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Using safety stocks and simulation to solve the vehicle routing problem with stochastic demands

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ABSTRACT

After introducing the Vehicle Routing Problem with Stochastic Demands (VRPSD) and some related work, this paper proposes a flexible solution methodology. The logic behind this methodology is to transform the issue of solving a given VRPSD instance into an issue of solving a small set of Capacitated Vehicle Routing Problem (CVRP) instances. Thus, our approach takes advantage of the fact that extremely efficient metaheuristics for the CVRP already exists. The CVRP instances are obtained from the original VRPSD instance by assigning different values to the level of safety stocks that routed vehicles must employ to deal with unexpected demands. The methodology also makes use of Monte Carlo simulation (MCS) to obtain estimates of the reliability of each aprioristic solution – that is, the probability that no vehicle runs out of load before completing its delivering route – as well as for the expected costs associated with corrective routing actions (recourse actions) after a vehicle runs out of load before completing its route. This way, estimates for expected total costs of different routing alternatives are obtained. Finally, an extensive numerical experiment is included in the paper with the purpose of analyzing the efficiency of the described methodology under different uncertainty scenarios.

1. Introduction

The Vehicle Routing Problem with Stochastic Demands (VRPSD) is a well-known NP-hard problem in which a set of customers with random demands must be served by a fleet of homogeneous vehicles departing from a depot, which initially holds all available resources. Obviously, there are some tangible costs associated with the distribution of these resources from the depot to the customers. In particular, it is usual for the model to explicitly consider costs due to moving a vehicle from one node –customer or depot – to another. These costs are often related to the total distance traveled, but they can also include other factors such as number of vehicles employed, service times for each customer. The classical goal here consists of determining the optimal solution (set of routes) that minimizes those tangible costs subject to the following constraints: (i) all routes begin and end at the depot; (ii) each vehicle has a maximum load capacity, which is considered to be the same for all vehicles; (iii) all (stochastic) customer demands must be satisfied; (iv) each customer is supplied by a single vehicle; and (v) a vehicle cannot stop twice at the same customer without incurring in a penalty cost.

The study of the VRPSD is within the current popularity of introducing randomness into combinatorial problems as a way of describing new real problems in which most of the information and data cannot be known beforehand. This tendency can

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be observed in Van Hentenryck and Bent (2010), which provides an interesting review of many traditional combinatorial problems with stochastic parameters. Thus, those authors studied Stochastic Scheduling, Stochastic Reservations and Stochastic Routing in order to make decisions on line, i.e., to re-optimize solutions when their initial conditions have changed and, therefore, are no longer optimal. This type of analysis has designed the Online VRP in which re-optimization is needed apart from a previous situation. This set of routing problems seems to be well analyzed with the use of stochastic hypothesis in their definitions (Bent and Van Hentenryck, 2007) providing more reality in their formulation. Another routing field in which randomness has also been developed is the resolution of inventory routing problems where the product usage is stochastic (Hemmelmayr et al., 2010). Bianchi et al. (2009) have written an interesting survey of the appropriate metaheuristics to solve a wide class of combinatorial optimization problems under uncertainty. This survey is a good reference for obtaining an appropriate list of articles regarding the use of metaheuristics in VRPSD and other related problems.

Furthermore, it is important to recall that Vehicle Routing Problems (VRPs) constitute a relevant topic for current researchers and practitioners. In fact, according to Eksioglu et al. (2009), the number of related articles published in refereed journals has experienced an exponential grown in the last 50 years. Notice that the main difference between the Capacitated Vehicle Routing Problem (CVRP) and the VRPSD is that in the former all customer demands are known beforehand, while in the latter the actual demand of each customer has a stochastic nature, i.e., its statistical distribution is known beforehand, but its exact value is revealed only when the vehicle reaches the customer. For the CVRP, a large set of efficient optimization methods, heuristics and metaheuristics have been already developed (Laporte, 2007; Golden et al., 2008). However, this is not yet the case for the VRPSD, which is a more complex problem due to the uncertainty introduced by the random behavior of customer demands. Therefore, as suggested by Novoa and Storer (2009), there is a real necessity for developing more efficient and flexible approaches for the VRPSD. On one hand, these approaches should be efficient in the sense that they should provide optimal or near-optimal solutions to small and medium VRPSD instances in reasonable times. On the other hand, they should be flexible in the sense that no further assumptions need to be made concerning the random variables used to model customer demands, e.g., these variables should not be assumed to be discrete neither to follow any particular distribution – to the best of our knowledge, most of the existing approaches to the VRPSD do not satisfy this flexibility requirement.

The random behavior of customer demands could cause an expected feasible solution to become infeasible if the final demand of any route exceeds the actual vehicle capacity. This situation is referred to as "route failure", and when it occurs, some corrective actions must be introduced to obtain a new feasible solution. For example, after a route failure, the associated vehicle might be forced to return to the depot in order to reload and resume the distribution at the last visited customer. Of course, it is also possible to consider preventive vehicle reloads even before the actual route failure occurs – e.g., when the expected demand of the next customer exceeds the current load of the vehicle. While some authors have mainly focused on modeling the costs associated with these route failures (Tan et al., 2007), our approach also aims at reducing the probability of occurrence of such undesirable situations to a reasonable value – according to the decision-maker's utility function. In other words, our methodology proposes the construction of reliable solutions, i.e., solutions with a low probability of suffering route failures. This is basically attained by constructing routes in which the associated expected demand will be somewhat lower than the vehicle capacity. Particularly, the idea is to keep a certain amount of surplus vehicle capacity (safety stock or buffer) while designing the routes so that if the final routes' demands exceed their expected values up to a certain limit, they can be satisfied without incurring a route failure. Of course, a trade-off exists between reliability and costs minimization and, therefore, an optimal balance must be set for these two factors, which is one of the main goals of the methodology presented here. The idea itself is not new in the literature. Sungur et al. (2008), for instance, built a robust solution approach for the VRPSD using adequate management of the remaining vehicle capacity compared to a uniform and non-uniform distribution of that slack over all the considered vehicles. However, while their goal is to find a robust solution "that optimizes the worst case value over all data uncertainty", our goal is to find reliable solutions with optimal or pseudo-optimal total expected costs under different uncertainty scenarios.

The rest of the paper is organized as follows: Section 2 contains a detailed formulation of the VRPSD, and Section 3 provides a literature review. Section 4 describes the main ideas behind our approach, and Section 5 discusses the advantages that our methodology offers over previous approaches. Section 6 presents numerical experiments that illustrate our methodology, followed by a discussion of results in Section 7. Section 8 explains future work that we plan to perform on related topics. Finally, Acknowledgement summarizes the main contributions of this paper.

2. Basic notation and assumptions

The Stochastic (or Probabilistic) Vehicle Routing Problem (SVRP or PVRP) is a family of well-known vehicle routing problems characterized by the randomness of at least one of their parameters or structural variables (Bastian and Rinnooy Kan, 1992). This uncertainty is usually modeled by means of suitable random variables which, in most cases, are assumed to be independent. A related problem having only one route is the Stochastic Traveling Salesman Problem (Balaprakash et al., 2010). Nevertheless, the Vehicle Routing Problem with Stochastic Demands (VRPSD) is among the most popular routing problems within the SVRP family. There are two other classical problems belonging to that family: the Vehicle Routing Problem with Stochastic Customers (VRPSC) (Jézéquel, 1985; Bent and Van Hentenryck, 2004) which was solved by Gendreau et al. (1996a) using an adapted Tabu Search, and the Vehicle Routing Problem with Stochastic Times (VRPST) (Verweij et al., 2003), but their applications are rather limited in comparison with the VRPSD, which is described in detail next.

Consider a complete network constituted by $n + 1$ nodes, $V = \{0, 1, 2, \ldots, n\}$, where node 0 symbolizes the central depot and $V^* = V \setminus \{0\}$ is the set of nodes or vertices representing the n customers. The costs associated with traveling from node i to node j are denoted by $c(i, j)\ \forall\ i, j \in V$, where the following assumptions hold true: (i) $c(i, j) = c(j, i)$ (i.e., costs are symmetric); (ii) $c(i, i) = 0$, and (iii) $c(i,j) \leqslant c(i,k) + c(k,j) \forall k \in V$ (i.e., the triangle inequality is satisfied). These costs are usually expressed in terms of traveled distances, traveling plus service time or a combination of both. Let the maximum capacity of each vehicle be $VMC \ll \underset{i \in V^*}{Max}\{D_i\}$, where $D_i \geqslant 0 \forall i \in V^*$ are the independent random variables that describe customer demand – it is assumed that the depot has zero demand. For each customer, the exact value of its demand is not known beforehand but it is only revealed once the vehicle visits. No further assumptions are made on these random variables other than that they follow a well-known theoretical or empirical distribution – either discrete or continuous – with existing mean denoted by $E[D_i]$. In this context, the classical goal is to find a feasible solution (set of routes) that minimizes the expected delivery costs while satisfying all customer demands and vehicle capacity constraints. Even when these are the most typical restrictions, other constraints and factors are sometimes considered, e.g., maximum number of vehicles, maximum allowable costs for a route, costs associated with each delivery, time windows for visiting each customer, solution attractiveness or balance, environmental costs, and other externalities.

