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High Roughness Time Series Forecasting Based on Energy Associated of Series

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Abstract: In this study, an algorithm to adjust parameters of high roughness time series based on energy associated of series using a feed-forward NN-based model is presented. The criterion for adjustment consists of building time series values from forecasted time series area and taking into account the roughness of series. These values are approximated by the NN to make a primitive calculated as an area by the predictor filter used as a new entrance. A comparison between this work and another that involves a similar approach to test time series prediction, indicates an improvement for certain sort of series. The NN filter output is intended to approximate the current value available from the series which has the same Hurst Parameter as the real time series. The proposed approach is tested over five time series obtained from samples of Mackey-Glass delay differential equations (MG). Therefore, these results show a model performance for time series forecasting and encourage to be applied for meteorological variables measurements such as soil moisture series, daily rainfall and monthly cumulative rainfall time series forecasting..

Key words: Neural networks, time series forecast, primitive, Hurst's parameter, Mackey-Glass.

1. Introduction

In order to use and model time series for control problems in agricultural activities such as the availability of estimated scenarios for water availability [1], seedling growth [2] and decision-making, natural phenomena prediction is a challenging topic.

Hence, to attain an expected production at the end of the campaign for farmers, such issue may be accomplished with certain accuracy by taking an ensemble of measurement points [3-4]. Here, the proposed approach is based on the classical non linear autoregressive (NAR) filter using time lagged feed-forward neural networks (FFNN), where the data are taken from the MG benchmark equation whose forecast is simulated by a Monte Carlo [5] approach. The number of filter parameters is set in function of the roughness of the time series, in such a way that the error between the smoothness of the time series data and the forecasted data (energy associated of series) modifies the number of the filter parameters.

2. Theory and Proposed Framework

2.1 Samples of Mackey Glass Equation

The MG equation serves to model natural phenomena and has been used by different authors to

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perform comparisons between different techniques for prediction and regression models [6-7].

Here we propose an algorithm to predict the energy associated to data time series taken from the solution of the MG equation [8]. The MG equation is explained by the time delay differential equation defined as,

$$\dot{y}(t) = \frac{\alpha y(t-\tau)}{1+y^c(t-\tau)} - \beta y(t), \quad (1)$$

where α, β, and c are parameters and τ is the delay time.

According as τ increases, the solution turns from periodic to chaotic. The equation is solved by a standard fourth order Runge-Kutta integration step. By setting the parameter β ranging between 0.1 and 0.9 the stochastic dependence of the deterministic time series obtained varies according to its roughness. The performance of the proposed method is tested with the SMAPE index and it is compared with a traditional NN based predictor.

2.2 Fractional Brownian Motion

This process was introduced by Kolmogorov [9] and studied by Mandelbrot [10] and Van Ness in Ref. [11], where a stochastic integral representation in terms of a standard Brownian motion was established. The parameter H is called Hurst index from the statistical analysis, developed by the climatologist Hurst [12], of the yearly water run-offs of Nile River.

In this work due to the random process of the time series, it is proposed to use the Hurst's parameter in the learning process to modify on-line the number of patterns, the number of iterations, and the number of filter's inputs. This H serves to have an idea of roughness of a signal, and also to determine its stochastic dependence. The definition of the Hurst's parameter appears in the Brownian motion from generalizing the integral to a fractional one. Brownian motion played a central role in the modeling of many random behaviors in nature and in stochastic analysis and formed the basis for the development of an enormous branch of applications.

The Fractional Brownian Motion (fBm) is defined through its stochastic representation:

$$B_H(t) = \frac{1}{\Gamma\left(H+\frac{1}{2}\right)}\left(\int_{-\infty}^{0}\left((t-s)^{H-\frac{1}{2}} - (-s)^{H-\frac{1}{2}}\right)dB(s) + \int_0^t (t-s)^{H-\frac{1}{2}} dB(s)\right) \quad (2)$$

where, $\Gamma(\cdot)$ represents the Gamma function

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx \quad (3)$$

and 0 < H < 1 is called the Hurst parameter. The integrator B is a stochastic process, ordinary Brownian motion. Note, that B is recovered by taking H = 1/2 in Eq. (2) and B is defined on some probability space (Ω, F, P). Thus, an fBm is a time continuous Gaussian process depending on the so-called Hurst parameter 0 < H < 1. The ordinary Brownian motion is generalized to H = 0.5 and its derivative is the white noise. The fBm is self-similar in distribution and the variance of the increments is defined by

$$Var\left(B_H(t) - B_H(s)\right) = v\left|t-s\right|^{2H} \quad (4)$$

where *v* is a positive constant.

This special form of the variance of the increments suggests various ways to estimate the parameter H. In fact, there are different methods for computing the parameter H associated to Brownian motion [13]. In this work, the algorithm uses a wavelet-based method for estimating H from a trace path of the fBm with parameter H [14].

