Chern-Simons Modified RPA-Eliashberg Theory of the $\nu = \frac{1}{2} + \frac{1}{2}$ Quantum Hall Bilayer

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The $\nu = \frac{1}{2} + \frac{1}{2}$ quantum Hall bilayer has been previsously modeled using Chern-Simons-RPA-Eliashberg (CSRPAE) theory to describe pairing between the two layers. However, these approaches are troubled by a number of divergences and ambiguities. By using a "modified" RPA approximation to account for mass renormalization, we can work in a limit where the cyclotron frequency is taken to infinity, effectively projecting to a single Landau level. This, surprisingly, controls the important divergences and removes ambiguities found in prior attempts at CSRPAE. Examining BCS pairing of composite fermions we find that the angular momentum channel $l = +1$ dominates for all distances d between layers and at all frequency scales. Examining BCS pairing of composite fermion electrons in one layer with composite fermion *holes* in the opposite layer, we find the $l = 0$ pairing channel dominates for all d and all frequencies. The strength of the pairing in these two different descriptions of the same phase of matter is found to be almost identical. This agrees well with our understanding that these are two different but dual descriptions of the same phase of matter.

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Quantum Hall bilayers have been a subject of intense investigation since the first experiments on these systems almost thirty years ago (see Refs. [1–3] for reviews). Conceptually, these are simple systems: a pair of parallel two-dimensional electron gases separated by a distance d placed in a magnetic field at low temperature. Nonetheless, they show a vast variety of fascinating phenomena. Perhaps the problem that has attracted the most interest in this field has been the case of the balanced bilayer with Landau level filling fraction $\nu = \frac{1}{2} + \frac{1}{2}$. In the limit of small distance d between the layers (compared to the magnetic length ℓ_B) the system forms an exciton condensate [1] (alternately called a quantum Hall ferromagnet [4] or the Halperin "111" state [5]). In this state one can think of each electron in one layer being bound to a correlation hole in the opposite layer, hence forming an exciton. In contrast, in the limit of large d/ℓ_B the system can be considered as two independent $\nu = \frac{1}{2}$ quantum Hall systems, which are known to be composite Fermi liquids [6–8].

The composite Fermi liquid may be described either with a Jain wave function approach [8], by attaching two Jastrow factors to the position of each electron, or in a Chern-Simons field theory [6,7,9] approach, where a singular gauge transformation is made to attach two infinitely thin flux tubes to each fermion. One can also consider the state as being described by a Fermi liquid of composite fermion *holes*. That is, one thinks of the holes in a filled Landau level as being the fundamental degrees of freedom, and attaches flux quanta (or Jastrow factors) to these holes. For clarity if we mean composite fermion holes we will abbreviate them as CH, whereas when we mean conventional composite fermions, where flux quanta, or Jastrow factors, are attached to the original electron coordinate, we will abbreviate this as CE. The distinction between the CE and CH Fermi liquids, two states that are related to each other by particle-hole conjugation within a single Landau level, is discussed in some detail by Refs. [10,11]. While the CE and CH trial wave functions do not precisely preserve particle-hole symmetry of the half-filled Landau level, which is expected of the ground state in the absence of Landau level mixing in the clean limit, they are numerically exceedingly close to particle-hole symmetric: for 10,11,12 electrons on a torus, the overlap of the CE wave function state [11,12] with its particle-hole conjugate (the CH wave function) is above 97%.

The CE and CH approaches to the half-filled Landau level are supplemented by the Dirac composite fermion approach [13], which explicitly preserves particle-hole symmetry of the half-filled Landau level. Neglecting this minor distinction that the Dirac approach precisely respects particle-hole symmetry whereas the CE and CH approaches only approximately respect this symmetry, all three approaches (CE, CH, and Dirac) are believed to correctly represent the universal long wavelength physics of the composite Fermi liquid [14–16].

