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Dephasing in matter-wave interferometry

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Abstract

We review different attempts to show the decoherence process in double-slit-like experiments both for charged particles (electrons) and for neutral particles with permanent dipole moments. Interference is studied when electrons or atomic systems are coupled to classical or quantum electromagnetic fields. The interaction between the particles and time-dependent fields induces a time-varying Aharonov phase. Averaging over the phase generates a suppression of fringe visibility in the interference pattern. We show that, for suitable experimental conditions, the loss of contrast for dipoles can be almost as large as the corresponding one for coherent electrons and therefore can be observed. We analyse different trajectories in order to show the dependence of the decoherence factor on the velocity of the particles.

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1. Introduction: decoherence and the quantum to classical transition

Quantum mechanics is one of the most successful theories in physics. It can be applied to describe the behaviour of solids, the structure and operation of DNA, and the properties of superfluids, just to mention a few practical applications. Despite all its virtues, quantum mechanics is still a controversial theory. Its description of the physics phenomena is often confronted with our perceptions of reality and, sometimes, leads to predictions that can be considered paradoxical. So, at the root of this ‘uneasiness’ with quantum theory lie the superposition principle ruling nature’s behaviour and the classical everyday world which seems to infringe this superposition principle.

Usually, classicality is identified with the macroscopic world. However, there are many examples in which we can realize that this association is not strictly true: Josephson junctions or nonclassical squeezed states with macroscopic numbers of photons. Therefore, we cannot systematically treat macroscopic systems as classical. Might classicality then be an acquired property of the system? How can we explain our perception of only one outcome out of the many possibilities that might be? Why do we not observe quantum interference effects between macroscopic distinguishable states? Why do we perceive a reality with different alternatives instead of a coherent superposition of alternatives?

Macroscopic quantum states are never isolated from their environments [1]. They are not closed quantum systems, and therefore they cannot behave according to the unitary quantum-mechanical rules. Consequently, these so often called 'classical' systems suffer a loss of quantum coherence that is absorbed by the environment. This *decoherence* destroys quantum interferences. For our everyday world, the time scale at which the quantum interferences are destroyed is so small that, in the end, the observer is able to perceive only one outcome. As far as we see, decoherence is the main process behind the quantum to classical transition. Formally, it is the dynamic suppression of the interference terms induced on subsystems due to the interaction with an environment.

Closed quantum systems are bound to have a unitary evolution in which the system's purity is preserved and the superposition principle can be applied. On the other hand, open quantum systems offer a different scenario. As they are in interaction with an environment (defined as any set of degrees of freedom coupled to the system which can entangle its states), a 'degradation' of pure states into mixtures takes place (sometimes, this degradation may be low). These mixture states will often turn out to be diagonal in the set of 'pointer states' [1] which are selected by the crucial help of the interaction Hamiltonian. They are stable subject to its action, i.e., the interaction between the system and the environment leaves them unperturbed. That is exactly what makes them a 'preferred' basis.

Let us take for example an interference experiment. The experiment starts by the preparation of two electron wavepackets in a coherent superposition, assuming that each of the charged particles follows a well-defined classical path (C_1 and C_2, respectively), as

$$\Psi(t=0) = (\varphi_1(x) + \varphi_2(x)) \otimes \chi_0(y), \tag{1}$$

where $\chi_0(\vec{y})$ represents the initial quantum state of the environment (whose set of coordinates is denoted by $\vec{y}$). Due to the interaction between the system and the environment, the total wavefunction at a later time t is

$$\psi(t) = \varphi_1(\vec{x},t) \otimes \chi_1(\vec{y},t) + \varphi_2(\vec{x},t) \otimes \chi_2(\vec{y},t). \tag{2}$$

It is easy to note that the electrons' states φ_1 and φ_2 became entangled with two different states of the environment. Therefore, the probability of finding a particle at a given position at time t (for example when the interference pattern is examined) is

$$\mathrm{Prob}(\vec{x},t) = |\varphi_1|^2 + |\varphi_2|^2 + 2\,\mathrm{Re}\left(\varphi_1\varphi_2^* \int \mathrm{d}^3y\, \chi_1^*(\vec{y},t)\chi_2(\vec{y},t)\right). \tag{3}$$