2.3 NN-Based NAR Model

One of the motivations for this study follows the closed-loop control scheme [15] where the controller considers meteorological future conditions for designing the control law as shown in Fig. 1. In that scheme the controller considers the actual state of the crop by a state observer and the meteorological variables, referred by x(*k*) and R_o, respectively. However, in this paper only the controller's portion concerning with the prediction system is presented by using a benchmark time series.

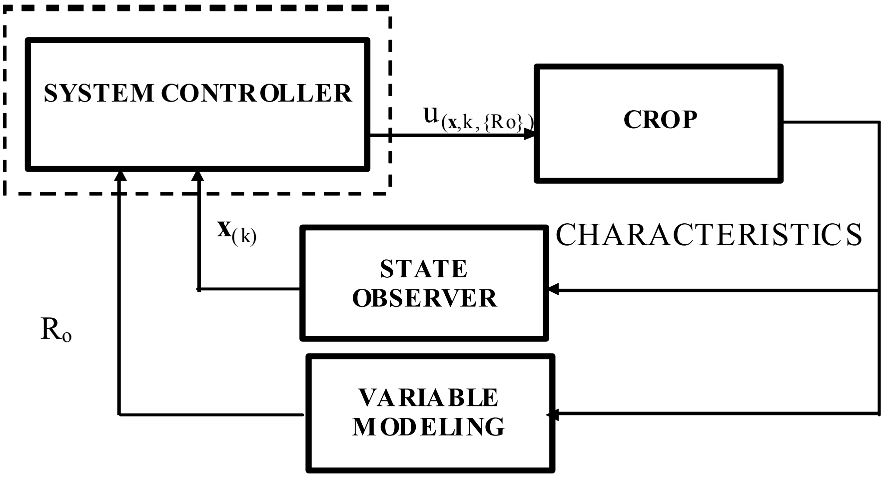

Fig. 1 Closed-loop system control approach.

In this paper, one of the main purposes is to develop an NN filter coupled with appropriate data-preprocessing to improve the accuracy of time series and rainfall forecasting. The learning process, which employs the Levenberg-Marquardt rule, considers the long or short term stochastic dependence of passed values of the time series to adjust at each time-stage the number of patterns, the number of iterations, and the length of the tapped-delay line, in function of the Hurst's value, H of the time series.

According to the stochastic characteristics of each series, H can be greater or smaller than 0.5, which means that each series tends to present long or short term dependence, respectively. In order to adjust the design parameters and show the performance of the proposed prediction model, solutions of the MG equation are used. The NN-based nonlinear filter is applied to the time series obtained from MG to forecast the next 18 values out of a given historical data set of 102 values.

Some results had been obtained from a linear autoregressive approach, which are detailed on Ref. [1]. These results were promising and deserve to be improved by more sophisticated filters. Here, a NN-based NAR filter model [2, 16-17] is proposed. The NN used is a time lagged feed-forward networks type.

The NN's topology consists of l_x inputs, one hidden layer of H_o neurons, and one output neuron as shown Fig. 2. The learning rule used in the learning process

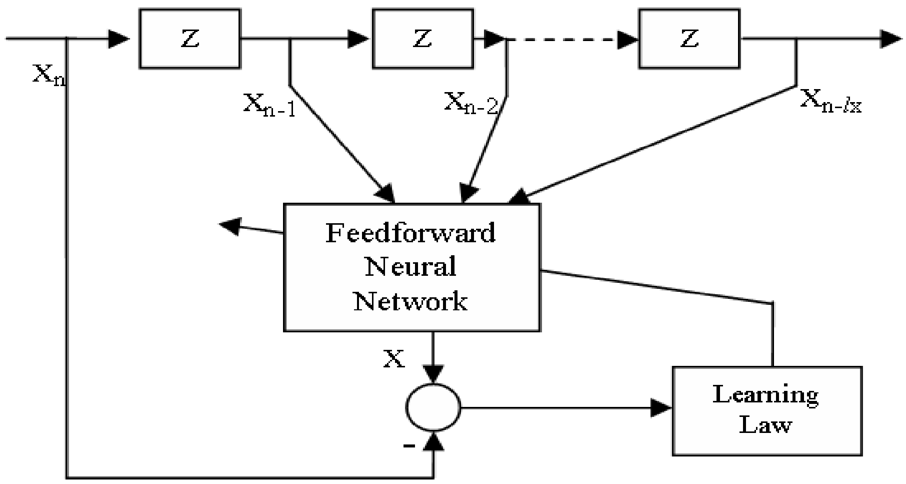

Fig. 2 Neural network-based nonlinear predictor filter.

is based on the Levenberg-Marquardt method [18]. However, if the time series is smooth or rough then the tuning algorithm may change in order to fit the time series. So, the learning rule modifies the number of patterns and the number of iterations at each time-stage according to the Hurst's parameter H, which gives short or long term dependence of the sequence $\{x_n\}$. From a practical standpoint, it gives the roughness of the time series.

