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ORIGINAL PAPER

# Interdependency, alternative forms of mathematical agency and joy as challenges to ableist narratives about the learning and teaching of mathematics

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**Abstract**
Catering for the mathematical needs of disabled learners equitably and productively requires the anti-ableist preparation and professional development of teachers. In CAPTeaM (Challenging Ableist Perspectives on the Teaching of Mathematics), we design tasks that emulate inclusion-related challenges from the mathematics classroom, and we engage teachers with these tasks in workshop settings. In this paper, we focus on evidence from one type of task in which participants engage in small groups with solving a mathematical problem while at least one of them is temporarily and artificially deprived of access to a sensory field or familiar channel of communication. In this paper, we focus on evidence of emerging resignification – discursive and affective shifts in the participating teachers' sense-making about what makes the construction of mathematical meaning possible and valuably different – as they work on the tasks. By linking Vygotsky's vision about the educational changes required to empower and include disabled learners with more contemporary ideas from embodied cognition and disability studies, our analyses show how engagement with the tasks affects participants' realisation and appreciation of interdependencies between learners, teacher, resources, and emotions, highlights alternative forms of mathematical agency and gives opportunities to turn initial sense of impasse and despair into joy.

**Keywords** Disability · Inclusion · Ableism · Mathematical agency · Vygotsky · Embodied cognition

## 1 Introduction

Learners with disabilities, neurodiverse learners and those who experience difficulties in learning are amongst those groups whose needs are yet to be catered for equitably, in many countries they still face blatant educational exclusion:

> Children with disabilities are over-represented in the population of those who are not in education. […] Children and youth with sensory, physical, or learning disabilities are two-and-a-half times more likely than their peers to never go to school. Where disability intersects with other barriers, such as gender, poverty, or remoteness, the risk of exclusion is greater still (UNESCO, 2023).

Ensuring the catering of these learners' needs is a *sine qua non* of an inclusive education system – and merely securing access to schooling is far from a guarantee for inclusive education. In this paper, we focus on one of the challenges in fostering an inclusive mathematics education. We do so through reporting from a study, the CAPTeaM project (Challenging Ableist Perspectives on the Teaching of Mathematics) in which we work with practising and future teachers to identify and challenge the ableist assumptions that currently mediate our interpretations of mathematics teaching and learning.

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## 2 Mathematics teachers' narratives about disability and inclusion: an emerging field of research

Pervasive narratives about what a "normal" and ideal student is – a by-product of the medical model of disability (LoBianco & Sheppard-Jones, 2007) according to which disability is a medical condition which needs to be fixed so that a person can keep up with, and fit in, society – are not conducive to introducing, and implementing, inclusive educational policies and can often legitimise marginalisation (Healy & Powell, 2013). Such narratives can also normalise taking disabled students' underperformance in mathematics as an uncritically accepted consequence of their disability (Gervasoni & Lindenskov, 2011), fomenting deficit models of disabled students[1] (Tan et al., 2019). Indeed, students labelled as "special learners" have often been offered a mathematics education in which attention to number and operations tends to dominate and only limited opportunities to engage in other areas of mathematics are offered (Woodward & Montague, 2002; McKenna et al., 2015).

Whilst disabled students are positioned as in need of remediation, as lacking when compared to their peers, their experience of mathematics education appears to be one of exclusion from many (even most) areas of mathematical practices. This has left cultural and institutional practices which present barriers to an equitable mathematics learning underexamined (Lalvani, 2015). That these students had not received much attention in mathematics education literature related to equity and social justice until recently (Tan & Kastberg, 2017) is perhaps surprising and certainly concerning.

Over the last decade or so, this has begun to change, and ableist narratives – which assume that deviations from a socially constructed ideal body standard makes people unfit for activities in society (Campbell, 2001) – are starting to be problematised and challenged within mathematics education (Tan et al., 2022). Alongside this problematisation, and in contrast to said medical model, models that posit disability as a historical, social, and political phenomenon (Hall, 2019) are starting to gain space in the field of mathematics education (D'Souza, 2020; Lambert, 2019).

Much work so far is framed as exploring the experiences of disabled learners and the associated pedagogies that co-determine these experiences (Roos, 2023). Studies in this emerging field investigate the mathematics of disabled students, focus on how their mathematical agency is shaped by the different ways through which they interact with, express and experience mathematics (Healy & Fernandes, 2011; Figueiras & Arcavi, 2014; Lambert, 2015) and offer counter-narratives to views of disability as deficit along with glimpses of creativity and brilliance (Tan & Kastberg, 2017).