The last term in the above expression represents quantum interferences. We define the overlap factor as $F = \int \mathrm{d}^3y\, \chi_1^*(\vec{y},t)\chi_2(\vec{y},t)$. This factor is responsible for two separate effects. Its phase generates a shift of the interference fringes, and its absolute value is responsible for the decay in the interference fringe contrast. Of course, in the absence of an environment the overlap factor is not present in the interference term. When the two environmental states do not overlap at all, the final state of the path identifies the path the electron followed. There is no uncertainty with respect to the path. Decoherence appears as soon as the two interfering partial waves shift the environment into states orthogonal to each other.

The loss of quantum coherence can alternatively be explained by the effect of the environment on the partial waves, rather than how the waves affect the environment. As has been noted, when a static potential $V(x)$ is exerted on one of the partial waves, this wave acquires a phase

$$\phi = -\int V[x(t)]\,\mathrm{d}t, \tag{4}$$

and therefore the interference term appears multiplied by a factor $\mathrm{e}^{\mathrm{i}\phi}$. This is a possible agent of decoherence. The effect can be directly related to the statistical character of ϕ, in particular in situations where the potential is not static. Yet more importantly, any source of stochastic noise would create a decaying coefficient. For a general case, ϕ is not totally defined, i.e. it is described by means of a distribution function $P(\phi)$. From this statistical point of view, the phase can be written as

$$\langle \mathrm{e}^{\mathrm{i}\phi}\rangle = \int \mathrm{e}^{\mathrm{i}\phi} P(\phi)\,\mathrm{d}\phi. \tag{5}$$

In this way, the uncertainty in the phase produces a decaying term that tends to eliminate the interference pattern. This dephasing is due to the presence of a noisy environment coupled to the system and can also be represented by the Feynman–Vernon influence functional formalism. It is easy to prove that equation (5) is the influence functional generated after integrating out the environmental degrees of freedom of an open quantum system. Therefore, the formal equivalence between the two ways of studying dephasing was shown in [2]

$$\langle \mathrm{e}^{\mathrm{i}\phi}\rangle = F = \int \mathrm{d}^3y\, \chi_1^*(\vec{y},t)\chi_2(\vec{y},t). \tag{6}$$

The overlap factor F encodes the information about the statistical nature of noise. Therefore, noise (classical or quantum) makes F less than 1, and the goal is to quantify how it slightly destroys the particle interference pattern.

2. Double-slit-like interference experiments

In many cases, the interaction with the environment cannot be switched off. Quantum electromagnetic field fluctuations are well known for being responsible for the Casimir forces. Less known is the role of these fluctuations in the possible destruction of electron coherence. Thus, for charged particles or neutral atoms with dipole moment, the interaction with the electromagnetic field is crucial because it induces a reduction of fringe visibility.

In [3], authors have analysed the influence of a conducting boundary on the decay of the visibility of interference fringes in a double-slit experiment performed with charged particles (or neutral particles with dipole moment). They considered the zero-point fluctuations as an external environment. As the presence of a conducting boundary modifies the properties of the vacuum, it is not surprising that they could also affect the interference pattern. They assumed an initial state $|\Psi(0)\rangle = (|\phi_1\rangle + |\phi_2\rangle) \otimes |E_0\rangle$, where $|E_0\rangle$ is the initial (vacuum) state of the field and $|\phi_{1,2}\rangle$ are two states of the electron localized around the initial point. Therefore, the probability of finding a particle at a given position was also given by equation (3), but with the overlap factor defined by $F = \langle E_2(t)|E_1(t)\rangle$, simply the overlap between two states of the field that arise from the vacuum under the influence of two different sources (the two electron currents $J^{\mu}_{1,2}$). Not only did they compute it in the presence of a conducting boundary, but also did it in vacuum in order to compare both results. In the end, they showed that the presence of the conducting plane may produce more decoherence in some cases and less decoherence in others (in relation to the fringe visibility in the vacuum case). This difference has to do with the orientation of the conducting plate and the trajectories of the interfering particles. Thus, the effect of the boundaries does not have a well-defined sign and may produce either more decoherence or complete recoherence (i.e., smaller or higher fringe visibility than in the absence of the conducting boundary). Nonetheless, even in the case where the fringe visibility is destroyed the most, the quantification of the decoherence suffered by the open system scales as v^2, where v is the velocity of the particles in the non-relativistic limit. Therefore, its magnitude is too small to be measured in the laboratory.

