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LETTERS TO THE EDITOR

TRANSVERSE VIBRATIONS OF A CLAMPED RECTANGULAR PLATE OR SLAB WITH AN ORTHOTROPIC PATCH

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1. INTRODUCTION

The problem under discussion is of basic interest in two technological situations: (1) the detection of damage in a vibrating isotropic plate-like structure when it is assumed that the deteriorated portion acquires orthotropic characteristics [1, 2]; (2) the effect of repairing an isotropic panel replacing a damaged portion by an orthotropic patch [3]*. The present study deals with a clamped, rectangular isotropic plate with a centrally located (a) circular (Figure 1) and (2) rectangular (Figure 2) patch.

The optimized Rayleigh–Ritz method is used to determine the fundamental frequency coefficient and in the case depicted in Figure 2 an independent solution is obtained by means of the finite element technique.

2. APPROXIMATE ANALYTICAL SOLUTION

In the case of normal vibrational modes the plate amplitude, $W(\bar{x}, \bar{y})$, must satisfy the energy functional

$$J(W) = D \iint_{\bar{P}_1} [(W_{\bar{x}^2} + W_{\bar{y}^2})^2 - 2(1-\nu)(W_{\bar{x}^2} W_{\bar{y}^2} - W^2_{\bar{x}\bar{y}})]\,\mathrm{d}\bar{x}\,\mathrm{d}\bar{y}$$
$$+ \iint_{\bar{P}_2} (D_1 W^2_{\bar{x}^2} + 2D_1 \nu_2 W_{\bar{x}^2} W_{\bar{y}^2} + D_2 W^2_{\bar{y}^2}) + 4D_k W^2_{\bar{x}\bar{y}})\,\mathrm{d}\bar{x}\,\mathrm{d}\bar{y} - \rho h \omega^2 \iint_{\bar{P}} W^2\,\mathrm{d}\bar{x}\,\mathrm{d}\bar{y} \tag{1}$$

and appropriate boundary conditions and where Lekhnitskii's classical notation [4] has been used for the orthotropic component of the functional. It must be pointed out that a similar approach has been used recently in the case of a vibrating circular plate with a concentric circular patch of polar anisotropy [5].

Introducing the dimensionless variables $x = \bar{x}/a$ and $y = \bar{y}/b$ and substituting into equation (1) one obtains

$$\frac{\lambda a^2}{D} J(W) = \iint_{P_1} [(W_{x^2} + \lambda^2 W_{y^2})^2 - 2(1-\nu)\lambda^2 (W_{x^2} W_{y^2} - W^2_{xy})]\,\mathrm{d}x\,\mathrm{d}y$$

* This reference deals with the analysis of a simply supported isotropic plate with an orthotropic patch.

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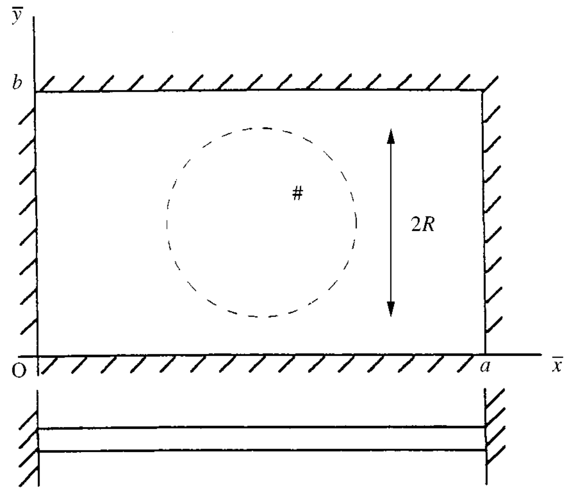

Figure 1. Clamped rectangular plate with a circular orthotropic patch.