This recent upturn in attention to the practices and experiences of disabled learners places questions of social justice centre-field. It signals how a commitment to inclusive education requires that those in the teaching profession resist any belief system that regards disabled students as "deficient and therefore beyond fixing" (European Agency for Development in Special Needs Education, 2010, p. 30). This too is a growing a concern of those researching in the field of disability and mathematics teacher education, not least because – while a recent meta-analysis of teacher attitudes to inclusion suggests teachers tend to express favourable views (Guillemot et al., 2022) – many teachers, from primary to tertiary education, across all subject areas and throughout the world, do not feel adequately prepared to teach disabled students (Sharma, 2018).

A range of strategies aimed at preparing practicing and future mathematics teachers to create mathematical learning scenarios based on respect and justice has been examined. To some extent, all require some disruption of hegemonic discourses and practices that position those who deviate from socially constructed norms as problematic. Evidence from research-based teacher development programmes has revealed, for example, how engagement with culturally responsive pedagogies can support teachers in providing equitable and inclusive mathematics instruction (Abdulrahim & Orosco, 2020). Most such studies have focused on including ethnically and/or linguistically diverse students (e.g., Grant & Sleeter, 2007; Moschkovich & Nelson-Barber, 2009), but a small number have focused on teaching mathematics to disabled students (Shumate et al., 2012; Healy & Santos, 2014).

More emphasis on subject-specific teacher preparation for inclusive classrooms has also been explored, particularly in the case of prospective teachers, given the tendency of pre-service teacher education courses to focus only on general pedagogical issues of inclusion (Troll et al., 2019). Tan et al. (2022) suggest that research in this vein indicates the role of mathematical understanding in supporting educators to recognise how students' mathematical ideas develop differently, if and when they are offered learning opportunities appropriate to their bodyminds[2].

As attention to social justice in teacher education for inclusive mathematics teaching has grown, research from the area of disability studies is beginning to find a place in mathematics teacher education. The study by Tan and

[1] We use "disabled person" to refer to neurodivergent learners and those with disabilities and learning differences as what they have in common are the ableist constraints of a world that does not accommodate them and limits their opportunities to participate and flourish.

[2] Price (2015, p.270) defines this term as "the imbrication (not just the combination) of the entities usually called 'body' and 'mind.'".

Padilla (2019) is one of still only a few examples. In this case-study, participants were prospective primary (elementary) teachers who were invited to incorporate "the principle of disability as difference instead of deficit" in their lesson plans for mathematics. A particular aim was to motivate the participants to "unlearn" ableist conceptions of ability in relation to knowledge construction and acquisition. Their findings indicate that, while the prospective teachers did adopt approaches to planning consistent with the principals of social models of disability, they also experienced tensions in unlearning aspects associated with the medical model that were contrary to these principles. The authors end by underscoring the need for further research to document and create concrete ways of involving teachers in challenging ableist assumptions and to support them in hearing and enabling the voices of disabled students, whilst also recognising the constraints imposed by current power structures within the education system.

Our own research in the CAPTeaM project can be seen as a contribution to this call for further research. We now briefly introduce its theoretical underpinnings and then outline its aims and the research question this paper explores.

## 3 Linking the embodied, social and political: our vygotskian inspired approach

Our interpretations of the historical-cultural perspective of Vygotsky, and especially his work with disabled learners (Vygotsky, 1993), sits at the heart of the theoretical framework that underpins the CAPTeaM project. For us, a concern for social justice permeates his perspective and, by treating disability as a potential strength rather than an inevitable deficit, he advocated radical educational changes aimed at empowering learners to develop capacities for a satisfying and constructive life (Stetsenko & Selau, 2018).

We re-vision his work from the 1920s and 30s through the lenses of recent commentators on his ideas (e.g. Roth & Jornet, 2016), along with contemporary views from both embodied cognition (Barsalou, 2008) and disability studies (Valle & Connor, 2011).

This revisioning has foregrounded Vygotsky's monist tendency, in which there is no separation of mind from body, or of intellect from affect. It also motivated us to include tools of the body alongside the material and semiotic tools he argued to shape activity. This underscores how the mathematics we do and know depends on the tools we use to practise it. In relation both to such shaping processes and to operationalising the unity he ascribes to cognition and emotion, we find his distinction between "meaning" – how concepts are configured and conveyed in a particular sociocultural context – and sense – "the aggregate of all the psychological facts that arise in our consciousness" in relation to a concept (Vygotsky, 1987, p. 275) – useful. Meaning can be thought of as a subset of sense, with the appropriation of mathematical meaning involving sensing and making-sense of both a culturally endorsed body of knowledge and oneself in relation to it.

From the field of embodied cognition, we borrow the notion of simulation as a way to explain how we become able to act in the present by drawing on our senses of past activities. Simulation involves the re-enactment of actions, emotions and sensations "acquired during experience with the world, body, and mind" Barsalou (2008, p. 618). In the context of mathematical activity, this would suggest that as we reuse a previously experienced concept, it is not just some decontextualised version of its meaning that is recalled. We also re-enact other aspects associated with our sense of the concept, the processes by which we came to know it and how we felt during these processes.

