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ABSTRACT

An IAHR Working Group was formed to study multidirectional wave simulation, generation and analysis techniques: this paper focuses on comparison of analysis methods used by different Group Members.

Twelve sets of multidirectional wave data were produced, all with the same incident wave height, period and direction, but corresponding to diverse and realistic situations in laboratory wave basins. Two different directional spreads were used; two different numerical synthesis methods plus wave basin measurements; four data sets included reflected or cross-wave components; and three data sets incorporated common recording faults.

The data sets were analysed separately by each of the Group Members using their own preferred methods, and the results were then collated. The main comparisons are presented in terms of the mean and sprêad of wave direction, reflection coefficients, and the distribution of wave energy as a function of direction.

The tests served to validate Group Members' existing analysis techniques, and they now provide a high standard against which methods can continue to be

tested in the future. Some generic conclusions are also drawn, relating to the ability of different types of method to cope with different situations such as measurement error, reflections or cross-waves.

1 INTRODUCTION

An IAHR Working Group on Multidirectional Waves was established in 1993 by the IAHR Maritime Hydraulics Section. The Group consisted of ten representatives from some of the world's leading hydraulics laboratories (Briggs, 1997). The Group met at roughly 6-monthly intervals during its four year existence: it presented six papers at the IAHR Speciality Seminar on Multidirectional Waves and their Interaction with Structures at the IAHR 1997 Congress in San Fransisco. The present paper compares the results of the many different methods used by the different laboratories to analyse common sets of synthetic and measured multidirectional wave basin data. A related paper (Miles *et al*, 1997) applies a common analysis method to wave data recorded in several different laboratories. Another Group paper (Benoit *et al*, 1997) reviews and classifies the different analysis methods.

The comparative analysis of common multidirectional wave data sets had several purposes:

- to validate Group Members' analysis methods, and to compare results with those of other Members;

- to provide a high standard against which non-Members could test their own methods;

- to test the impact of faulty measurements, due to gain, cross-talk or noise, on analysis results;

- to determine which types of method worked best in different situations;

- to determine which methods could cope with identification of reflected or cross-wave components;

- to test the ability of methods to correctly analyse the incident wave condition, in the face of cross-waves, reflections or measurement errors;

- to look at the effect of different directional spreads, both in simulated and measured wave data.

A total of twelve data sets were produced and analysed, all with the same incident wave height, period and direction. Two different directional spreads were used, and two different numerical synthesis methods plus wave basin measurements; four data sets included reflected or cross-wave components; and three data sets incorporated common recording faults. Details are given in

Section 2.1.

Since all methods were successful in analysing the wave energy spectrum as a function of frequency, this was not pursued in any detail. Instead, the analyses by different laboratories and different methods are compared with each other and with target values in several different ways, but mainly in terms of:

- the mean and spread of wave direction as a function of frequency;

- the mean and spread of wave direction averaged over all frequencies;

- reflection coefficients as a function of frequency and overall;

- the distribution of wave energy as a function of direction.

The casual reader may wish to focus his attention on the results for Tests A1 and B1, consisting of idealised wave spectra with broad and narrow directional spreads, respectively, and no reflected component. Tests D1 and E1 are the equivalent test cases recorded in a laboratory basin. The remaining eight cases are of more specialised interest.

In keeping with normal Working Group practice, the separate laboratories are not identified by name, but by a reference number between 1 and 10, in no particular order. This is to reflect the intention to carry out an objective scientific comparison, rather than to conduct a competition between the different laboratories.

The test data sets are freely available from Group Members.

2 THE TEST CASES

2.1 DESCRIPTION OF THE TESTS

Tests A, B and C were based on numerical simulations provided by Delft Hydraulics and Marintek using the summation of sine waves method. The methods for synthesising the data are discussed in Section 2.2. Copies of the synthetic wave data were sent to all ten laboratories and the results of the directional wave analysis were collated and analysed by HR Wallingford.

The simulations were performed for the following gauge arrangements:

- linear surface elevation signals at the centre (η_6) of a pentagon and at the five vertices (η_1-η_5) located 0.5m from its centre, as shown in Figure 1;

- linear horizontal particle velocities (u, v) in the x- and y-directions (at the centre of the pentagon but 0.2m below the still water level).

The input to the simulations consisted of a JONSWAP spectrum with a significant wave height of $H_s = 0.12$m, a peak period of $T_p = 1.8$s and a peak enhancement factor of $\gamma = 3.3$. This spectrum was also used by a previous IAHR Working Group on Wave Generation and Analysis (Briggs, 1997) and was the same for all the tests.

A model form of $\cos^{2s}\frac{1}{2}(\theta-\theta_0)$ was chosen for the directional distribution, where θ is the direction relative to the normal to the wavemaker. Two basic tests were considered, corresponding to s = 6 and s = 40, with spreads of 31.8° and 12.7°, respectively. The mean wave direction (θ_0) was 30° in all of the tests (see Figure 1).

Additional Tests D and E used equivalent measured wave spectra provided by the Canadian Hydraulics Centre as input to the comparative analyses.

Table 1 lists and briefly describes the tests.

Table 1: List of comparative analysis tests

Test	T_p	H_{m0}	θ_0	s	d	Comments
A1	1.8s	12cm	30°	6	3.0m	Standard test
A2	1.8s	12cm	30°	6	3.0m	Double-sum
A4	1.8s	12cm	30°	6	3.0m	20% noise (all gauges)
A5	1.8s	12cm	30°	6	3.0m	1.1 gain (η_1 and u)
A6	1.8s	12cm	30°	6	3.0m	Cross talk (η_1/η_2 and u/v)
B1	1.8s	12cm	30°	40	3.0m	Narrow spread
C1	1.8s	12cm	30°	6	3.0m	20% refn, wall parallel to wavemaker at 10m
C2	1.8s	12cm	30°	6	3.0m	70% refn, wall parallel to wavemaker at 1m
C3	1.8s	12cm	30°	6	3.0m	70% "reflection" from wall parallel to wavemaker
C4	1.8s	12cm	30°	6	3.0m	cross-resonant modes: 4cm at -50°; 4cm at +130°
D1	1.8s	12cm	30°	6	0.5m	*Measured* at Ottawa
E1	1.8s	12cm	30°	40	0.5m	*Measured*, narrow spread

Test A1 is considered to be the standard case. A2 is the same test but with simulation derived using the "double-summation" technique, which involved several times more sine wave components than used for the standard case. Tests A4, A5 and A6 are the same as Test A1, but with noise, amplification on one channel, and cross-talk between channels, respectively.

Test B1 is a simulation with a narrow directional spread corresponding to s = 40.

Tests C1-C3 include reflected wave components: C1 has reflected waves with 20% of the height of the incident waves reflected from a wall ten metres from the centre of the pentagon, and parallel with the wavemaker; C2 with 70% reflection from a wall one metre away; C3 also includes a 70% reflected wave height, but without phase-locking of the waves between the incident and reflected components. Test C4 is for a tri-modal sea state consisting of incident waves and two cross-modes.

Finally, Tests D1 (s = 6) and E1 (s = 40) are equivalent to Tests A1 and B1 but measured in a laboratory basin.

The record length for all tests was 1500s with sampling at 20Hz.

Figure 2 illustrates the target directional spectra for Tests A1 (standard test), B1 (narrow spread) and C2 (70% reflection).

2.2 METHOD OF DATA SYNTHESIS

The synthetic wave elevation signals $\eta(x, y, t)$ at the given array locations $\mathbf{r} = (x, y)$ were numerically generated as discrete time series by inverse FFT, as follows:

$$\eta(x, y, t_i) = \sum_{n=-NF}^{NF} N(f_n; x, y) \exp(i\, 2\pi\, f_n\, t_i) \tag{1}$$

where $N(f_n; x, y)$ is a random, complex elevation amplitude component assigned to the discrete wave frequency number n, f_n, at the location $\mathbf{r} = (x, y)$, and i is the imaginary unit. In these tests, the Fourier amplitudes N are calculated in two different ways, according to whether single-summation or double-summation is applied. NF is the number of equidistant (and positive) FFT frequencies, which was chosen to be 16384. The time increment $\Delta t = t_i - t_{i-1}$ was 0.050s; the total record duration used in the analysis was 1500 seconds, which is slightly shorter than the FFT repetition cycle of 1638.4 seconds.

For single-summation (i.e. for all cases except Case A2), N was generated as:

$$N(f_n; x, y) = F_n \cdot \exp[-i(\mathbf{k}_n \cdot \mathbf{r} - \phi_n)] \tag{2}$$

where the modulus F_n is a random, real-valued variable selected from a Rayleigh distribution such that its expected square value is equal to $\frac{1}{2}S(f_n)\Delta f$. Here S is the specified one-sided scalar (frequency) wave energy spectrum, and Δf is the frequency increment $f_n - f_{n-1}$. $\mathbf{k}_n = [k_n\cos\theta_n; k_n\sin\theta_n]$ is the wave vector, k_n is the angular wave number connected with the frequency f_n through the linear dispersion relation, θ_n is a wave direction randomly selected for the frequency f_n according to a probability distribution equal to the actual $\cos^{2s}½(\theta - \theta_0)$ distribution, and ϕ_n is a random phase angle uniformly distributed in $[-\pi, +\pi]$. Thus there is only one random direction assigned to each frequency. For this numerical test, there was no constraint on the upper and lower direction limits, but practically all the energy was seen to be distributed in the 180 degree half-plane around the mean direction.

For double-summation (i.e. for Case A2), N was generated as:

$$N(f_n; x, y) = \sum_{m=1}^{ND} F_{mn} \cdot \exp\left[-i(\mathbf{k}_{mn} \cdot \mathbf{r} - \phi_{mn})\right] \qquad (3)$$

where F_{mn} is the amplitude of a plane harmonic wave component with frequency f_n and direction θ_m. The value of F_{mn} is given from the specified spectrum as $[½\, S(f_n)\, D(\theta_m)\, \Delta f\, \Delta\theta]^{½}$, where $S(f_n)$ and $D(\theta_m)$ are the one-sided wave energy spectrum and the directional distribution, respectively. Δf and $\Delta\theta$ are the frequency and directional increments, respectively. Furthermore, the wave vector $\mathbf{k}_{mn}$ is defined as $[k_n\cos\theta_m; k_n\sin\theta_m]$. Thus for each frequency f_n (or wave number $k_n = k(f_n)$) there is a summation over ND discrete equidistant directions θ_m, which for this numerical test were distributed over the 180 degree half-plane around the mean direction. To each plane wave component F_{mn} there was assigned a random phase ϕ_{mn}, uniformly distributed in $[-\pi, +\pi]$. The number of discrete directions, ND, was 100, and they were the same for all frequencies.

The corresponding synthetic horizontal velocity vector signals $\mathbf{u}(x, y, z, t) = \mathbf{u}(\mathbf{r}, z, t) = [u; v]$ were generated in a similar manner, at the position $[x_0; y_0; z_0] = [0; 0; -0.20\text{m}]$. Thus they were numerically generated as described by the procedures above, except that the amplitudes F_n in eq. (2) were replaced by $F_n \cdot H_{\mathbf{u}}(f_n,z) \cdot G(\theta_n)$, and F_{mn} in eq. (3) by $F_{mn} \cdot H_{\mathbf{u}}(f_n,z) \cdot G(\theta_m)$. Here:

$$H_{\mathbf{u}}(f_n, z) = 2\pi\, f_n \cosh[k_n(h+z)] \,/\, \sin(k_n h) \qquad (4)$$

where h = water depth and $G(\theta_k) = \cos(\theta_k)$ or $\sin(\theta_k)$, for the x-velocity u or the y-velocity v, respectively.

2.3 TARGET VALUES OF MEAN WAVE DIRECTION AND SPREAD

The calculations of mean wave direction and spread were made over the range $\theta = \pm 90°$, ie the half-plane of directions away from the wavemaker. For the spectral peak direction of $\theta_0 = 30°$ used in the tests, this means that generated energy was spread over 90° on one side of the peak direction but over only 60° on the other. Because of this directional "cut-off" for broad spectra (s = 6), the target values of mean direction and spread are 27.96° and 29.72° respectively (and not 30° and 31.8° as would be expected for full ±90° spreading). (For Tests C with reflected wave energy, the target mean direction for the reflected component is 152.04° with the same spread as the incident wave, that is 29.72°.)

For Tests B1 and E1, with narrow directional distributions (s = 40), there is no effect due to directional "cut-off", and the target mean direction and spread are 30° and 12.7°, respectively.

In order to be able to evaluate the nature of statistical scatter expected in the results, sample target mean and spread values were also worked out as a function of frequency, based on the actual random sine components and directions used in these numerical realisations. Thus the "sample targets" were calculated based on the directional energy distribution within each frequency band of 0.039Hz (see Chapter 3) in the actual realisations of Tests A1, A2 and B1. The results of this exercise are described together with the comparative analysis in Chapter 4.

As no truncation was applied in the numerical simulation to remove wave energy that would be generated backwards from the wavemaker, about 3% of the incident energy in the broad spectrum tests spills over to the reflected wave half-plane. For Test C1 this means that the reflected wave half-plane contains a total of 7% (not 4%) of the energy present in the incident wave half-plane. Consequently the "sample reflection coefficient" for Test C1 becomes $\sqrt{0.07}$, ie 0.26 instead of the expected 0.20. A similar argument applied to Tests C2 and C3 means that the sample reflection coefficient is 0.72 instead of the nominal target value of 0.70.

3 COLLATION OF RESULTS

The test data sets were analysed by Group Members using the same methods and assumptions as they would apply in the normal course of research and consultancy studies. The essential features of the different methods used are summarised in Table 2, which also provides a key to the labelling used in Tables 3-5 and Figures 3-24. The methods and their underlying assumptions are described in more detail in Benoit *et al* (1997).

Preliminary results for the non-reflection cases suggested that all methods were successful in analysing the frequency spectrum, significant wave height, and mean and peak wave periods: comparison of these parameters was therefore not pursued in any detail. Instead, the analyses by different laboratories and different methods are compared with each other and with target values mainly in terms of the directional parameters.

Analysis results were produced in the same format and over the same range of frequencies and directions for all laboratories and for all test cases. The two-dimensional (frequency - direction) matrices supplied by participants were for frequencies between 0.430Hz and 1.133Hz (0.039Hz interval) and for directions from θ = -90° to θ = +90° (2° interval). For Tests C, with wave reflection, data was supplied over the complete circular range from -90° to +270°. The frequency cut-offs (and to a slight extent the direction cut-offs) had the effect of reducing wave heights by about 2%, but produced more reliable directional results over the retained range of frequencies.

Table 2: Key to labelling of analysis methods

Lab	Label	Description or aspect of method or gauge
All	huv	velocity and surface elevation gauge η_6-u-v
All	5probe	gauge consisting of the five outer surface elevation probes
All	6probe	gauge consisting of all six surface elevation probes
1	multi-3probe	results averaged over five 3-probe triangle gauges
All	MEM	Maximum Entropy Method
1, 7	EMEP	Extended Maximum Entropy Method
1, 3	MLM	Maximum Likelihood Method
7	EMLM	Extended Maximum Likelihood Method
5	MLM2	MLM modified to search for two distinct directional peaks
2	SDA	Single Direction Analysis Method
2	DDA	Double Direction Analysis Method
2	DDAC	Double Direction Analysis (with Constraint) Method
3	Fourier	Fourier Analysis Method
3	short	analysis based on a record length of 819.2s
3	long	analysis based on a full 1500s record
5, 6	BDM	Bayesian Method
9	circular	Circular Analysis Method

The mean direction ($\theta_m(f)$) and spread ($\sigma_\theta(f)$) for each frequency (f) were computed (by HR Wallingford) from the spectral density information ($S(f, \theta)$) for each analysis method over the frequency range 0.430Hz to 1.133Hz. The upper and lower frequencies correspond to $2f_p$ and $0.8f_p$ respectively, where f_p (= 0.556Hz) is the spectral peak of the simulated data. The integration range for incident wave direction was $-90° < \theta < +90°$; in addition for Tests C1-C3, a separate second range for reflected wave direction was used, namely $90° < \theta < 270°$.

$$\theta_m(f) = \arctan\left(\int S(f, \theta)\sin\theta\, d\theta, \int S(f, \theta)\cos\theta\, d\theta\right) \tag{5}$$

$$\underline{\theta} = \int[\theta_m(f)\, S(f)\, df] \,/\, \int[S(f)\, df] \tag{6}$$

$$\sigma_\theta^2(f) = \int S(f, \theta)\,(\theta - \theta_m(f))^2\, d\theta \tag{7}$$

$$\underline{\sigma} = \int[\sigma_\theta(f)\, S(f)\, df] \,/\, \int[S(f)\, df] \tag{8}$$

Effectively the results are energy-weighted averages and standard deviations of wave direction, averaged over all directions in each half-plane, for each frequency in turn ($\theta_m(f)$ and $\sigma_\theta(f)$) and then overall ($\underline{\theta}$ and $\underline{\sigma}$); a circular representation of θ was used for the mean, and a linear representation of θ for spread. The equations used are discussed further in Frigaard *et al* (1997).

