**TABLE 1 Comparison of the Results of Using the Present Method, the Feed-Forward Backpropagation Algorithm, and Experimental Resonant Frequency**

| Patch No. | Experimental Resonant Frequency (GHz) | Resonant Frequency (GHz) (Present Method) | Resonant Frequency (GHz) (Backpropagation Algorithm) |
|---|---|---|---|
| 1 | 5.820 | 5.82515 | 5.79649 |
| 2 | 4.660 | 4.67353 | 4.52594 |
| 3 | 3.980 | 3.95329 | 3.93908 |
| 4 | 3.900 | 3.87665 | 3.91498 |
| 5 | 2.980 | 3.02413 | 2.99279 |

cally, the hidden-node redundancy into consideration. In a gradient-descent feed-forward backpropagation method, there is a chance that the solution may be trapped by local minima, which does not happen in the case of the GA. Hence, the present algorithm of training ANNs by using a GA takes advantage of the population-to-population GA search. Hidden-node redundancy has been handled by taking different values of the steepness of activation function. Applying two-point crossover or uniform crossover and replacing simple the GA by a micro-GA, the computational time may be reduced. Further improvement can be done by considering architecture optimization. This model can be used as a CAD model for antenna design.

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# A NEW CONDITION TO IDENTIFY ISOTROPIC DIELECTRIC-MAGNETIC MATERIALS DISPLAYING NEGATIVE PHASE VELOCITY

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**ABSTRACT:** *The derivation of a new condition for characterizing isotropic dielectric-magnetic materials exhibiting negative phase velocity, and the equivalence of that condition with previously derived conditions, are presented.* © 2004 Wiley Periodicals, Inc. Microwave Opt Technol Lett 41: 315–316, 2004; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.20127

**Key words:** *negative phase velocity; power flow*

## 1. INTRODUCTION

Nondissipative media with both simultaneously negative permittivity and permeability were first investigated by Veselago [1] in 1968. These media support electromagnetic-wave propagation, in which the phase velocity is antiparallel to the direction of energy flow, and other unusual electromagnetic effects, such as the reversal of the Doppler effect and Cerenkov radiation. After the publication of Veselago's work, more than three decades went by before the actual realization of artificial materials that are effectively isotropic, homogeneous, and possess negative real permittivity and permeability in some frequency range [2, 3].

A general condition for the constitutive parameters of an isotropic dielectric-magnetic medium to have phase velocity directed oppositely to the power flow, when dissipation is included in the analysis, was reported about two years ago [4]. Most importantly, according to that condition, the real parts of both the permittivity and the permeability need not be both negative.

In this paper, we derive a new condition for characterizing isotropic materials with negative phase velocity. Although this new condition looks very different from its predecessor [4], we also show the equivalence between both conditions.

## 2. THE NEW CONDITION

Let us consider a linear isotropic dielectric-magnetic medium characterized by complex-valued relative permittivity and relative permeability scalars $\varepsilon = \varepsilon_r + i\varepsilon_i$ and $\mu = \mu_r + i\mu_i$. An $\exp(-i\omega t)$ time dependence is implicit, with $\omega$ as the angular frequency.

The wave equation gives the square of the complex-valued refractive index $n = n_r + in_i$ as

$$n^2 = \varepsilon\mu \Rightarrow n_r^2 - n_i^2 + 2in_rn_i = \mu_r\varepsilon_r - \mu_i\varepsilon_i + i(\mu_i\varepsilon_r + \mu_r\varepsilon_i). \tag{1}$$

The sign of $n_r$ gives the phase-velocity direction, whereas the sign of the real part of $n/\mu$, given by

$$\mathrm{Re}\left(\frac{n}{\mu}\right) = n_r\mu_r + n_i\mu_i, \qquad (2)$$

gives the direction of power flow [4]. Therefore, for this medium to have negative phase velocity and positive power flow, the following conditions should hold simultaneously:

$$n_r < 0, \qquad (3)$$

$$n_r\mu_r + n_i\mu_i > 0, \qquad (4)$$

Eq. (1) yields the following biquadratic equation:

$$n_r^4 - (\mu_r\varepsilon_r - \mu_i\varepsilon_i)n_r^2 - \frac{1}{4}(\mu_i\varepsilon_r + \mu_r\varepsilon_i) = 0. \qquad (5)$$