In short, in learning mathematics, cognition and emotion are inevitably entwined. This raises questions about sense-making that occurs when learners' experiences of mathematics are accompanied by negative emotions, by feelings of inadequacy or exclusion. Here, we see a meeting of critical disability theory with both embodied and sociocultural approaches. We can expect such feelings to become part of how mathematics is experienced and of a learner's sense of themselves as a mathematics learner: the disabling discourses and practices that accompany and frame mathematics learning experiences also become part of the sense refracted from them. The process of learning mathematics can hence be seen as an ongoing process of refraction (Vygotsky, 1994), through which past and current experiences are enacted, re-enacted and interpreted, either enabling or constraining our visions of the possibilities for future mathematical experiences. Since learning occurs in social settings, our own sense-making also affects not only the sense-making of others, but also of what de Freitas and Sinclair (2014) describe as the body of mathematics and the ways in which it is produced.

We argue that teachers' senses of teaching mathematics develop in a similar manner that involves them in sensing mathematics and mathematical pedagogies, making sense of their objects and relations, whilst also making sense of themselves as teachers. For educating mathematics teachers, and for practising mathematics teaching, a framework combining historical-cultural, embodied, and critical perspectives has significant implications. If our perceptual-motor, social and cultural experiences are tied inextricably with knowing as tool-mediated, then curriculum, pedagogy and assessment priorities in mathematics education must surely follow suit in the ways in which they attend to appropriateness of tools for all, including those whose tool-dependence diverges from what is seen as the norm. Building an inclusive mathematics education thus necessitates: problematising this norm; understanding how each student may construct mathematical meaning through engaging with different tools in diverse ways; and, designing/enacting/promoting optimal practices that take on board, and respect, student diversity.

CAPTeaM aims to contribute to building an inclusive mathematics education in precisely this manner. We now outline the project's aims and the research question this paper explores.

## 4 The rationale and aims of the CAPTeaM project

In CAPTeaM, we seek to explore how engaging teachers of mathematics with challenges they are likely to face in class regarding the inclusion of disabled learners might serve as an effective professional development approach. CAPTeaM – a collaboration that involves researchers and pre- and in-service teachers in Brazil and the UK and combines the different research foci and methodological expertise of two research teams – sets out from the assumption that, rather than being the consequence of internal, individual factors, disabled students' oft-reported underperformance in mathematics can result from explicit or implicit exclusion from mathematics learning. In CAPTeaM, we design situation-specific tasks (Biza et al., 2018) which challenge ableist assumptions about the teaching and learning of mathematics and we engage teachers with these tasks in reflective workshop settings.

In contrast to narratives that emphasise the mathematical difficulties experienced by disabled students, our prior studies demonstrate what students can achieve when working in appropriately designed learning situations (e.g., Fernandes & Healy, 2016). Innovative mathematical contributions emerge from such work that question and often surpass teacher expectations (Healy & Santos, 2014). These studies have suggested that "by promoting meetings between differences, possibilities emerge to experience differences as similarities and to learn to inhabit bodies with capacities that might differ from our own." (Nardi et al., 2018, p. 150). The overarching aim of the tasks we design, then, is to affect teachers' senses of how dis/ability is constructed in the processes of mathematics education. We do so by engineering encounters with lessons we have learnt from our own research and by stimulating shifts in the participating teachers' senses about the enabling and constraining of mathematical activity in the tasks they were working on and beyond – this is a process we call resignification.

Specifically, to explore how pre- and in-service teachers can be encouraged to recognise and challenge ableism, and develop pedagogies that empower rather than disable learners, we design and deploy two types of research-informed tasks. Type I tasks provide opportunities for teachers to reflect on episodes in which disabled students engage successfully with mathematics. Teacher engagement with Type I tasks has been shown to encourage participants to discern the potency of these students' mathematical productions, recognise that they are not mathematically deficient (Nardi et al., 2018) and even to transform their own thinking about the mathematical objects at play (Batista et al., 2019).

Engagement with Type I tasks has also illustrated the reservations, of at least some – and mostly the practising teachers – about the viability of incorporating the less conventional (and perhaps most creative) productions given institutional constraints such as curriculum and assessment demands. These findings resonate with those of Tan and Padilla (2019) in that, while the notion of disability as deficit was challenged, the (exclusionary) nature of curricular and assessment structures create tensions. Rather than a process of unlearning, we suggest that experiencing the contradictions these tensions cause may be integral to problematising them.

In this paper, we concentrate on teachers' work with Type II tasks – in which, to motivate the experiencing of said contradictions, we invite participants to engage in mathematical activities without using some of the sensory or communication means that might usually be available to them. We do so to locate evidence of resignification regarding teachers' sense-making of dis/ability in mathematics education that surfaces when they engage with tasks that ask them to produce solutions to mathematical problems in collaborative settings in which one or more participants are temporarily deprived of access to at least one sensory or communication means. We thus explore the following research question:

> How does the experience of doing mathematics without access to familiar bodily tools (such as eyes for seeing or mouths for speaking) affect the teachers' sense of the roles that tools and teaching play in enabling/disabling mathematical practices?