4 COMPARISON OF RESULTS

The results are presented in three ways for each test and for each laboratory: firstly, as a comparison of the mean direction and spread; secondly, as a comparison of the reflection coefficients (for Tests C1-C3); and thirdly, as a comparison of the distribution of wave energy (integrated over all frequencies) against wave direction (for Tests C1-C4). They are shown in a series of tables and figures, in which the results from the different analysis methods are compared with each other and with target values. (The ten laboratories are referenced anonymously by a code number between 1 and 10, in no particular order, but used consistently throughout the paper.)

4.1 MEAN DIRECTION AND SPREAD

Figures 3-17 show the results for mean direction and spread, with summary results being given in Tables 3 and 4. (Some of the tests were not analysed by individual laboratories and therefore appear as blanks in the tables and diagrams.) The nominal (ie theoretical, as opposed to sample) target values of mean direction and spread for each test are indicated by horizontal lines over all frequencies.

A useful overall comparison of the results is shown in the inset diagram at the foot of each page where the overall mean wave direction and spread are compared with the target values. Also included in these diagrams are the means and standard deviations of the results of the various individual analyses. For Tests C1-C3 there are two figures for each test, corresponding to the incident and reflected half-planes.

<u>Test A1 (Figure 3, single summation)</u>

General comments:

- The mean direction is "less" than the nominal target value for frequencies below about 0.8Hz ($1.4f_p$); at higher frequencies the mean direction oscillates about the target value.
- Results for the spread show values close to the target value below f_p; at higher frequencies there is a tendency for the spread to decrease.
- The inset figure at the foot of the page shows that the overall mean wave direction is, for almost all the laboratories, "lower" but within one standard deviation of the target value.
- For the overall spread, most of the laboratories are lower, but again within one standard deviation of the target value.

Particular comments:

- There is not much difference between results from different methods used (see for example Lab 2 (3 methods) and Lab 7 (2 methods)).
- There are minor differences between results from apparently similar systems used by individual laboratories (see η_6-u-v (MEM) results for Labs 2, 4, 6, 8 and 10).
- Different record length has only a very small influence (see Lab 3).

It is interesting to note that the "sample target" values worked out on the basis of the actual set of sine components and directions showed roughly the same variations with frequency as typically observed in Figure 3. Thus a significant part of the observed deviations from the "nominal" targets is due to the random realisation, rather than to the analysis itself.

Test A2 (Figure 4, double-summation)
Results and comments are quite similar to those for A1, but results for both mean direction and spread are closer to target values. The statistical variation with frequency is lower than for Test A1. This agrees well with observations from the "sample target" values that were worked out, where a variability level of about ±1-2 degrees was found.

Test A4 (Figure 5, 20% noise)
Noise has little effect on mean wave direction except in the results from Lab 5, where the modified Maximum Likelihood Method (MLM2) gives poor results for both mean wave direction and spread. The spread for other laboratories is also influenced to some extent by noise.

Test A5 (Figure 6, 1.1 gain: η_1 and u)
The mean direction is relatively robust to the addition of 10% gain, but in some cases the spread is degraded by this amount of gain, compared to results for Test A1. Results for Labs 4, 8, 9 and 10 and for Labs 2 (SDA), 3 (Fourier), 5 (BDM) and 6 (MEM) are little affected by the gain.

Particular comments:

- The spread for Lab 1 approaches the target value only for frequencies beyond 0.8Hz ($1.4f_p$).
- Results for Lab 3 from the 6-probe array are degraded at low frequencies.
- The modified MLM method gives poor results at low and high frequencies.

Test A6 (Figure 7, cross-talk: η_1/η_2 and u/v)
Both mean direction and spread tend to be degraded by cross-talk, compared to results for Test A1. Results for Labs 2, 4, 6 and 8, and for Lab 3 (Fourier method) are little affected by the gain.

Test B1 (Figure 8, narrow spread)
Most laboratories achieve good results for this more long-crested wave test. The exception is the modified MLM method of Lab 5.

As in Test A1, the "sample target" values worked out on the basis of the sine wave components used to generate the data showed roughly the same variations with frequency as seen in Figure 8. This supports the view that a significant part of the discrepancy between the analysis results and the "nominal" targets is due

Test C2 (Figure 22, 70% reflection, wall at 1m)
Reflected wave energy is clearly seen in the results from Labs 2, 4, 5 and 6, and from Lab 8 (η_6-u-v). The other laboratories were unable to detect the presence of a separate reflected wave component. Again this supports the conclusions drawn in Section 4.1.

Test C3 (Figure 23, 70% reflection)
Reflected wave energy is evident in results from all of the laboratories shown, including those which failed to detect the reflected energy in Test C2.

Test C4 (Figure 24, cross-resonant modes)
The cross-resonant modes are shown for M1 (at -50°) and M2 (at 130°). Labs 5 and 6 (6-probe array), both using the Bayesian method, show the best evidence of the two cross-modes in their results (although these same methods are amongst the poorest at reproducing the target incident spectrum alone, see Figure 15). The other laboratories are unable to distinguish clearly the two modes, although most of them show some sideband energy.

The selection of a "best" method for a sea state with reflections or cross-modes depends very much on what is required from the analysis, for example detection of secondary peaks, representation of the incident spectrum, and/or estimation of reflection coefficients.

5 CONCLUSIONS

Results for Tests A1 (standard test) and A2 (double-summation) showed that, in general, all the laboratories produced results close to the target values for both mean wave direction and spread. There was no significant and consistent difference in the results obtained from different analysis methods and/or different systems of measurement. Average deviations from the nominal targets of mean and spread, seen to be more or less common to most laboratories, are considered to be mainly due to the random nature of the record realisations, and not to the analysis. Results for Test D1 (standard test, measured) gave similar conclusions to those for Test A1.

Test A4 showed that the addition of 20% noise had a small effect on the results. Test A5 showed that a 10% increase in gain on one channel had a small influence on mean wave direction but degraded the spread. Test A6 showed that cross-talk has an effect on both mean direction and spread.

Most laboratories achieved good results for Tests B1 and E1 with a narrow directional spread. The mean wave direction and spread are closer to the target values than for Tests A.

For Test C1 (20% reflection, wall at 10m) the results for the reflected wave direction and spread for all laboratories appear poor (see Figure 10) but this is as much due to sampling effects as to the analysis itself.

For Test C2 (70% reflection, wall at 1m) the results for the reflected waves are closer to the target values for mean wave direction and spread than for Test C1. The reflected wave energy is clearly seen in the energy against direction graphs for some, but not all laboratories. Meanwhile, results for the incident wave component are generally not quite as good as for Test C1. These observations were to be expected, as the reflected component is much larger in this test than in Test C1.

Test C3 (still 70% reflection, but now without phase-locking) gave consistent results for the reflected wave direction and spread for all laboratories; the graphs of wave energy against direction show the second (reflected) peak more clearly than for Test C2.

The analysis of reflection coefficients showed the results to be independent of frequency, as expected from the test data set. The results for Test C3 were closer to the target values than for Tests C1 and C2, although results for Test C1 (20% reflection) were noticeably closer to the sample reflection coefficient of 0.26 than to the nominal target value of 0.20.

Most analysis methods "survived" the presence of the cross-modes of Test C4, in the sense that they continued to detect the incident wave conditions reasonably well. The two cross-resonant modes of Test C4 were detected only by methods used in two laboratories.

Analyses of Test A1, and to a lesser extent related Tests A4-6 and Tests C, are consistently in error relative to the nominal target values, with a repeatable pattern of error: the mean wave direction tends to be less than the target value for frequencies below about 0.8Hz ($1.4f_p$); at higher frequencies, the direction oscillates about the target value. Analysis of the sine wave components used to generate the data for Tests A1, A4-A6 and C1-C4 (carried out after the authors' analyses of the test data sets) showed a rather similar pattern of deviation from nominal target values. The overall mean wave direction is, in general, within 3° of the target value for all laboratories. Values of spread are more variable: at higher frequencies there is a tendency for the spread to decrease.

Conversely, analyses of Test A2, which was produced from a much larger group of sine wave components derived using the "double-summation" technique, match the target values much better. This is not surprising, as the larger number of components permit the target spectrum to be more finely resolved in Test A2 than in the other synthetic data cases. Results for Test B1 also match the target values better than in most of the other tests. The reason here is that for the single-summation method a rather small number of directions are used (one per frequency) and thus a narrow distribution is represented better than a wide distribution.

There does not appear to be much consistent difference between results from to sampling rather than to analysis effects.

Test C1 (Figures 9 and 10, 20% reflection, wall at 10m)
Results for the incident waves have similar characteristics to Test A1. Results from all laboratories for reflected waves (nominally 4% of incident wave energy) appear poor for both mean direction and spread. However, in the numerical simulation some of the incident wave energy spilled over into the reflected wave half-plane. As a result, the apparently "low" reflected wave directions and high spreads seen in the analyses are closer to sample target values than to the nominal target values shown in Figure 10.

Test C2 (Figures 11 and 12, 70% reflection, wall at 1m)
The results for incident waves show greater variability in both mean wave direction and spread, than for Test C1. Results for reflected waves (nominally 49% of incident wave energy, but actually about 52%) are closer to the target values than in Test C1 for both mean direction and spread.

Test C3 (Figures 13 and 14, 70% reflection)
Results for this test are much less variable than Tests C1 and C2, with incident and reflected wave direction and spread closer to the target values.

Test C4 (Figure 15, cross-resonant modes)
In general the results for incident waves show considerable variability: the mean wave direction is "below" the target value and the spread is greater than the target value. For the reflected half-plane, there is no dominant wave direction because of the existence of two oblique cross-modes: comparison with target values would therefore not be helpful.

Test D1 (Figure 16, as A1, but d = 0.5m and *measured* at Ottawa)
Satisfactory results were obtained by all laboratories except for Lab 3 (η_6-u-v system) and Lab 5 (modified MLM method).

Test E1 (Figure 17, as B1, narrow spread, but d = 0.5m and *measured* at Ottawa)
Most of the laboratories obtained good results for this test with a narrow directional distribution. Exceptions are Lab 1 (spread), Lab 3 (spread with η_6-u-v) and Lab 5 (modified MLM).

4.2 REFLECTION COEFFICIENT

The reflection coefficient (R) was computed for Tests C1-C3 (with reflection) as a function of wave frequency, from:

$$R^2 = \text{(energy in reflected half-plane) / (energy in incident half-plane)} \qquad (9)$$

The results are plotted in Figures 18-20 as a function of frequency for all methods used; the nominal target values (0.2 for Test C1, and 0.7 for Tests C2 and C3) are also shown. The corresponding incident and reflected significant wave heights are listed in Table 5.

Test C1 (Figure 18, 20% reflection, wall at 10m)
The reflection coefficient results from all laboratories are higher than the nominal target value of 0.20 (except for the method SDA of Lab 2), but most are quite close to the sample target value of 0.26. The reflection coefficient is almost independent of wave frequency, as would be expected since this was a feature of the test data set. (Rounding errors at frequencies where there is little wave energy in the spectrum give erroneous results for Lab 3 beyond 0.9Hz.)

The incident significant wave heights (see Table 5) are slightly below the target value and the reflected wave heights a little above. However, after allowance for the frequency cut-offs used, the total energy of the two is about right. It is not clear whether this is a genuine feature of the data for Test C1, or due to smoothing effects within the analysis methods.

Test C2 (Figure 19, 70% reflection, wall at 1m)
Reflection coefficient results are closer to the target value than for Test C1, with the exception of Lab 5 (MLM2 method), Lab 6 (6-probe array) and Lab 8 (η_6-u-v). The reflection coefficient is more variable with frequency compared to Test C1. There does not appear to be a consistent conclusion common to either analysis method or system used. Both the incident and reflected wave heights are about 10% below target values. Apart from a small reduction in wave height due to the frequency cut-offs used, this effect remains unexplained.

Test C3 (Figure 20, 70% reflection)
The results for most of the labs lie closer to the target values than for Tests C1 and C2: this was to be expected as the lack of phase-locking between the incident and reflected waves makes the two components easier to separate out. One case where these comments do not apply is the deterministic methods of Lab 2: these methods do not make any assumption about phases, and results for Tests C2 and C3 are equally good. The incident significant wave heights are a little below the target value, whilst the reflected wave heights are consistently close to target; both are closer to target values than in Test C2.

4.3 WAVE ENERGY AGAINST DIRECTION
Graphs of wave energy (integrated over frequency) against direction can reveal the existence or otherwise of genuine peaks in wave direction for Tests C1-C4. Figures 21-24 show the results for the participating laboratories.

Test C1 (Figure 21, 20% reflection, wall at 10m)
There is no evidence of a separate reflected wave energy peak from any of the laboratories except for a distinct peak at 150° for Lab 2 (DDAC method, which uses its knowledge of the reflecting structure's orientation to search for direction peaks at 30° and at 150°). This is consistent with comments made in Section 4.1.

different methods, or between results from different gauges (6-probe or η_6-u-v). One exception is Lab 5's modified MLM method (modified specifically to detect two directional peaks) which performs less well than expected. This is thought to be due to a programming bug rather than an inherent flaw in the method, but the results have been retained since the method is in normal use at Lab 5.

The use of a different record length has little influence on the results, as shown by Lab 3's results using both shortened and full length data sets.

As the directional resolution of an analysis method increases, the statistical variability also increases. Some methods having few degrees of freedom (eg Maximum Entropy Methods fitting the directional spreading function to a 2-peaked curve) give reliable results provided that the true directional spreading function has no more than two peaks. Some methods having many degrees of freedom (eg Bayesian methods fitting the directional spreading function to a 72-peaked curve) are capable of detecting more complex phenomena such as the multi-peaked directional spreading function associated with cross-modes.

Lab 2's DDAC (Double Direction Analysis with Constraint) was amongst the most successful methods for analysis of the reflection Tests C1-C3, and was the only method to detect a distinct second peak for Test C1. It uses its knowledge of the existence and orientation of a reflecting structure to search for a second peak from an appropriate direction.

The choice between a high resolution method and a more reliable low resolution method will depend on the expected wave field. Ideally an analysis should begin with a high resolution method designed to detect all the relevant physical phenomena, and then proceed to a low resolution method designed to give statistically more reliable results.

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M Miles *et al* (1997). "A comparative study of multidirectional waves generated in laboratory basins." IAHR Speciality Seminar on Multidirectional Waves and their Interaction with Structures, IAHR Congress, San Fransisco.