Finally, it is interesting to notice that our approach does not require one to assume symmetry in the costs, even when this is a usual assumption in most VRPs approaches.

3. The state-of-the-art of the VRPSD and the related work

Since customer demands are only revealed after each stop during the distribution process, the VRPSD has been often modeled as a two-stage problem. In stage one (aprioristic design), a route for each vehicle is planned. When routes fail, recourse actions must be implemented to serve any remaining customers. Therefore, stage two specifies the actual route of each vehicle taking into account these recourse actions. These recourse actions usually imply the vehicle return to the depot to load or unload the commodities needed by the customers. Nevertheless, there are some models in which re-stocking on route is allowed (Yang et al., 2000). Thus, the usual goal is to build a solution (set of planned routes) that minimizes the expected total costs, i.e., the sum of the costs due to the routes planned in stage one and the expected costs due to possible recourse actions. Examples of approaches where routes are re-optimized after a failure can be found in Secomandi (2001, 2003) or in Secomandi and Margot (2009). Also, Campbell and Thomas (2008) provide a comprehensive review of advances and challenges in aprioristic routing.

The VRPSD is related to many practical situations. Psaraftis (1995) and Chepuri and Homem de Mello (2005) investigated the delivery of petroleum products, industrial gases, and home heating oil. Shen et al. (2009) presented an example regarding the delivering of supplies to cities under a state of emergency. Markovic et al. (2005) suggested that the VRPSD could be used to model the delivery and pickup processes of mail, packages, and recycled material from offices and industrial plants.

From an historical perspective, the seminal work of Tillman (1969) is considered the first paper to analyze the VRPSD problem considering more than one depot. That author built a heuristic approach using the saving concept developed by Clarke and Wright (1964) in their well-known paper. The next significant contribution on the VRPSD was developed by Golden and Stewart (1978), which presented a chance constrained model and two penalty models to discuss a potential assimilation of the VRPSD as a CVRP. Dror and Trudeau (1986) contributed another important milestone in the VRPSD literature, highlighting the asymmetric nature of distance costs. Nevertheless, it was Bertsimas who, at the end of the eighties and beginning of the nineties, made very significant contributions in the solution of stochastic routing problems. Bertsimas (1988, 1992), Bertsimas et al. (1990) and Bertsimas and Howell (1993) established the most important definitions in the area, providing most of the knowledge we have about the VRPSD. In parallel, Dror et al. (1989) contributed some key results for the VRPSD based on previous work developed by Jaillet (1985, 1988) and Jaillet and Odoni (1988) to solve the Traveling Salesman Problem. Those results are traditionally summarized in the following way: (1) routes in an optimal solution could intersect themselves; (2) in a Euclidean space, the convex hull of vertices does not univocally define the visiting order; and (3) sections of an optimal route are not necessarily optimal when considered locally. The latter property is considered the most representative of a VRPSD solution (Cordeau et al., 2007).

Laporte and Louveaux (1993) and Gendreau et al. (1995, 1996b) were among the first authors to develop exact approaches for the VRPSD. To that end, the former proposed the use of an integer L-shaped method for solving a stochastic linear program, which had been previously introduced by Teodorovic and Pavkovic (1992), whereas the latter wrote interesting reviews to the routing problems with stochastic assumptions. However, the number of customers allowed by their approaches is limited. Bertsimas and Simchi-Levi (1996) offer an early review of stochastic vehicle routing research and a comprehensive set of references. Another important milestone is the Hadjiconstantinou and Roberts (2002), which is devoted to routing under uncertainty.

Other relevant approaches are those from Séguin (1994), Hjorring and Holt (1999), and Laporte et al. (2002), all of which are based on the concept of (expected) vehicle filling rate – which is basically defined as the quotient between the (expected) total customer demand and the (expected) total vehicle capacity. In these works, the authors show how an increase in this rate raises the intrinsic difficulty of the VRPSD. Additionally, they are able to obtain promising results for low values of this rate (e.g., 0.3) and also for larger values (e.g., 1.0) when Poisson or Normal distributions are assumed for modeling customer demands. More recently, Mendoza et al. (2010) defined a variant of the VRPSD with multi-compartment constraints, and designed a memetic algorithm to solve that problem, providing a good approximation to properly analyze the VRPSD. Similarly, Rei et al. (2010) developed a hybrid Monte Carlo local branching algorithm for the VRPSD that obtained the same quality of results as the exact L-shaped algorithm with less computation time. Perhaps, this has been the most recent paper to make connections between Monte Carlo techniques and the VRPSD.

4. Fundamentals of our methodology

As already suggested in the Introduction section, our approach is inspired by the following facts: (a) the VRPSD can be seen as a generalization of the CVRP or, to be more specific, the CVRP is just a VRPSD with constant demands – random demands with zero variance – and (b) while the VRPSD is yet an emerging research area, extremely efficient metaheuristics do already exists for solving the CVRP; in fact, state-of-the-art metaheuristics based on the use of Genetic Algorithms, Tabu Search, Simulated Annealing, Ant Colony Optimization or Hybrid GRASP are able to provide near-optimal solutions for most known CVRP benchmarks. Thus, one key idea behind our approach is to transform the issue of solving a given VRPSD instance into a new issue which consists of solving several "conservative" CVRP instances, each characterized by a specific risk (probability) of suffering route failures. The term conservative refers here to the fact that only a certain percentage of the vehicle total capacity will be considered as available during the routing design phase. In other words, part of the total vehicle capacity will be reserved for attending possible "emergencies" caused by under-estimated random demands during the actual distribution (routing execution) phase. This part can be considered as a safety stock since it reflects the level of extra stock that is maintained to buffer against possible route failures. In fact, we have adapted some ideas from the Juan et al. (2009b) along with the reliability concepts to be developed in this case for the VRPSD.

Next, the specific steps of our methodology are described in detail (Fig. 1):

1. Consider a VRPSD instance defined by a set of n customers with stochastic demands $D_i \geqslant 0$ $(1 \leqslant i \leqslant n)$, where each D_i follows a well-known statistical distribution – either theoretical or empirical as long as its mean exists. Let the vehicle maximum capacity be VMC.
2. Set a value for k $(0 < k \leqslant 1)$, the percentage of the maximum vehicle capacity that will be used during the routing design stage, and calculate $VMC^* = k \cdot VMC$.
3. Consider the CVRP(k) defined by a total vehicle capacity of VMC^* and by the deterministic demands $d_i^* = E[D_i]$, where $E[D_i]$ symbolizes the mean or expected value of each random demand.
4. Solve the CVRP(k) by using any efficient CVRP methodology. Notice that the solution of this CVRP is also an aprioristic solution for the original VRPSD. Moreover, it will be a feasible VRPSD solution as long as there will be no route failure, i.e., as long as the extra demand that might be originated during execution time in each route does not exceed the vehicle reserve capacity (safety stock) $VRC^* = (1 - k) \cdot VMC$. Notice also that the cost given by this solution, $C_{CVRP}(k)$, can be considered as a base or fixed cost of the VRPSD solution, i.e., the cost of the VRPSD in case that no route failures occur. Chances are that some route failures occur during the execution phase – these chances increase as the value of k gets closer to 1. If so, corrective actions – such as returning to the depot for a reload before resuming distribution – and their corresponding variable costs, $C_{RF}(k)$, will need to be considered. Therefore, for a given value of k, the total costs of the corresponding VRPSD solution will be the sum of the CVRP fixed costs and the variable costs due to the corrective actions, i.e., $C_{VRPSD}(k) = C_{CVRP}(k) + C_{RF}(k)$. Notice that, on average, low values of k (close to 0) will be associated with relatively high fixed costs (more routes will be needed to satisfy total demand) and relatively low variable costs (route failure is less likely to occur). On the contrary, high values of k (close to 1) will have the opposite effect.
5. Using the aprioristic solution with m routes, estimate the expected (average) costs due to possible failures in the jth route, $E[C_{RF}^j(k)], \forall j = 1, 2, \ldots, m$. This can be done by using Monte Carlo simulation, i.e., random demands are generated and whenever a route failure occurs (or just before it happens), a corrective policy is applied and its associated costs are registered (in the experimental section of this paper, every time a route fails we consider the costs of a round-trip from the current customer to the depot; but, since we are using simulation, other alternative policies and costs could also be considered in a natural way). After iterating this process for some hundred/thousand times, a random sample of observations regarding these variable costs are obtained and an estimate for its expected value can be calculated. Then, the expected total costs due to possible route failures in the aprioristic solution is given by the following expression: $E[C_{RF}(k)] = \sum_{j=1}^{m} E[C_{RF}^j(k)]$.
6. Using the aprioristic solution with m routes, obtain an estimate for the reliability of each route, R_j $(1 \leqslant j \leqslant m)$. In this context, R_j is defined as the probability that the jth route will not suffer any failure during the distribution phase, i.e., that the jth vehicle will not run out of load before attending to all customer demands on its route. This reliability value can be estimated by direct Monte Carlo simulation using the statistical distributions that model the customer demands in each route – observe that in each route over-estimated demands could sometimes be compensated by under-estimated demands. To this end, a number of trials – between several hundreds and several hundred thousands depending of the desired accuracy – can be randomly generated. Each of these trials will provide a random value for the total demand in a given route. Then, the relative frequency of trials in which that total demand has not exceeded VMC can be used as an estimate of the route's reliability. Notice that $F_j = 1 - R_j$ represents the probability that the jth route will fail during the distribution phase. Notice also that if the variances associated with customer demands are not too large, it seems natural to expect no more than one failure per route. Otherwise, more than a failure per route could occur.