## 5 CAPTeaM workshops: Data collection and analysis

Type II tasks aim to elicit reflections on how access to mediational tools differently shapes mathematical activity. In this sense, they are what Sannino et al. (2016) call "formative interventions" which offer participants the opportunity to engage in activities that "can lead to generative, novel outcomes" (p. 606)– in this case, discoveries about the capacity of our bodies to participate in mathematical activity in many and varied ways. Also resonant with this notion of formative experience in designing Type II tasks is the participatory, situated notion of "embodiment" in activities for the preparation of teachers (Ord & Nuttall 2016) as a means to bypass the alienation often generated by the theory-practice divide.

Type II task participants work in groups of three. One group member acts as observer. A second group member has a student role and is asked to solve a mathematical problem whilst, temporarily and artificially, deprived of using a particular sensory field and/or communicational mode (in the task in this paper: seeing). The third member has a teacher role, communicating the problem and intervening as judged necessary, but without access to another sensory field or communicational mode (in the task in this paper: speaking).

The data we draw on in this paper originate in datasets collected in Brazil and the UK from workshops in four different universities with a total of 91 pre- and in-service teacher-participants (70 from Brazil and 21 from the UK). Bar a small number of in-service mathematics teachers– none with Special Educational Needs and Disability (SEND) coordinator responsibilities– participants in the UK were pre-service mathematics teachers, enrolled on a Secondary Mathematics Post-Graduate Certificate in Education (PGCE) programme. Participants in Brazil included four practising teachers with some Special Education responsibilities, 10 teachers who were also undertaking a two-year Masters in Mathematics Education course, 38 undergraduate students on a four-year course in Mathematics Education (future mathematics teachers) and 18 undergraduate students studying on a four-year course in Education (to become primary teachers).

In the workshops we draw on in this paper, participants completed four tasks (three Type I; one Type II) in three-hour sessions. In the Type II task, participants were asked to communicate and carry out a multiplication of two numbers, a three-digit number and a two-digit number (e.g. $347 \times 36$).

The data consist of audio / video recordings from four different institutions, three in Brazil and one in the UK: 27 small-group Type II sessions and four plenary discussions. Data collection was carried out once ethical approval by the Research Ethics Committees in both the UK and Brazil institutions had been granted. We note that the participants whose photographic images are used in this paper consented also, and specifically, to this use.

Analysis of the data aimed to identify how working on a mathematical activity in the absence of familiar mediational tools affected participants' sense-making about teaching mathematics to disabled learners. In our search for participants' strategies for coping with the task, we explored the types of bodily involvement observed in their interactions and what communicational channels they deployed during these interactions. We identified four strategies (counting fingers; tracing the sum; negotiating signs to indicate place value; decomposing numbers into hundreds, tens, and ones[3]) all involving haptic constructions of number in communicating and carrying out the calculation. As we coded forms of bodily involvement, evidence of resignification started to emerge. Such shifts were observable in two forms:

- discursive (language, visual or other mediation, engagement with mathematical routines and comments about mathematics, its teaching and dis/ability of self and others).
- affective (movements between negative and positive emotive states (Liljedahl & Hannula, 2016), such as manifestations of helplessness, disempowerment, despair, amusement, elation, and joy).

We then systematically searched for indications of discursive and affective shifts. All three authors independently recorded these indications in all of the UK data, before reconvening to triangulate and discuss how the evidenced shifts might be grouped into themes[4]. Three themes emerged from these discussions:

- alternative forms of mathematical agency (including collaborations between humans as well as between humans and tools)
- realisation and appreciation of interdependencies between teachers, learners and tools
- the entwinement of cognition and emotion (as emotional engagement affected mathematical strategies and/or reflection on inclusive teaching practices– and vice versa).

To present our findings, we zoom in on two episodes from the data that evidence resignification. We do so with caution not to overclaim resignification from singular episodes and while attending to frequency of occurrence across our datasets. Our selection of episodes resonates with what Coles and Sinclair (2019) call "telling" episodes, namely episodes that aim to "sensitis[e] the reader to new possibilities […] rather than asser[t] causal connections […] looking in detail at particular cases in order to draw out more general principles" (ibid., p. 182). We see these episodes as "paradigmatic" examples (Nardi, 2008, p. 18–23): while they brim with the contextual specificity, situational particularity and fluidity afforded by narrative approaches, they also mirror

[3] For example, representing 300 with three fingers followed by two fists.

[4] The data collected in Brazil were subsequently analysed by the first author.