In order to predict the sequence $\{x_e\}$ one-step ahead, the first delay is taken from the tapped-line x_n and used as input. Therefore, the output prediction can be denoted by:

$$(x_i, y_i) \quad i = 1,2,...., \quad N_p$$

where F_p is the nonlinear predictor filter operator, and $x_e(n + 1)$ is the output prediction at n + 1.

3. Problem Statement

The best prediction of the present values from a random (or pseudo-random) time series is desired. The predictor system may be implemented using an autoregressive model-based nonlinear adaptive filter. In this work, time lagged feed-forward neural networks are used. The adaptive filter output will be the one-step prediction signal. In Fig. 3 the block diagram of the nonlinear prediction scheme based on an NN filter is shown.

Here, a prediction device is designed such that starting from a given sequence $\{x_n\}$ at time n corresponding to a time series it can be obtained the best prediction $\{x_e\}$ for the following sequence of 18 values. Hence, it is proposed a predictor filter with an input vector l_x, which is obtained by applying the delay operator, Z^{-1}, to the sequence $\{x_n\}$. Then, the filter output will generate x_e as the next value, that will be equal to the present value x_n. So, the prediction error at time k can be evaluated as:

$$e(k) = x_n(k) - x_e(k) \qquad (5)$$

which is used for the learning rule to adjust the NN weights.

The coefficients of the nonlinear filter are adjusted on-line in the learning process, by considering a criterion that modifies at each pass of the time series the number of patterns, the number of iterations and the length of the tapped-delay line, in function of the Hurst's value H calculated from the time series. According to the stochastic behavior of the series, H can be greater or smaller than 0.5, which means that the series tends to present long or short term dependence, respectively [19].

4. Proposed method

4.1 The Proposed Learning Process

The NN's weights are tuned by means of the Levenberg-Marquardt rule, which considers the long or short term stochastic dependence of the time series measured by the Hurst's parameter H. The proposed learning approach consists of changing the number of patterns, the filter's length and the number of

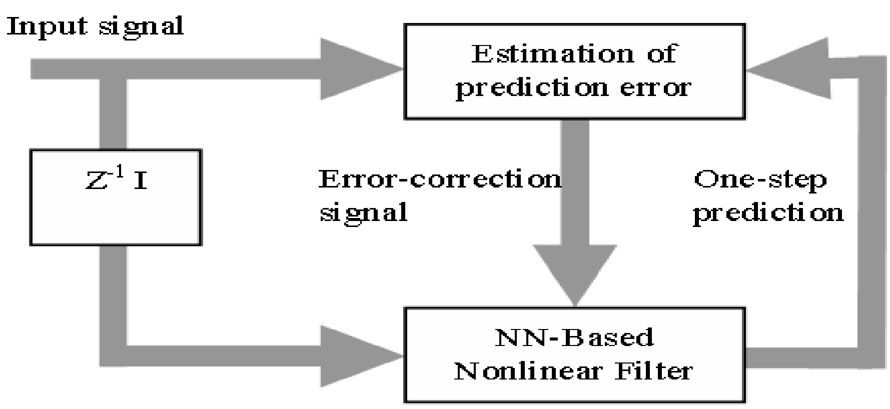

Fig. 3 Block diagram of the nonlinear prediction.

iterations in function of the parameter H for each corresponding time series. The learning process is performed using a batch model. In this case the weight updating is performed after the presentation of all training examples, which forms an epoch. The pairs of the used input-output patterns are:

$$(x_i, y_i) \quad i = 1,2,....,N_p \qquad (6)$$

where, x_i and y_i are the corresponding input and output pattern respectively, and N_p is the number of input-output patterns presented at each epoch. Here, the input vector is defined as:

$$X_i = Z^{-1}I(\{x_i\}) \qquad (7)$$

and its corresponding output vector as:

$$Y_i = x_i. \qquad (8)$$

Furthermore, the index i is within the range of N_p given by

$$l_x \leq N_p \leq 2 \cdot l_x \qquad (9)$$

where l_x is the dimension of the input vector.

In addition, the number of iterations performed by each epoch it is given by

$$l_x \leq i_t \leq 2 \cdot l_x. \qquad (10)$$

The proposed criterion to modify the pair (i_t, N_p) is given by the statistical dependence of the time series $\{x_n\}$, supposing that it is an fBm. The dependence is evaluated by the Hurst's parameter H, which is computed by a wavelet-based method [14-20]. Then, a heuristic adjustment for the pair (i_t,N_p) in function of H according to the membership functions shown in Fig. 4 is proposed.

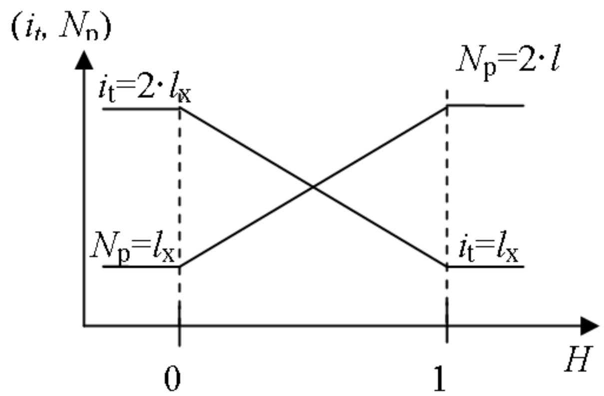

Fig. 4 Heuristic adjustment of (i_t,N_p) in terms of H after each epoch.

