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COMMENT

Comment on 'On scaling solutions with a dissipative fluid'

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Abstract

In this paper we show that the claims in Ibáñez *et al* (2002 *Class. Quantum Grav.* **19** 3067) related to our analysis in Chimento *et al* (2000 *Phys. Rev.* D **62** 063508) are wrong.

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As is well known, current observational evidence strongly favours an accelerating and spatially flat Friedmann–Robertson–Walker universe [3, 4]. Since normal matter obeys the strong energy condition and cannot drive accelerated expansion, recourse is often made either to a small cosmological constant or to an almost evenly distributed source of energy—termed 'quintessence'—in the form of a self-interacting scalar field with equation of state $p_\phi = (\gamma_\phi - 1)\rho_\phi$ (where $0 \leqslant \gamma_\phi < 1$) such that it provides a negative pressure high enough to render the deceleration parameter negative (i.e., $q \equiv -\ddot{a}/(aH^2) < 0$); see, e.g., [5]. As usual, a denotes the scale factor of the FRW metric and $H \equiv \dot{a}/a$ the Hubble factor. Owing to the fact that, in general, the scalar field and matter energy are expected to scale differently with expansion, the following question arises: 'why should the ratio between matter and quintessence energies be of precisely the same order today?' Put another way: 'where does the relationship $(\Omega_m/\Omega_\phi)_0 \simeq \mathcal{O}(1)$ come from?' This is the so-called *coincidence problem* [6]. Here the zero subindex means present time and the dimensionless density parameters of matter and quintessence are defined by

$$\Omega_m \equiv \frac{\rho_m}{3H^2}, \qquad \Omega_\phi \equiv \frac{\rho_\phi}{3H^2} \qquad (c = 8\pi G = 1),$$

respectively. Note that the cosmological constant case is recovered by setting $\gamma_\phi = 0$.

Obviously, solving the coincidence problem in an accelerating universe amounts to showing that the ratio Ω_m/Ω_ϕ tends to a constant for large times with $q < 0$. Then, one may choose the free parameters of the model to fix the constant ratio to order unity. In a recent paper we demonstrated that the coincidence problem and an accelerated expansion phase of FRW cosmologies cannot be *simultaneously* addressed simply by the combined effect of a perfect matter fluid and a quintessence scalar field. Nonetheless, if the matter fluid is not

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perfect but dissipative, both problems can find a simultaneous solution for FRW spatially flat and open FRW universes [2].

A key point in our derivation was the stationary condition

$$\gamma_m + \frac{\pi}{\rho_m} = \gamma_\phi = -\frac{2\dot{H}}{3H^2}, \tag{1}$$

as well as the violation of the strong energy condition

$$\pi < \left(\tfrac{2}{3} - \gamma_m\right)\rho_m \tag{2}$$

(cf [2]), where, as usual, γ_m denotes the baryotropic index of matter, i.e., $p_m = (\gamma_m - 1)\rho_m$ thereby $1 \leqslant \gamma_m \leqslant 2$. Likewise π stands for the dissipative stress which is negative for expanding universes. It may be associated with particle production [7], understood as frictional effects arising in mixtures [8] or even model other kinds of sources (e.g., a string-dominated universe as described by Turok [9], or a scalar field).

Equation (1) expresses the condition for Ω_m and Ω_ϕ to be constants. It can be straightforwardly derived by combining the conservation equations for the matter fluid and quintessence field

$$\dot{\rho}_m + 3\left(\gamma_m + \frac{\pi}{\rho_m}\right)\rho_m H = 0 \qquad \text{and} \qquad \dot{\rho}_\phi + 3\gamma_\phi\rho_\phi H = 0, \tag{3}$$

with the definitions of the density parameters Ω_m and Ω_ϕ, respectively. Thus, the condition $\dot{\Omega}_m = \dot{\Omega}_\phi = 0$ translates into (1). Equation (2) expresses the asymptotic stability of the solution $\Omega = \Omega_m + \Omega_\phi = 1$. By slightly perturbing the solutions $\Omega_i = \text{constant}$ $(i = m, \phi)$ one finds they are stable for spatially flat and open accelerating universes—see equations (18) and (22) of [2]. At this stage we would like to emphasize two points. (i) Our derivation shows that a dissipative stress negative enough to satisfy equations (1) and (2) is required to provide a coincidence-solving attractor solution. This derivation does not hinge on the specific choice of the potential of quintessence field. (ii) Our proof is meant just for universes under accelerated expansion, i.e., for $q = \frac{3}{2}\gamma_\phi - 1 < 0$. Clearly, the conditions expressed by equations (1) and (2) define the set of transport equations for π or, equivalently, the set of expressions of the bulk viscosity coefficient, leading to accelerated stable solutions (see section IV of [2] for some examples).

However, Ibáñez *et al* recently studied the autonomous system of equations of a FRW universe filled with a dissipative matter fluid and a self-interacting scalar field $V(\phi) \propto \exp(k\phi)$ and found stable and unstable equilibrium points with $\Omega_i = \text{constant}$ [1]. The stability of these points seems to depend on the specific equation of state of π as well as on the values assumed by different parameters of their model. Since this outcome looks at variance with point (i) of the preceeding paragraph, Ibáñez *et al* erroneously claimed to have found examples that contradict our findings. The fact is, however, that they overlooked point (ii): all the unstable solutions in [1] correspond to non-accelerating universes. To be specific, the equilibrium point of equations (29) has $q > 0$ (cf equation (32)). The set of equilibrium points associated with a massless scalar field ($\Gamma = 0$) has also $q > 0$. Likewise, the equilibrium point of equations (40)–(42) is either stable and accelerating or unstable and decelerating (see equation (48) and figure 4). Again, the equilibrium point of equations (49)–(52) is non-accelerating (see equation (53)). Finally, the equilibrium point (54)–(56) has $q > 0$.