Returning now to the $\nu = \frac{1}{2} + \frac{1}{2}$ bilayer, although the two limits of large and small distance d between the layers have

been quite well understood for some time, the question that has occupied the community for years [1,3,4,17–51] is what happens for intermediate d/ℓ_B. Only recently a clear picture has finally emerged as to the physics of this regime. Based partially on the Dirac composite fermion picture [13], Sodemann *et al.* [40] proposed that the two CE Fermi liquids, when weakly interacting with each other, should BCS pair in the $l = +1$ angular momentum channel (chiral p wave), and they further proposed that this phase of matter is continuously connected to the exciton condensate at $d = 0$. The idea of BCS pairing of CEs in such bilayers was not new [19,36,37,50,52], but it was not previously clear that the BCS paired state of CEs could be the same phase of matter as the exciton condensate.

Inspired by new experiments in bilayers built from graphene [17,18], an alternative picture was recently constructed. In this picture one imagines condensing BCS pairs made from a CE in one layer bound to a CH of the other layer in the $l = 0$ angular momentum channel (s wave). This then gives an apparently different picture of a paired state. What is emphasized in Ref. [17] (See the supplementary material of that Ref.) is that in this picture, in the limit of tightly bound pairs, projection to the lowest Landau level gives precisely the exciton condensate, or Halperin 111 state.

To test these two pictures of interlayer pairing, Wagner *et al.* [62] (see also Ref. [63]) constructed Jain style [8] trial wave functions for BCS paired states both for $l = +1$ CE-CE pairing and for $l = 0$ CE-CH pairing. (These constructions were both based on earlier work of Möller, Simon, and Rezayi [36].) Both approaches were found to be extremely accurate for all values of d when compared with exact diagonalizations on small systems (square overlaps $\gtrsim 0.97$ for system sizes of $6 + 6$ electrons on a sphere where the symmetry-reduced Hilbert space is 252 dimensional), and the two approaches were essentially indistinguishable in how well they performed. We conclude that both approaches are describing the same physics—although the mapping between the two approaches is nontrivial.

To try to access the thermodynamic limit, and in order to gain more physical intuition, one can attempt to address the pairing between the two layers analytically. Very early in the history of the field, Bonesteel, Macdonald, and Nayak [19] described the CEs in each layer using the Halperin-Lee-Read (HLR) Chern-Simons field theory [6]. The bosonic "glue" that pairs the fermions together between the two layers is the Chern-Simons RPA screened Coulomb interaction. Reference [19] then used Eliashberg theory to evaluate the pairing instability, the result of which we call Chern-Simons RPA Eliashberg theory (CSRPAE). Although such calculations are plagued with divergences, these authors were nonetheless able to argue that the system would be unstable to pairing at any finite d, although at the level of this calculation all angular momentum channels of pairing are degenerate.

A more detailed version of this CSRPAE calculation was attempted much later by Isobe and Fu [64] (other versions were attempted by Refs. [50,65]). To control infra-red diveregences, Isobe and Fu introduced a wave vector cutoff q_c which is taken to be a very small fraction of the Fermi momentum. There are two coupling constants that are calculated in this Eliashberg theory: $\lambda_Z(\omega_m)$, the prefactor of the nonanomalous electron self-energy, which in this calculation diverges as $1/q_c$ at any (fermion) Matsubara frequency ω_m and is independent of the pairing channel, and $\lambda_\phi^{(l)}(\omega_m)$, the prefactor of the anomalous self-energy, which in this calculation diverges as $\log(q_c)$ at any nonzero Matsubara frequency and depends on the pairing channel l. (There are additional, but integrable, divergences as ω_m goes to zero, which do not need to be regularized). Despite these divergences, it was found that the *difference* between the coupling constants $\lambda_\phi^{(l)}(\omega_m)$ in different pairing channels l is nondivergent, so that the arbitrary cutoff need not be implemented when comparing different channels to each other, thus suggesting that the arbitrary cutoff may not be problematic. In particular, the claim of Isobe and Fu was that the pairing angular momemtum channel $l = +1$ is always favored, i.e., $\lambda_\phi^{(l)}(\omega_m)$ is always most negative for $l = +1$. This pairing channel agreed with earlier trial wave function work of Möller, Simon, Rezayi [36] as well as with the more recent predictions of Sodemann *et al.* [40]. However, upon repeating this calculation we found that while $l = +1$ often minimizes $\lambda_\phi^{(l)}$, it can sometimes (depending on ω_m and d) be minimized instead with $l \neq +1$ (see examples of this in the Supplemental Material [53], Sec. I), making it hard to draw conclusions confidently as to which pairing channel is actually favored.