In [4], authors have calculated the effect of the classical and quantum noise in external nonclassical microwaves on the phase factor that describes electron interference. In the presence of such external nonclassical electromagnetic fields, the phase factor is a quantum-mechanical operator. Using this phase factor operator, they studied the effect on mesoscopic Josephson junctures and on time-dependent Aharonov–Bohm and Aharonov–Casher devices. In addition to the presence of quantum noise, one can see the importance of classical fields in the destruction of the interference pattern as well. Classical fields' effects seem to be of the same magnitude or even bigger than those generated by nonclassical fields.

A very innovative study of the loss of contrast in the interference pattern of two electron beams was brought to light by Hsiang and Ford in [5]. They studied the effect of time-varying electromagnetic fields on electron coherence, including the statistical origin of the Aharonov–Bohm (AB) [6] phase ϕ. However, they considered the phase ϕ originated in neither quantum fluctuations nor a time-dependent field. They included a random variable t_0, which was defined as the electron emission time. This variable produces a fluctuating phase, and an average over it is required in order to obtain the result of the double-slit interference experiment. In this simple version of decoherence, the role of a quantum environment is replaced by a time-dependent external field which gives a time-varying AB phase. In the end, they studied the 'dephasing' process and computed the reduction of the interference oscillations, evaluating the overlap factor given in equation (6). They considered the case of a linearly polarized monochromatic electromagnetic wave, propagating in a direction orthogonal to the plane containing two electron beams. Surprisingly this time, the effect on the fringes seems to be sufficiently large to be observable.

In [7], we followed the last idea. We evaluated the overlap factor for coherent neutral particles with permanent (electric and magnetic) dipoles that are affected by time-varying external fields (linearly polarized electromagnetic plane waves and fields inside a waveguide). This is the generalization of the results of [5] to the case of Aharonov–Casher (AC) [8] in atomic systems, where coherent dipoles follow a closed path around an external field. As we will discuss, not all these effects foresee an observable displacement of the interference pattern related to the phase shift of the wavefunction of the system. Dependence on the velocity of the interfering particle is crucial. We will show different configurations where the decoherence factor depends on the particle's speed in a different way.

3. Aharonov phases in external time-dependent classical fields

The AB phase, known to arise when two coherent electrons traverse two different paths C_1 and C_2 in the presence of an electromagnetic field, is $(c = \hbar = 1)$

$$\phi = -e \oint_{\delta\Omega} \mathrm{d}x_\nu A^\nu(x), \tag{7}$$

where $\delta\Omega = C_1 - C_2$ is a closed spacetime path. If the electromagnetic field fluctuations happen on a time scale shorter than the total time of the experiment, this shift of phase results in a loss of contrast in the interference fringes. Then, the overlap factor (or decoherence factor) is given by equation (6) [2, 5]. In that expression, angular brackets denote either an ensemble of quantum noise or a time average over a random variable.

The phase shift that two neutral particles with electric and magnetic dipole moments experience due to a classical time-dependent electromagnetic field is known as the Aharonov–Casher phase and is defined by

$$\phi = -\oint_{\delta\Omega} a_\nu(x)\,\mathrm{d}x^\nu, \tag{8}$$

where $a_\nu(x) = (-\mathbf{m}\cdot\mathbf{B} - \mathbf{d}\cdot\mathbf{E}, \mathbf{d}\times\mathbf{B} - \mathbf{m}\times\mathbf{E})$ plays the role of A_ν in the AB case [9].

In order to evaluate the integral in equation (8), we consider the case of a linearly polarized monochromatic wave of frequency ω propagating in the $\hat{y}$ direction, with an electric and a magnetic field in the $\hat{z}$ and $\hat{x}$ directions, respectively (different external fields have been analysed in [7]). We will also assume that the particles' paths are confined to the $\hat{x}$–$\hat{z}$ plane. We can write the plane wave as $\mathbf{E}(x) = E_0\sin(wt - ky)\hat{z}$, $\mathbf{B}(x) = E_0\sin(wt - ky)\hat{x}$ and compute a_ν.