**Fig. 1** Daniela on the left (not allowed to see), Juliana on the right (not allowed to speak), initial sense of despair

patterns in our participants' narratives about inclusion and disability which signal evidence of the resignification our analysis aims to explore.

## 6 Teachers' shifting sense-making about disability and inclusion: two "telling" resignification episodes

The two episodes we present in 6.1 and 6.2 are drawn from the same workshop, but we signal the representativity of the approaches involved in them across the data set as a whole, by identifying the frequency of the coded strategies.

### 6.1 Desperation turns to joy

In this episode, Juliana assumed the role of teacher and Daniela the role of student. Their exchanges were filmed by Wellington. The calculation given to this group was $305 \times 67$.

Juliana was very unsure how to start. This is evidenced in expressions of despair and insecurity in the first three minutes (Fig. 1).

After these initial displays, Juliana took hold of Daniela's hand and separated out three fingers. Finger counting was an extremely common strategy, used in 25 of the 27 Type II sessions, although in this particular case, Juliana changed almost immediately, as she was unsure about how to deal with zero. Her second strategy involved tracing out the first number in the calculation on Daniela's arm (a strategy observed in nine sessions overall). Daniela complained: "*wait, here…go slower, it's too fast*". Following this instruction, Juliana slowly traced out the number 305 and was delighted when Daniela correctly identified it, putting both her thumbs up and waving them in the air. Although Daniela could not see the gesture, she sensed from Juliana's jubilation that this was the right response, and then suggested that Juliana tap her back whenever she made a correct interpretation. The negotiation of shared signs such as this one occurred in all but three of the 27 sessions that were videoed and seemed to have been a critical factor for successful completion. In this case, as in eleven others, the sign was suggested verbally by the student, while in the remaining thirteen cases, initiation of the sign came from the teacher.

**Fig. 2** Juliana's jubilation upon Daniela's verbalising the requested calculation correctly

To communicate the times symbol, Juliana offers her two index fingers in the shape of a cross, which made sense to Daniela. Daniela then suggested that Juliana holds out her own fingers for the digits of the next number, as she found the tracing on her arm difficult to decipher.

Following Daniela's lead, Juliana easily communicated 67 and her joy when Daniela verbalised the requested calculation was palpable (Fig. 2).

Daniela immediately tried to effectuate the calculation as if she was (mentally) completing the paper and pencil algorithm– by far the most common calculation strategy emerging in some form at some point in 22 of the 27 sessions.

Daniela started her attempt to perform the algorithm, quietly muttering the steps. But, as invariably happened in the sessions in which this was attempted, she kept losing track of the numbers (Fig. 3).

As in many other sessions, having communicated the required calculation, Juliana's first reaction was to sit back, passing the responsibility to Daniela. In three cases, the person acting as a teacher made a conscious decision not to intervene once the student knew the requested calculation, even following requests for help. This was justified in the one of the post-task discussions with the argument that it is

**Fig. 3** Daniela struggles to remember the steps of her calculation as Juliana sits back

**Fig. 4** Desperation has turned to joy

important for a student to do the calculation on their own. In most sessions, however, in the face of their students' struggles, the teacher did intervene.

In this case, Juliana's first intervention was to place a pen into Daniela's hand, an action that prompted her to attempt to complete the calculation by writing out an algorithm. Again, this was not uncommon and appeared in ten sessions. Daniela worked without help for some time, writing out the steps that she could not see. When she did arrive at an answer, it was incorrect. Unsure how to convey this, Juliana began to communicate the calculation again and Daniela quickly interpreted this to indicate that her answer is wrong. She tried to perform the algorithm again before lifting her head towards Juliana and noting: "*I think I need a different way*". After a moment's pause, Juliana offered her hands to suggest that Daniela calculates 7 times 300, now using her fist for the two zeros. Daniela then spontaneously calculated $60 \times 300$ and added 2100 and 18,000 resulting in 20,100. They both initially thought they had the answer before Juliana realised they had one more step. She tapped Daniela's head to suggest that she remembers

**Fig. 5** Iara on the left (not allowed to see), Alberto on the right (not allowed to speak), initial sense of impasse

20,100. Daniela chose to write it down, to "*hold onto it*". Juliana then finger-communicated the final step of $5 \times 67$, which Daniela calculated again by writing the sum on paper and pencil and verbalising each step to arrive at 335. She finally added 335 to the now familiar 20,100 to obtain the final result of 20,435, leaving both equally pleased (Fig. 4).

## 6.2 You helped me more than I helped you

Alberto (in the role of teacher) had the task of communicating and assisting Iara (in the role of student) to solve $347 \times 36$. They were observed by Zaíra. Alberto also used the popular strategy of counting fingers to indicate the digits in the first number. It took some time for Iara to grasp that the 3, 4 and 7 composed a three-digit number, initially distracted by the fact that $3+4=7$. This provoked Alberto to offer thumbs up and thumbs down as signs for right / wrong. Eventually, after numerous repetitions of the sequence "3, 4, 7, x", Iara identified the first number as 347. The second number (36) was then quickly communicated.