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Lab No	Method	Overall mean wave direction (deg)															
		A1	A2	A4	A5	A6	B1	C1 (inc)	C1 (refl)	C2 (inc)	C2 (refl)	C3 (inc)	C3 (refl)	C4 (inc)	C4 (refl)	D1	E1
	Target	**27.96**	**27.96**	**27.96**	**27.96**	**27.96**	**30.00**	**27.96**	**152.04**	**27.96**	**152.04**	**27.96**	**152.04**	**27.96**	**152.04**	**27.96**	**30.00**
1	6probe (EMEP)	23.94	25.98	22.98	22.56	13.30	26.65	25.13	137.92	46.46	130.92	25.66	149.11	17.71	175.84	23.80	27.51
1	multi-3probe array (MLM)	31.67	30.80				29.80										
2	huv (SDA)	26.63	27.15	26.40	24.76	30.35	29.41	28.80	111.13	31.06	136.53	32.20	141.87	24.26	113.51	27.31	31.25
2	huv (DDA)	26.22	27.87	26.15	24.98	29.07	29.54	26.59	122.27	29.52	145.59	26.04	151.15	21.57	124.73	26.25	30.84
2	huv (MEM)	26.63	28.18	26.38	24.14	30.52	29.43	26.61	125.50	28.16	147.75	24.74	153.78	20.73	119.65	27.43	31.09
2	huv (DDAC)							24.29	157.16	27.77	154.83	22.37	159.81				
3	6probe,short (MLM)	26.65	28.65	25.80	25.30	15.86	29.40	28.04	120.29	52.06	118.30	28.06	147.47	20.42	116.80	26.71	30.53
3	6probe,long (MLM)	26.81	28.48	26.23	25.26	16.33	29.32	28.08	120.66	51.46	118.75	28.05	147.73	20.72	116.59	27.20	30.84
3	huv,short (Fourier)	25.61		25.27	25.50	26.49	29.39	26.97	129.54	41.17	133.75	25.70	150.12	21.49	134.49	24.67	29.10
3	huv,long (Fourier)	25.56		25.31	25.36	26.33	29.10	26.95	128.68	40.91	133.59	26.43	149.61	21.99	136.66	25.53	30.06
4	huv (MEM)	26.89	28.18	26.02	24.36	30.76	29.37	26.90	126.11	28.31	147.31	25.15	153.57				
5	6probe (MLM2)	21.99		19.77	25.27	21.70	16.11			19.81	133.69			17.75	118.00	19.08	23.48
5	6probe (BDM)	25.35		25.34	25.24	22.31	28.94	26.06	117.76	32.02	134.68	24.33	151.26	18.66	117.73	28.11	28.22
6	6probe (BDM)	24.40	27.93	25.76	27.00	24.53	29.60	26.67	121.34	27.01	149.17	24.78	153.33	19.38	121.50	39.81	50.78
6	huv (MEM)	24.71	27.59	26.05	23.85	30.38	29.79	26.51	126.25	28.18	148.53	24.44	154.07	21.28	121.78	37.20	48.96
7	huv (EMLM)	27.81	30.63				32.71										
7	6probe (EMEP)	26.95	31.10														
8	huv (MEM)	25.29	28.04	26.60	24.35	30.72	29.25	26.84	123.37	28.51	146.76	25.24	152.39	20.50	124.93	27.46	31.09
8	5probe (MEM)	24.87	27.75	26.50	26.60	24.60	29.46	27.16	120.87	32.68	135.04	25.88	151.60	19.52	119.31	26.50	29.80
9	6probe (circular)	23.82	25.59	25.41	25.56	23.49	28.29	25.42	122.01	19.41	148.87	20.99	150.58	20.11	127.01	26.25	29.08
10	5probe (MEM)	24.13		24.20	24.13	23.29	28.27	27.59		48.82		38.36				17.17	17.65
10	huv (MEM)	26.56		26.42	24.95	28.70	29.26	29.73		52.14		39.56				27.82	29.55
	Average	**25.83**	**28.26**	**25.37**	**24.95**	**24.93**	**28.65**	**26.91**	**125.68**	**35.02**	**139.06**	**27.11**	**151.09**	**20.41**	**125.90**	**26.96**	**31.17**
	Standard Deviation	**1.88**	**1.53**	**1.62**	**0.98**	**5.31**	**3.06**	**1.25**	**9.89**	**10.47**	**10.35**	**4.77**	**3.73**	**1.64**	**14.74**	**5.14**	**7.59**

Table 3: Summary of overall mean directions for all tests

Lab No	Method	Overall mean spread (deg)															
		A1	A2	A4	A5	A6	B1	C1 (inc)	C1 (refl)	C2 (inc)	C2 (refl)	C3 (inc)	C3 (refl)	C4 (inc)	C4 (refl)	D1	E1
	Target	**29.72**	**29.72**	**29.72**	**29.72**	**29.72**	**12.70**	**29.72**	**29.72**	**29.72**	**29.72**	**29.72**	**29.72**	**29.72**	**29.72**	**29.72**	**12.70**
1	6probe (EMEP)	31.05	30.10	32.53	40.02	41.18	12.92	32.66	40.99	42.84	43.08	37.16	37.54	37.01	31.60	30.91	21.33
1	multi-3probe array (MLM)	31.86	31.12				15.59										
2	huv (SDA)	30.01	32.02	30.52	28.91	26.55	12.17	26.80	24.32	28.62	28.66	31.77	36.63	31.98	30.60	29.43	13.90
2	huv (DDA)	31.73	31.01	31.90	27.27	27.45	11.60	31.29	43.93	29.18	33.59	34.54	41.54	32.91	35.82	31.30	17.48
2	huv (MEM)	30.45	32.17	30.42	28.23	27.76	11.90	28.99	38.17	24.90	26.61	30.01	33.61	36.90	40.97	29.26	13.31
2	huv (DDAC)							27.21	43.97	30.30	34.70	28.88	33.11				
3	6probe,short (MLM)	30.59	29.52	31.91	39.96	40.66	12.76	31.92	30.02	42.21	41.08	35.49	38.23	36.50	22.62	30.50	20.12
3	6probe,long (MLM)	30.50	29.85	31.83	39.72	40.59	12.89	31.72	29.29	41.98	40.75	35.34	37.99	36.59	22.01	30.44	20.15
3	huv,short (Fourier)	32.39		32.65	29.78	30.53	14.17	34.99	41.46	39.57	41.82	40.56	44.92	40.00	48.35	35.78	27.10
3	huv,long (Fourier)	32.36		32.62	29.75	30.54	14.32	35.09	41.35	39.30	41.70	40.91	45.13	40.31	48.75	35.73	26.84
4	huv (MEM)	30.20	32.12	30.35	28.09	27.42	12.15	28.89	38.54	24.37	26.02	29.55	32.69				
5	6probe (MLM2)	31.24		39.10	36.31	38.73	21.85			32.88	27.85			34.54	17.37	41.69	37.92
5	6probe (BDM)	29.36		27.08	26.93	30.04	12.23	27.29	36.04	31.91	37.16	26.46	31.15	35.58	22.25	16.01	12.97
6	6probe (BDM)	30.19	30.41	30.17	32.38	32.16	13.68	29.07	36.98	23.91	28.05	28.91	32.47	35.34	21.98	37.04	15.95
6	huv (MEM)	30.34	33.09	30.17	27.94	27.52	11.93	28.94	38.21	25.23	27.38	29.66	32.88	37.61	41.18	38.65	17.39
7	huv (EMLM)	30.94	30.13				13.88										
7	6probe (EMEP)	30.35	29.57														
8	huv (MEM)	30.50	32.13	30.42	28.22	27.71	12.23	29.00	38.68	25.19	27.37	30.01	34.14	37.56	39.71	29.33	13.32
8	5probe (MEM)	30.47	33.17	30.82	30.59	29.39	13.07	29.24	37.88	34.66	36.05	29.50	33.91	38.09	28.96	29.52	14.36
9	6probe (circular)	31.29	31.92	31.45	31.29	35.08	14.54	29.66	38.63	27.98	37.34	31.03	41.29	38.82	31.06	31.79	17.80
10	5probe (MEM)	28.95		29.09	28.95	27.83	13.11	30.58		38.61		46.77				16.51	7.86
10	huv (MEM)	28.17		29.03	28.69	23.63	12.24	31.73		52.48		47.36				27.45	13.33
Average		**30.62**	**31.22**	**31.23**	**31.28**	**31.38**	**13.46**	**30.28**	**37.40**	**33.48**	**34.07**	**34.11**	**36.70**	**36.65**	**32.22**	**30.67**	**18.30**
Standard Deviation		**1.01**	**1.20**	**2.36**	**4.39**	**5.33**	**2.18**	**2.35**	**5.18**	**7.83**	**6.10**	**6.05**	**4.36**	**2.26**	**9.65**	**6.47**	**6.88**

Table 4: Summary of overall spread of directions for all tests

Lab No	Method	Wave height (cm)					
		C1 (inc)	C1 (refl)	C2 (inc)	C2 (refl)	C3 (inc)	C3 (refl)
	Target	**12.0**	**2.4**	**12.0**	**8.4**	**12.0**	**8.4**
1	6probe (EMEP)	11.4	3.7	10.8	8.1	11.0	8.2
1	multi-3probe array (MLM)						
2	huv (SDA)	11.7	2.1	10.9	6.1	11.8	7.4
2	huv (DDA)	11.3	3.3	11.2	8.2	10.9	8.0
2	huv (MEM)	11.5	3.2	10.1	7.3	11.3	8.1
2	huv (DDAC)	11.1	3.4	11.1	7.9	10.8	7.7
3	6probe,short (MLM)	11.6	3.8	9.9	8.2	11.0	8.6
3	6probe,long (MLM)	11.4	3.7	9.8	8.1	11.2	8.6
3	huv,short (Fourier)	11.1	4.4	11.2	9.2	10.8	8.8
3	huv,long (Fourier)	11.1	4.1	11.2	9.1	10.9	8.9
4	huv (MEM)	11.4	3.1	10.1	7.3	11.3	8.1
5	6probe (MLM2)						
5	6probe (BDM)	11.9	3.4	11.3	7.3	11.8	8.4
6	6probe (BDM)	11.6	2.9	10.7	6.4	11.3	8.1
6	huv (MEM)	11.5	3.2	10.1	7.1	11.2	8.1
7	huv (EMLM)						
7	6probe (EMEP)						
8	huv (MEM)	11.6	3.3	10.2	7.4	11.3	8.2
8	5probe (MEM)	11.6	3.0	10.5	6.9	11.4	8.1
9	6probe (circular)	11.5	3.1	10.3	7.0	11.2	8.2
10	5probe (MEM)						
10	huv (MEM)						
Average		**11.46**	**3.36**	**10.59**	**7.60**	**11.20**	**8.22**
Standard Deviation		**0.22**	**0.51**	**0.51**	**0.84**	**0.29**	**0.37**

Table 5: Incident and reflected wave heights for Tests C1 - C3

Figures

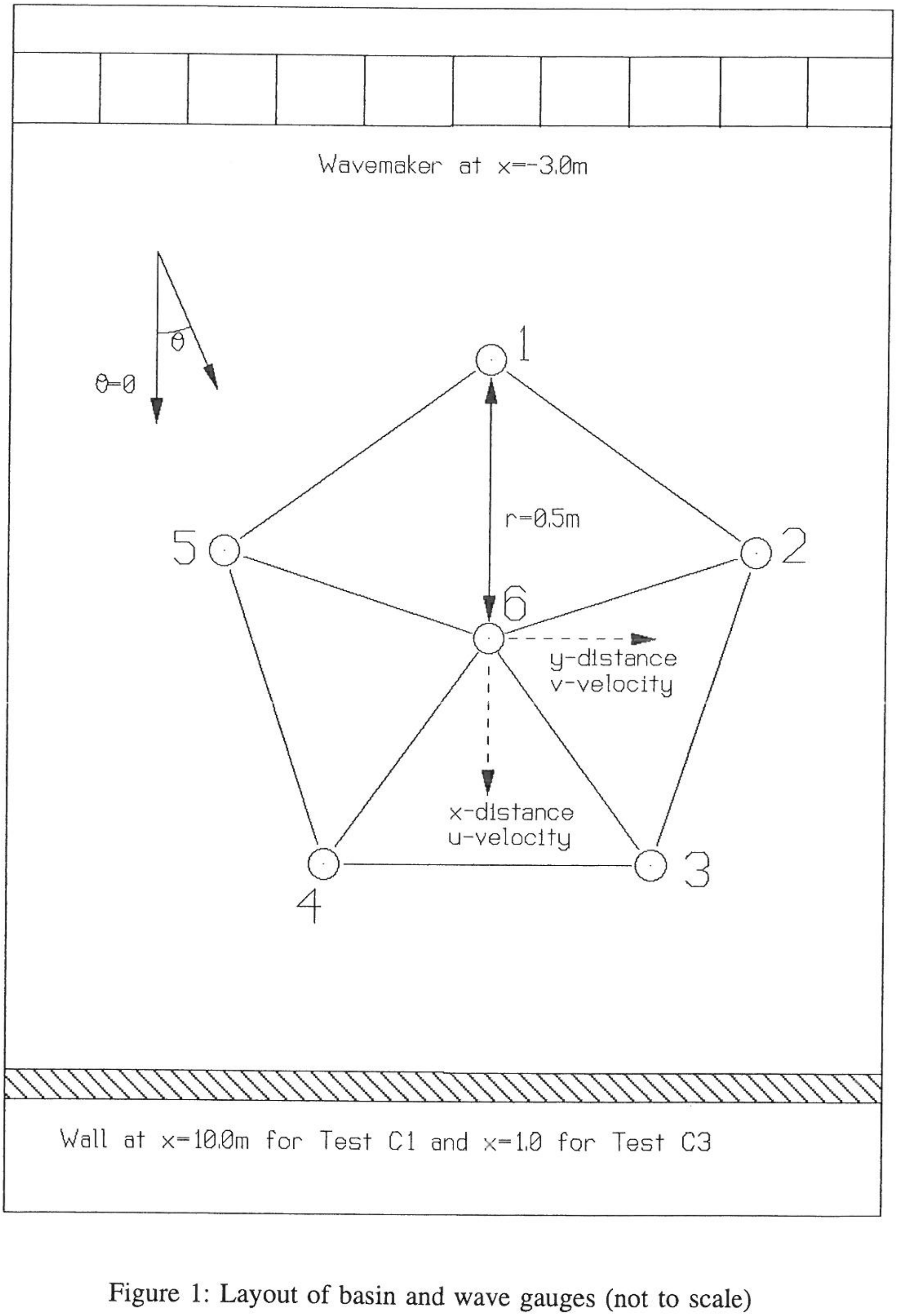


Figure 1: Layout of basin and wave gauges (not to scale)

Figure 4:Test A2 (Double sum)

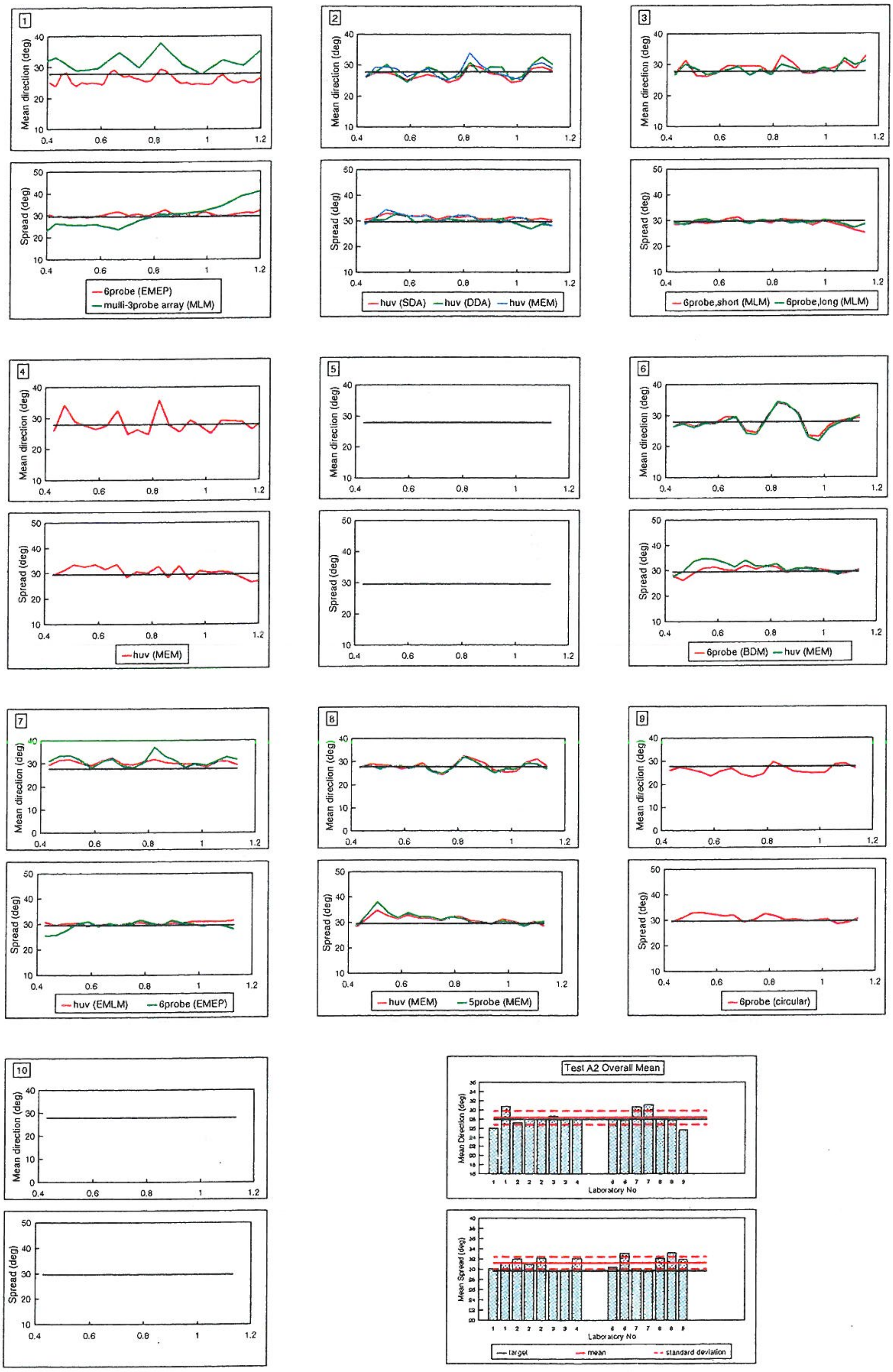


Figure 5:Test A4 (20%noise (all gauges))