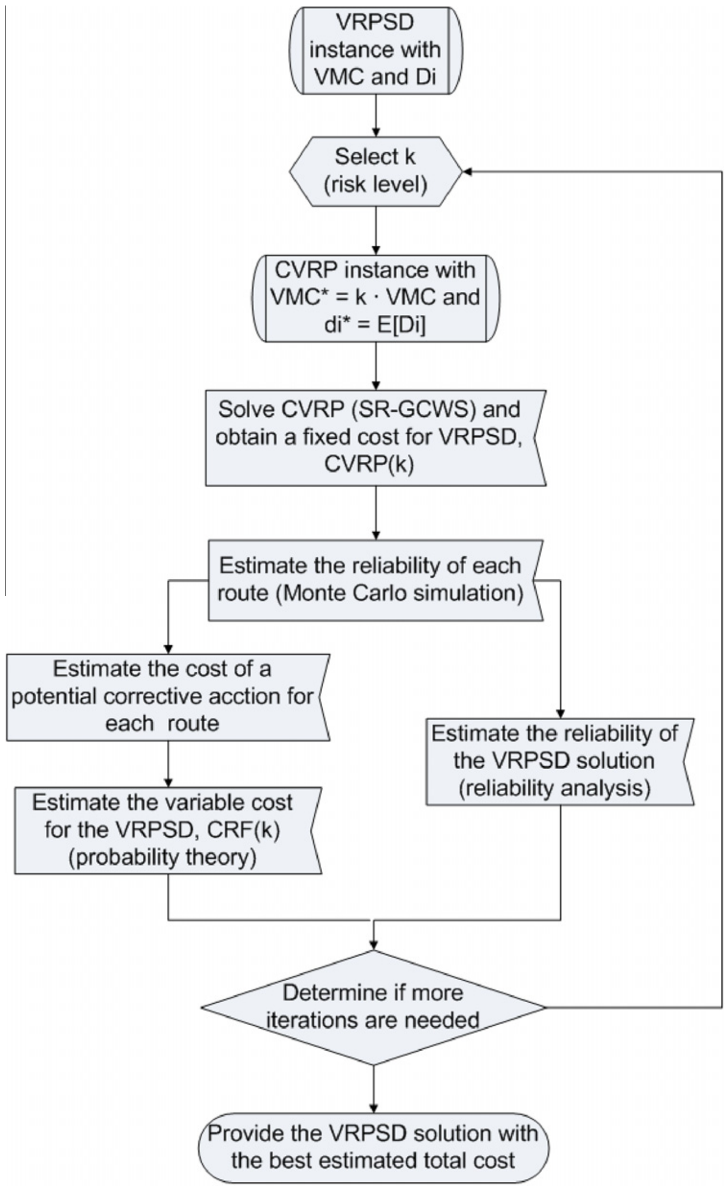

Fig. 1. Flow diagram for the proposed methodology.

7. Obtain an estimate for the reliability index associated with the aprioristic solution. Under the assumption that customer demands are independent – which is a reasonable hypothesis, as discussed earlier – this can be attained by simply multiplying the reliabilities of each route. A solution reliability level can be considered as a measure of the feasibility of that solution in the VRPSD context.
8. Depending on the total costs and the reliability indices associated with the solutions already obtained, repeat the process from Step 1 with a new value of the parameter k – i.e., explore different scenarios to check how different levels of safety stock affect the expected total cost of the VRPSD solution.
9. Finally, provide a sorted list with the best VRPSD solutions found so far as well as their corresponding properties (fixed costs, expected variable costs, expected total costs and reliability index), as shown in Table 1.

5. Advantages and original contributions of our approach

The idea of solving the VRPSD through solving a related CVRP is not new. As a matter of fact, several authors have proposed approaches which transform the VRPSD into an equivalent CVRP and then apply an existing VRP heuristic to the resulting problem (Stewart and Golden, 1983; Laporte et al., 1989). However, our approach differs from those others in sev-

Table 1
Example of a possible output offered to the decision-maker. The data in italics represent the most significative and meaningful case of all options depicted in this table.

Solution ID	# Routes	k	Base cost	Expected variable cost	Expected total cost	Solution Reliability
1	11	0.9	3200	1700	4900	0.76
2	*12*	*0.8*	*3520*	*1200*	*4720*	*0.83*
3	13	0.7	4020	950	4970	0.95

eral aspects: (i) while previous approaches propose to find an equivalent CVRP for a given VRPSD, our approach contemplates the analysis of different risk scenarios, each of them based on a different CVRP as defined by the value of k; (ii) while previous approaches are founded on theoretical chance-constrained models, our approach uses a more practical perspective that combines Monte Carlo simulation and reliability analysis; and (iii) while previous approaches need to assume a particular behavior for the random variables that model customers' demands, our approach offers more flexibility and does not require these assumptions which, in some cases, might be rather artificial or restrictive. Thus, as far as we know, the methodology presented here offers some unique advantages over other existing approaches for solving the VRPSD. Some of the potential benefits of our approach are discussed next.

- The methodology is valid for any statistical distribution with a known mean, either theoretical – e.g., Normal, Log-Normal, Weibull, Gamma, etc. – or experimental – in which case bootstrapping techniques can be used to generate the random values. Being mostly associated with non-negative and continuous values, real-life demands are not likely to follow a Normal or a Poisson distribution – as it is often assumed in the VRPSD literature. On the contrary, they should be modeled by using any theoretical – both discrete and continuous – or experimental distribution offering either non-negative values or asymmetries generated by long right-hand tails – e.g., Log-Normal, Weibull, Gamma, etc.
- In some sense, the methodology is "reducing" a complex VRPSD – where no efficient metaheuristics have been developed yet – to a limited set of more tractable CVRPs where excellent, fast and extensively tested metaheuristics exists. This adds credibility to the quality of the final solution or solutions provided to the decision-maker.
- Moreover, the fact that the decision-maker can consider different scenarios –each of them based on a different level of risk or reliability – makes the methodology more flexible in order to satisfy the utility function of each decision-maker – at the end, he/she will be able to choose from a list of Pareto optimal solutions.
- By using simulation, the methodology can be naturally extended to consider a different distribution for each customer demands, possible dependences among these demands, multiple failures in a single route – which could be generated by occurrence of extreme demands in scenarios with high variability levels – and even different types of recourse actions.
- Finally, notice that the methodology exposed here can be applied to CVRPSD instances of virtually any size (e.g., problems with hundreds or even thousands of nodes) since complexity due to size can be managed by efficient CVRP metaheuristics and Monte Carlo simulation.

6. A numerical example

The methodology described in this paper has been implemented as a C/C++ application. A standard personal computer, Intel® Core™2 Quad CPU at 1.6 GHz and 2 GB RAM was used to perform the experiments described in this section.

In the CVRP literature, there exists a classical set of very well-known benchmarks commonly used to test their algorithm. However, as noticed by Bianchi et al. (2006), there are no commonly used benchmarks in the VRPSD literature and, therefore, each paper presents a different set of randomly generated benchmarks – which, in our opinion, reveals the immatureness of the VRPSD knowledge area when compared with the CVRP area. Unfortunately, most authors only provide details regarding the parameters used to randomly generate their instances, but they do not provide the exact coordinates of the nodes (which are necessary to calculate the traveling costs) or the exact parameters of the distributions that model customer demands. This situation makes it extremely difficult to compare the performance of different approaches. Consequently, we decided to employ a natural generalization of several classical CVRP instances by using random demands instead of constant ones. This approach has at least three advantages: (1) all data details (including nodes coordinates and random demands) are clearly given, so that other authors can use the same data sets for verifying and benchmarking purposes; (2) we are using a well-known set of instances which includes a diversity of clustered and disperse problems of different sizes; and (3) as discussed bellow, our CVRPSD results for each instance can be compared with the corresponding CVRP best-known solution (BKS) – ideally, our results should converge to the CVRP BKS as variances in customers demands tend to zero.