Therefore, they do not invalidate in any way whatsoever the findings of [2]. We believe this is more than enough to dismiss the claims of Ibáñez *et al*. However, we have found inconsistencies in their analysis. In the rest of this reply we concentrate on bringing them to the fore.

The starting equations of Ibáñez *et al* are

$$\dot{H} = -H^2 - \frac{1}{6}(\rho_m + 3p_m + 3\pi + 2\dot{\phi}^2 - 2V(\phi)) \tag{4}$$

$$3H^2 = \rho_m + \frac{1}{2}\dot{\phi}^2 + V(\phi) - \frac{3K}{a^2} \qquad (K = \pm 1, 0) \tag{5}$$

$$\dot{\rho}_m = -3H(\rho_m + p_m + \pi) \tag{6}$$

$$\ddot{\phi} = -3H\dot{\phi} - \frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi}. \tag{7}$$

The dissipative pressure π is assumed to satisfy the truncated Israel–Stewart equation [10]

$$\pi + \tau\dot{\pi} = -3\zeta H, \tag{8}$$

where ζ is the coefficient of bulk viscosity and τ is the relaxation time ($\zeta > 0, \tau > 0$). Strictly speaking, equation (8) is valid only when the fluid is close to equilibrium. However, let us assume it holds even when the fluid is far from equilibrium. (A more rigorous and comprehensive analysis would make use of the full transport equation of the Israel–Stewart theory.) Ibáñez *et al* also assumed the linear baryotropic equation of state $p_m = (\gamma_m - 1)\rho_m$, but with $\gamma_m =$ constant, and two different relations for the coefficient of bulk viscosity and relaxation times. To translate (3)–(8) into an autonomous system of differential equations Ibáñez *et al* introduced the set of variables

$$\begin{aligned} &\Omega_m = \frac{\rho_m}{3H^2}, \qquad &&\Sigma = \frac{\pi}{H^2}, \qquad &&\Psi = \frac{1}{\sqrt{6}}\frac{\dot{\phi}}{H}, \\ &\Gamma = \frac{1}{3}\frac{V(\phi)}{H^2}, \qquad &&h = H^{1-2n}, \qquad &&n \neq \frac{1}{2}, \end{aligned} \tag{9}$$

as well as a new time parameter τ defined by

$$\mathrm{d}\tau = H(t)\,\mathrm{d}t. \tag{10}$$

The Friedmann equation (4) now reads

$$1 - \Omega_m^2 - \Psi^2 - \Gamma = -\frac{K}{H^2a^2}, \tag{11}$$

and the autonomous system takes the form

$$\Omega_m' = \Omega_m(-3\gamma_m + 2x) - \Sigma \tag{12}$$

$$\Sigma' = -9\Omega_m + \Sigma\left[-\frac{1}{\alpha}(3\Omega_m)^{(1-n)}h + 2x\right] \tag{13}$$

$$\Psi' = \Psi(x - 3) - \frac{3k}{\sqrt{6}}\Gamma \tag{14}$$

$$\Gamma' = \Gamma(2x + k\sqrt{6}\Psi) \tag{15}$$

$$h' = -(1 - 2n)hx, \tag{16}$$

where the prime denotes the derivative with respect to τ, and $x \equiv 1 + \frac{1}{2}(3\gamma_m - 2)\Omega_m + \frac{1}{2}\Sigma + 2\Psi^2 - \Gamma$.

Note that equation (13) is the truncated Israel–Stewart transport equation, while equation (16) is just the derivative of the change of variables introduced in (9). In equation (13), first case of Ibáñez *et al*, use has been made of the relationships $\zeta = \alpha\rho_m^n, \tau = \alpha\rho_m^{n-1}$, introduced by Belinskii *et al* [11]. In their second case, somewhat modified relationships were used for ζ and τ (introduced by their equations (37)), amounting to replacing (13) by a similar equation.

The equilibrium points are found by setting $\Omega_m' = \Sigma' = \Psi' = \Gamma' = h' = 0$. As a consequence, $\rho_m, \pi, \dot{\phi}, V(\phi)$ and H are constants there (use of the variables introduced in equations (9) and (10) excludes $H = 0$). Then, by virtue of equation (7) we have $\mathrm{d}V(\phi)/\mathrm{d}\phi = 0$ and $\dot{\phi} = 0$, i.e., $V(\phi)$ must have an extremum at the equilibrium points, and $\gamma_\phi = 0$. Potentials having this feature yield de Sitter solutions on the equilibrium points and for them, as noted above, we could replace the quintessential field with an effective cosmological constant.

From equation (4), with $H, \rho, V(\phi)$ constants and $\dot{\phi} = 0$, we get $K = 0$. Moreover, from (5) it follows that $\pi = -\gamma_m \rho_m$. This is independent of the specific form of $V(\phi)$. Note that this is a particular case of the result already found in [2], as follows from (1) and the assumption $H =$ constant.

The setting $h' = 0$ by Ibáñez *et al* (their equation (18)) leads to wrong conclusions. In the first case (Belinskii *et al* relationships), they erroneously state that its solution is $h = 0$ rather than $h = -3\alpha(3\Omega_m)^n/\Sigma$ (with $x = 0$), as follows from their equations (15) and (19); and $h =$ constant in the second case. Besides, their relationship $h' = 0$ neither belongs to the set of Einstein equations nor describes any property of the sources of the gravitational field.

Altogether, the work of Ibáñez *et al* misinterprets our results rendering the claims in [1] void. Further, even if the analysis of Ibáñez *et al* were correct, it would fail to disqualify our findings in [2] as all their counterexamples correspond to non-accelerated cosmic expansions.

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