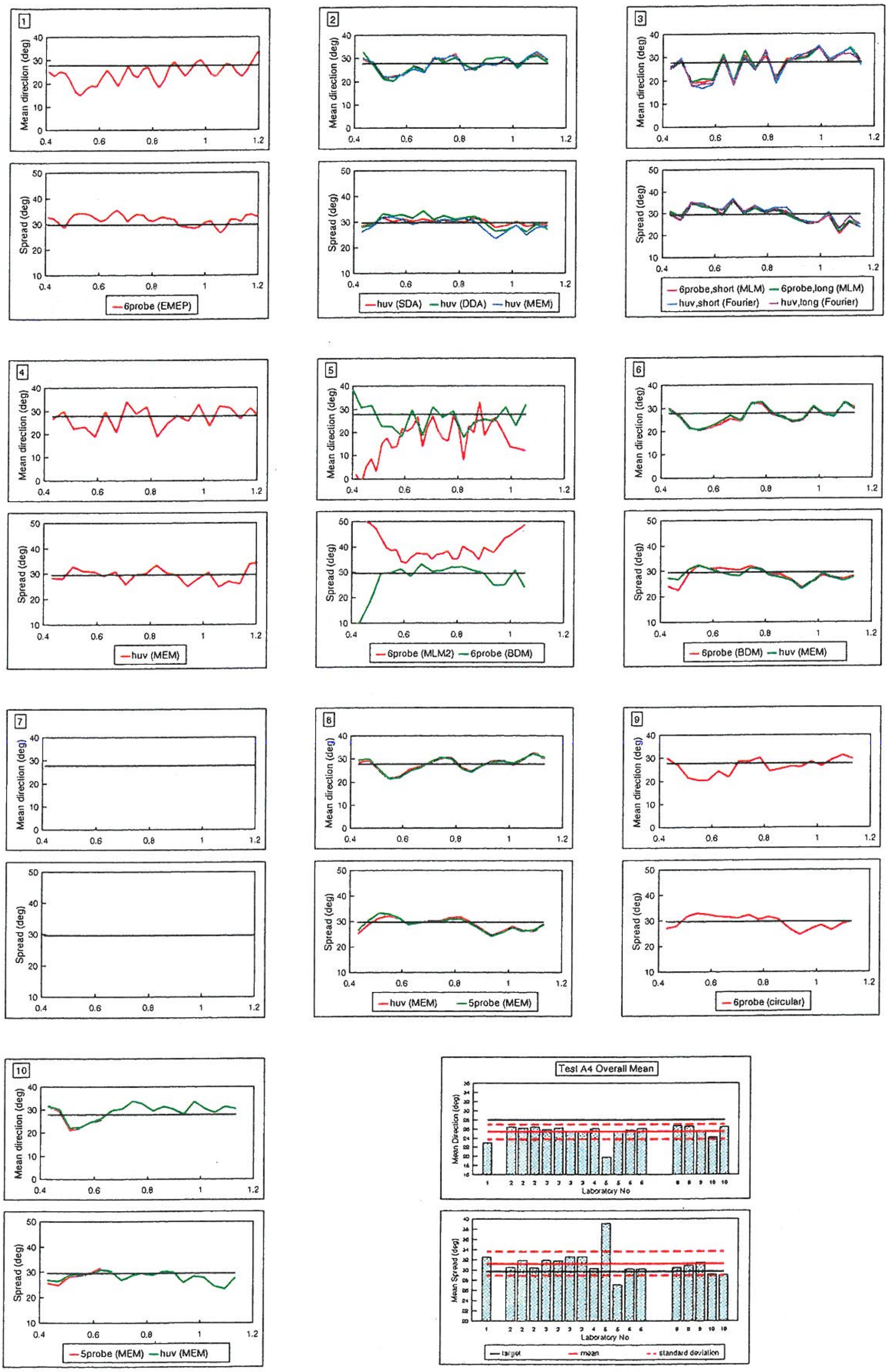


Figure 6:Test A5 (1.1 gain (h1 and u))

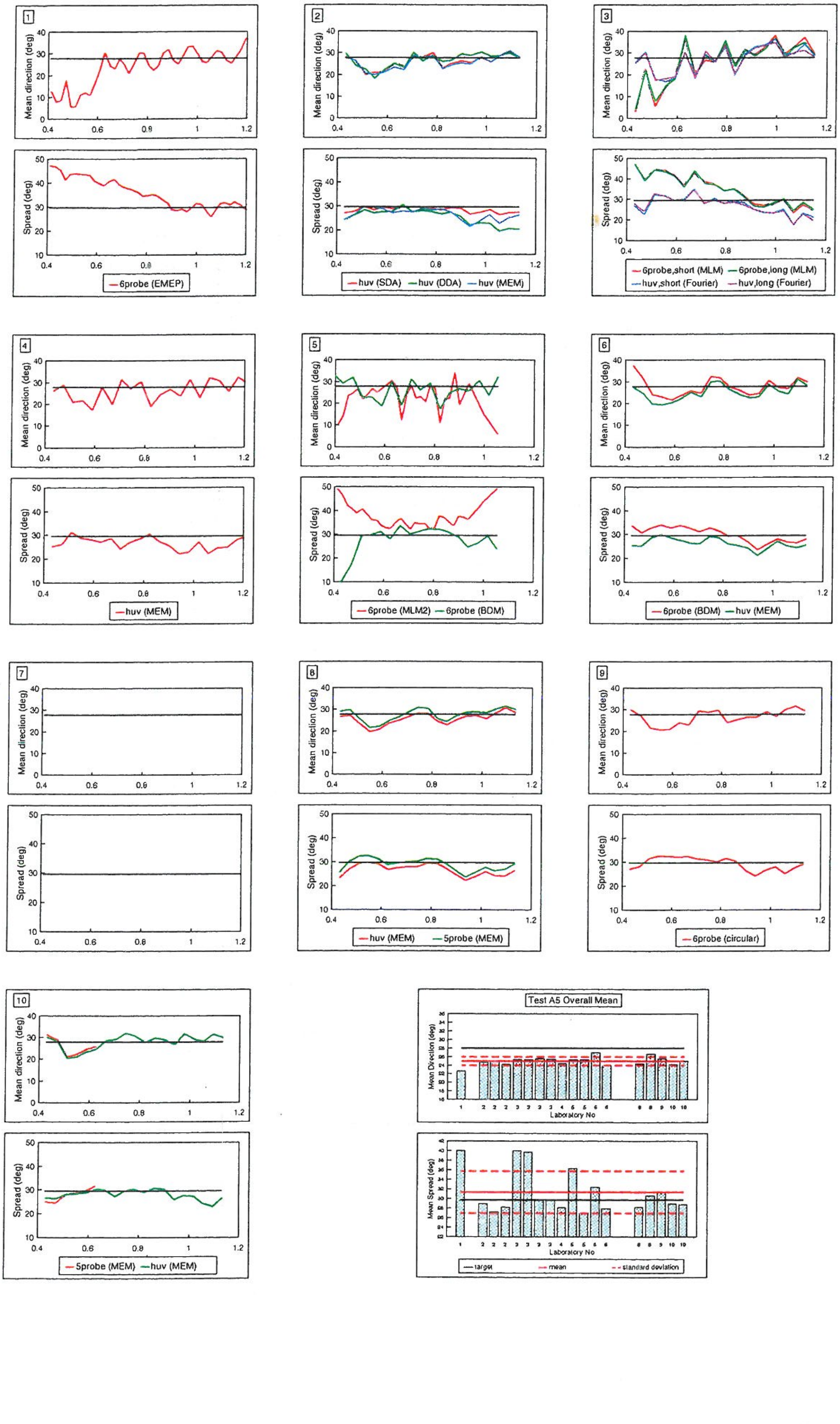


Figure 7:Test A6 (Cross talk (h1/h2 and u/v))

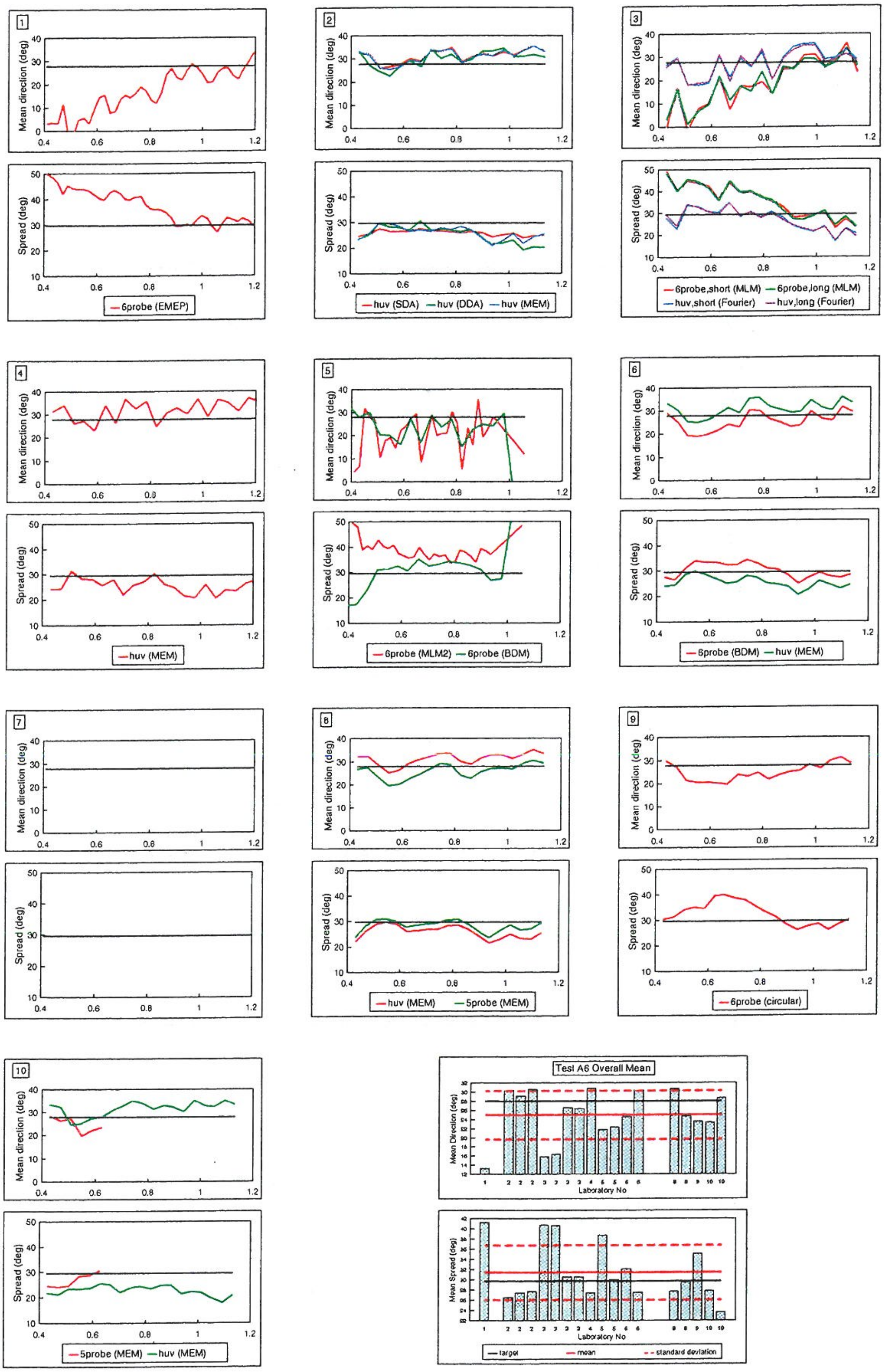


Figure 8:Test B1 (Narrow spread)