In order to test our methodology, we generalized a set of 55 classical CVRP instances, which details (in terms of nodes coordinates, deterministic demands and vehicle capacity), can be found at http://www.branchandcut.org/. So, for each instance, while we decided to keep all node coordinates and vehicle capacities, we changed d_i, the deterministic demands of client $i(\forall i \in \{1, 2, \ldots, \#nodes - 1\})$ to stochastic demands D_i with $E[D_i] = d_i$. In other words, we considered the demand of each client as a random variable following a well-known statistical distribution with a given mean and a given variance. As stated before, one of the original contributions of our methodology is that it does not assume that demands must necessarily follow either a Normal or a Poisson distribution. To illustrate this, we selected a Log-Normal distribution for modeling demands, although any other distribution with a known mean could have been used instead – notice that in a real-world case, historical data would be used to model each client's demands by a different statistical distribution. The Log-Normal distribution – which is a more natural choice than the Normal distribution when modeling non-negative customers' demands – has two parameters: the location parameter, μ_i, and the scale parameter, σ_i. According to the properties of the Log-Normal distribution, these parameters will be given by the following expressions:

$$\mu_i = \ln(E[D_i]) - \frac{1}{2} \cdot \ln\left(1 + \frac{Var[D_i]}{E[D_i]^2}\right)$$

$$\sigma_i = \left|\sqrt{\ln\left(1 + \frac{Var[D_i]}{E[D_i]^2}\right)}\right|$$

Finally, we considered three different scenarios regarding the variance levels of each instance assuming relatively low variances, specifically $Var[D_i] = 0.05 \cdot d_i$, medium variances, specifically $Var[D_i] = 0.25 \cdot d_i$, and relatively high variances, specifically $Var[D_i] = 0.75 \cdot d_i$, respectively. Fig. 2 illustrates these three scenarios for the case of a node i with an expected demand $E[D_i] = 19$. While in the low-variance scenario 95% of the actual demands fall between 17.2 and 21.0, in the high-variance scenario 95% of the actual demands fall between 12.7 and 27.4. Obviously, as uncertainty (variance) in node demands increases, total expected costs will also tend to increase since more reliable or robust solutions will need to be employed in order to avoid unnecessary route failures and recourse actions. Also, notice that the original CVRP instances are a particular case of these new defined instances with $Var[D_i] = 0, \forall i \in \{1, 2, \ldots, \#nodes - 1\}$.

Before providing and discussing the results for all 55 instances, we will use one in order to better illustrate our approach. Given the classical A-n80-k10 instance, we generalized it by considering customer demands, D_i, as a Log-Normal random variable with $E[D_i] = d_i$ and $Var[D_i] = 0.05 \cdot d_i$ (low-variance scenario) for all $1 \leqslant i < 80$. Next, we set $k = 0.95$ – i.e., we reserved 5% of total vehicle capacity for attending to potential route failures during the actual execution of the delivery stage – and used the SR-GCWS algorithm (Juan et al., 2009a) to solve the CVRP(0.95) instance defined by the following parameters: $d_i^* = E[D_i] = d_i$ and $VMC^* = 0.95 \cdot VMC = 95$. The SR-GCWS algorithm is a multi-start search procedure that uses Monte Carlo simulation to add randomness to the Clarke & Wright savings heuristic. In particular, the procedure makes use of different biased probability distributions – such as the geometric or the triangular – to select the next edge during the solution construction process. In some sense, this algorithm behaves in a manner similar to GRASP procedures (Feo and Resende, 1995). Also, the computational efficiency of this algorithm can be improved by adding some cache and splitting techniques as ex-

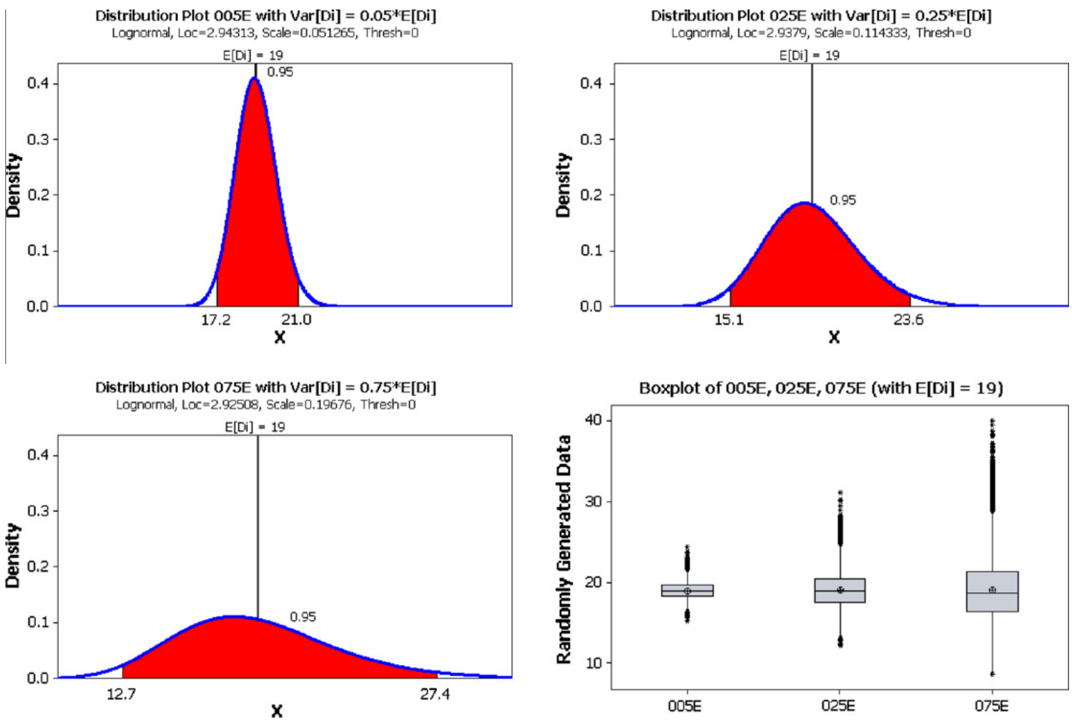

Fig. 2. Different levels of uncertainty (variance) in the Log-Normal customer demands.

plained in Juan et al. (2010). Of course, any other efficient method to solve CVRP instances could have been used instead of the SR-GCWS with similar results.

After a few seconds of execution, we obtained an 11-route solution with a base cost C_{CVRP} = 1838.51. Fig. 3 shows a graphical representation of this solution containing 11 routes together with the best-known solution for the original CVRP instance – which contains only 10 routes due to the fact that no safety stock is needed. Similarities between both solutions can be