In order to predict the sequence $\{x_e\}$ one-step ahead, the first delay is taken from x_n data and used as input. Therefore, the output prediction can be denoted by

$$x_e(+1) = F_p(Z^{-1}I(\{x_n\})) \qquad (11)$$

where F_p is the nonlinear predictor filter operator, and $x_e(n + 1)$ is the output prediction at n + 1.

$$I \in \Re^{lx} \times \Re^{lx} \qquad (12)$$

4.2 Approximation by Primitive

The area resulting of integrating the data time series of MG equation is the primitive that is obtained by considering each value of time series its derivate;

$$\int_{t_k}^{t_{k+1}} y_t dt \cong y_t(t_{k+1} - t_k) \qquad (13)$$

where y_t is the original value time series. The area approximation by its periodical primitive is:

$$I_{t_n} = \int_{t_n}^{t_{n+p}} y_t dt = Y_t\Big|_{t_n}^{t_{n+p}}, n = 1,2,..N. \qquad (14)$$

During the learning process, those primitives are calculated as a new entrance to the NN, in which the prediction attempts to even the area of the forecasted area to the primitive real area predicted. The real primitive integral is used in two instances, firstly from the real time series an area is obtained and run by the algorithm proposed. The H parameter from this time series is called H_A. On the other hand, the data time series is also forecasted by the algorithm, so the H parameter from this time series is called H_S. Finally, after each pass the number of inputs of the nonlinear filter is tuned—that is the length of tapped-delay line, according to the following heuristic criterion. After the training process is completed, both sequences -$\{\{I_n\}, \{I_e\}\}$ and $\{\int\{y_n, y_e\}\}$, in accordance with the hypothesis that should have the same H parameter. If the error between H_A and H_S is greater than a threshold parameter θ the value of l_x is increased (or decreased), according to $l_x \pm 1$. Explicitly,

$$l_x = l_x + sign(\theta) \qquad (15)$$

Here, the threshold θ was set about 1%.

5. Main Results

5.1 Generations of Areas from MG Benchmark

Primitives of time series are obtained from sampling the MG equations with parameters shown in Table 1, with τ = 100, c = 10 and varying β, α. This collection of coefficients was chosen to generate time series whose H parameters vary between 0 and 1. In fact, the chosen one was selected in accordance with its high roughness.

5.2 Model Startup and Learning Process

A key point is that every single filter has the initial conditions. The learning algorithm is shown in Table 2. Note that the first number of hidden neurons and iteration are put as a function of the input number. These initiatory conditions of the learning algorithm were used to forecast the primitive of the time series, whose length is 102 values.

5.3 Performance Measure for Forecasting

In order to test the proposed design procedure of the NN-based nonlinear predictor, an experiment with time series obtained from the MG solution was performed. The performance of the filter is evaluated using the Symmetric Mean Absolute Percent Error (SMAPE) proposed in the most of metric evaluation, defined by

$$SMAPE_S = \frac{1}{n}\sum_{t=1}^{n}\frac{|X_t - F_t|}{(X_t + F_t)/2}\cdot 100 \qquad (16)$$

Table 1 Parameters to generate the times series.

Series No.	β	α	c
1	1.6	20	10
2	1.8	40	10
3	1.9	30	10
4	2.1	40	10

Table 2 Initial conditions of the parameters.

Variable	Initial Condition
l_x	15
H_o	7
i_t	l_x
H	0.5

where t is the observation time, n is the size of the test set, s is each time series, X_t and F_t are the actual and the forecasted time series values at time t respectively. The SMAPE of each series s calculates the symmetric absolute error in percent between the actual X_t and its corresponding forecast value F_t, across all observations t of the test set of size n for each time series s.

5.4 Forecasting Results

Each time series is composed by sampling the MG solutions. However, there are three classes of data sets: one is the original time series used for the algorithm in order to give the forecast, which comprises 102 values. The other one is the primitive obtained by integrating the values of original time series and the last one is used to compare if the forecast is acceptable or not where the 18 last values can be used to validate the performance of the prediction system, which 102 values form the data set, and 120 values constitute the Forecasted and the Real ones. A comparison is made between nonlinear NN dependent-independent filter and an ARMA predictor filter.

The Monte Carlo method was used to forecast the next 18 values from MG time series and its primitive acquired by integration. Such outcomes are shown from Fig. 4 to Fig. 15. The plot shown in Fig. 4 is provided for a linear ARMA filter outcome.