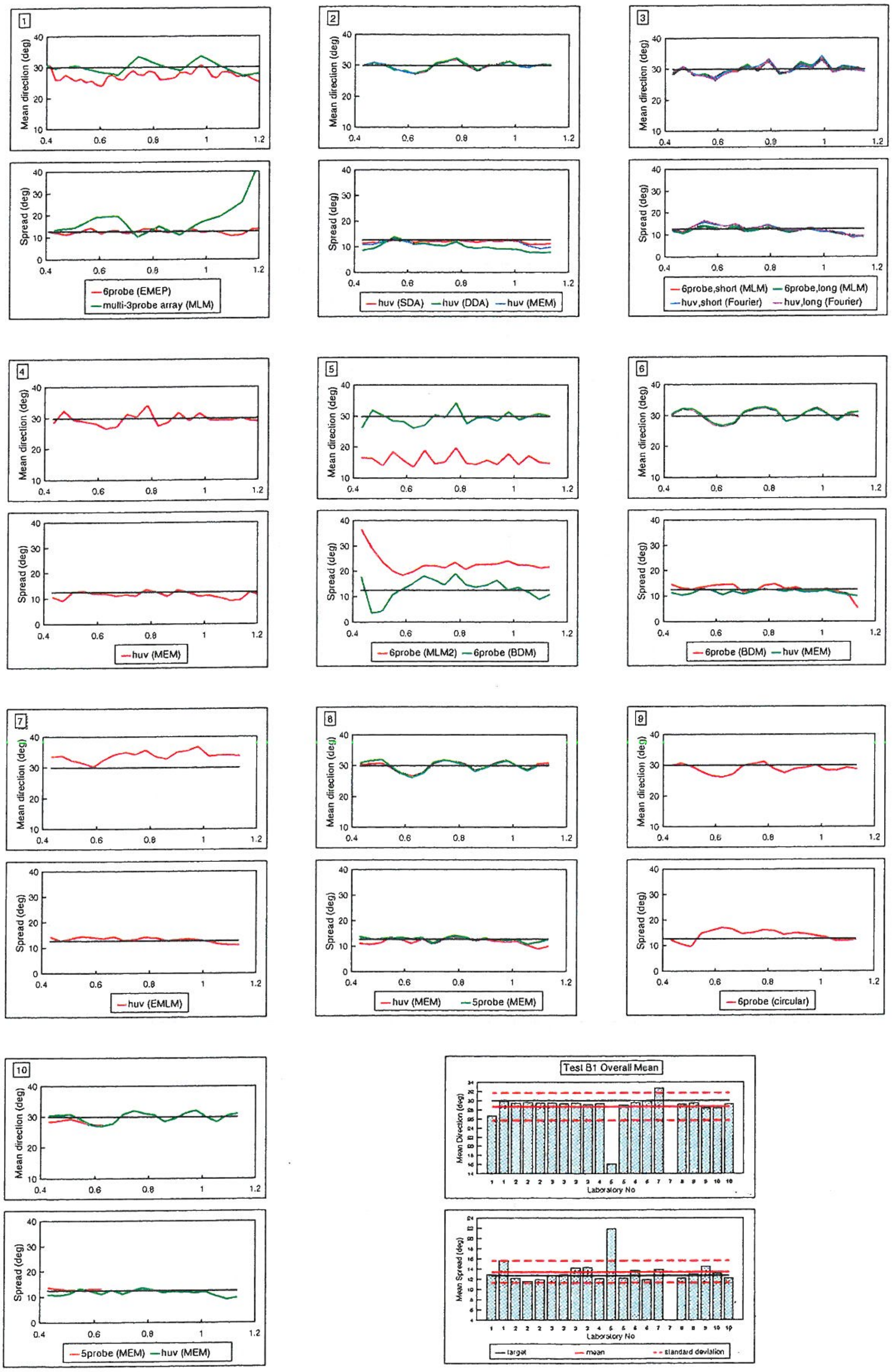


Figure 9:Test C1 (20% refn, wall at 10m)
Incident waves

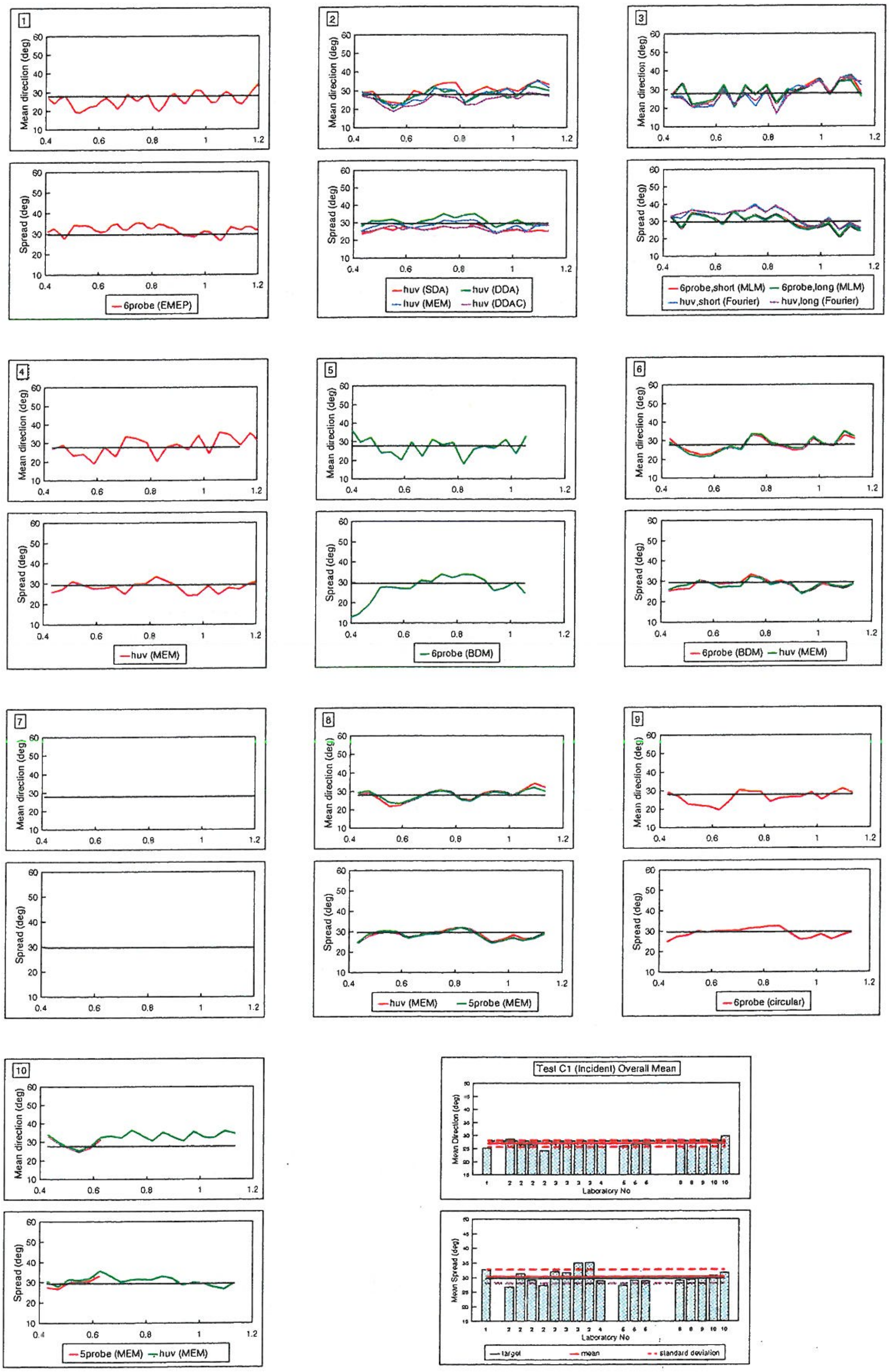


Figure10:Test C1 (20% refn, wall at 10m)
Reflected waves

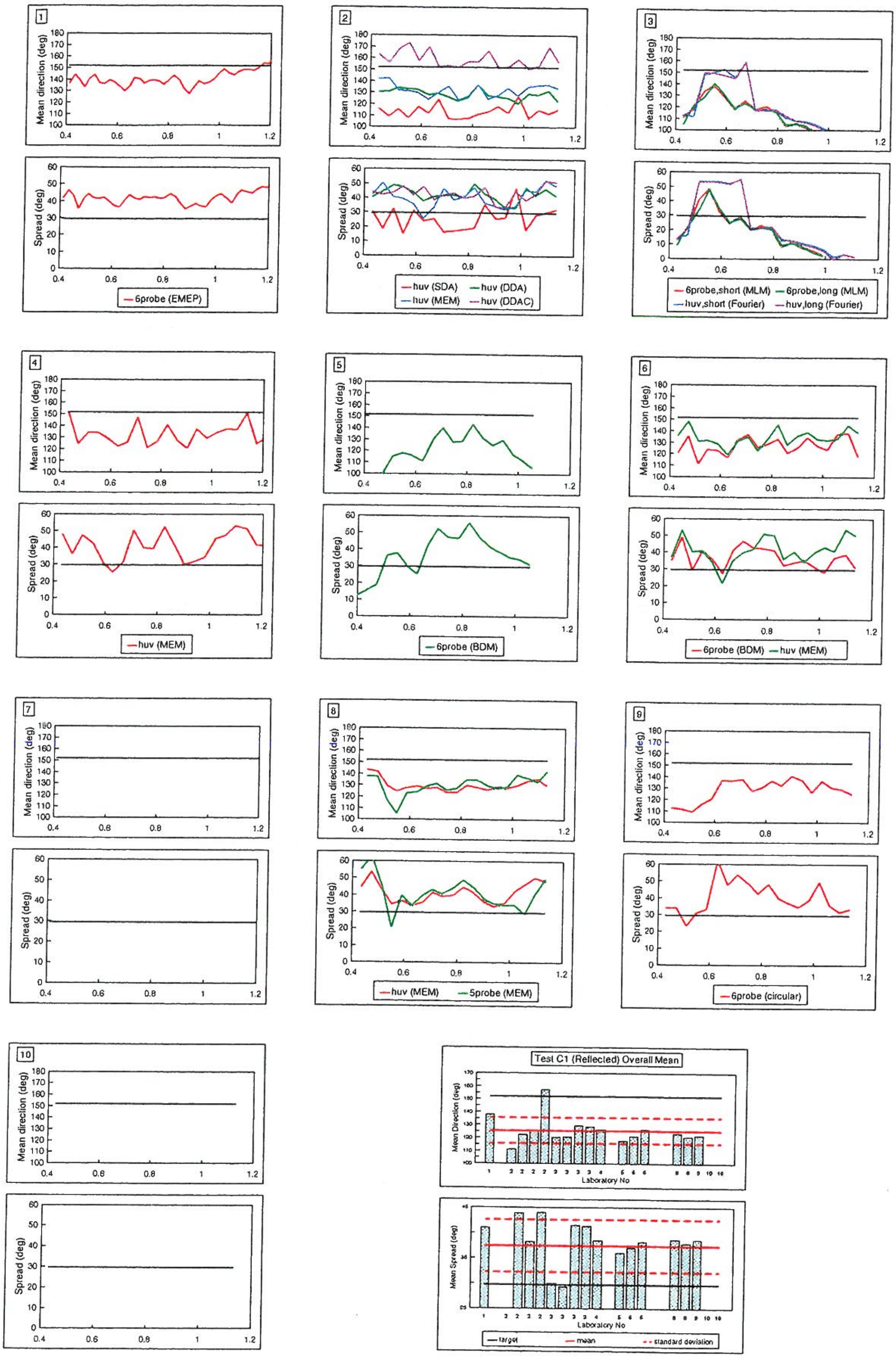


Figure11:Test C2 (70% refn, wall at 1m)
Incident waves

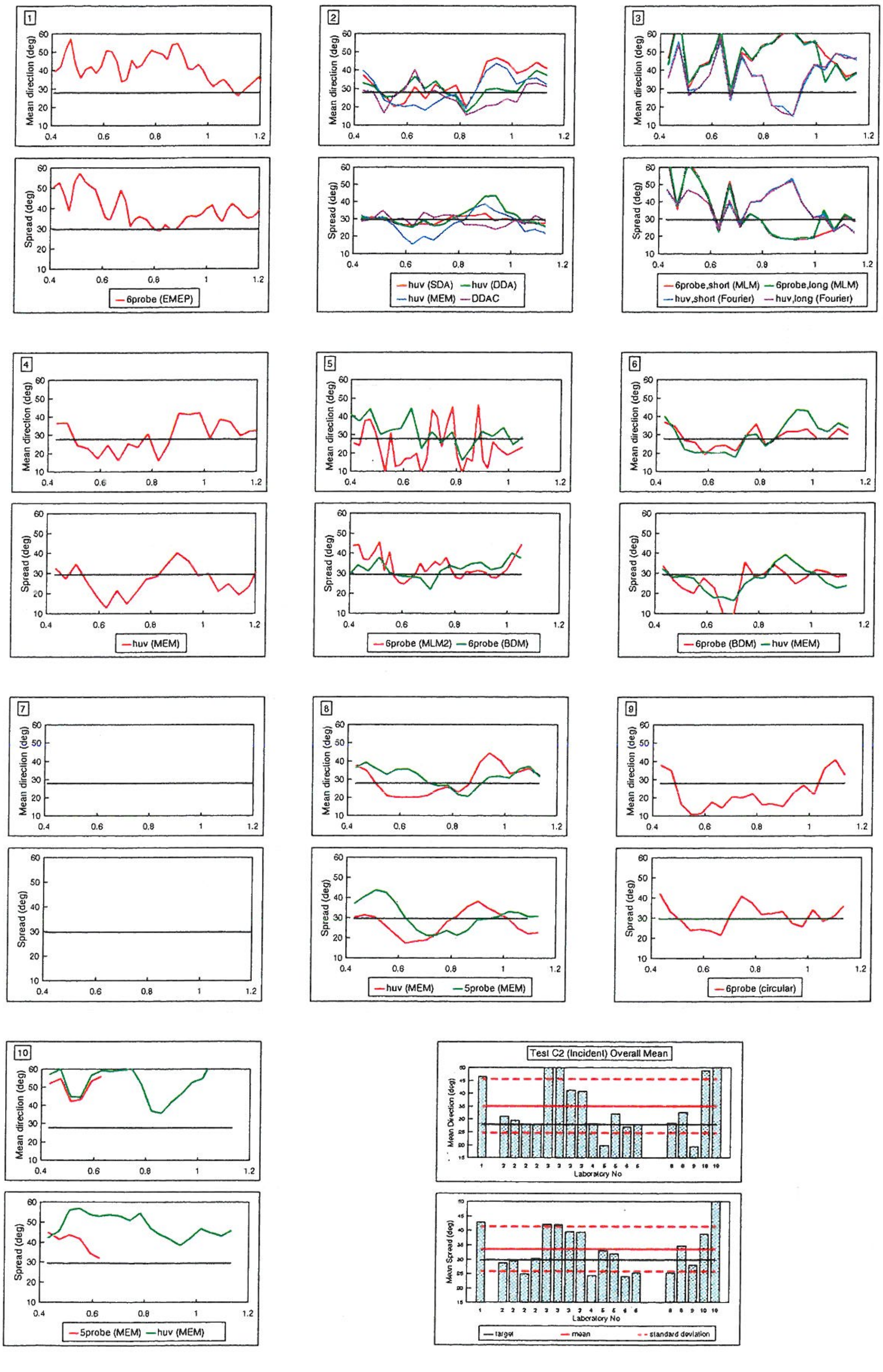


Figure12:Test C2 (70% refn, wall at 1m)
Reflected waves

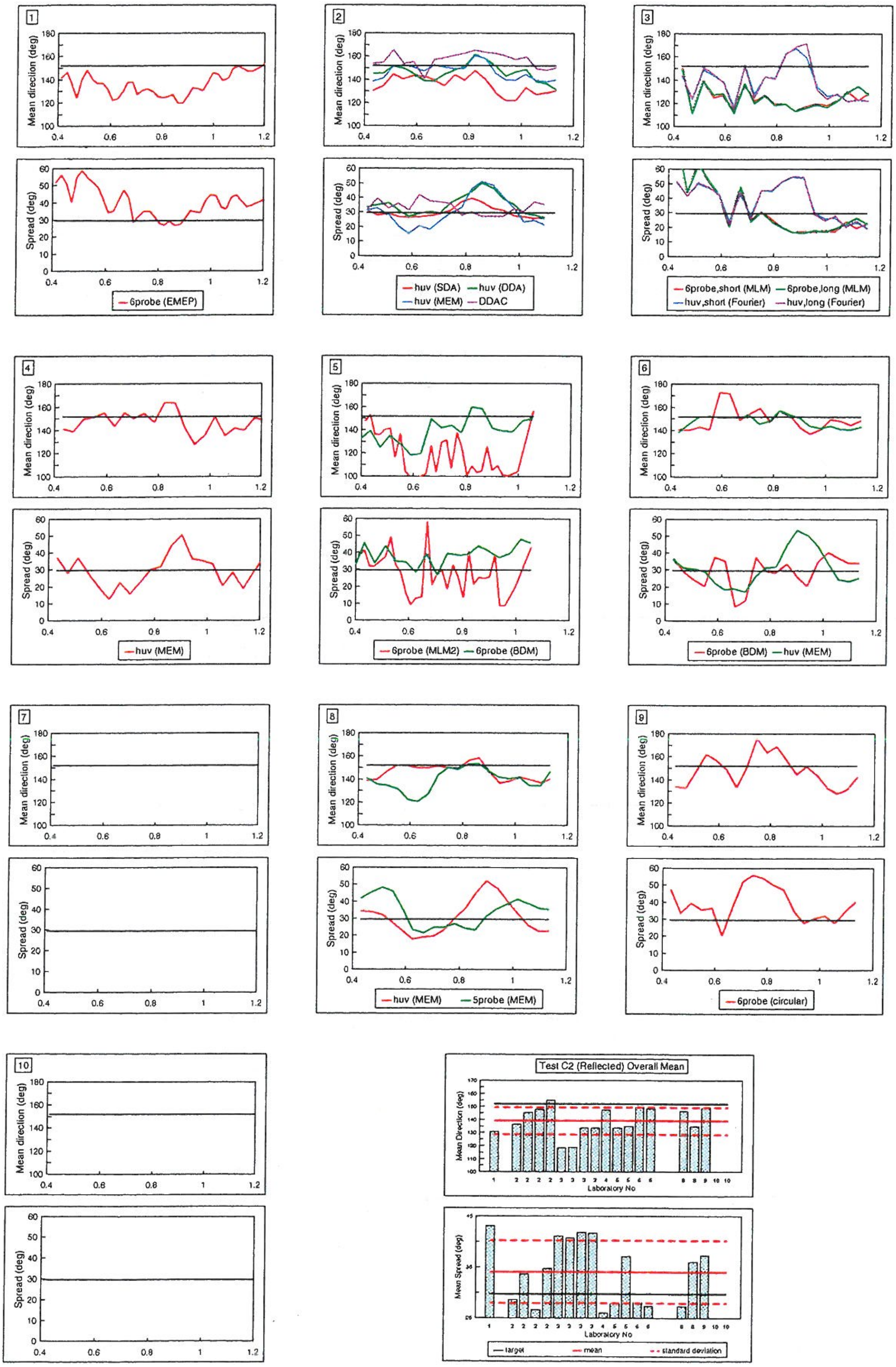


