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High depth of focus by combining annular lenses

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Abstract

In some technological applications, optical systems that produce a high depth of focus and superresolving transversal responses are required. In this paper we present a pupil design consisting in a phase pupil with binary amplitude, that added to a conventional optical system, can accomplish these goals. The pupil function is characterized by a complex amplitude that consists basically in combining two annular lenses with different focal length. Meanwhile the central portion of the pupil has an amplitude equal to 0, the external portion is modulated with two quadratic phases each one covering an annular zone. One of the phases corresponds to a convergent lens and the other to a divergent lens. The effect on the incident wavefront is to redirect the light in front of and behind the best image plane (BIP) producing a widened focus. The evolution of the transverse gain for the extended focus is also studied. Experimental results are given, and they confirm the extended focus and the superresolving behavior of the proposed pupil function.

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1. Introduction

In many optical systems a high depth of focus (DOF) is needed. For instance, this feature could be required in photolithography [1,2] or in optical data storage [3]. Another very common goal is also to obtain lateral superresolution [4].

Different strategies have been used to obtain high DOF and/or superresolution. Annular pupils are very well known for their capability to produce superresolution and to increase focal depth [4–6]. Also annular color pupil functions have been used to modify both the illuminance and chromaticity of the polychromatic response [7].

Other amplitude only pupil functions have also been used to extend the focal depth [8,9]. Ojeda-Castañeda et al. increased the focal depth by creating quasi-bifocus [10] or by means of a complex pupil function that replicates several times the axial amplitude response [11]. Campos et al. [12] showed that it is possible to find families of amplitude pupil functions that produce the same axial response, but with different transverse response, for instance a superresolving response or an apodizing one. The main disadvantage of amplitude only pupil functions is the loss of light.

Another line of research is the use of diffractive optical elements to produce the so-called non-diffracting beams [13,14]. These techniques are used to produce an specific transverse response, for instance a Bessel profile, along an axial zone as large as possible. Different methods to create these diffractive elements have been proposed, but in many cases complicated diffractive phase elements are needed [15].

There is an increasing interest in using phase pupils functions which are easier to be produced to obtain high focal depth [2,16–19]. In some cases annular phase pupil functions have been proposed [16], in other cases continuously

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varying phase pupil functions are used [17,20]. In [21] we showed several symmetry properties which may help to design pupil phase functions.

Recently, it has been shown that amplitude [22,23] or phase filters [24] may be implemented in spatial light modulators (SLM). In [25] it has been reported the use of a programmable apodizer to optimize the chromatic aberration effects using a liquid crystal spatial light modulator working with polychromatic light. Also, liquid crystal SLM and deformable mirrors have been used to represent adaptative optics in order to correct aberrations in human eyes [26–29].

In this paper we propose a complex pupil function that consists in an external ring where two portions of lenses are combined. One of the lenses is convergent and the other is divergent. The effect of this combination is to shift the focus in front of and behind the best image plane (BIP), defined for the clear pupil system. It should be noted, that the superposition of the complex amplitudes could produce effects of destructive interferences along the axis. By selecting appropriate focal distances, this effect is minimized and an almost flat appearance for a wide range of distances is obtained. With the proposed pupil function we obtain a high focal depth and transverse superresolution along the extended focus.

In Section 2 we show the formulation to calculate the transverse and axial distribution, and also the transverse axial gain. In Section 3 we propose the combined annular lenses to produce high focal depth and superresolution, and some numerical results are shown. In Section 4, we present experimental results where the performance of the pupil functions is clearly observed. Finally, we summarize some conclusions.

2. Theory

In the scalar approximation, the amplitude of the electromagnetic field produced by a pupil with radial symmetry can be written as [9,20]:

$$U(v,u) = 2\int_0^1 P(\rho)J_0(v\rho)\exp[\mathrm{i}u\rho^2/2]\rho\,\mathrm{d}\rho, \tag{1}$$

where v and u are optical dimensionless coordinates in a transverse plane and along the axis respectively, ρ is the normalized radial coordinate in the pupil plane, $P(\rho)$ is the pupil function and J_0 is the Bessel function of the first kind and zero order.