Following [5], we will assume that the phase ϕ depends on a random variable $\xi = \omega t_0$ given by the emission time of the particles. It is the time t_0 at which the centre of a localized wavepacket is emitted. When the measuring time is longer than the flight time, we will observe a result which is the temporal average over t_0. Thus, t_0 is a random variable by which ϕ has to be averaged. We can write the AC phase ϕ as

$$\phi(t_0) = -\oint_{\delta\Omega} \tilde{a}_\nu \sin(\omega t - ky + \omega t_0)\,\mathrm{d}x^\nu = A\cos(\omega t_0) + B\sin(\omega t_0), \tag{9}$$

where $\tilde{a}_\nu$ are the spatial components of $a_\nu(x)$.

The average over the random phase (generating a classical noise) produces a decoherence factor

$$F = \langle \mathrm{e}^{\mathrm{i}\phi}\rangle = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\mathrm{d}t_0\exp\{\mathrm{i}[A\cos(\omega t_0) + B\sin(\omega t_0)]\} = J_0(|C|),$$

where J_0 is the Bessel function. The modulus of complex number $C = A + \mathrm{i}B$ measures the degree of decoherence. The overlap factor F decreases from 1 to 0 as $|C|$ varies between zero and the first zero of J_0. For larger values of $|C|$, the overlap factor oscillates with decreasing amplitude.

A characteristic feature of the usual AB and AC effects is that the phase shift is independent of the velocity of the particle and there is no force on the particle [10, 11]. Moreover, the phase shift depends only on the topology of the closed spacetime path $\delta\Omega$. Of course, these properties are no longer valid when the external field is time dependent because the particle does suffer a net force applied on it. Thus, in order to analyse the dependence upon the trajectory, we will evaluate equation (8) for different paths and will find that the phase's dependence on the velocity is strongly related to the trajectory the particles follow.

3.1. Overlap factors for AC phases

In this subsection, we will estimate the AC phase acquired by two neutral particles with electric and magnetic dipole moments when they follow two different trajectories. To begin with, we will consider an elliptic path (figure 1(*a*)). Each particle traverses a half ellipse ($\mathcal{C}_1$ and $\mathcal{C}_2$) sharing the initial and final points. Therefore, the elliptic closed path is symmetric with respect to the particles' directions of propagation. This is what we will call *symmetric* trajectory. The quantity $\left|C^{\mathrm{d}}_{\mathrm{ellip}}\right|$ is given by

$$\left|C^{\mathrm{d}}_{\mathrm{ellip}}\right| = 2\pi\alpha E_0 d_y J_1[\omega\tau] \approx \frac{\sqrt{2}\pi\alpha E_0 d_y}{(\omega\tau)^{1/2}} = \sqrt{\pi}\alpha e E_0 L\left(\frac{v\lambda}{s'}\right)^{1/2}, \tag{10}$$

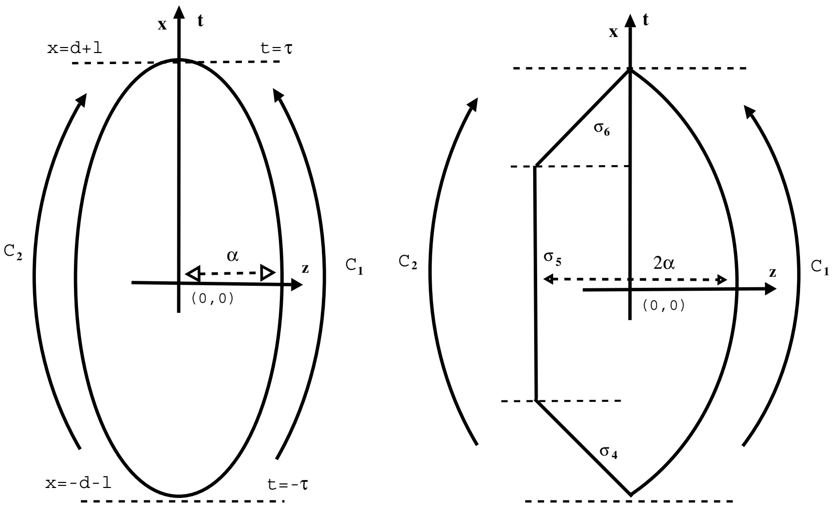

Figure 1. Paths $\mathcal{C}_1$ and $\mathcal{C}_2$ are shown for (*a*) the elliptic trajectory and (*b*) the asymmetric one.