Figure13:Test C3 (70% "reflection")
Incident waves

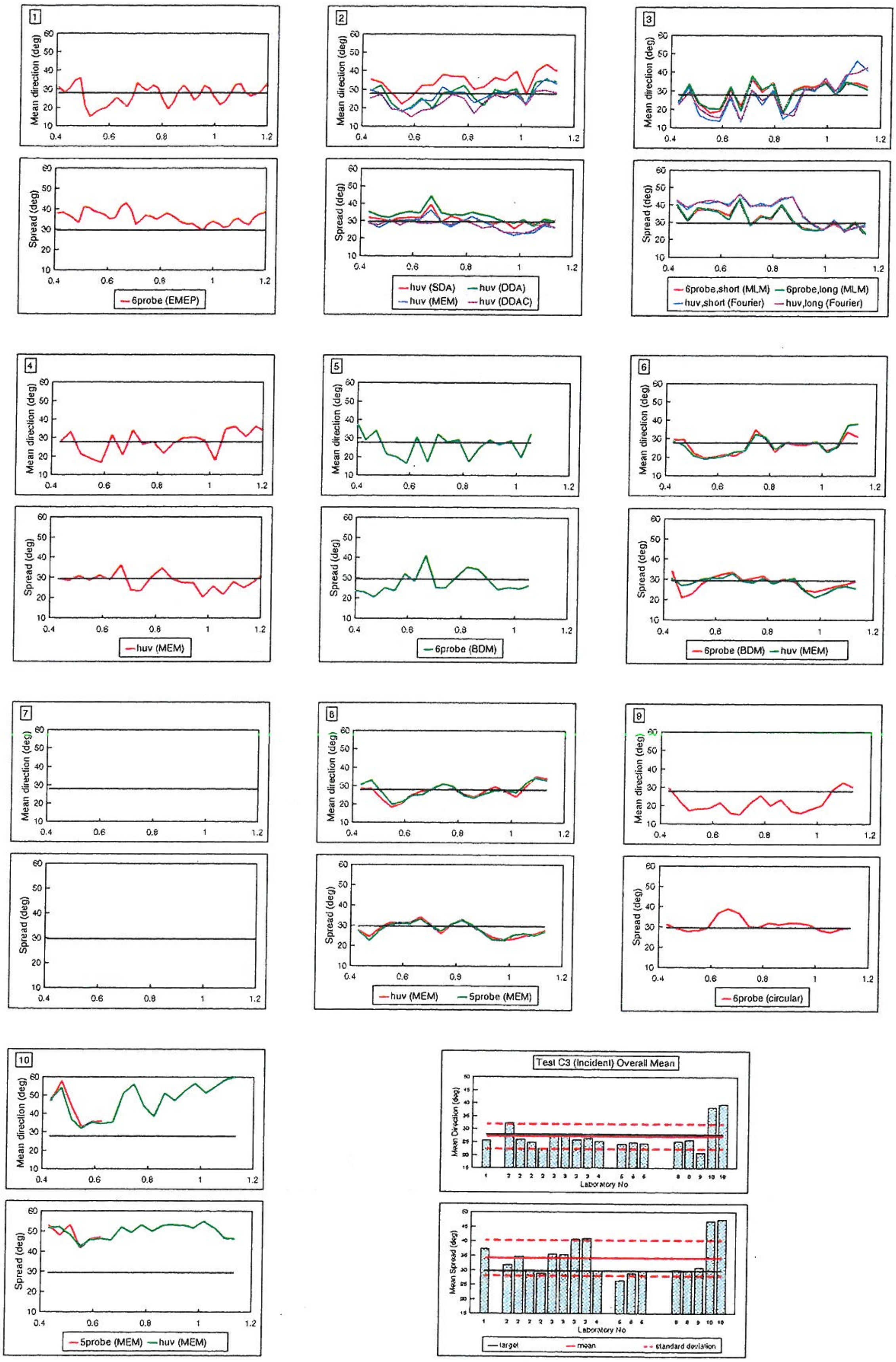


Figure 14:Test C3 (70% "reflection")
Reflected waves

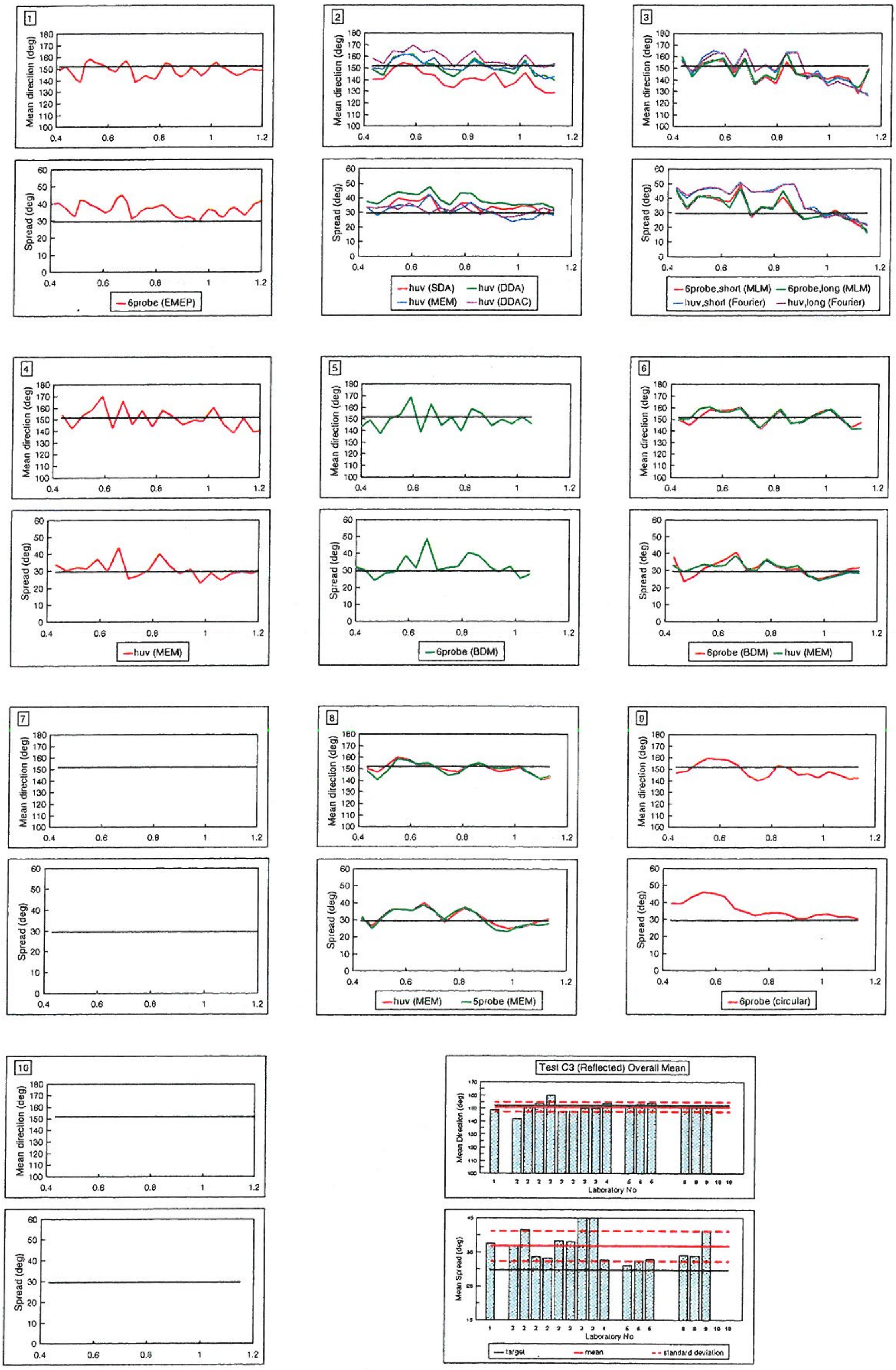


Figure15:Test C4 (cross-resonant modes: 4cm at -50deg; 4cm at +130deg)
Incident waves

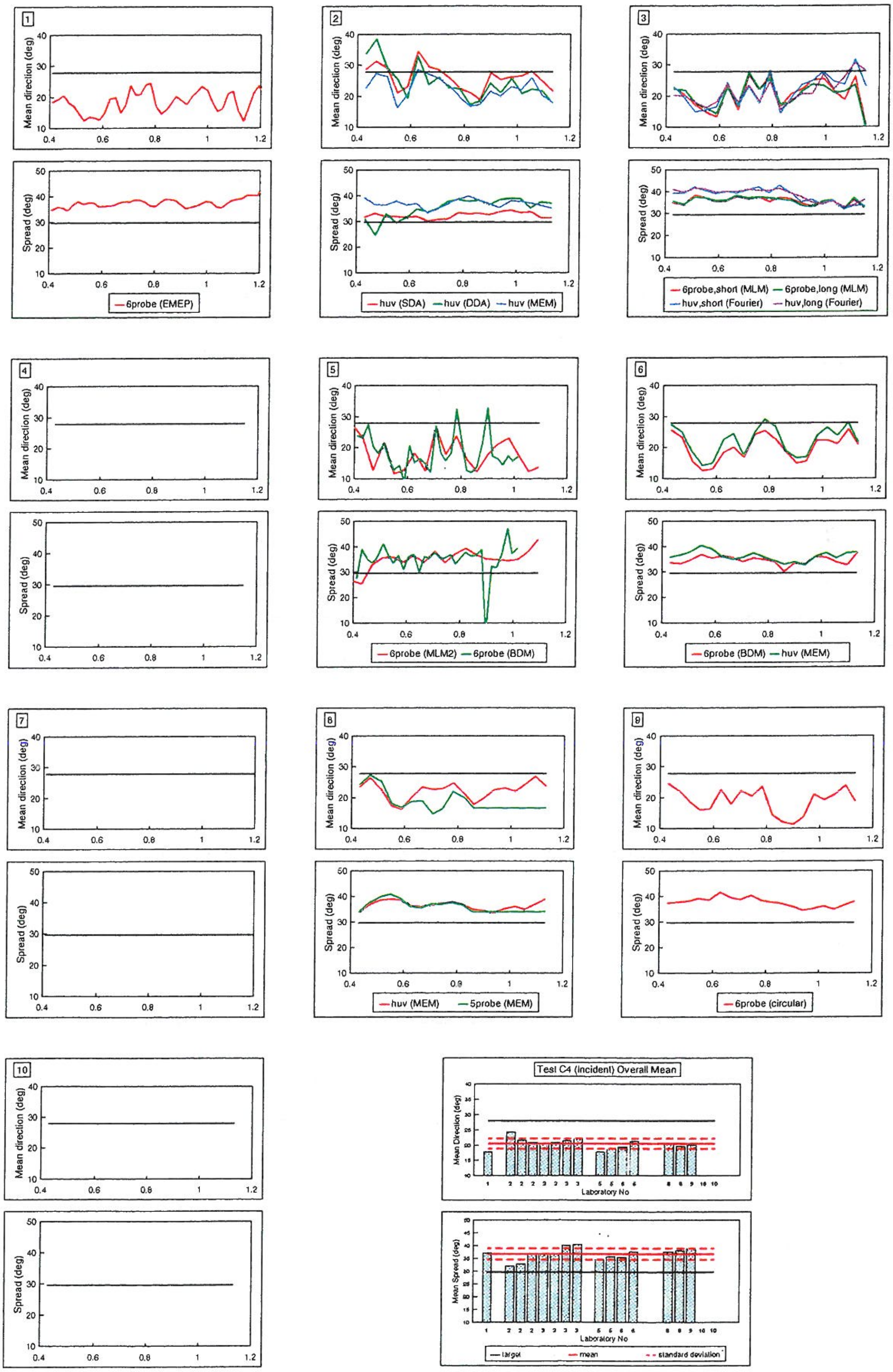


Figure 16:Test D1 (Standard test, measured at Ottawa)

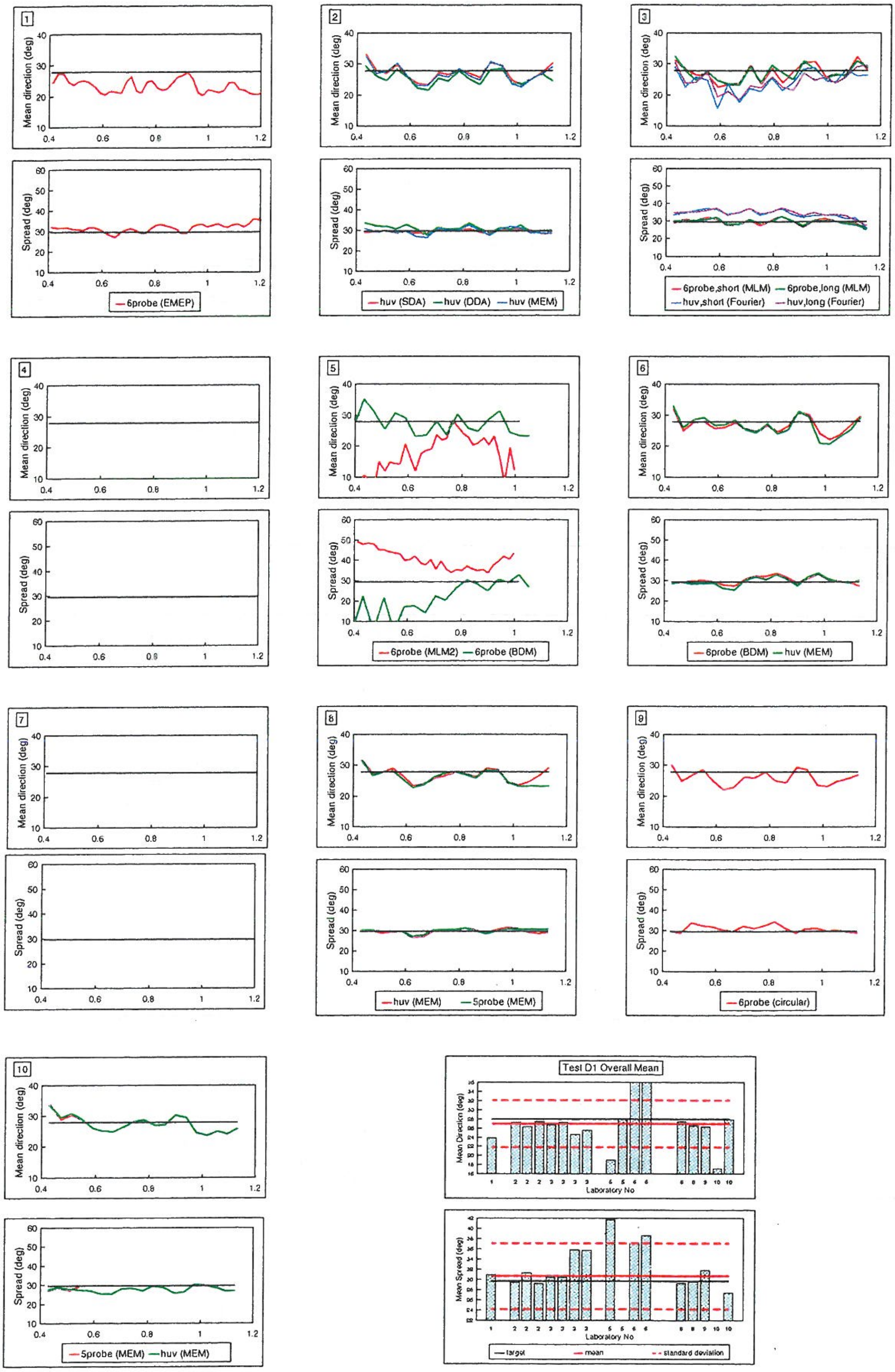
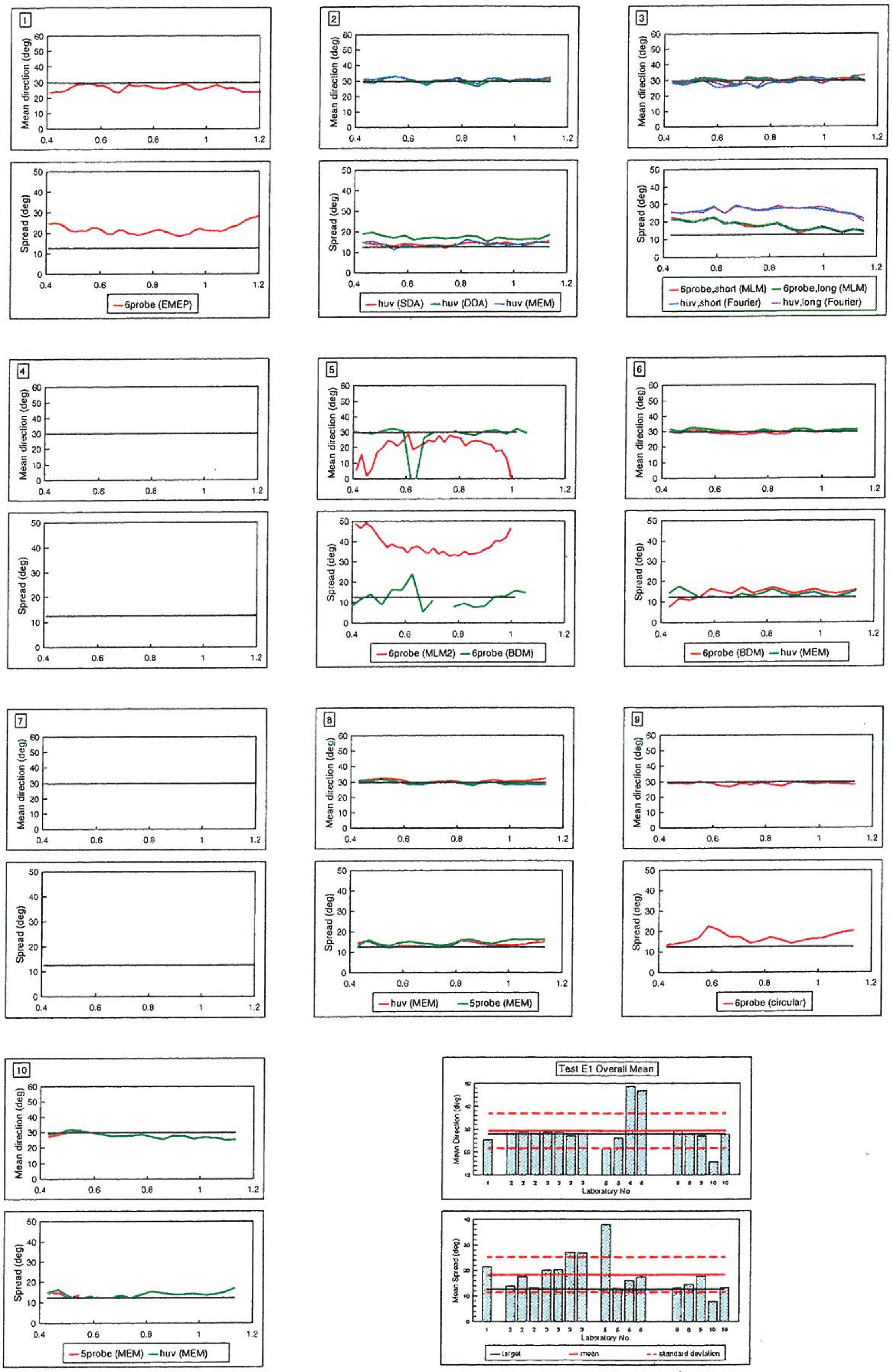