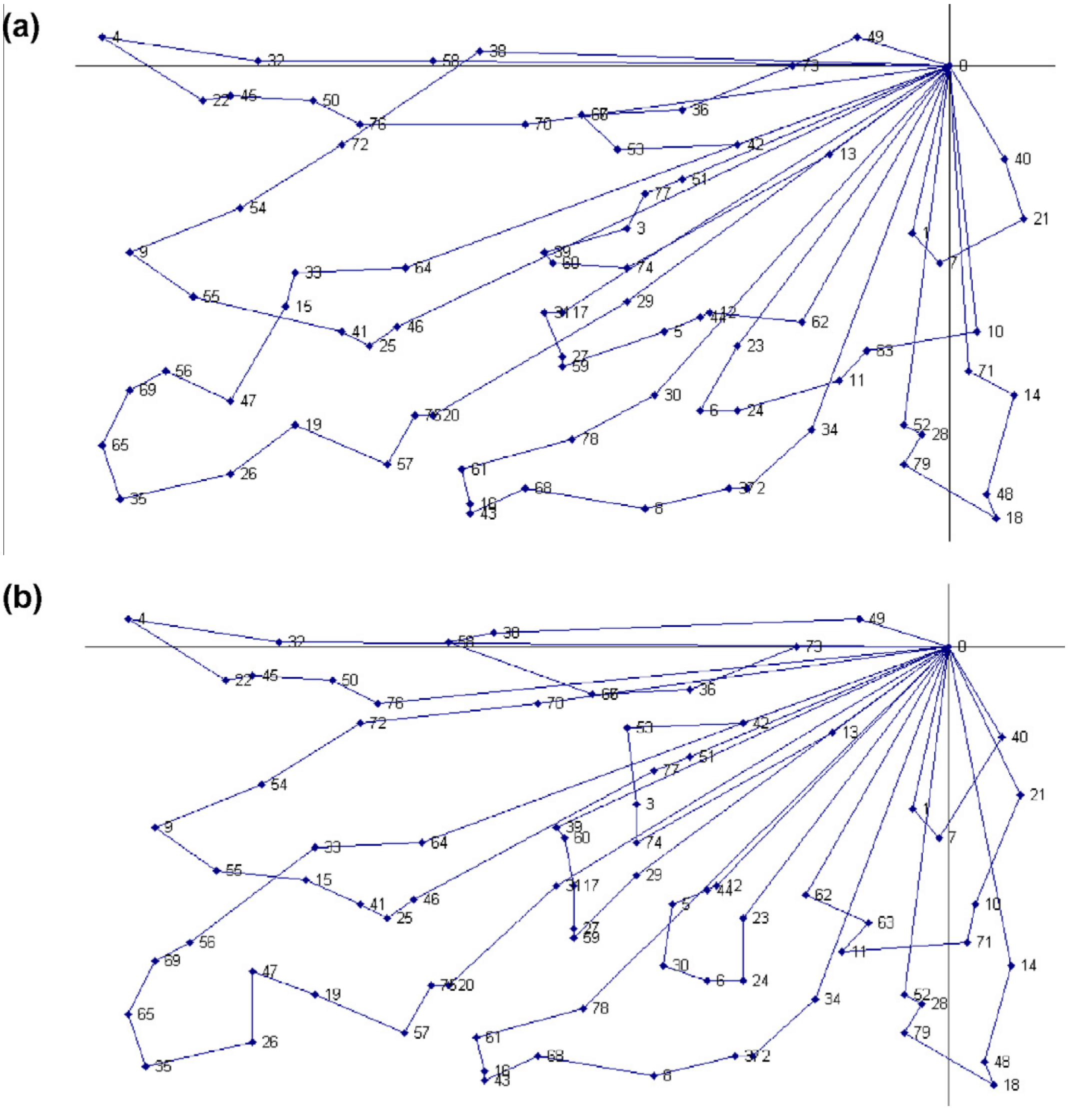

Fig. 3. Solutions for A-n80-k10 (top, 10 routes) and for stochastic A-n80-k10 with k = 0.95 (bottom, 11 routes).

Table 2
Costs and Reliability levels for different A-n80-k10 VRPSD solutions. The data in italics represent the most significative and meaningful case of all options depicted in this table.

k	# Routes	Base costs	Expected variable costs[a]	Expected total costs[a]	Solution reliability[a]
0.92	11	1902.23	0.07	1902.30	0.99
0.93	11	1883.47	0.41	1883.89	0.99
0.94	11	1845.25	1.69	1846.93	0.99
0.95	11	1838.55	4.70	1843.26	0.96
0.96	*10*	*1809.50*	*18.90*	*1828.40*	*0.86*
0.97	10	1803.01	37.96	1840.97	0.73
0.98	10	1785.38	141.95	1927.33	0.30

[a] Estimated values obtained by Monte Carlo simulation.

Table 3
Results for the 55 instances in a low-variance scenario $Var[D_i] = 0.05 * d_i$.

#	Instance	BKS costs (1)	# Routes (1)	BKS expected costs (2)	Gap (1)–(2)	Reliability (1)	OBS expected costs (3)	# Routes (3)	k	Gap (1)–(3)	Reliability (3)
1	A-n32-k5	787.08	5	828.24	5.2%	0.55	797.74	5	0.96	1.4%	1.00
2	A-n33-k5	662.11	5	689.28	4.1%	0.60	678.14	5	0.97	2.4%	0.90
3	A-n33-k6	742.69	6	770.58	3.8%	0.57	762.02	6	0.98	2.6%	0.67
4	A-n37-k5	672.47	5	687.68	2.3%	0.78	685.36	5	0.96	1.9%	0.93
5	A-n38-k5	733.95	5	773.46	5.4%	0.41	754.04	6	0.95	2.7%	0.99
6	A-n39-k6	833.21	6	940.65	12.9%	0.16	839.12	6	0.96	0.7%	0.96
7	A-n45-k6	944.88	6	1076.45	13.9%	0.10	975.12	7	0.96	3.2%	0.93
8	A-n45-k7	1146.77	7	1299.53	13.3%	0.20	1169.85	7	0.95	2.0%	0.95
9	A-n55-k9	1074.46	9	1205.57	12.2%	0.04	1106.31	9	0.96	3.0%	0.87
10	A-n60-k9	1355.80	9	1517.79	11.9%	0.12	1383.90	9	0.95	2.1%	0.94
11	A-n61-k9	1039.08	9	1145.39	10.2%	0.09	1073.99	10	0.97	3.4%	0.75
12	A-n63-k9	1622.15	9	1884.98	16.2%	0.08	1682.37	10	0.95	3.7%	0.95
13	A-n65-k9	1181.69	9	1352.64	14.5%	0.06	1241.51	10	0.96	5.1%	0.99
14	A-n80-k10	1766.50	10	2028.28	14.8%	0.08	1828.40	10	0.96	3.5%	0.86
15	B-n31-k5	676.09	5	726.15	7.4%	0.50	682.65	5	0.95	1.0%	0.99
16	B-n35-k5	956.29	5	1068.48	11.7%	0.45	986.58	5	0.94	3.2%	1.00
17	B-n39-k5	553.16	5	589.74	6.6%	0.49	564.47	5	0.96	2.0%	0.96
18	B-n41-k6	834.92	6	912.97	9.3%	0.23	858.91	7	0.95	2.9%	0.99
19	B-n45-k5	754.22	5	801.66	6.3%	0.23	764.27	6	0.94	1.3%	1.00
20	B-n50-k7	744.23	7	800.04	7.5%	0.31	752.54	7	0.95	1.1%	1.00
21	B-n52-k7	749.96	7	826.93	10.3%	0.33	762.26	7	0.95	1.6%	0.99
22	B-n56-k7	712.92	7	830.88	16.5%	0.17	739.23	7	0.95	3.7%	0.98
23	B-n57-k9	1602.28	9	1922.49	20.0%	0.06	1626.89	9	0.95	1.5%	0.98
24	B-n64-k9	868.31	9	1070.90	23.3%	0.02	903.31	10	0.95	4.0%	0.98
25	B-n67-k10	1039.36	10	1207.88	16.2%	0.08	1096.78	10	0.96	5.5%	0.92
26	B-n68-k9	1276.20	9	1525.45	19.5%	0.04	1311.58	9	0.94	2.8%	0.99
27	B-n78-k10	1227.90	10	1370.36	11.6%	0.11	1299.00	10	0.97	5.8%	0.76
28	E-n22-k4	375.28	4	375.28	0.0%	1.00	375.28	4	1.00	0.0%	1.00
29	E-n30-k3	505.01	4	505.01	0.0%	1.00	505.01	4	1.00	0.0%	1.00
30	E-n33-k4	837.67	4	838.80	0.1%	0.99	838.80	4	1.00	0.1%	1.00
31	E-n51-k5	524.61	5	539.21	2.8%	0.27	537.11	5	0.98	2.4%	0.65
32	E-n76-k7	687.60	7	703.46	2.3%	0.58	695.49	7	0.97	1.1%	1.00
33	E-n76-k10	835.26	10	905.54	8.4%	0.07	861.77	11	0.98	3.2%	0.70
34	E-n76-k14	1024.40	15	1121.13	9.4%	0.04	1056.36	15	0.96	3.1%	0.90
35	F-n45-k4	723.54	4	757.83	4.7%	0.37	729.97	5	0.96	0.9%	1.00
36	F-n72-k4	241.97	4	247.76	2.4%	0.70	247.29	4	0.99	2.2%	1.00
37	F-n135-k7	1164.73	7	1263.53	8.5%	0.46	1191.26	7	0.99	2.3%	0.99
38	M-n101-k10	819.56	10	933.10	13.9%	0.07	866.83	10	0.95	5.8%	0.99
39	M-n121-k7	1043.88	7	1101.50	5.5%	0.08	1074.85	8	0.98	3.0%	0.83
40	P-n19-k2	212.66	2	217.48	2.3%	0.85	216.96	2	0.98	2.0%	0.85
41	P-n20-k2	217.42	2	233.06	7.2%	0.65	220.89	2	0.99	1.6%	0.93
42	P-n22-k2	217.85	2	226.34	3.9%	0.65	224.13	2	0.97	2.9%	0.96
43	P-n22-k8	588.79	9	619.74	5.3%	0.50	589.39	9	0.97	0.1%	1.00
44	P-n40-k5	461.73	5	466.25	1.0%	0.77	464.51	5	0.97	0.6%	1.00
45	P-n50-k8	632.69	9	697.56	10.3%	0.10	646.65	9	0.97	2.2%	0.91
46	P-n50-k10	699.56	10	753.27	7.7%	0.16	721.91	10	0.96	3.2%	0.90
47	P-n51-k10	741.50	10	828.90	11.8%	0.05	761.62	11	0.96	2.7%	0.90
48	P-n55-k7	570.27	7	588.59	3.2%	0.39	578.61	7	0.95	1.5%	1.00
49	P-n55-k15	944.56	16	1033.66	9.4%	0.04	991.85	17	0.95	5.0%	0.79
50	P-n60-k10	748.07	10	790.46	5.7%	0.20	761.49	10	0.97	1.8%	0.85
51	P-n65-k10	795.66	10	854.65	7.4%	0.13	811.83	10	0.97	2.0%	0.92
52	P-n70-k10	829.93	10	906.87	9.3%	0.05	853.83	11	0.96	2.9%	0.94
53	P-n76-k4	598.19	4	611.81	2.3%	0.49	609.31	5	0.98	1.9%	0.97
54	P-n76-k5	633.32	5	651.51	2.9%	0.34	648.86	5	0.98	2.5%	0.85
55	P-n101-k4	691.29	4	693.52	0.3%	0.96	693.52	4	0.98	0.3%	0.96
Averages		7.1		8.3%	0.34		7.4	0.96	2.4%	0.93	

easily observed. Notice that, by keeping a 5% security stock, the resulting solution will be more reliable – and therefore will have lower variable costs – than the pseudo-optimal solution obtained for the original CVRP, even when the base costs of the later might be somewhat lower (1766.50 in the case of the A-n80-k10).