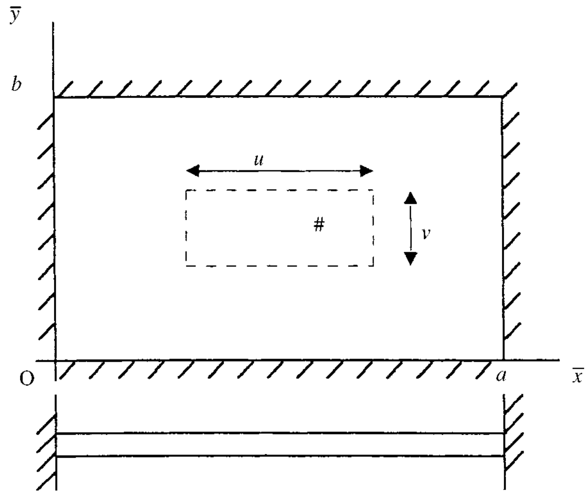

Figure 2. Clamped rectangular plate with a rectangular orthotropic patch.

$$+ \iint_{\bar{P}_2} (D_1' W_{x}^2 + 2D_1' \nu_2 \lambda^2 W_{x^2} W_{y^2} + D_2' \lambda^4 W_{y^2}^2 + 4D_k' \lambda^4 W_{xy}^2)\, \mathrm{d}x\, \mathrm{d}y$$

$$- \Omega^2 \iint_{\bar{P}} W^2\, \mathrm{d}\bar{x}\, \mathrm{d}\bar{y}, \qquad (2)$$

where $\lambda = a/b$, $D_1' = D_1/D$, $D_2' = D_2/D$, $D_k' = D_k/D$, $\Omega^2 = (\rho h a^4/D)\,\omega^2$.

The following approximating expression has been used:

$$W_a = \sum_{j=1}^{N} C_j \varphi_j(x, y), \qquad (3)$$

where

$\varphi_j(x, y) = [x^{p+j-1} + (3 - p - j)\,x^3 + (p + j - 4)x^2]\,[y^{p+j-1} + (3 - p - j)y^3 + (p + j - 4)y^2]$

and p is the Rayleigh's optimization parameter.

Substituting equation (3) into equation (2) and applying Ritz minimization condition one obtains

$$\frac{1}{2}\frac{\lambda a^2}{D}\frac{\partial J}{\partial C_i} = \sum_{j=1}^{N} \left\{ \iint_{P_1} [(\varphi_{jx^2} + \lambda^2 \varphi_{jy^2})(\varphi_{ix^2} + \lambda^2 \varphi_{iy^2}) \right.$$
$$- (1-\nu)\lambda^2 (\varphi_{jy^2}\varphi_{ix^2} + \varphi_{jx^2}\varphi_{iy^2} - 2\varphi_{jxy}\varphi_{ixy})]\,\mathrm{d}x\,\mathrm{d}y$$
$$+ \iint_{P_2} [D_1' \varphi_{jx^2}\varphi_{ix^2} + D_1' \nu_2 \lambda^2 (\varphi_{jy^2}\varphi_{ix^2} + \varphi_{jx^2}\varphi_{iy^2})$$
$$\left. + D_2' \lambda^4 \varphi_{jy^2}\varphi_{iy^2} + 4D_k' \lambda^4 \varphi_{jxy}\varphi_{ixy}]\,\mathrm{d}x\,\mathrm{d}y - \Omega^2 \iint_{P} \varphi_j \varphi_i\,\mathrm{d}x\,\mathrm{d}y \right\} C_j = 0, \tag{4}$$

which, following well-established procedures, leads to a determinantal equation whose lowest root constitutes the fundamental frequency coefficient $\Omega_1 = \sqrt{\rho h/D}\;\omega_1 a^2$. All the numerical determinations have been performed for $N = 4$.