Figure 17:Test E1 (Measured, narrow spread)



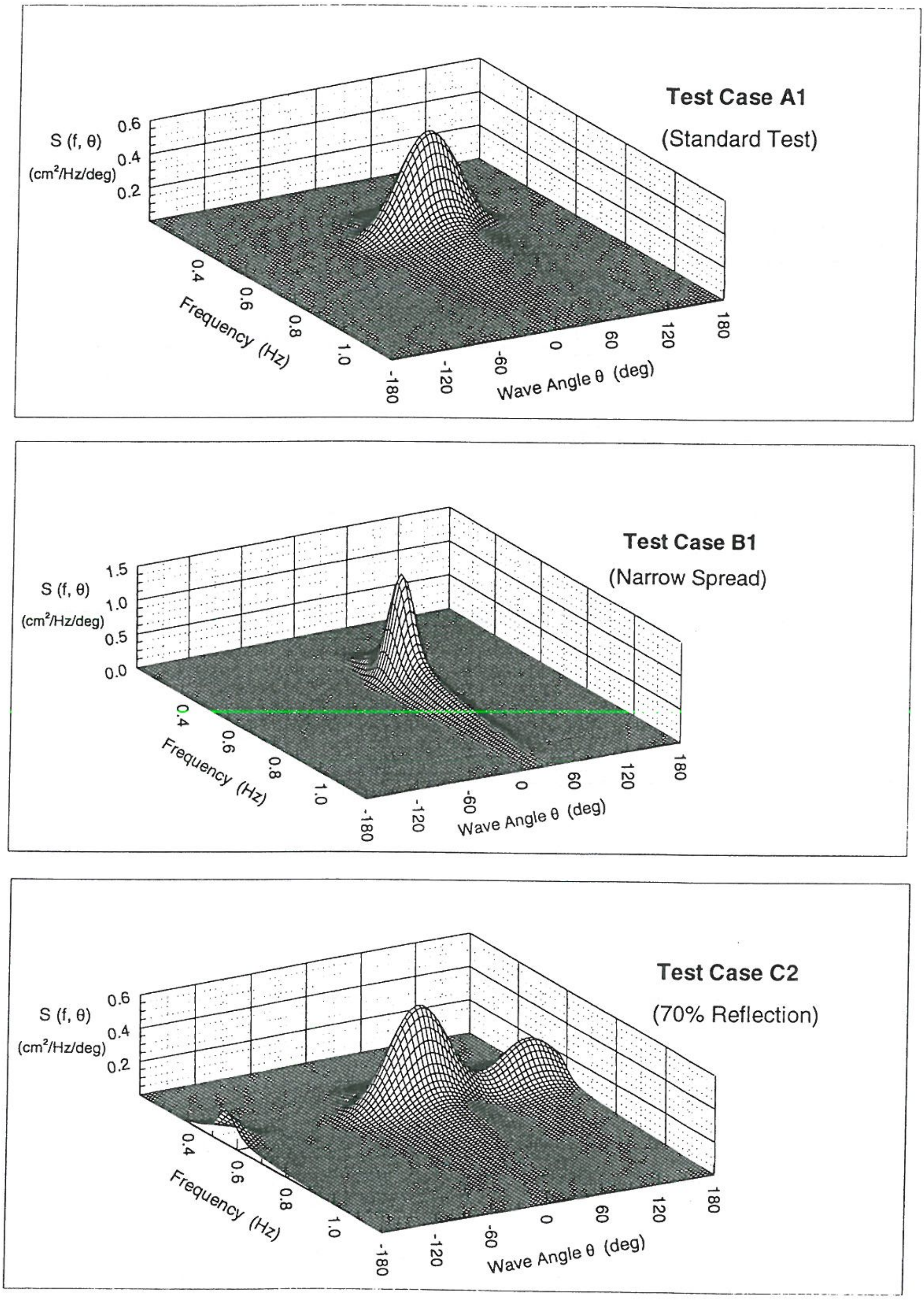


Figure 2: Target directional spectra for Tests A1, B1 and C2

Figure 3:Test A1 (Standard test)

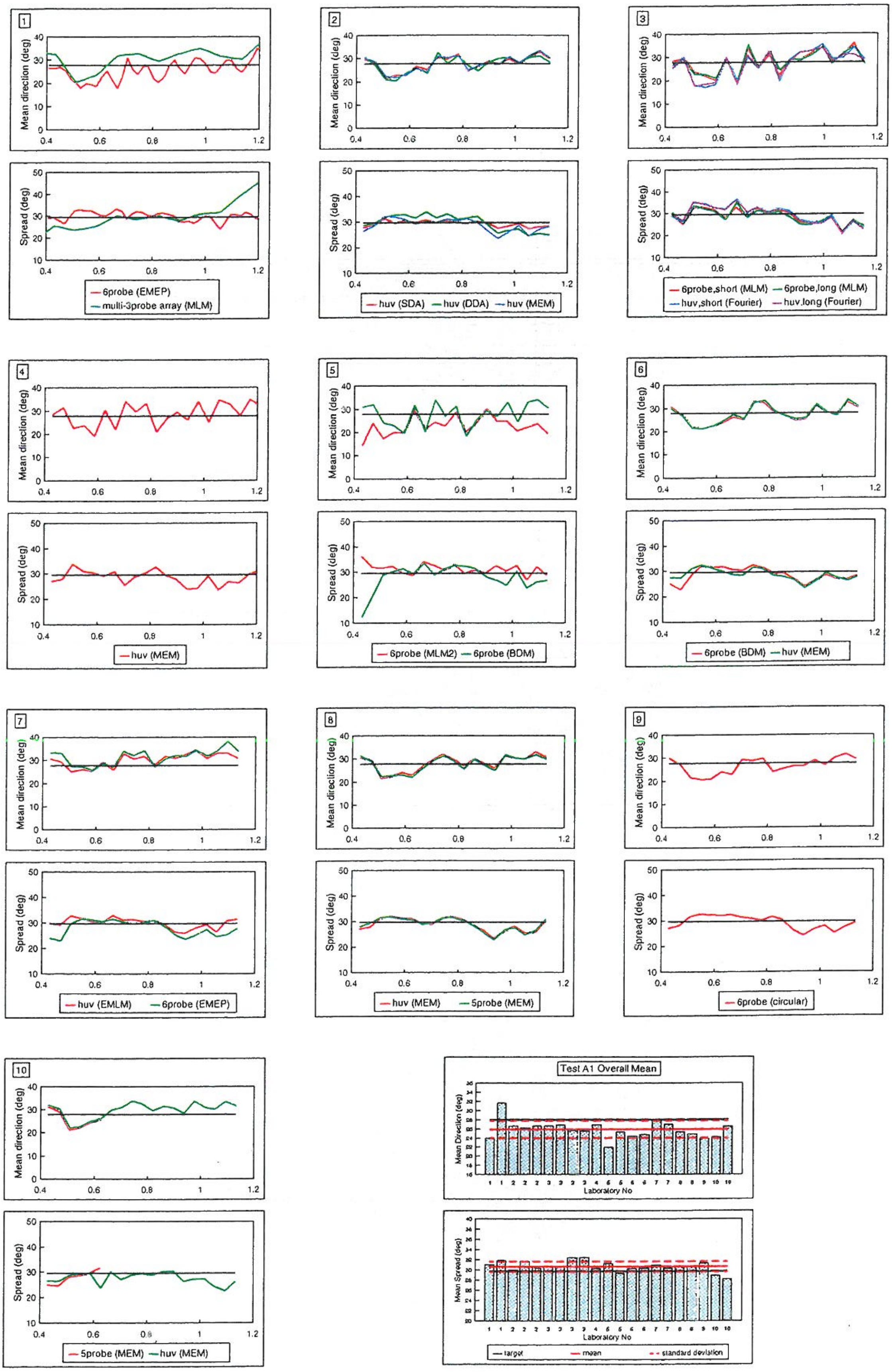


Figure18: Reflection Coefficients Test C1 (20% refn, wall at 10m)

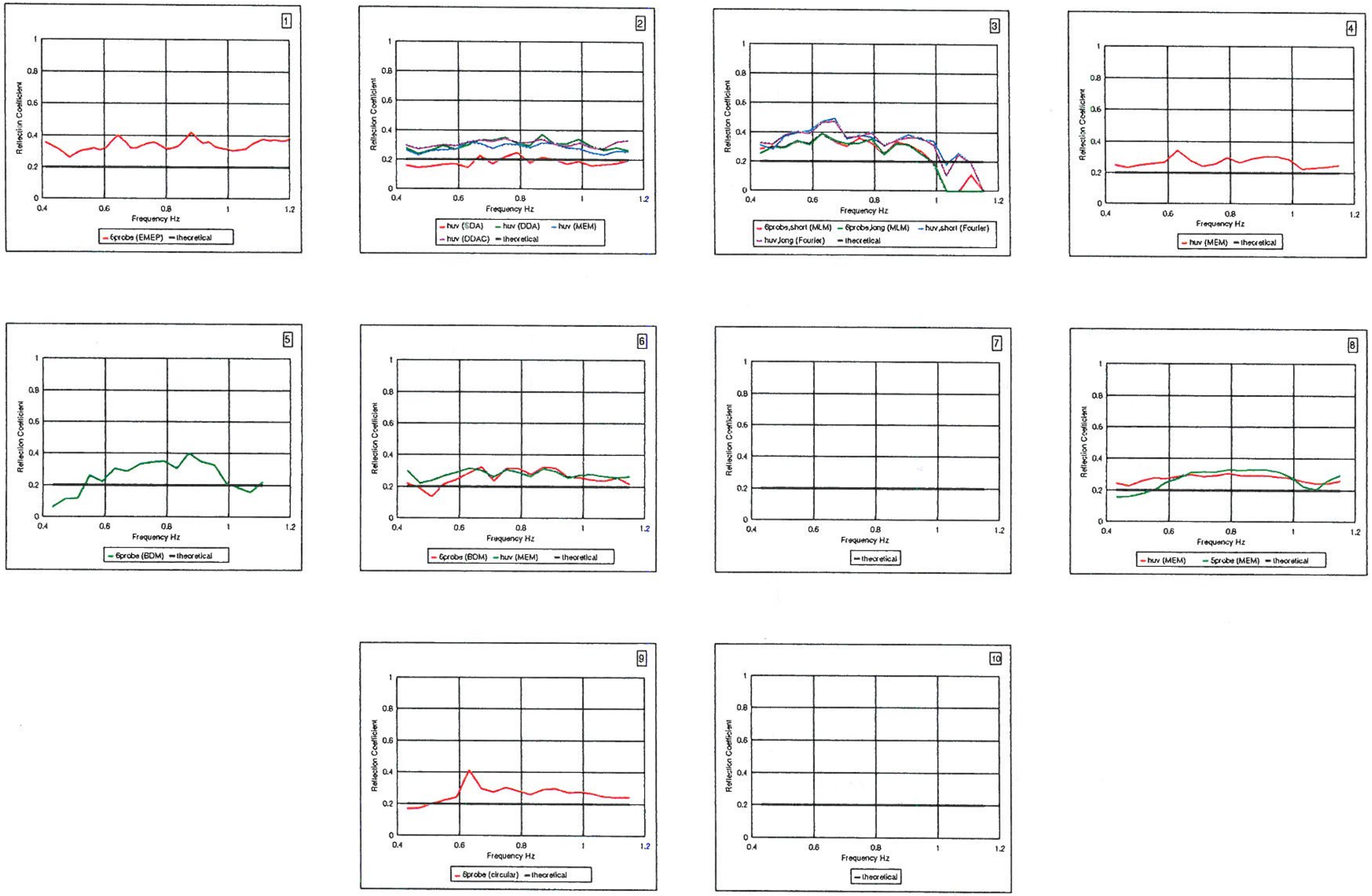


Figure19: Reflection Coefficients Test C2 (70% refn, wall at 1m)

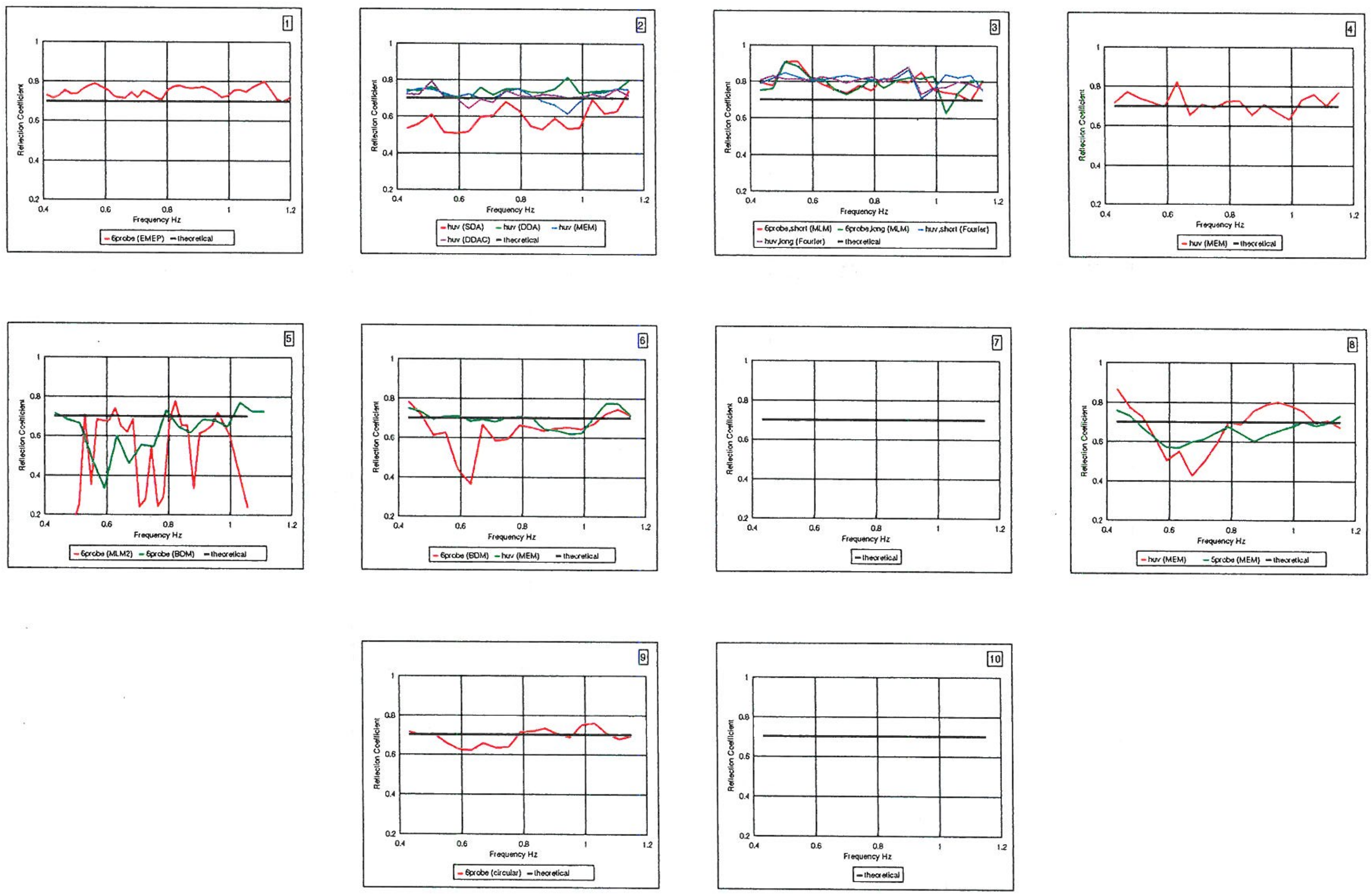


Figure 20: Reflection Coefficients Test C3 (70% "reflection")