Immediately following the confirmation that her task was to multiply 347 by 36, she says "*How am I going to do this, I am hopeless at calculating mentally, I have a really bad memory*". She starts trying to multiply 347 by 6 but quickly loses track and asks for help: "*how are you going to teach me to do this, Alberto*?". This request was unusual in that, in most sessions, it seems to be initially accepted by both participants that the calculation would be something the student would be able to do, and, in the first instance, attempt on their own.

In this case, as a response to Iara's request for help, Alberto gave her a piece of paper and put a pen in her hand. She rejected both, laughing while saying "*What's this Alberto? Alberto, I can't see.*" It seems that Iara wanted to encourage Alberto to use a strategy other than one commonly used, one that would be appropriate for someone who might never had had access to the visual field. Alberto's gestures (Fig. 5) demonstrated that he had no idea how to help Iara and he resorted to repeating the calculation 347 × 36 over and over, until Iara said "*I know it's 347 × 36, I need you teach me how to do it*".

After a short pause, she joked "*you could just show me the numbers for the answer*" and they both laughed, breaking the tension a little. Iara then went back to mental calculations, whispering to herself before stopping when she became unable to hold all the numbers in her memory. Perhaps because Iara's joke suggested to him it might help if he did know the answer, Alberto sat back, losing tactile contact with Iara, and worked through the calculation on paper. Iara sensed Alberto's distancing and asked "*Alberto, what are you doing, have you given up on me?*". His gestures now directed at the observer suggested that he was indeed on the point of desisting (Fig. 6).

In response to Alberto's gestures, Zaíra (observer) tells him not to give up. Upon hearing this, Iara reiterates "*Are you giving up on me? Don't give up. You can't give up*". They sit without attempting to communicate for a short while, after which Alberto breaks the condition of not being allowed to speak, saying "*This is really difficult, I don't know what to do, you need to tell me how I can help you*". The tone of his voice indicates to Iara, who remained with her eyes shut, his discomfort and insecurity– a contrast with the generally light-hearted tone of the exchanges between them before this point.

His plea turns the interaction around: Iara begins to verbalise more clearly the calculations as she performs them and explicitly solicits that he both help her remember the numbers and indicate when she was correct. She starts by multiplying 347 by 30, letting go of the sequence usual in performing the paper and pencil algorithm and, having ascertained her answer of 10410 was correct, she instructs

> 10410, 10410, 10410. I am never going to forget this number 10410, but write it down, my head is already hurting.

By now, the pace of the interaction had changed as they seemed to be working in tandem. Figure 7 shows how, as Iara verbalised the steps in her calculation and Alberto recorded them, they maintained physical contact.

Rather than attempting to multiply 347 by 6, Iara notices two number facts that she could draw on: 6 is two times 3 and 30 is three times 10. This avoided using the more traditional algorithm, and she explained "*30 was 10410, so 3 times would be 1041 and 6 would be double, 2082*". Alberto vibrates with excitement as he feels the answer is in sight. Having indicated that 2082 is correct, he places Iara's fingers in form of the addition sign and she adds 10,410 to 2082. Alberto raises her hands in triumph when the correct response is obtained (Fig. 8).

**Fig. 6** Alberto appears to be about to give up

**Fig. 7** Iara and Alberto resume working together

**Fig. 8** Iara and Alberto complete the task

They are both pleased. Alberto's expression especially closely resembles pure joy. Iara opens her eyes, and they laugh together as Alberto says "*You helped me more than I helped you*" to which Iara responds "*but it shows the necessity of getting the student to verbalise more their point of view, you had to tell me to do that, but you couldn't, but we can and should do that when we are teaching*".

## 7 Mathematical agency, interdependence and joy in collective mathematical activity

The “telling” episodes we report in 6.1 and 6.2 originate in the same workshop. Following the small group interactions on the Type II task, the whole group reconvened for the post-task discussion. There was a sense of excitement, and everyone was eager to share their experience and their strategies. We note that the same excitement was evident in the workshops that took place in the other three institutions. As we report upon these discussions, we return to our research question, and we reflect on how participants' sense of the roles of tools and teachers in enabling/disabling mathematical practices was affected in relation to each of the three themes that emerged.

*Mathematical agency*. Perhaps not surprisingly, in post-task discussions, a major focus was the algorithm and the difficulties of carrying this out without recourse to the visual field. Wellington (observer in 6.1) noted: “*when she started to calculate, she wrote out the algorithm and then the problem became the algorithm itself*”. All participants in the student role in this workshop (there were four groups) had attempted the algorithm and all had found it difficult. Melanie (observer in another group) noted:

> “I think in all the groups, the person with their eyes shut tried to reproduce the algorithm in their head. So, I am wondering, how does a person who is blind imagine this calculation?”.