From (1) we can obtain the distribution along the axis by putting:

$$U(0,u) = 2\int_0^1 P(\rho)\exp[\mathrm{i}u\rho^2/2]\rho\,\mathrm{d}\rho. \tag{2}$$

We can rewrite Eq. (2) as a function of a new coordinate $t = \rho^2 - 0.5$ [8,9] as follows:

$$U(0,u) = \int_{-0.5}^{0.5} Q(t)\exp\left[\mathrm{i}u\frac{t+0.5}{2}\right]\mathrm{d}t, \tag{3}$$

where $Q(t)$ is the pupil function $P(\rho)$ written as a function of t. Then, the intensity along the optical axis can be expressed as

$$I(0,u) = \left|\int_{-0.5}^{0.5} Q(t)\exp[\mathrm{i}ut/2]\,\mathrm{d}t\right|^2, \tag{4}$$

which is the square of the modulus of the 1-D Fourier transform of the pupil function written as a function of t.

To find the amplitude in a given point (u,v) we approximate Eq. (1) up to second order in the coordinate v. The nth order moments of $Q(t)$ [7,15] can be defined as

$$I'_n(u) = \int_{-0.5}^{0.5} Q(t)(t+0.5)^n\exp[\mathrm{i}ut/2]\,\mathrm{d}t. \tag{5}$$

Finally, the intensity can be approximated as follows:

$$I(v,u) = \left|I'_0(u)\right|^2 - \frac{1}{2}\mathrm{Re}\left(I'_0(u)I'^*_1(u)\right)v^2. \tag{6}$$

In order to analyze the behavior of the transverse response along the axial axis we use the transverse gain over the axial coordinate u, already defined in [30]. Eq. (6) represents a parabola in v. The roots of the parabola are compared with those corresponding to the clear pupil system. We define the transverse gain $G_\mathrm{T}(u)$ as the ratio between the square of the roots for the clear pupil system and the square of the roots obtained with filters, i.e.

$$G_\mathrm{T}(u) = \frac{2\,\mathrm{Re}\left(I'_0(u)I'^*_1(u)\right)}{\left|I'_0(u)\right|^2}. \tag{7}$$

This parameter is easier to be calculated than the complete 3-D response of the system. Additionally in cases where the axial response has not great fluctuations the transverse gain gives a useful information about the pupil behavior.

3. Pupil design

Taking into account that the axial response is associated to the 1-D Fourier transform of the pupil function $Q(t)$, as it is shown in Eq. (4), we propose to add two linear phases in t in order to get a high depth of focus. The basic idea consists on the fact that a linear phase with positive slope will move the maximum to an axial coordinate behind of the BIP meanwhile a negative slope linear phase will move the maximum to an axial coordinate in front of the BIP. The combination of both linear phases will result in an extended axial response. From another point of view, the addition of a linear phase in t is equivalent to a positive quadratic phase in ρ, which describes a lens in the paraxial approximation. The positive slope linear phase in t produces a divergent lens. Its effect will be to decrease the convergence of light coming from the clear pupil system. On the contrary, a negative slope linear phase in t represents a convergent lens that produces an increasing of the convergence of the beam coming from the optical system alone. By combining these two effects an extended focus can be obtained.

It should be noted that there are many combinations that can be used to obtain this effect. In order to optimize the pupil response, we have made some additional considerations. First, we have selected only those combinations that give a symmetrical axial response. Secondly, we have chosen pupils that produce a transverse superresolving effect.

To illustrate the process, we select two annular lenses and afterwards we combine them. For the annular convergent lens we select the following pupil function:

$$Q_1(t) = \begin{cases} e^{[i(-bt+c)]} & \text{if } 0.25 \leqslant t \leqslant 0.5, \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

For the annular divergent lens we chose the following the pupil function $Q_2(t)$ as follows:

$$Q_2(t) = \begin{cases} e^{[i(bt)]} & \text{if } 0 \leqslant t \leqslant 0.25, \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

As an example, we show in Figs. 1 and 2 the effect produced on the axial response when only one of these filters is placed on the pupil plane. For all the following cases, the intensities are normalized to the response for the clear aperture system. In Fig. 1 we show the axial intensity when a divergent lens is added to the outer portion of the pupil. We can observe the displacement to the right or to the zone behind the BIP for the clear pupil system. In Fig. 2 the effect of an annular convergent lens added to the system is shown. It can be observed that this pupil moves the focus in front of the BIP or to the left. Obviously, the values of b and c determine the displacement with respect to the BIP.