The algorithm attempts to achieve the long or short term stochastic dependence measured by the Hurst parameter in order to make more precisely the prediction. The forecasted time series area is put as a new entrance to the NN and serves to be compared with the real area of the time series.

The next figures show a class of high roughness time series selected from a benchmark of MG Equation and serve to be applied for meteorological measurement.

These are classified by its statistically dependency, the algorithm is adjusted by implementing an independent and dependent of the H parameter.

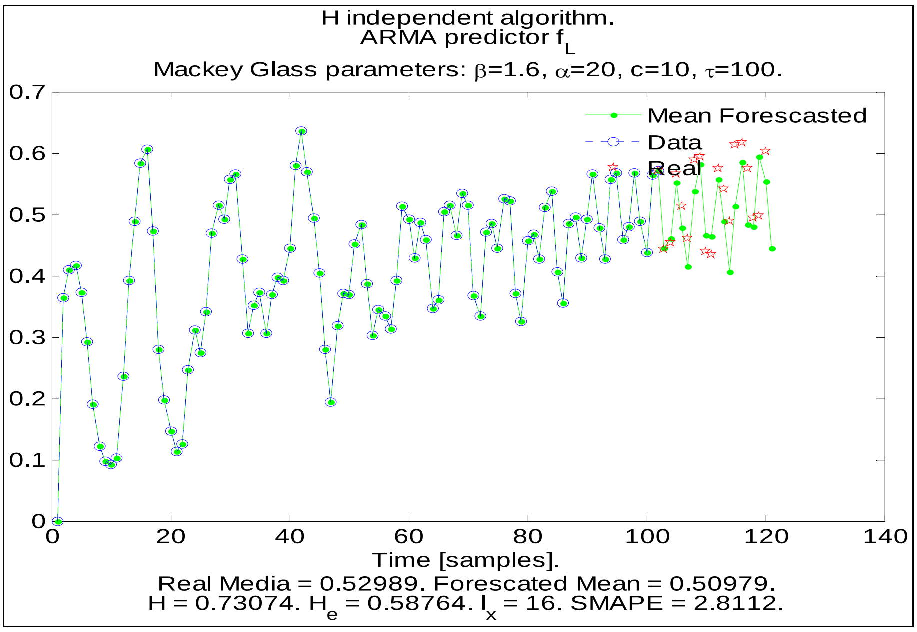

Fig. 5 Primitive of MG time series obtained from ARMA predictor filter.

It can be seen that the NN shows a good performance when it takes into account the roughness of the series considering the use of the stochastic dependence measured by the H parameter.

In the figure, the legend "Data" represents the values obtained by Eq. (11), and the legend "Real" denotes the actual values-not available in practice-used here for verification purposes only. From time k equal 103 to time 120 the inputs of Eq. (11) include the outputs delayed one time interval. The obtained time series has a mean value, which is indicated at the foot of the figure by "Forecasted Mean". The "Real Mean" it is not available at time 102. In the learning process, the primitive is calculated as area by the predictor filter, in which each value of time series represents its derivate.

The algorithm proposed to predict the area related with the energy of the time series is shown from Fig. 11 until Fig. 15. These are calculated by a dependent and independent NN filter.

5.5 Comparative Results

The performance of the stochastic NN-based predictor based on energy associated of series is evaluated through the SMAPE index—Eq. (16), shown in Table 3 and 4 along the time series from MG solution with β ranging between 1.6 and 2.1, in which it can be seen a sort of rough time series.

The comparison between the deterministic [7], the stochastic approach [21-22] and the present forecasted time series is shown through Figs. 6 to 15. In addition, an area of a primitive value acquired of MG time series was incorporated in order to use the proposed approach. The SMAPE index for each time series is obtained by a deterministic NN-based filter, which uses the Levenberg–Marquardt algorithm with fixed parameters (i_t, N_p). In addition, the results of the SMAPE obtained by the stochastic NN-based filter proposed here are also shown in Fig. 6 to Fig. 11 where the Forecasted mean is closer the Real media due to the fact that H is dependent of the algorithm. Furthermore, the SMAPE value is improved of order of 10^{-12} for a class of time series with high roughness of the signal, in this case with β ranging between 1.6 and 2.1 which is one of worst condition for signal prediction compared with earlier work [22]. The algorithm shows a better performance shown from Fig. 13. To Fig 16, where it can be seen that the real and forecasted mean in Fig. 13 is improved against Fig. 12, both have the same MG parameters.

Table 3 Figures obtained by the proposed approach.

Series No.	H	H_e	Real mean	Forecasted mean	SMAPE
Fig.5	0.730	0.5876	0.5298	0.5097	2.8112
Fig.6	0.002	0.0039	0.10307	0.10594	0.034
Fig.7	0.002	0.0336	0.10307	0.10204	$2.90\ 10^{-5}$
Fig.8	0.002	0.037	0.10307	0.10357	$5.66\ 10^{-12}$
Fig.9	0.022	0.0177	0.15597	0.16871	$1.08\ 10^{-12}$
Fig.10	0.242	0.2186	0.10534	0.11893	$1.16\ 10^{-10}$
Fig.11	0.142	0.1413	0.1122	0.0989	$1.25\ 10^{-12}$

Table 4 Comparisons obtained by the proposed approach.