where τ is the time of flight of the dipoles, α is the maximum distance between the dipoles, d_y is the electric dipole moment in the $\hat{y}$ direction and J_1 is the Bessel function of first order. In the last term, we have used the asymptotic expansion of the Bessel function for $\omega\tau \gg 1$ (which is expected for non-relativistic particles). L is the characteristic length of an atom with electric dipole $d = eL$ ($L \approx 10^{-9}$ m), λ is the wavelength of the plane wave and s' is the length travelled by the neutral particles at a speed v and in a time τ. It is important to note that for this symmetric trajectory $\left|C^{\rm d}_{\rm ellip}\right|$ scales as $\sqrt{v}$.

As a second case, we study the decoherence suffered by two neutral particles that transverse different paths, giving rise to the asymmetric closed path shown in figure 1(*b*) (see [7] for details)). Velocity dependence will be different from the case of the symmetric trajectory.

After performing the corresponding integrations in equations (8) and (9), $\left|C^{\rm d}_{\rm asym}\right|$ can be approximated (in the slow velocity limit) by

$$\left|C^{\rm d}_{\rm asym}\right| \approx \frac{e}{\pi} E_0 L \lambda, \tag{11}$$

being independent of the velocity.

3.2. *Overlap factors for AB phases*

It is interesting to check whether the same velocity dependence applies to the case of the AB phase for charged particles or not. We will consider the case of electron wavepackets travelling across the above symmetric and asymmetric trajectories. Therefore, in order to compare the results for charged and neutral particles with dipole moments, we will compute the AB phase for both cases using equations (7) and (9).

If the charged particles traverse the elliptic trajectory, the quantity $\left|C^{\rm e}_{\rm ellip}\right|$ is

$$\left|C^{\rm e}_{\rm ellip}\right| = 2\pi\alpha e E_0 \lambda J_1[\omega\tau] \approx \sqrt{\pi}\alpha e E_0 \lambda \left(\frac{v\lambda}{s'}\right)^{1/2}, \tag{12}$$

showing that the dependence on the velocity is similar for both neutral and charged particles travelling the same trajectory (although the velocities differ in magnitude).

The last case we must consider is that of charged particles performing the asymmetric trajectory. The decoherence factor can now be computed as

$$\left|C^{\mathrm{e}}_{\mathrm{asym}}\right| \approx \frac{e}{\sqrt{2\pi}} E_0 \alpha \left(\frac{v\lambda^3}{s'}\right)^{1/2}, \qquad (13)$$

which depends on the velocity similarly to the preceding case. What is worth noting is the fact that while this result does depend on the velocity of the electrons, the decoherence factor for the dipoles applied to the same trajectory does not.

4. Numerical estimations and final remarks

In [7], we have shown that, contrarily to what might be naively expected, the loss of contrast of the interference fringes for dipoles might be as big as for the case of electrons. By the way, if one introduces real numbers into our analytical results for electrons and dipoles, the results are quite interesting. In electron interferometry, the wavepackets can be moved apart up to 100 μm [12]. A typical non-relativistic velocity is $v_{\mathrm{e}} \sim 0.1$. This yields a relation $\omega\tau \sim 10$ for a field that has a wavelength of about 100 μm. On the other hand, in atomic interferometry, two neutral particles can be separated up to 1mm [13, 14]. Typical speeds are of the order $v_{\mathrm{d}} \sim 10^{-5}$ [15]. We will assume an energy flux of 10 W cm^{-2}, approximately.