Next, we used Monte Carlo simulation to estimate the reliability of each route in the obtained solution: For each client i in a route r, we used the corresponding statistical distribution – Log-Normal with mean d_i and variance $0.05 \cdot d_i$ in this case – to generate a random value for D_i and then we checked whether or not the total demand in r exceeded the real vehicle capacity $VMC = 100$. Observe that in this example we are assuming independence among different customer demands; however, since our approach uses simulation, it could also consider dependences among different customer demands as well as different statistical distributions for each node demand and multiple route failures due to extreme values in actual demands. By repeating this process a pre-defined number of times (e.g., 10,000 times) we obtained a confidence interval around the probability that each route can be implemented without suffering a route failure, R_j ($1 \leqslant j \leqslant 11$). This same Monte Carlo simulation process was also used to obtain estimates for the variable costs of the aforementioned solution, C_{RF}; whenever a vehicle ran out of load due to a larger-than-expected accumulated random demand and needed to come back to the depot for a refill, the costs of a round-trip from the current node to the depot were computed and averaged as variable costs. Finally, we estimated the solution's reliability by simply multiplying reliabilities of each route since a solution will be successfully implemented without route failures if, and only if, each and every route in that solution is implemented without route failures, i.e., a solution can be seen as a series system of routes. In summary, after setting $k = 0.95$, our method was able to

Table 4
Results for the 55 instances in a medium-variance scenario $Var[D_i] = 0.25 * d_i$.

#	Instance	BKS costs (1)	# Routes (1)	BKS expected costs (4)	Gap (1)–(4)	Reliability (1)	OBS expected costs (5)	# Routes (5)	k	Gap (1)–(5)	Reliability (5)
1	A-n32-k5	787.08	5	867.93	10.3%	0.29	815.78	5	0.92	3.6%	0.86
2	A-n33-k5	662.11	5	726.04	9.7%	0.30	693.09	5	0.95	4.7%	0.81
3	A-n33-k6	742.69	6	792.49	6.7%	0.29	775.19	7	0.95	4.4%	0.89
4	A-n37-k5	672.47	5	707.31	5.2%	0.53	701.38	5	0.93	4.3%	0.79
5	A-n38-k5	733.95	5	801.86	9.3%	0.19	762.87	6	0.93	3.9%	0.93
6	A-n39-k6	833.21	6	963.15	15.6%	0.08	858.78	6	0.92	3.1%	0.92
7	A-n45-k6	944.88	6	1115.53	18.1%	0.04	990.74	7	0.94	4.9%	0.78
8	A-n45-k7	1146.77	7	1375.16	19.9%	0.09	1208.11	7	0.93	5.3%	0.72
9	A-n55-k9	1074.46	9	1214.44	13.0%	0.06	1137.69	10	0.94	5.9%	0.70
10	A-n60-k9	1355.80	9	1576.73	16.3%	0.07	1452.36	10	0.93	7.1%	0.74
11	A-n61-k9	1039.08	9	1193.77	14.9%	0.03	1105.31	10	0.92	6.4%	0.77
12	A-n63-k9	1622.15	9	2002.01	23.4%	0.03	1769.05	10	0.93	9.1%	0.70
13	A-n65-k9	1181.69	9	1403.74	18.8%	0.02	1274.47	10	0.96	7.9%	0.62
14	A-n80-k10	1766.50	10	2126.74	20.4%	0.03	1923.30	11	0.89	8.9%	0.91
15	B-n31-k5	676.09	5	753.82	11.5%	0.39	695.57	5	0.91	2.9%	0.93
16	B-n35-k5	956.29	5	1115.03	16.6%	0.33	992.01	5	0.89	3.7%	0.97
17	B-n39-k5	553.16	5	608.82	10.1%	0.37	579.60	5	0.94	4.8%	0.78
18	B-n41-k6	834.92	6	969.74	16.1%	0.13	884.28	7	0.92	5.9%	0.86
19	B-n45-k5	754.22	5	827.48	9.7%	0.13	767.74	6	0.90	1.8%	0.94
20	B-n50-k7	744.23	7	827.32	11.2%	0.20	761.54	7	0.89	2.3%	0.94
21	B-n52-k7	749.96	7	848.27	13.1%	0.25	774.01	7	0.89	3.2%	0.97
22	B-n56-k7	712.92	7	856.15	20.1%	0.11	755.90	7	0.89	6.0%	0.94
23	B-n57-k9	1602.28	9	2003.64	25.0%	0.03	1686.40	9	0.91	5.2%	0.84
24	B-n64-k9	868.31	9	1086.15	25.1%	0.01	929.61	10	0.92	7.1%	0.81
25	B-n67-k10	1039.36	10	1277.39	22.9%	0.04	1121.49	11	0.93	7.9%	0.85
26	B-n68-k9	1276.20	9	1634.31	28.1%	0.01	1381.21	9	0.95	8.2%	0.44
27	B-n78-k10	1227.90	10	1522.01	24.0%	0.01	1333.02	11	0.91	8.6%	0.85
28	E-n22-k4	375.28	4	375.49	0.1%	1.00	375.49	4	1.00	0.1%	1.00
29	E-n30-k3	505.01	4	505.01	0.0%	1.00	505.01	4	1.00	0.0%	1.00
30	E-n33-k4	837.67	4	858.91	2.5%	0.88	844.08	4	0.99	0.8%	0.99
31	E-n51-k5	524.61	5	551.21	5.1%	0.17	548.63	6	0.97	4.6%	0.60
32	E-n76-k7	687.60	7	717.42	4.3%	0.37	700.55	7	0.96	1.9%	0.80
33	E-n76-k10	835.26	10	935.03	11.9%	0.03	873.81	11	0.92	4.6%	0.89
34	E-n76-k14	1024.40	15	1169.79	14.2%	0.02	1087.01	16	0.93	6.1%	0.64
35	F-n45-k4	723.54	4	769.79	6.4%	0.34	729.97	5	0.96	0.9%	1.00
36	F-n72-k4	241.97	4	249.92	3.3%	0.56	247.29	4	0.99	2.2%	1.00
37	F-n135-k7	1164.73	7	1271.72	9.2%	0.43	1200.91	7	0.97	3.1%	1.00
38	M-n101-k10	819.56	10	939.56	14.6%	0.06	886.16	10	0.98	8.1%	0.62
39	M-n121-k7	1043.88	7	1178.76	12.9%	0.06	1140.95	8	0.96	9.3%	0.60
40	P-n19-k2	212.66	2	228.35	7.4%	0.59	222.37	3	0.94	4.6%	0.96
41	P-n20-k2	217.42	2	238.83	9.8%	0.53	232.49	2	0.97	6.9%	0.63
42	P-n22-k2	217.85	2	229.08	5.2%	0.54	229.08	2	1.00	5.2%	0.54
43	P-n22-k8	588.79	9	619.77	5.3%	0.50	589.41	9	0.98	0.1%	1.00
44	P-n40-k5	461.73	5	470.99	2.0%	0.60	466.92	5	0.96	1.1%	0.90
45	P-n50-k8	632.69	9	711.47	12.5%	0.04	655.67	9	0.92	3.6%	0.89
46	P-n50-k10	699.56	10	783.61	12.0%	0.05	736.71	11	0.92	5.3%	0.82
47	P-n51-k10	741.50	10	851.78	14.9%	0.02	785.64	11	0.96	6.0%	0.39
48	P-n55-k7	570.27	7	608.59	6.7%	0.18	582.94	7	0.93	2.2%	0.90
49	P-n55-k15	944.56	16	1114.75	18.0%	0.00	1032.11	18	0.93	9.3%	0.55
50	P-n60-k10	748.07	10	821.07	9.8%	0.07	790.12	11	0.92	5.6%	0.93
51	P-n65-k10	795.66	10	854.79	7.4%	0.11	835.04	10	0.95	4.9%	0.42
52	P-n70-k10	829.93	10	946.00	14.0%	0.02	868.20	11	0.93	4.6%	0.90
53	P-n76-k4	598.19	4	616.10	3.0%	0.33	616.07	5	0.98	3.0%	0.69
54	P-n76-k5	633.32	5	656.29	3.6%	0.20	651.26	6	0.96	2.8%	0.82
55	P-n101-k4	691.29	4	698.25	1.0%	0.74	695.92	4	0.96	0.7%	0.95
Averages		7.1		11.8%	0.25		7.6	0.94	4.6%	0.81	

provide a VRPSD solution with an estimated reliability level of 0.96 (i.e., a solution which will only suffer from route failures 4% of the times) and a total estimated cost given by $C_{VRPSD} = C_{CVRP} + C_{RF} = 1838.55 + 4.70 = 1843.26$.

