is engaged in the development of power and ac–dc transfer standards. He has been a Guest Scientist at the Physikalisch-Technische Bundesanstalt, Braunschweig, and the National Institute of Standards and Technologies, Gaithersburg, MD, working on highly accurate low-frequency ac measurements. Since June 1999, he has been the Deputy Director of Metrology at INTI, dealing with scientific and legal metrology.