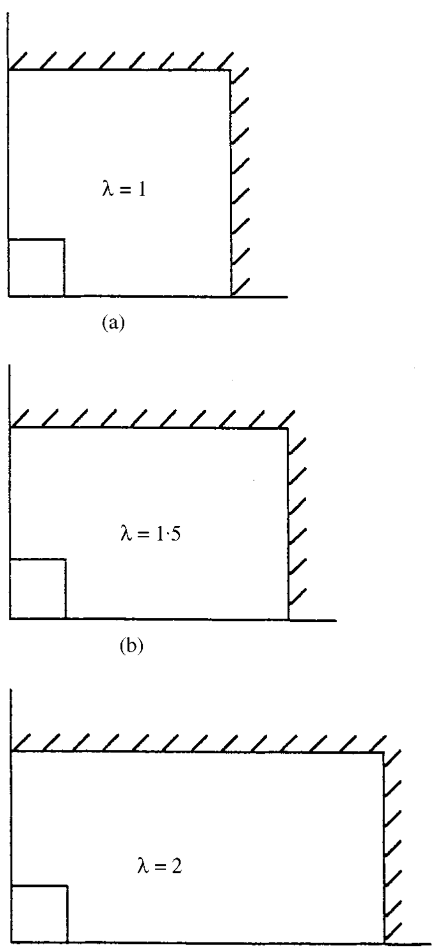

Figure 3. Finite element analysis of one-quarter of a clamped rectangular plate with a centrally located square orthotropic patch: (a) 20 × 20 elements, 1521 equations: (b) 30 × 20 elements, 2301 equations; (c) 40 × 20 elements, 3081 equations.

3. FINITE ELEMENT SOLUTION

A finite element analysis of the problem was performed in the case of the configuration shown in Figure 2 using the algorithm developed in reference [6]. The determinations of fundamental frequency coefficients were performed for $\lambda = 1$, 3/2 and 2 and centrally located orthotropic square patches; see Figure 3.

4. NUMERICAL RESULTS

Determination of fundamental frequency coefficients were performed for the following constitutive characteristics of the materials: (1) Isotropic plate: the Poisson ratio $(\nu) = 0{\cdot}30$. (2) Orthotropic patch: $\nu_2 = \nu$; $D_2/D_1 = 1/2$, $D_k/D_1 = 1/3$, $D_1/D = 0{\cdot}8 = D'_1$. Consequently, $D'_2 = D_2/D = (D_2/D_1)\ 0{\cdot}8 = \frac{1}{2}\ 0{\cdot}8$ and $D'_k = D_k/D = (D_k/D_1)\ 0{\cdot}8 = \frac{1}{3}\ 0{\cdot}8$.

In the case of the circular patch, calculations of Ω_1 were made for $r = 2R/b = 0$ (fully isotropic), 0·2, 0·4, 0·6 and 0·8. Table 1 shows fundamental frequency coefficients for the case of the centrally located circular patch. For $r = 0$ the calculated eigenvalue is in excellent agreement with very accurate results available in the literature [7].

Table 2 depicts values of Ω_1 for the case of a centrally located square patch ($u/v = 1$) and for $\beta = v/b = 0$ (fully isotropic), 0·2, 0·4, 0·6 and 0·8. Results obtained by means of the finite element method are also shown in the table. One concludes that there is a very good engineering agreement between the analytical predictions and the values obtained by means of the finite element method (the maximum difference is of the order of 6% for $\lambda = 1{\cdot}5$ and $\beta = 0{\cdot}8$).

TABLE 1

Fundamental frequency coefficient of a clamped isotropic rectangular plate with a centrally located circular orthotropic patch

λ	$r = 0$	0·2	0·4	0·6	0·8
1	35·998	35·382	34·207	33·396	32·681
1·5	60·843	59·884	57·745	56·208	55·274
2	98·514	97·448	94·249	91·959	89·522

TABLE 2

Fundamental frequency coefficient of a clamped isotropic rectangular plate with a centrally located square orthotropic patch

λ	$\beta = 0$	0·2	0·4	0·6	0·8	
1	35·998	35·334	33·978	33·174	32·150	(1)
	35·985	35·092	33·773	33·040	31·755	(2)
1·5	60·843	59·744	57·449	56·221	55·183	(1)
	60·761	59·305	56·902	55·202	51·874	(2)
2	98·514	96·811	93·270	91·867	88·932	(1)
	98·311	96·283	92·775	90·016	84·066	(2)

Note: (1) analytical results, (2) finite element values.

The approach presented in this study can be extended in a straightforward fashion to the case of patches of generalized anisotropy and also to more complicated geometries.

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