From a computational point of view, the whole process described can be completed in just a few seconds (less than 10 s in our standard personal computer). Therefore, it seems computationally affordable to repeat these steps for other values of the parameter k. Table 2 shows alternative solutions (for the A-n80-k10 VRPSD with low-variances) that we obtained using different values of k. For each of the considered solutions, the following additional information is provided: number of routes employed, base costs (those from the associated CVRP(k) instance), estimates for the variable and total costs, and an estimate of the solution reliability index.

Table 5
Results for the 55 instances in a high-variance scenario $Var[D_i] = 0.75 * d_i$.

#	Instance	BKS costs (1)	# Routes (1)	BKS expected costs (6)	Gap (1)–(6)	Reliability (1)	OBS expected costs (7)	# Routes (7)	k	Gap (1)–(7)	Reliability (7)
1	A-n32-k5	787.08	5	889.99	13.1%	0.23	853.63	5	0.92	8.5%	0.55
2	A-n33-k5	662.11	5	751.24	13.5%	0.20	715.18	6	0.89	8.0%	0.77
3	A-n33-k6	742.69	6	814.24	9.6%	0.18	803.96	7	0.95	8.2%	0.58
4	A-n37-k5	672.47	5	732.29	8.9%	0.36	719.09	5	0.85	6.9%	0.91
5	A-n38-k5	733.95	5	820.56	11.8%	0.13	777.18	6	0.90	5.9%	0.86
6	A-n39-k6	833.21	6	969.20	16.3%	0.07	890.16	6	0.93	6.8%	0.60
7	A-n45-k6	944.88	6	1142.27	20.9%	0.03	1026.08	7	0.95	8.6%	0.44
8	A-n45-k7	1146.77	7	1411.09	23.0%	0.06	1279.20	7	0.93	11.5%	0.35
9	A-n55-k9	1074.46	9	1259.62	17.2%	0.04	1189.56	10	0.87	10.7%	0.71
10	A-n60-k9	1355.80	9	1631.51	20.3%	0.04	1539.37	10	0.94	13.5%	0.30
11	A-n61-k9	1039.08	9	1223.96	17.8%	0.01	1154.93	10	0.93	11.1%	0.34
12	A-n63-k9	1622.15	9	2047.45	26.2%	0.01	1867.97	10	0.89	15.2%	0.47
13	A-n65-k9	1181.69	9	1424.04	20.5%	0.01	1321.27	10	0.91	11.8%	0.43
14	A-n80-k10	1766.50	10	2207.69	25.0%	0.02	1994.10	11	0.89	12.9%	0.53
15	B-n31-k5	676.09	5	775.84	14.8%	0.32	713.90	5	0.89	5.6%	0.86
16	B-n35-k5	956.29	5	1145.13	19.7%	0.22	1036.03	5	0.91	8.3%	0.66
17	B-n39-k5	553.16	5	655.32	18.5%	0.27	602.72	6	0.88	9.0%	0.78
18	B-n41-k6	834.92	6	1008.46	20.8%	0.09	929.01	7	0.88	11.3%	0.86
19	B-n45-k5	754.22	5	847.67	12.4%	0.09	790.20	6	0.92	4.8%	0.61
20	B-n50-k7	744.23	7	853.48	14.7%	0.14	790.54	7	0.89	6.2%	0.61
21	B-n52-k7	749.96	7	881.49	17.5%	0.16	794.33	7	0.89	5.9%	0.66
22	B-n56-k7	712.92	7	886.93	24.4%	0.08	774.66	8	0.88	8.7%	0.80
23	B-n57-k9	1602.28	9	2059.75	28.6%	0.02	1793.16	9	0.91	11.9%	0.38
24	B-n64-k9	868.31	9	1103.81	27.1%	0.01	968.50	11	0.84	11.5%	0.85
25	B-n67-k10	1039.36	10	1287.63	23.9%	0.02	1170.91	11	0.88	12.7%	0.63
26	B-n68-k9	1276.20	9	1660.42	30.1%	0.01	1453.30	10	0.90	13.9%	0.46
27	B-n78-k10	1227.90	10	1582.81	28.9%	0.01	1383.07	12	0.86	12.6%	0.69
28	E-n22-k4	375.28	4	378.28	0.8%	0.93	378.25	4	0.99	0.8%	0.93
29	E-n30-k3	505.01	4	505.01	0.0%	1.00	505.01	4	0.88	0.0%	1.00
30	E-n33-k4	837.67	4	883.62	5.5%	0.73	844.13	4	0.98	0.8%	1.00
31	E-n51-k5	524.61	5	561.35	7.0%	0.12	555.26	6	0.94	5.8%	0.73
32	E-n76-k7	687.60	7	743.71	8.2%	0.09	704.48	7	0.93	2.5%	0.79
33	E-n76-k10	835.26	10	970.33	16.2%	0.01	894.96	11	0.92	7.1%	0.44
34	E-n76-k14	1024.40	15	1221.11	19.2%	0.00	1127.04	16	0.89	10.0%	0.43
35	F-n45-k4	723.54	4	772.90	6.8%	0.30	731.01	5	0.96	1.0%	0.98
36	F-n72-k4	241.97	4	252.15	4.2%	0.45	247.51	4	0.99	2.3%	0.99
37	F-n135-k7	1164.73	7	1280.75	10.0%	0.40	1208.99	7	0.97	3.8%	0.91
38	M-n101-k10	819.56	10	943.69	15.1%	0.06	922.16	10	0.96	12.5%	0.30
39	M-n121-k7	1043.88	7	1230.53	17.9%	0.03	1218.10	8	0.98	16.7%	0.14
40	P-n19-k2	212.66	2	236.63	11.3%	0.46	227.41	3	0.95	6.9%	0.85
41	P-n20-k2	217.42	2	244.14	12.3%	0.44	239.34	2	0.99	10.1%	0.48
42	P-n22-k2	217.85	2	232.30	6.6%	0.47	232.30	2	1.00	6.6%	0.47
43	P-n22-k8	588.79	9	620.24	5.3%	0.49	591.05	9	0.98	0.4%	0.97
44	P-n40-k5	461.73	5	477.88	3.5%	0.47	474.21	5	0.95	2.7%	0.64
45	P-n50-k8	632.69	9	733.49	15.9%	0.02	675.00	9	0.93	6.7%	0.47
46	P-n50-k10	699.56	10	806.07	15.2%	0.02	763.24	11	0.91	9.1%	0.44
47	P-n51-k10	741.50	10	876.30	18.2%	0.01	822.43	11	0.96	10.9%	0.14
48	P-n55-k7	570.27	7	625.80	9.7%	0.09	593.15	7	0.92	4.0%	0.79
49	P-n55-k15	944.56	16	1156.71	22.5%	0.00	1087.04	19	0.87	15.1%	0.52
50	P-n60-k10	748.07	10	869.55	16.2%	0.02	809.56	11	0.92	8.2%	0.54
51	P-n65-k10	795.66	10	904.90	13.7%	0.05	861.92	11	0.93	8.3%	0.33
52	P-n70-k10	829.93	10	959.11	15.6%	0.01	889.26	11	0.93	7.1%	0.47
53	P-n76-k4	598.19	4	622.71	4.1%	0.18	621.75	5	0.89	3.9%	0.99
54	P-n76-k5	633.32	5	660.43	4.3%	0.12	659.67	6	0.90	4.2%	0.96
55	P-n101-k4	691.29	4	703.87	1.8%	0.53	699.87	4	0.96	1.2%	0.78
Averages		7.1		14.8%	0.19		7.7	0.92	7.9%	0.64	

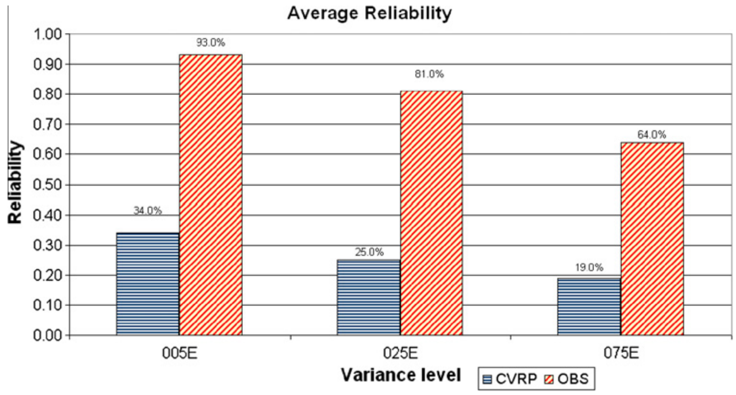

Fig. 4. Average reliability indices for different uncertainty levels and solutions.