This equation has only two real-valued solutions for $n_r$, namely,

$$n_r = \pm\left(\frac{|\varepsilon||\mu| + \mu_r\varepsilon_r - \mu_i\varepsilon_i}{2}\right)^{1/2}. \qquad (6)$$

Noting that the relation

$$\mu_i\varepsilon_i - \mu_r\varepsilon_r < \sqrt{(\mu_i\varepsilon_i - \mu_r\varepsilon_r)^2 + (\mu_i\varepsilon_r + \mu_r\varepsilon_i)^2} \qquad (7)$$

holds for all values of the constitutive parameters $\varepsilon_{r,i}$ and $\mu_{r,i}$, we see that

$$0 < |\varepsilon||\mu| + \mu_r\varepsilon_r - \mu_i\varepsilon_i; \qquad (8)$$

hence, the right side of Eq. (6) is always positive.

As the negative square root must be chosen in Eq. (6) in order to satisfy the condition (3), therefore

$$n_r = -\frac{1}{\sqrt{2}}(|\varepsilon||\mu| + \mu_r\varepsilon_r - \mu_i\varepsilon_i)^{1/2}, \qquad (9)$$

$$n_i = -\frac{1}{\sqrt{2}}\frac{\mu_i\varepsilon_r + \mu_r\varepsilon_i}{(|\varepsilon||\mu| + \mu_r\varepsilon_r - \mu_i\varepsilon_i)^{1/2}}. \qquad (10)$$

On using these expressions and Eq. (2) in condition (4), a condition for power flow and phase velocity in opposite directions is finally derived as follows:

$$\mu_r(|\varepsilon||\mu| + \mu_r\varepsilon_r - \mu_i\varepsilon_i)^{1/2} + \mu_i\frac{\mu_i\varepsilon_r + \mu_r\varepsilon_i}{(|\varepsilon||\mu| + \mu_r\varepsilon_r - \mu_i\varepsilon_i)^{1/2}} < 0. \qquad (11)$$

This condition can be rewritten in the very simple form

$$\varepsilon_r|\mu| + \mu_r|\varepsilon| < 0, \qquad (12)$$

which is the chief contribution of this paper.

## 3. EQUIVALENCE WITH PREVIOUSLY DERIVED CONDITION

The general condition for the phase velocity to be oppositely directed to the power flow, which was derived about two years ago [4], is as follows:

$$(|\varepsilon| - \varepsilon_r)(|\mu| - \mu_r) > \varepsilon_i\mu_i. \qquad (13)$$

Although it looks very different, this condition, which can be rewritten as

$$\varepsilon_r|\mu| + \mu_r|\varepsilon| < |\varepsilon||\mu| + \mu_r\varepsilon_r - \mu_i\varepsilon_i, \qquad (14)$$

is completely equivalent to the new condition (12).

Clearly, if (12) is satisfied, then, taking into account the validity of (8), (14) is also satisfied. To show that the reverse is also true, we start from (14) and assume that (12) does not hold. As the left side of the inequality (14) is nonnegative, squaring both sides does not change the sense of the inequality and we obtain

$$(\varepsilon_r|\mu| + \mu_r|\varepsilon|)^2 < (|\varepsilon||\mu| + \mu_r\varepsilon_r - \mu_i\varepsilon_i)^2. \qquad (15)$$

Simplification of this inequality leads to

$$\varepsilon_i\mu_i(|\varepsilon||\mu| + \mu_r\varepsilon_r - \mu_i\varepsilon_i) < 0. \qquad (16)$$

However, causality dictates that $\varepsilon_i \geq 0$ and $\mu_i \geq 0$; hence, we must conclude that

$$|\varepsilon||\mu| + \mu_r\varepsilon_r - \mu_i\varepsilon_i < 0, \qquad (17)$$

in contradiction with Eq. (8). Therefore, we must accept the validity of condition (12). This completes the demonstration of the equivalence between conditions (12) and (13).

## 4. CONCLUSION

We note in passing that both conditions (12) and (13) are also equivalent to the condition

$$\varepsilon_r\mu_i + \mu_r\varepsilon_i < 0, \qquad (18)$$

which was reported very recently [5]. This condition is due to R. Ruppin.

To conclude, in this work we have derived a simple new condition for the constitutive parameters of a linear isotropic dielectric-magnetic medium to have phase velocity that is opposite to the direction of power flow, and we have demonstrated its equivalence with previously derived conditions.

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