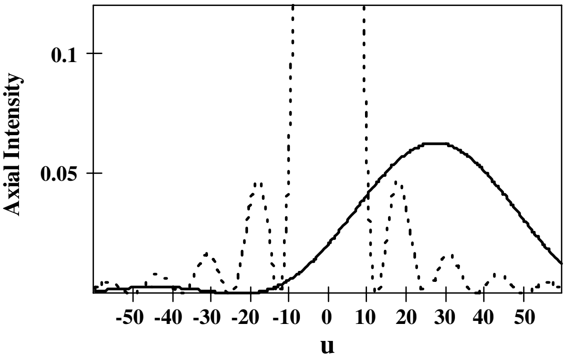

Fig. 1. Axial response for a pupil function as that described by Eq. (8). In dotted line, the response of the system for the clear aperture.

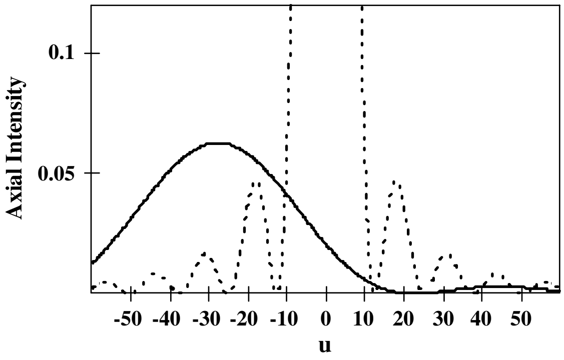

Fig. 2. Axial response for a pupil function as that described by Eq. (9). In dotted line, the response of the system for the clear aperture.

Based in two filters as those depicted in Figs. 1 and 2, we propose the use of a composed pupil function that combines the convergent and the divergent ring, with amplitude null outside the rings. In this case, the combination of the complex amplitudes corresponding to each pupil function, produces an interference pattern along the axis. Taking into account this effect, the values of the parameters b and c must be selected. On one hand, if we move the foci very far from the BIP, we will have two peaks instead of an almost constant function. On the other hand, if we design the pupil function to move the foci a small distance from the BIP, we will have along the axis a principal maximum centered at $u = 0$, and two low secondary maxima. So, we will obtain a low increase in the DOF. It should be noted that, if the amplitude of the pupil were not null outside the rings region, an additional term must be added in order to calculate the interference along the axis. This pupil will produce, in general, an asymmetrical axial response.

As we have mentioned, the another point to be considered in the pupil design, is the transverse superresolving effect. Different positions of the center of the combined ring could give different transverse responses and the same axial response. As we have discussed in [21] a pupil with real transmission will act as transverse apodizing filters if the center of mass is located in the $t < 0$ region and as transverse superresolving filter if it is located in the $t > 0$ region. For complex pupils, it is not easy to predict the behavior of the pupil and the transverse gain must be calculated for each particular case. In our case, we have verified that if the combined pupil function occupies the external portion of the clear aperture, then the transverse response is superresolving along all the extended focus. Additionally, a better illumination condition is obtained if a large area in the clear aperture is used. By taking into account these considerations, we have selected a combined pupil with size 0.5 and centered at $t = 0.75$. Then, we have chosen the parameters of the linear phases in t, in order to obtain an almost flat function. To this end, we have evaluated the axial response by changing the maximum dephasing in steps of $\pi/10$. We have found that by selecting $b = 4.4\pi$ and $c = 2.2\pi$ a good axial response is achieved, where the central peak approximately equals the other two foci.

In this way, we propose a composed pupil function that can be described by

$$Q(t) = \begin{cases} 0 & \text{if } -0.5 < t \leqslant 0, \\ e^{i4.4\pi t} & \text{if } 0 < t \leqslant 0.25, \\ e^{i(-4.4\pi t+2.2\pi)} & \text{if } 0.25 < t \leqslant 0.5. \end{cases} \tag{10}$$