With all these values, we can estimate the C factor for all the cases presented in the previous sections. The results for electrons are of order 1 or even bigger, which means that the effect is experimentally observable. Dipole results are smaller but not as much as one would naively expect. In the case of the elliptic trajectory, the decoherence factor for dipoles is $C^{\mathrm{d}}_{\mathrm{ellip}} \sim 10^{-3}$, while in the case of the asymmetric path one gets $C^{\mathrm{d}}_{\mathrm{asym}} \sim 10^{-1}$.

In electron interference experiments, we have shown that, in principle, it is possible to obtain a complete destruction of the interference pattern (setting the value of $|C|$ equal to a zero of the Bessel function J_0). On the other hand, in an interference experiment with dipoles, the best experimental setup would be the asymmetric trajectory. In this case, the effect is non-negligible, thanks to the fact that the C factor is independent of the velocity. Moreover, if one takes into account the result in [7], one is allowed to increase the intensity of the external field, since the scattering cross section for dipoles is much lower than for electrons, it still being possible to neglect the direct interaction with the electromagnetic field.

Matter-wave interferometry has been largely studied in the last few years. Many theoretical studies have been done around the *mesoscopic* systems [16, 17]. 'Mesoscopic' objects are neither microscopic nor macroscopic. They are generally systems that can be described by a wavefunction, yet they are made up of a significant number of elementary constituents, such as atoms. Well-known examples these days are fullerene molecules C_{60} and C_{70}. Most notably, the quantum interference of these molecules has been observed [18]. In this context, we may estimate the decoherence factor for dipoles using the experimental values of the fullerene experiments. Even though we know it is a toy model for the fullerene molecules, it provides a quantitative estimation about the possibility of measuring the effect of decoherence for neutral systems. These molecules have a speed similar to that of the dipoles we have considered. However, they travel longer distances. Therefore, it is compulsory to consider a bigger wavelength, at least of the same order of magnitude as the total distance the dipoles travel. It is important to note that, in the experiment described in [18], a laser beam with a power of 26 W cm^{-2} behaves as an external field. With all these new values in mind, the decoherence factor for dipoles when traversing the asymmetric trajectory is $|C_{\text{asym}}| \approx 1$, giving complete destruction of the interference pattern.

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References

[1] Zurek W 2003 *Rev. Mod. Phys.* **75** 715
[2] Stern A, Aharonov Y and Imry Y 1990 *Phys. Rev.* A **41** 3436
[3] Mazzitelli F D, Paz J J and Villanueva A 2003 *Phys. Rev.* A **68** 062106
[4] Chong C C, Tsomokos D I and Vourdas A 2002 *Phys. Rev.* A **66** 033813, and references therein
[5] Hsiang J-T and Ford L H 2004 *Phys. Rev. Lett.* **25** 250402
[6] Aharonov Y and Bohm D 1959 *Phys. Rev.* **115** 485
[7] Lombardo F C, Mazzitelli F D and Villar P I 2005 *Phys. Rev.* A **72** 042111
[8] Aharonov Y and Casher A 1984 *Phys. Rev. Lett.* **53** 319
[9] Anandan J 2000 *Phys. Rev. Lett.* **85** 1354
[10] Berry M V 1991 *Proc. R. Soc. Lond.* A **392** 45
[11] Aharonov Y, Pearle S A and Vaidman L 1988 *Phys. Rev.* A **37** 4052
[12] Hasselbach F 1988 *Z. Phys.* B **71** 443
[13] Keith D W *et al* 1991 *Phys. Rev. Lett.* **66** 2693
[14] Pfau T *et al* 1993 *Phys. Rev. Lett.* **71** 3427
[15] Greenberger D M, Atwood D K, Arthur J, Shull C G and Schlenker M 1981 *Phys. Rev. Lett.* **47** 751
[16] Facchi P, Pascazio S and Yoneda T 2005 *Preprint* quant-ph/0509032
[17] Brezger B, Hackermuller L, Uttenthaler S, Petschinka J, Arndt M and Zeilinger A 2002 *Phys. Rev. Lett.* **88** 100404
[18] Hornberger K, Uttenthaler S, Brezger B, Hackermuller L, Arndt M and Zeilinger A 2003 *Phys. Rev. Lett.* **90** 160401