Series No.	H_S	H_A
Fig.12	0.6543	0.6423
Fig. 13	0.6455	0.6423
Fig. 14	0.4420	0.4646
Fig. 15	0.5365	0.5593
Fig. 16	0.5039	0.4829

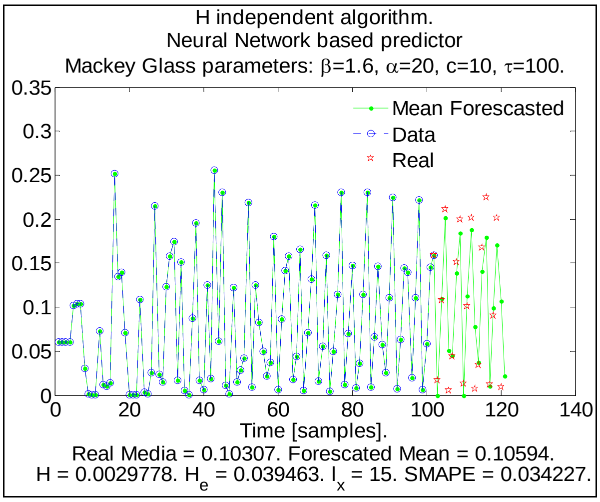

Fig. 6 H independent MG time series with β = 1.6.

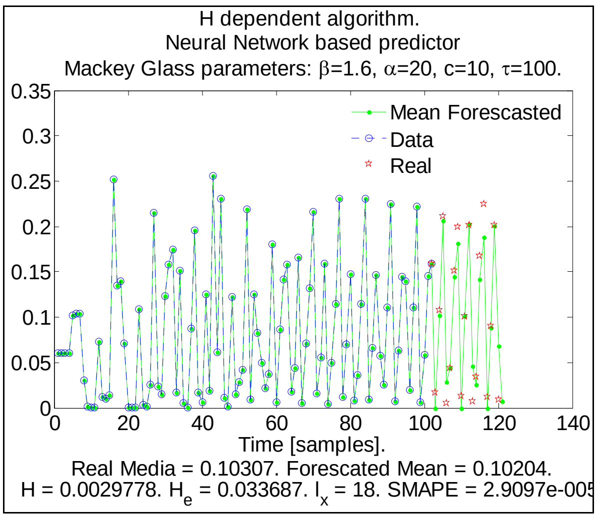

Fig. 7 H dependent MG time series with β = 1.6.

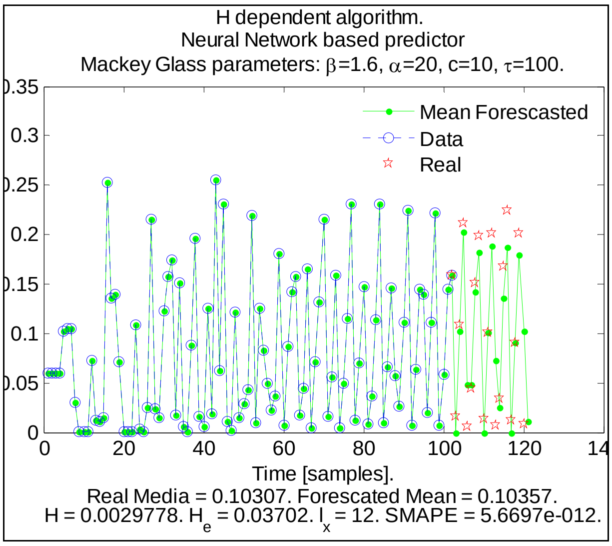

Fig. 8 H dependent MG time series with β = 1.6.

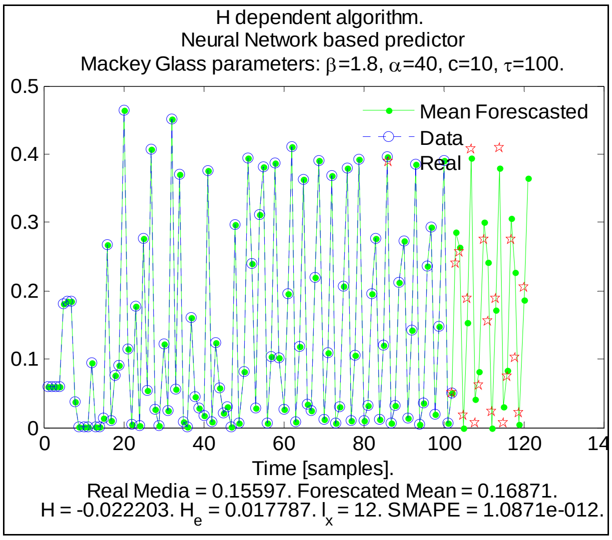

Fig. 9 H dependent MG time series with β = 1.8.