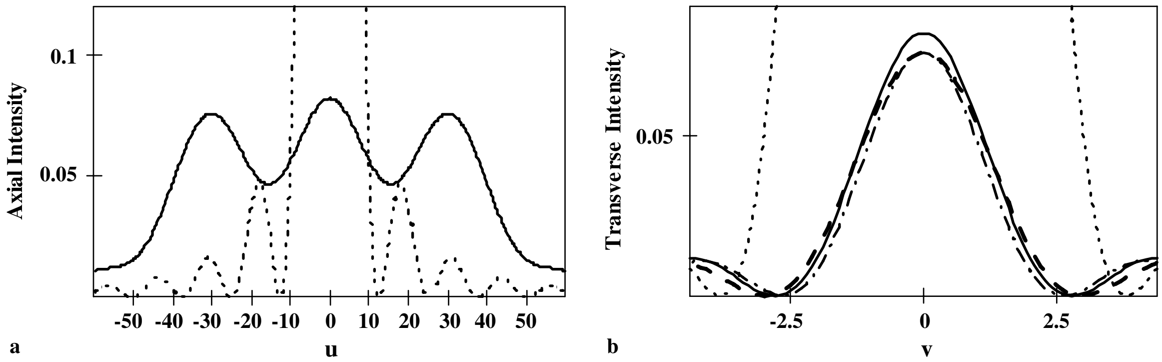

Fig. 3. Responses of the annular pupil described by Eq. (10): (a) axial distribution and (b) transverse intensity obtained at $u = -30$ (dashed line), $u = 0$ (fill line) and $u = 30$ (dashed–dotted line). In dotted line the response for the clear aperture.

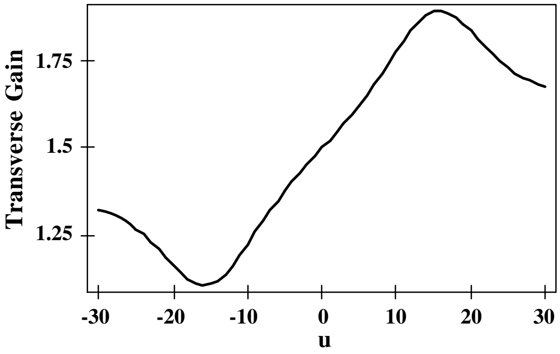

Fig. 4. Transverse gain factor of the proposed pupil as a function of the axial coordinate.

In Fig. 3(a) we show the axial response produced by this pupil function where an excellent DOF can be appreciated. We can estimate the depth of focus as the full width at half maximum (FWHM). By calculating the ratio of this parameter for this pupil function and for the clear pupil system we obtain a factor of approximately eight. We have studied the transverse response in planes along the interval where we define the FWHM, to be sure that superresolution is obtained in the extended focus.

As an example, we show in Fig. 3(b) the transverse response at coordinates $u = -30$, $u = 0$ and $u = 30$ along the axial axis. It can be observed that the transverse intensity is almost the same in these three planes and we have always a superresolving effect.

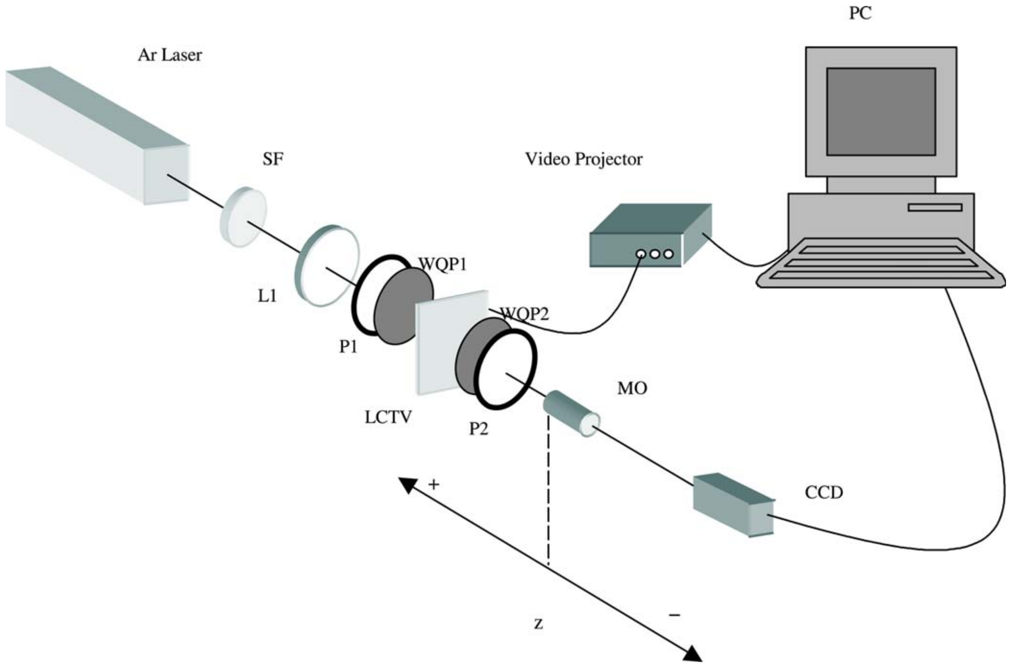

Fig. 5. Experimental set-up used to capture the transverse response at different axial positions.