The researcher (first author) explained that some schools have access to a grid and Braille pieces (Cubaritmo) which enables blind learners to set out the algorithm, but that some of the blind learners had described to her how they tend to use decomposition or rounding and adjusting strategies. This reminded Iara of her work with youth and adults in a type of schooling in Brazil (Educação de Jovens e Adultos, EJA) aimed at those who did not complete basic schooling with their peers. She described how some of those who worked as builders, for example, used such strategies to calculate with impressive speed.

Taking part in this activity highlighted the value and legitimacy of strategies other than the conventional algorithm, but also how dependent the participant themselves had become on its use. Zaíra summed this up as follows:

> I think this activity took us to an unknown place in the sense that we have to really think about the operations, perhaps we don't really know the operations, we just know the algorithm…The algorithm becomes more important than the operation, than multiplication.

Although alternatives to the algorithm were used in only ten of the 27 sessions, they were discussed in all post-task discussions. In one of the workshops, a participant, Bruno, explained how he had calculated $247 \times 35$ by first adding three to 247, so he could multiply 250 by 30, and adding to this the result of multiplying 247 by 5 before subtracting the extra $3 \times 30$. Though clearly impressed by the creativity of his method, another participant suggested that it worked because he had been lucky with the numbers, while students need to know the algorithm because “*it always works, it is general, efficient*”. To this, Bruno replied laughing “*not with my eyes shut*”. In the post-task discussion of another workshop, David, like Melanie, expressed concerns about the appropriateness of teaching this method to someone who is blind. He was questioned by another participant who was worried that, if the same thing was not taught to the blind student, then this was not inclusion. David's view was that the content, multiplication, could be the same but not necessarily the methods.

Constraining access to the visual field hence highlighted the algorithm (tool)-dependent nature of multiplication, as well as potential difficulties that learners might experience if expected to work with tools that were not congruent with their bodies. These are points which resonate with our Vygotskian interpretation of the roles of embodied, material, and semiotic tools in the practice of mathematics and hence that tools offered in learning situations need to be attuned to the bodyminds of the students.

The post-task discussion in all four institutions suggested that Type II interactions had provoked many to reassess aspects related to the teaching of multiplication, affecting their senses of both the algorithm and other ways of performing multiplication. Their initial approaches had been shaped by their own experiences with multiplication. The algorithm as a tool had become an integral part of their thinking, but there was an incongruency between this method and the resources they had available to perform it. The culturally shared mathematical meanings they had appropriated in their senses for the algorithm likely remained stable, but meanings associated with its role in the process of teaching, learning and practising mathematics were experienced as contested rather than fixed. While some held onto a view, generally supported in curricula sequencing – that the formal written method was in some respects more sophisticated than the other methods – for many students, participation in the task appeared to provoke discursive shifts and some resignification of their senses of multiplication methods. Some argued for the importance of doing things differently with different students, while others seemed to have reflected deeply on the alternative forms of mathematical agency provoked by the absence of access to the visual field, leading to shifts in their evaluations of the mathematical validity of such methods.

*Interdependence.* Another issue that was raised in post-task discussions was the extent of teacher help that was legitimate. We saw in the Alberto/Iara episode, Iara jokingly suggesting that Alberto communicated the answer at one point. In fact, the teacher directly giving the answer to the student was not observed in any of the Type-II sessions. Much more common was an initial expectation that the students would do the calculation on their own. The realisation that, under the conditions posed, it proved very difficult for students to succeed without support, provoked different reactions from those assigned to act as teachers. In three extreme cases, no help at all was offered, except to encourage and indicate correctness of particular calculations. In none of these cases was the result of the calculation obtained. In all other cases, upon perceiving difficulties, the teacher helped. In a small number of cases (in six of the 27 sessions), and always involving the algorithm, the teacher essentially took over, guiding the student through each step. Observers of these cases reported that, although the students correctly performed the steps, they could not tell if they were aware of where they were in the algorithm (in three cases, those acting as students agreed later that they were not). Indeed, in contrast to the shared joy evident in Figs. 4 and 8, expression of satisfaction on solving the task in these cases tended to be muted, with the teacher generally more pleased than the student (Fig. 9).

It could be argued that students' independence had been restricted by the communication conditions we imposed on the task, and that the relative lack of enthusiasm of the students when the teacher took over reflected an over-reliance on the teacher. We offer an alternative interpretation that challenges culturally sanctioned meanings which posit independence as desirable and dependence as a form of weakness. Disabled activists and critical disability scholars have decried the ableist myth of independence (Goodley, 2020), pointing to how all human beings are dependent on each

**Fig. 9** Raquel, in the role of teacher, seems more satisfied than Arthur

other and on the tools of our cultures. Dependencies of the disabled body tend to be more visible, since the dependencies of those with bodies labelled as able, as typical, have become so normalised as to disappear.