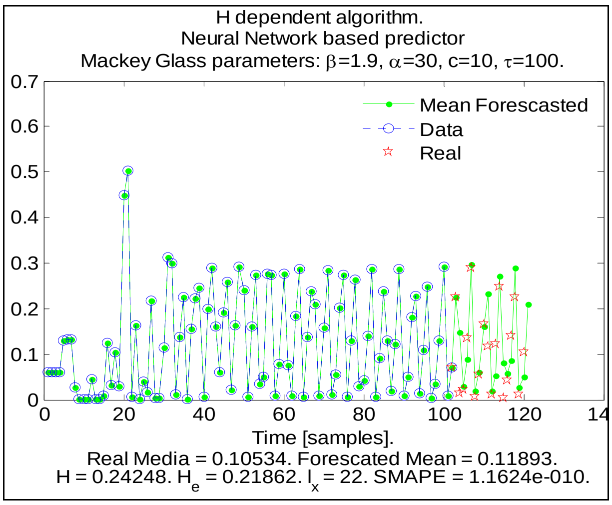

Fig. 10 H dependent MG time series with β = 1.9.

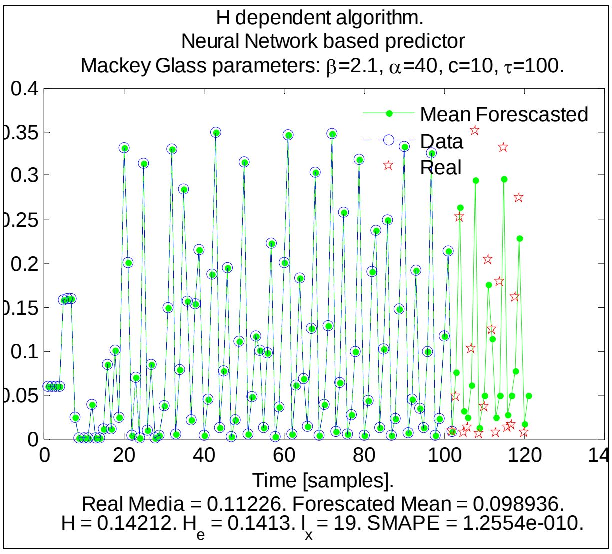

Fig. 11 H dependent MG time series with β = 2.1.

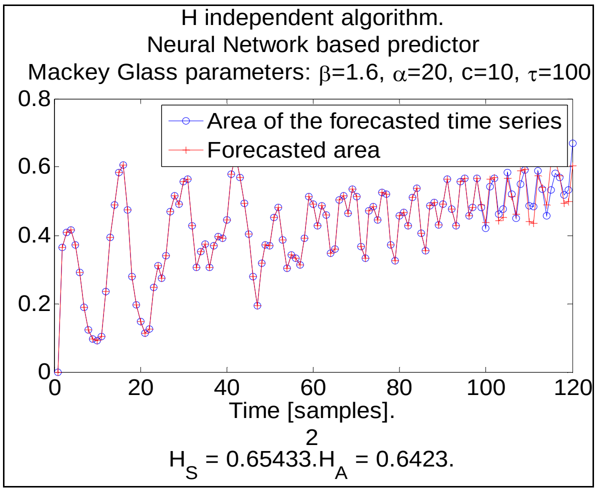

Fig. 12 Primitive of MG time series with parameter β = 1.6.

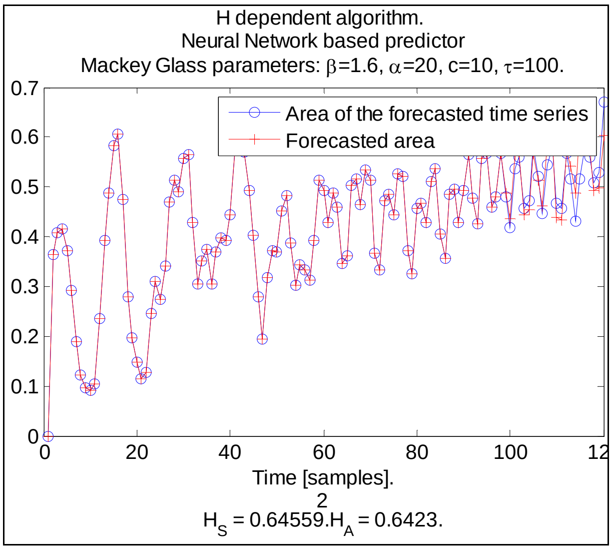

Fig. 13 Primitive of MG time series with parameter β = 1.6.