In Fig. 4 we show the evolution of the transverse gain defined in Eq. (7) along the axis. The transverse gain factor is always higher than one, which guarantees a superresolving effect for every transverse plane.

4. Experimental results

In this section, we show experimental results that illustrate the performance of the proposed pupil function. In order to physically implement the pupil, we have used a spatial light modulator (SLM) working in a phase mostly mode [23] in combination with an optical system. Fig. 5 shows the experimental arrangement. An Ar laser (457 nm) expanded and spatially filtered impinges on a photographic objective L_1 whose focal length is 135 mm. We place an SLM behind L_1 and before the focus to display the pupil function on it. We have used a Sony liquid crystal TV (LCTV) panel extracted from a video-projector Proxima with VGA resolution of 640×480 pixels driven by a PC. To control the polarization and to get phase only response for the SLM, a set composed by linear polarizers (P_1 and P_2) and quarter wave plates (QWP_1 and QWP_2) is placed in the input and the output of the LCTV. In our case the pupils have a diameter of 10.5 mm (or 256 pixels). A microscope objective (MO) is used to image the focal plane on a CCD. The coordinate z is measured from the BIP and it is defined positive from the focus to the lens and negative in the other direction, as is indicated in Fig. 5. For our experimental parameters, a value of $u = 1$ corresponds to $z = 1.7$ mm.

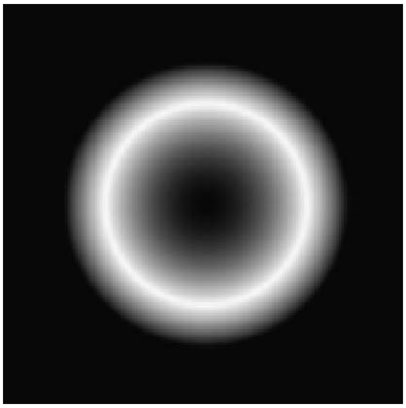

Fig. 6. Phase function represented onto the LCTV corresponding to the proposed pupil function.

In Fig. 6 the pupil represented onto the liquid crystal panel is shown. The different gray levels correspond to the different values of the phase. To represent the complex image on the SLM working in phase mode, we have used a codification of the binary amplitude [31]. The codification consists in adding a linear phase to the filter in the zones where the amplitude is 1, and no linear phase in the zones where the amplitude is null. The effect of this codification is redirect the light coming from the SLM. In this way, the regions on the SLM that represents the zones where the transmission function is null, contribute to the zero order. The regions on the SLM representing the zones with unitary amplitude are redirected to an off axis position. The images presented here correspond to the off axis direction. To measure the response for the clear pupil system, we have displayed a circle of 256 pixels of radius with a linear phase inside and phase zero outside.

In Figs. 7 and 8 we show the experimental results obtained with the described pupil functions. Fig. 7 resumes the behavior of the proposed pupil function in the most significant axial positions. To compare the transverse response when no pupil function is employed, in Fig. 7(a) the transverse response at the BIP for the clear pupil system is shown. In Fig. 7(b) we show five images obtained along the focus. The five positions were selected as follows. First, for the clear pupil system we have captured with the camera the intensity in the BIP. This position was labeled with $z = 0$ mm. Then, using the proposed pupil we moved the MO from the BIP to the lens and we found the position where the axial intensity has a local minimum ($z = 28$ mm).