What we evidenced in many of the Type II sessions was the emergence of productive interdependencies, with participants in both roles dependent on each other for different aspects of the interaction. Those assigned to be the teachers relied on instructions from those acting as their students, while the students counted on their teachers to serve, for example, as tools for remembering. These interdependent interactions supported autonomy, with autonomy viewed as "emancipation from hegemonic and hierarchical ideologies" rather than reduced to independence (Meekosha & Shuttlesworth, 2009, p. 52–53). While we do not contend that this meaning was explicitly appropriated by participants in this study, there were a number of expressions of interdependence both in the Type II tasks (emerging in 18 of the 27 sessions) and in all post-task discussions, evidencing discursive shifts in the participants' senses of teacher-student relationships. For example, in relation to the first episode, Juliana commented that Daniela (as student) "*showed me the way to interact with her*", while Daniela recognised how "*Juliana thought of decomposition, even then it was difficult to recollect, but she helped with this. I used her as my memory*". Alberto and Iara's interactions also affected their sense of interdependencies between teachers and learners as their last comments above illustrate.

*Entwinement of emotion and cognition*. The uncertainty and anxieties about how to proceed expressed by those assigned to act as the teacher in both the above episodes emerged at some point in all Type II interactions. As different forms of communicating numbers and operations were developed, this frequently turned to a shared sense of delight. Not all the interactions ended so positively: three participants in the teacher's role suspended the interaction before they had communicated the task; and, in a further seven groups, the calculation was only ever partly completed. But, in the cases in which student autonomy had been supported by the teachers' interventions, the sense of satisfaction of both participants was palpable– as the creative enactment of mathematical agency in forms not previously experienced was felt with enjoyment and pleasure. These affective shifts, we argue, are as significant to the resignification process as the discursive shifts related to agency and interdependency discussed in the other two themes.

We also observed how those acting as the student seemed able to sense how their teachers were feeling, even though they couldn't see them. Iara's awareness that Alberto was on the point of desisting affected how she felt in the moment. It also provoked her to identify with those students who might have difficulty in appropriating mathematical ideas communicated by her and impacted her sense of the role of the teacher in enabling or disabling mathematical practices:

> Iara: So, for me a sensation that perhaps a student would feel was when Alberto gave up on me. I felt so frustrated, and I said to him don't give up. I felt super sad with this and thought this could be how a student ends up feeling sometimes if we don't manage to help them, and it seems like we are losing our patience or we ourselves feel we are not capable of creating another strategy so the student can learn.
> Alberto: And this really does happen, especially to disabled students […] In a class of 40, without resources, a teacher ends up giving up.
> Iara: Well, we can't. We can't. I know how that feels. It made me feel I was a bit of a donkey, what if that kept happening to me? How would I feel about myself and about mathematics?

The overarching aim of the CAPTeaM project is to explore how practising and future teachers might come to recognise, feel and challenge the ableist practices currently associated with school mathematics and to imagine pedagogies that empower rather than disable learners. In this paper, we have examined how inviting them to interact differently as both teachers and learners of mathematics on what we have called Type II tasks might contribute to this aim.

Our analyses suggest that participating in such activities motivated both implicit and explicit questioning of some of the dominant narratives associated with success in school mathematics. The challenge of solving a familiar task in the face of an incongruency between their favoured method and the resources they had available to perform meant that it was necessary to seek new ways of expressing themselves mathematically and alternatives to the formal written method they could no longer successfully employ. The artificial restrictions frequently led to a blurring of conventional roles associated with being a teacher and being a learner– and the establishing of productive interdependencies in which autonomy, rather than independence, was supported. They also motivated counter-narratives to the privileging of solitary performance of mathematics and encouraged rethinking the relationships between formal written methods and other forms of mathematical agency.

We end though by returning to the question of affect. Iara's final contribution corroborates the embodied view that teaching is both intra- and inter-personal. It is both informed by, and informs, the actions, emotions, and senses of others and of ourselves. Like Iara, we feel passionately about dismantling the practices and discourses that marginalise and disable so many mathematics learners and we recognise that, both as teachers and as researchers, our own senses of mathematics and its teaching and learning have been strongly affected by the intimate connections of emotion and cognition played out in the research activities we have been involved in. By restricting participants from using the resources that they are accustomed to use to communicate and solve a mathematical task, the Type II tasks invite teachers to temporarily inhabit a body whose capacities are different from their own. Our evidence suggests that this experience can encourage them to feel for themselves how inclusion and exclusion might be sensed by their students. We hope too that the analyses presented in this paper open a window onto our feelings about inclusion, and about how– and why– rather than deficiency, we view difference as potential, renewal, change, resistance, and inspiration.

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