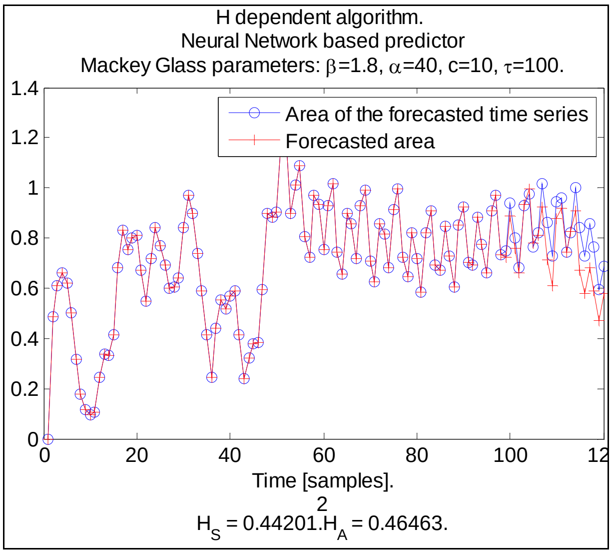

Fig. 14 Primitive of MG time series with parameter β = 1.8.

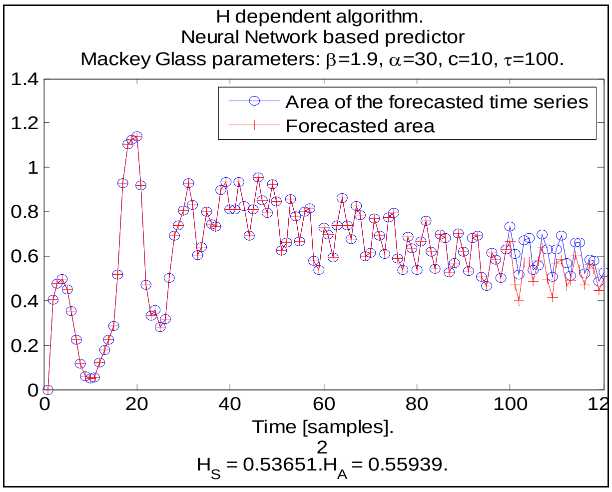

Fig. 15 Primitive of MG time series with parameter β = 1.9.

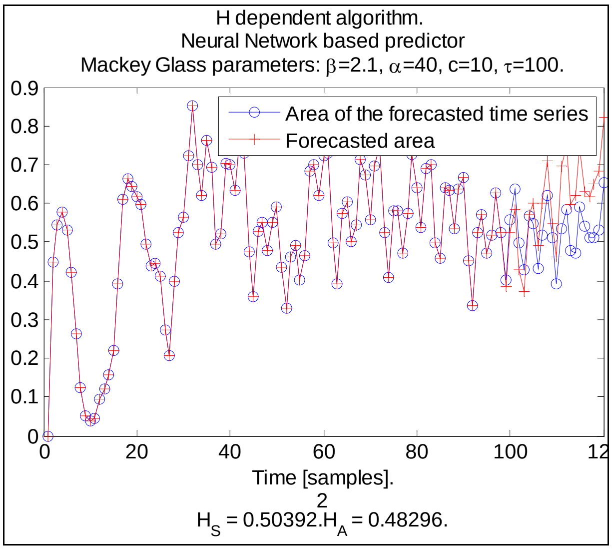

Fig. 16 Primitive of MG time series with parameter β = 2.1.

6. Discussion

The evaluations of the obtained results by comparing the performance of the proposed filter with the classic filter, both are based on NN. Although the difference between both filters resides only in the adjustment algorithm, the coefficients that each filter has, each one performs different behaviors. In the five analyzed cases of primitive of time series and original time series, the generation of 18 future values from 102 present values was made by each algorithm. The same initial parameters were used for each algorithm, but these parameters and the filter's structure are changed by the proposed algorithm that is not modified by the classic algorithm.

The adjustment of the proposed filter, the coefficients and the structure of the filter are tuned by considering their stochastic dependency. It can be noted that in each of the Figs. 5 to 16—the computed value of the Hurst's parameter is denoted either by H_e or H and H_S or H_A, both taken from the Forecasted time series or from the Data time series, respectively, since the Real time series (future time series) are unknown. Index SMAPE is computed between the complete Real time series (it includes the series Data) and the Forecasted one, as indicates the Eq. (16) for each filter. Note that there is no improvement of the forecast for any given time series, which results from the use of a stochastic characteristic to generate a deterministic result, such as a prediction.

7. Conclusions

In this work a high roughness time series forecasting based on energy associated of series was presented. The learning rule proposed to adjust the NN's weights is based on the Levenberg-Marquardt method. Likewise, in function of the long or short term stochastic dependence of the time series evaluated by the Hurst parameter H, an on-line heuristic adaptive law was proposed to update the NN topology at each time-stage. This work took into account a sort of series, rough times series measured by the Hurst Parameter, respectively. The major result shows that the area predictor system supplied to time series has an optimal performance from several samples of MG equations, in particular, those whose H parameter has a high roughness of signal, which is assessed by H_S and H_A, respectively. This fact encourages us to be applied for meteorological variables measurements such as soil moisture series, daily rainfall and monthly cumulative rainfall time series forecasting when the observations are taken from a single point.

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