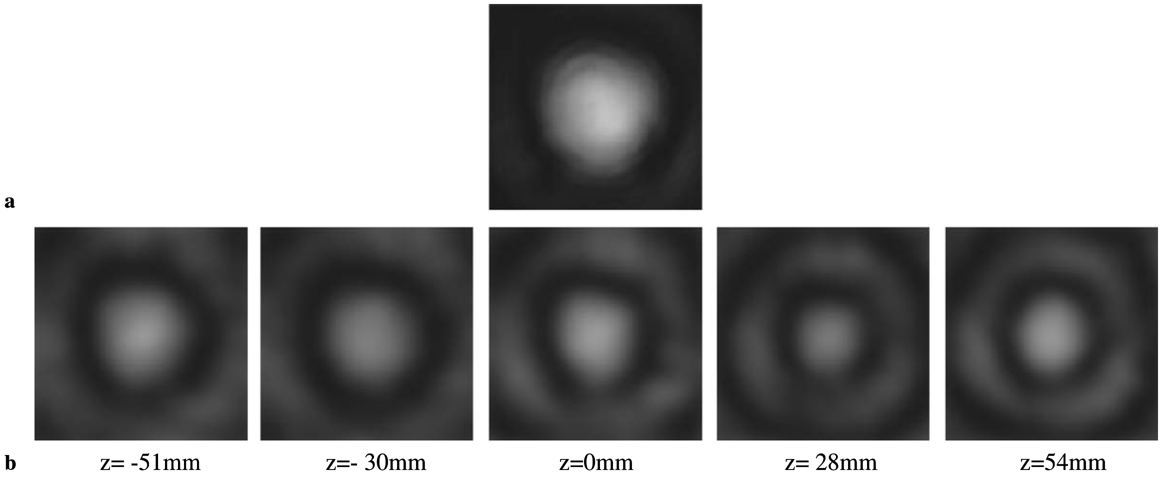

Fig. 7. Transverse responses: (a) the response at the BIP for the pupil without filter and (b) the responses at the different positions measured from the BIP for the proposed pupil.

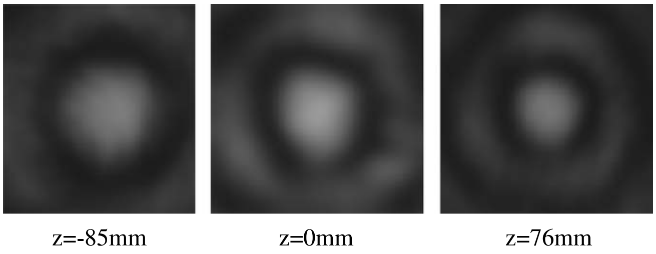

Fig. 8. Transverse responses obtained at the BIP ($z = 0$ mm) and at positions corresponding to the half of the maximum axial response behind the BIP ($z = -85$ mm) and in front of the BIP ($z = 76$ mm) for the proposed pupil.

This local minimum corresponds to the coordinates $u \approx 15$ in Fig. 3(b). Moving the MO in the same direction we found the position of the secondary maximum ($z = 54$ mm). This maximum corresponds to the axial coordinate $u \approx 30$ in Fig. 3(b). Then, we moved the MO starting from the BIP but in the opposite direction and we found the secondary minimum and maximum at positions $z = -30$ mm and $z = -51$ mm, respectively. We can observe in Fig. 7(b) that the proposed filter maintains its superresolving behavior along all the extended focus. Also we can see that a minimum of the transverse gain is produced at $z = -30$ mm and a maximum of the transverse gain is produced at $z = 28$ mm. These values are in a very good agreement with the simulated results shown in Fig. 4. Moreover, the measured variation in the axial intensity along the extended focus was <40%.

In Fig. 8 we show images of the central axial spot and the spots captured at a half of the maximum intensity when the proposed pupil is represented. We have measured a focal depth of 161 mm with the proposed pupil and a focal depth of 20 mm for the clear pupil system. This is in excellent agreement with the simulated results that predict for this filter a focal depth eight times higher than the obtained with the pupil without filter. Also we can observe the superresolving effect along all the extended focus.

5. Conclusions

Nowadays the development of new register technologies imposes different challenges to optical designers. One of them is to get high depth of focus and superresolving systems that allow high precision in the register step.

We have presented here a complex pupil function that accomplishes these goals. The proposed pupil function has a complex transmission that produces a high depth of focus and transverse superresolution.

The proposal consists in adding two quadratic phases in two annular region of the pupil. One of the quadratic phases corresponds to a convergent lens while the other corresponds to a divergent lens. These lenses have the effect of shifting the light in front of and behind the BIP corresponding to the system without filter. These shifts have been selected to have and extended DOF. With the position of the annuli, the transverse gain can be controlled. We have chosen the position of the filter in the outer part of the pupil in order to improve the transverse resolution in comparison with the system without filter.

We have implemented the pupil function by means of a SLM based on a LC panel working in phase mode. The experimental results are in very good agreement with the numerical results. The analyzed filter increase eight times the depth of focus of the clear pupil system. We have experimentally shown the possibilities of designing pupil functions with a very high depth of focus and a superresolving behavior along the extended focus.

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