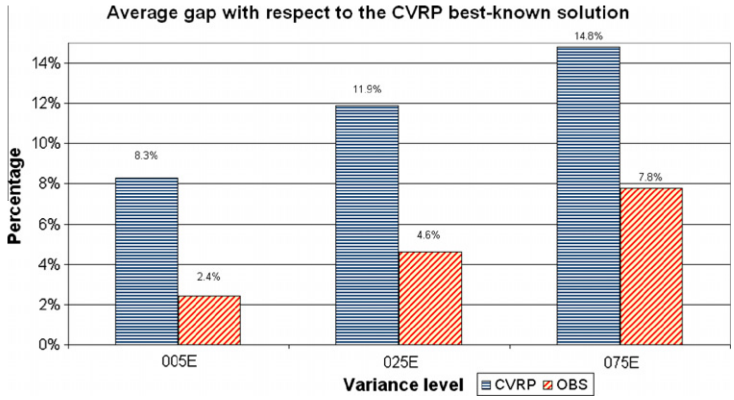

Fig. 5. Average gaps for different uncertainty levels and solutions.

As expected from the discussion developed during the methodology explanation, higher values of the parameter k tend to produce solutions with less routes and lower base costs, but they also tend to have lower reliability levels – i.e., they are less robust against variations in the demands – and, therefore, they tend to have higher variable costs, which can significantly affect the expected total costs of the associated VRPSD solution. Among the different alternatives presented in Table 2, a good candidate for most decision-makers would be the VRPSD solution obtained with $k = 0.96$, since it combines the lowest total expected costs (1828.40) with a reasonable reliability level (0.86). Note that if the decision-maker decides to use for the VRPSD the pseudo-optimal solution obtained for the original deterministic CVRP, he/she should need to consider the expected variable costs due to route failures (in the considered low-variance scenario, the total expected costs for this solution has an approximated value of 2028.28). In fact, note that the base costs of a pseudo-optimal solution s for the original CVRP can be considered as a lower bound for the CVRPSD optimal solution, while the total expected costs associated with s when demands are random represent an upper bound for the CVRPSD optimal solution.

Tables 3–5 show the complete results obtained for all 55 classical instances we generalized and tested. Each of these tables corresponds to one of the three uncertainty scenarios (low-variance, medium-variance and high-variance) which were previously described. As discussed in the next section, similar ideas and conclusions to the ones reached for the A-n80-k10 instance with low-variance also apply for other instances and uncertainty levels.

7. Discussion of results

Even in a low-variability scenario like the one defined by $Var[D_i] = 0.05 \cdot d_i$ (Table 3), results show that the best-known solutions (BKS) for the CVRP, marked as (1), tend to provide a poor average reliability level of 0.34 when applied to the VRPSD. As discussed before, lower reliability levels imply more route failures and, therefore, higher expected variable costs. This explains the average gap of 8.3% between (1) and (3), the later containing the total expected costs associated with the BKS when demands are stochastic instead of deterministic. Meanwhile, our methodology is able to provide alternative solutions for the VRPSD with a higher reliability level (0.93 on the average for this scenario) and an average gap of 2.4%, that is: total expected costs provided by our best solutions (OBS) for the VRPSD low-variance scenario are only 2.4% higher, on the average, than base costs provided by the BKS for the deterministic CVRP scenario. Notice also that the mean number of routes for the chosen solutions has slightly increased from 7.1 in (1) to 7.4 in (3) – use of safety stocks tends to increase somewhat the number of necessary routes, but the resulting solution tends to be more reliable or robust. Finally, in this low-variability scenario the average value for the recommended safety-stocks level is $k = 0.96$, that is: each vehicle will reserve up to 4% of its capacity to attend unexpected demands.

In the case of the medium-variability scenario with $Var[D_i] = 0.25 \cdot d_i$ (Table 4), the CVRP BKS (1) are even less reliable than before (the average reliability level is about 0.25) and, therefore, their associated variable costs tend to be higher in this new scenario characterized by a higher degree or uncertainty. Thus, the average gap between (1) and these same solutions when considering variable costs, (4), is about 11.9%. In contrast to this, our methodology is able to provide more reliable solutions (approximated average reliability of 0.81), with an average gap of 4.6% with respect to the BKS CVRP solutions. Notice that, even when the costs registered at (1) represent a lower bound of the costs associated to the VRPSD optimal solution, the later will be strictly larger than the former in most cases, especially for high uncertainty (variance) levels. Other interesting results in this second scenario are the average number of routes (7.6) and the average value of selected safety stocks, $k = 0.94$, which is slightly lower than before (as higher the uncertainty level, lower the safety stocks index).

Finally, the high-variability scenario (Table 5) reports an estimated average reliability of just 0.19 for the CVRP BKS (1), and a corresponding average gap of 14.8%. At the same time, our methodology generates solutions with an estimated reliability index of 0.64 and an approximated gap of 7.8% – once more, notice that this gap is not with respect to the optimal CVRPSD solution but with respect to the CVRP BKS – that is, the given gap is an upper bound for the real gap between our best solution and the CVRPSD optimal solution. Also, the average number of routes in our solutions is 7.7, and the average value of the selected safety stocks is $k = 0.92$ – again, slightly smaller than in previous scenarios.

Fig. 4 graphically summarizes the average reliability indices for each uncertainty level and solution (CVRP BKS vs. our best solution). Similarly, Fig. 5 shows the average gaps for each uncertainty level and solution. Notice that, for a given CVRPSD solution, s, the higher the variance, the lower the reliability index of s. Also, the higher the variance, the larger the gaps (in costs) between s and the CVRP BKS – again, it is importance to notice that this is value is an upper bound value for the gap between s and the (unknown) optimal CVRPSD solution.

8. Conclusions and future work

We have presented a hybrid approach to solving the Vehicle Routing Problem with Stochastic Demands (VRPSD). The approach combines Monte Carlo simulation with reliability indices and a well-tested metaheuristic for the Capacitated Vehicle Routing Problem (CVRP). One of the basic ideas of our methodology is to consider a vehicle capacity lower than the actual maximum vehicle capacity when designing VRPSD solutions. This way, this capacity surplus or safety stocks can be used when necessary to cover route failures without having to assume the usually high costs involved in vehicle restock trips. Another important idea is to transform the VRPSD instance to a limited set of CVRP instances – each of them defined by a given safety-stocks level – to which efficient solving methods can be applied. Our approach provides the decision-maker with a set of alternative solutions, each of them characterized by their total estimated costs and their reliability values – the former reflecting the probability of that solution being a feasible one – leaving to him/her the responsibility of selecting the specific solution to be implemented according to his/her utility function. Although other previous works have proposed to benefit from the relationship between the VRPSD and the CVRP, they usually require hard assumptions that are not always satisfied in realistic scenarios. On the contrary, our approach relaxes most of these assumptions and, therefore, it allows for considering more realistic customer demand scenarios. Thus, for example, our approach can be used to solve CVRPSD instances with hundreds of nodes in a reasonable time and, even more important, it is valid for virtually any statistical distribution – the one that best fits historical data on customer demands. Also, the methodology can be naturally extended to consider: (a) different distributions for different customer demands, (b) possible dependences among these demands, (c) multiple failures per route, and (d) multiple recourse strategies. A complete set of tests have been performed to illustrate the methodology and analyze its efficiency as well as its potential benefits over previous works.

As future work, we plan to compare the efficiency and robustness of the proposed approach against alternative optimization methods lying on mathematical or constraint programming models. More specifically, the main idea is to replace the metaheuristic used in the step 4 of the proposed methodology by a rigorous two-stage stochastic optimization approach. In this way, the solution generated will simultaneously consider multiple scenarios for the customer demands instead of the one based only on the expected value of each random demand. By considering the uncertain information of demands in a proactive way, we expect to be able to generate cost-effective solutions with higher reliability, although at the expense of a potential significant increase of the computational times. The trade-off between solution quality and computational times will be carefully evaluated. In addition, since routes failures may be reduced but never eliminated, we also plan to develop efficient dynamic optimization methods for quickly updating the original solution after the occurrence of route failures.

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