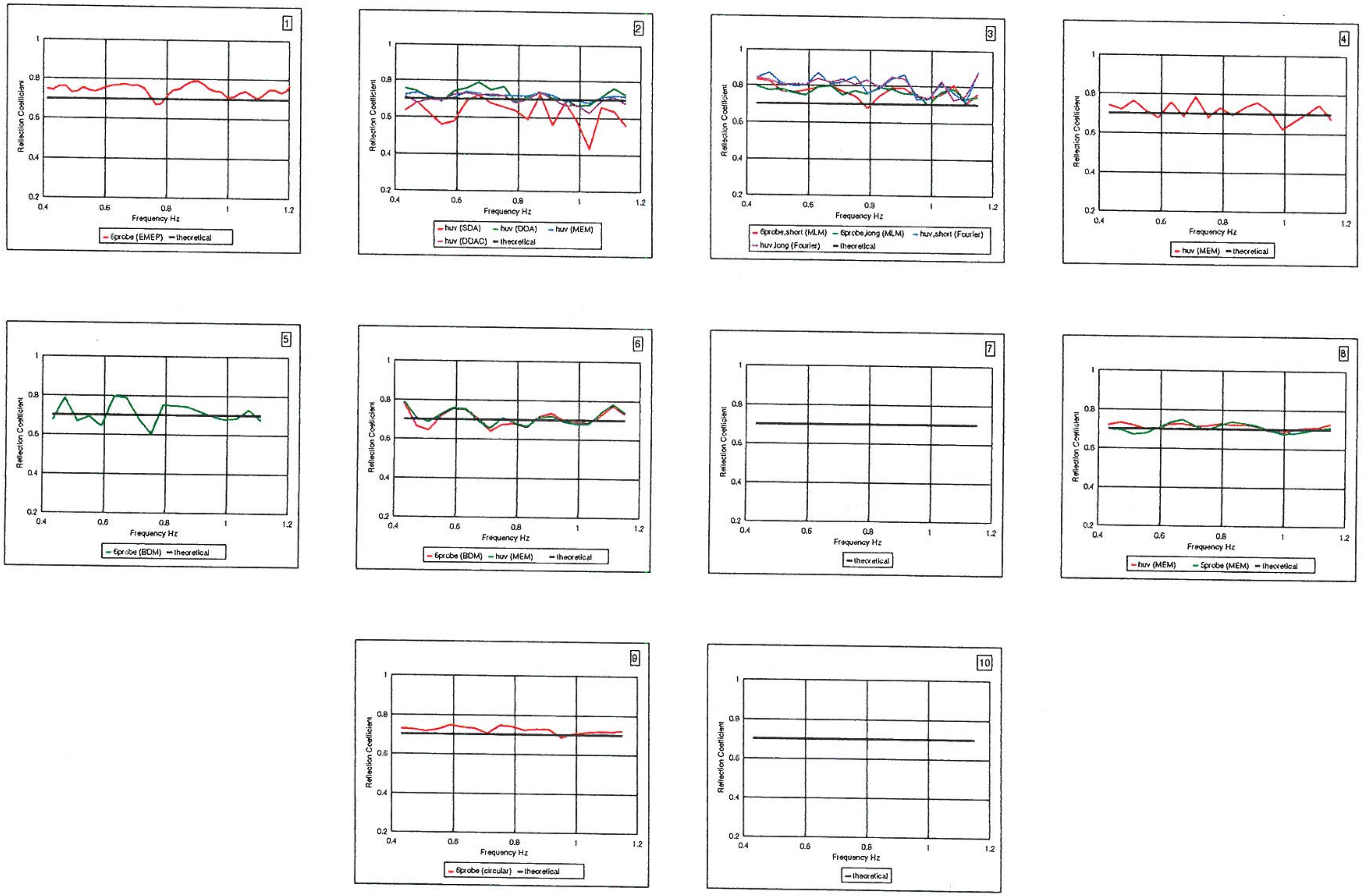


Figure 21: Wave Energy Density v Direction: Test C1 (20% refn, wall at 10m)

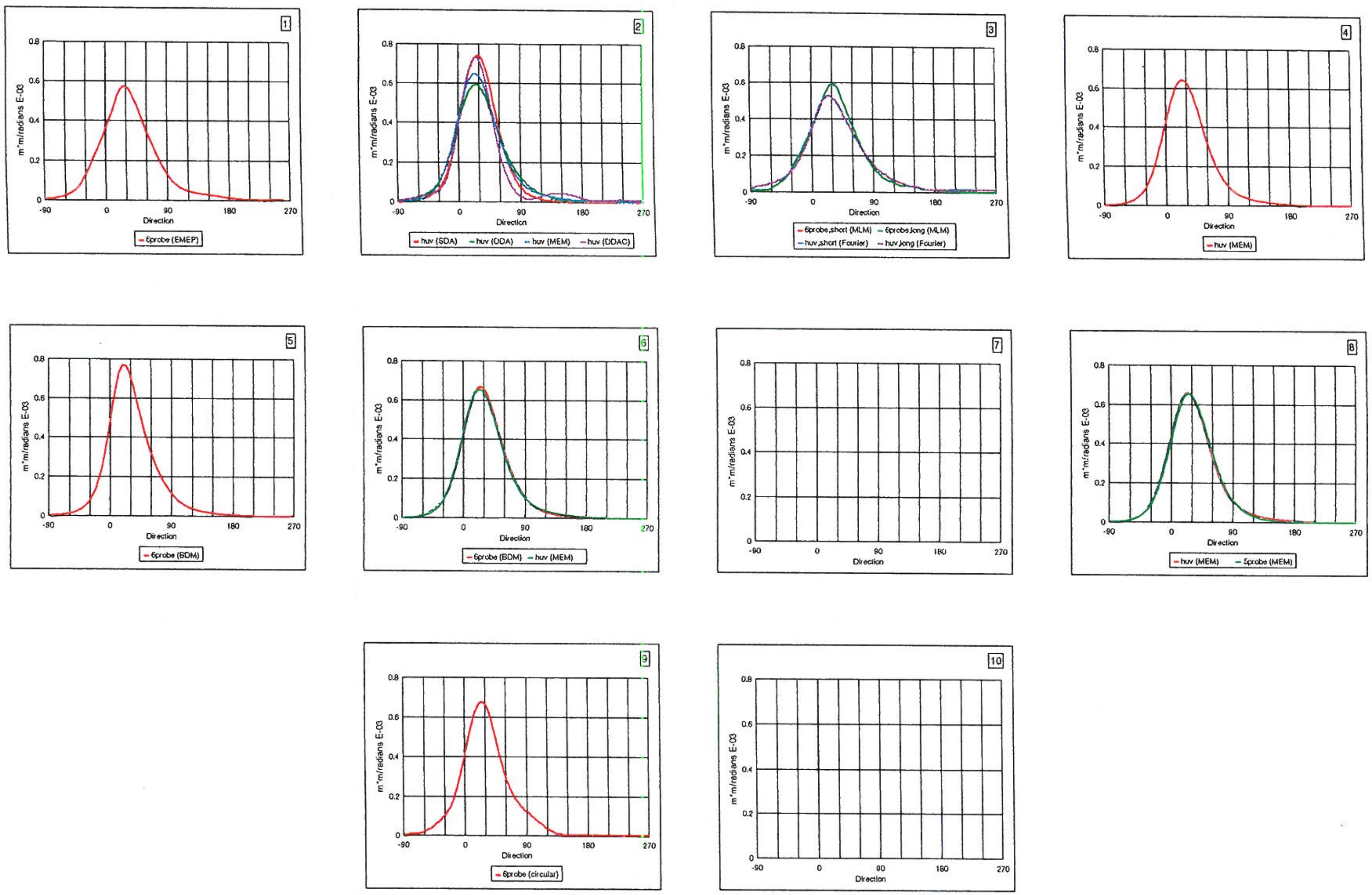


Figure 22: Wave Energy Density v Direction: Test C2 (70% refn, wall at 1m)

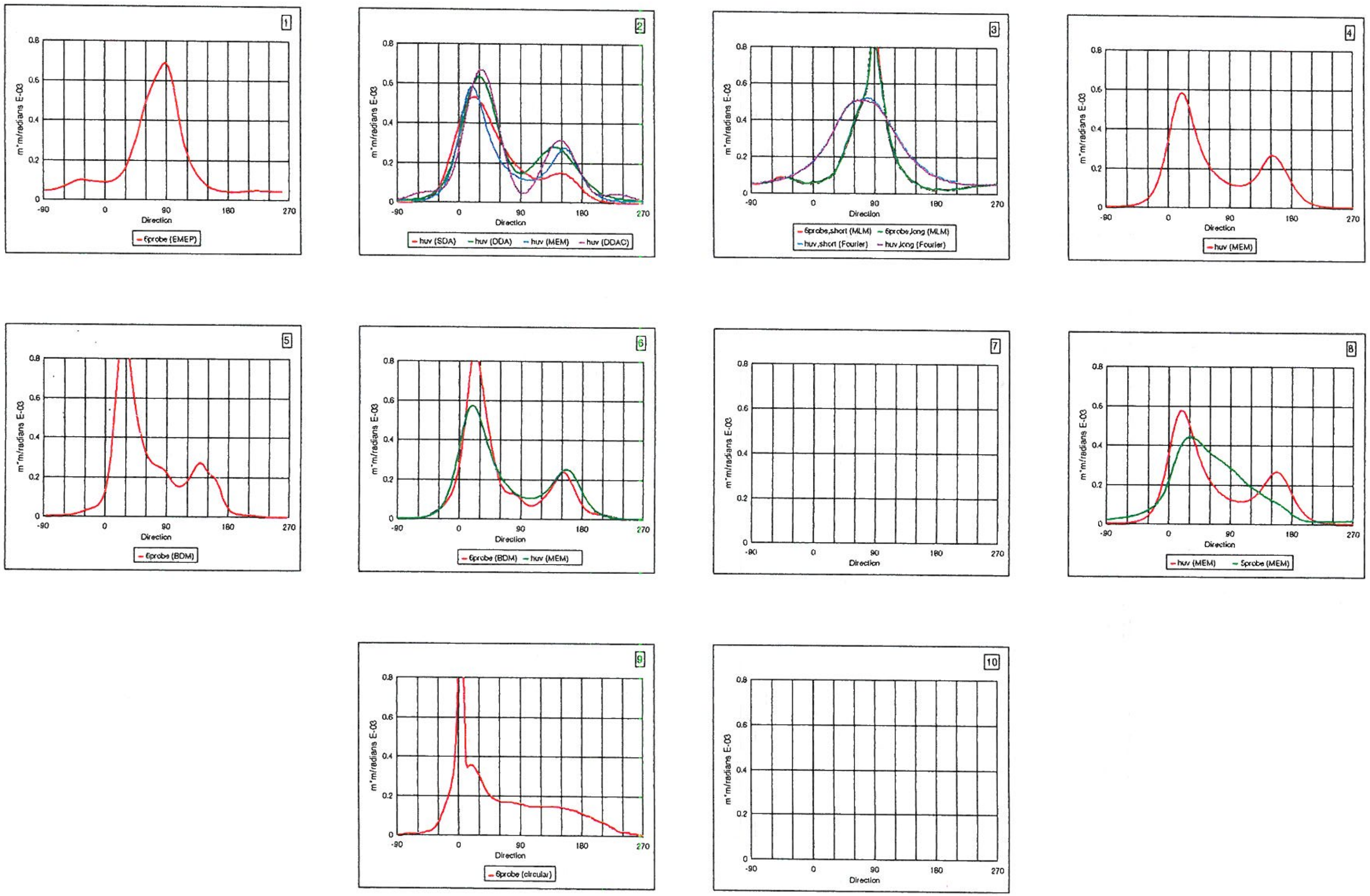


Figure 23: Wave Energy Density v Direction: Test C3 (70% "reflection")

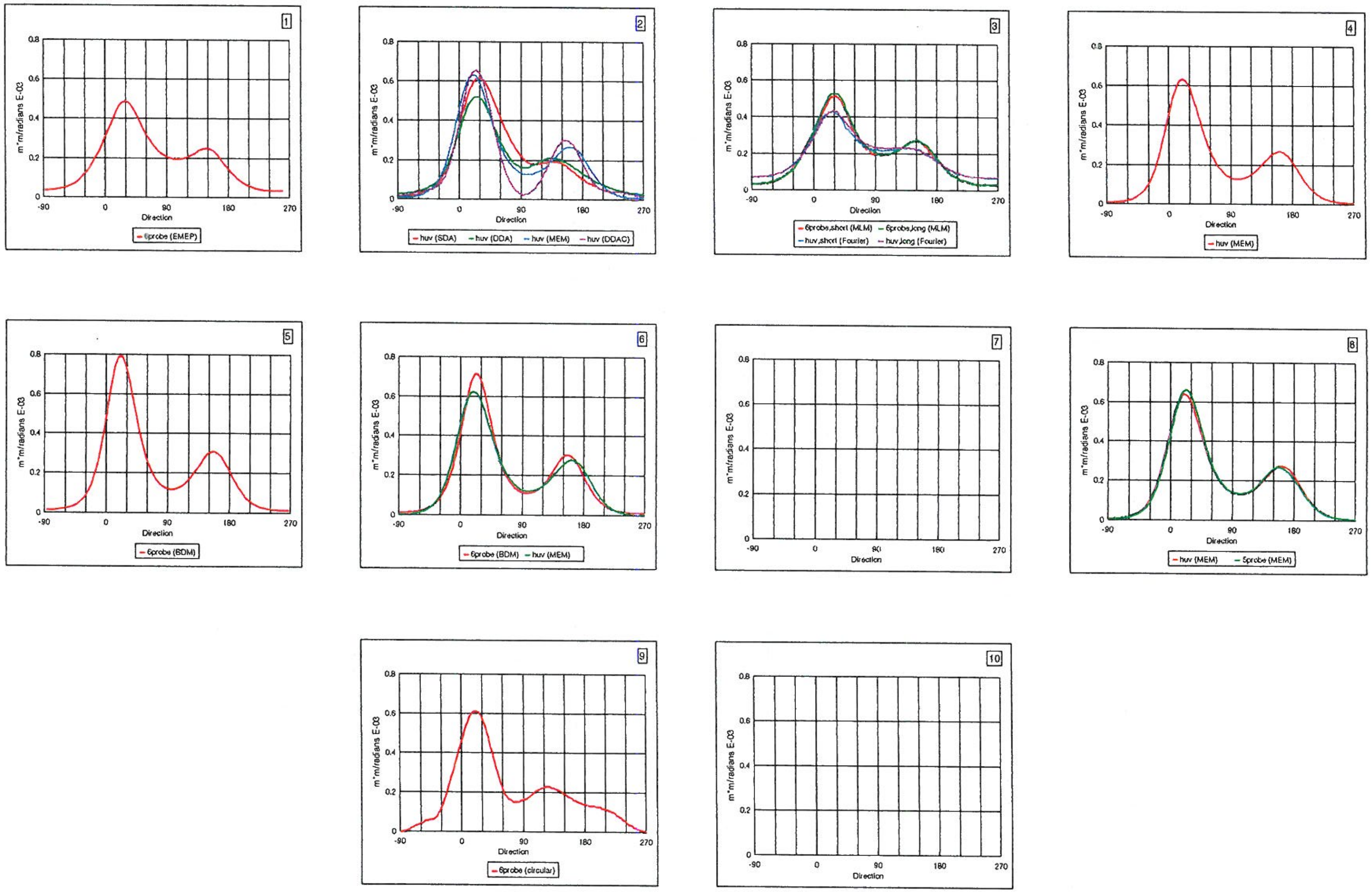


Figure 24: Wave Energy Density v Direction: Test C4 (Cross-resonant modes: 4cm at -50deg; 4cm at +130deg)