Recently the Isobe-Fu calculation [64] was generalized by Rüegg, Chaudhary, and Slager [66] to consider the alternative picture of CEs in one layer and CHs in the other layer. Again, within CSRPAE theory, the leading divergent terms are independent of the pairing channel and one relies on a cutoff to regularize the calculation, although differences in pairing strength are nondivergent and allows comparison between different pairings. The calculation found that the $l = 0$ pairing channel for CE-CH pairing is favored, in agreement with the trial wave functions of Ref. [62]. In addition, the authors claimed that the CE-CH pairing is stronger than the CE-CE pairing. This latter point is a somewhat curious result when compared to the trial wave function results of Wagner *et al.* [62], where both trial wave functions seem equivalently good. In fact, the comparison made by Rüegg, Chaudhary, and Slager [66] between CE-CE pairing and CE-CH pairing leaves much unclear because the two approaches have different divergent terms, so comparison of the coupling strengths depends on how these divergences are regularized (see Supplemental Material [53], Secs. II and IV). While it is always the case that CE-CH pairing is favored compared to

CE-CE pairing, i.e., $\lambda_\phi^{(l=0),\text{CE-CH}}(\omega_m) \leq \lambda_\phi^{(l=+1),\text{CE-CE}}(\omega_m)$, depending on the cutoff and ω_m this inequality may either be greatly unequal or may be very close to an equality.

The above-mentioned Chern-Simons Eliashberg calculations [19,64,66] are excellent starting points for further analytic work which we shall pursue here. These prior calculations, however, have a number of clear shortcomings: (i) As mentioned above, the CSRPAE calculations with CE-CE pairing are somewhat ambiguous in which the pairing channel is actually favored (Supplemental Material [53], Sec. I). (ii) The introduction of an arbitrary infrared cutoff is somewhat unsatisfying and gives room to doubt that the results are reliable. (iii) The fact that CE-CE and CE-CH pairing have different divergences makes it impossible to compare these two calculations in a cutoff-independent way (Supplemental Material [53] Secs. II and IV. (iv) The CSRPAE calculations are based on RPA evaluation of a propagator and RPA is known to have a number of problems—in particular, RPA does not correctly put the low energy physics on the interaction scale and the high energy physics on the cyclotron scale [6,67]. (v) In making comparison of the CSRPAE approach with the successful trial wave functions of Wagner *et al.* [62] one may also worry that the wave functions are strictly in the lowest Landau level, whereas Chern-Simons RPA theory is not. As detailed in Supplemental Material [53] Sec. VI, this is particularly concerning in the case of CH calculations where one cannot even use "hole" coordinates as the fundamental degrees of freedom unless the system has a finite Hilbert space dimension, such as when the system is projected to a single Landau level.

The purpose of this Letter is to repair the many problems of these previous works and for the first time obtain unambiguous results. This Letter will report our main findings with the calculational details relegated to the Supplemental Material [53]. Surprisingly, a single new physical ingredient added to the prior calculations can, to a large extent, address *all* of the above listed shortcomings. In this Letter we extend the CSRPAE calculations to use a so-called *modified* RPA (MRPA) approach developed by Simon and Halperin [67], rather than the pure RPA. This scheme, based on Landau Fermi liquid theory, puts the low energy physics on the interaction scale while pushing the cyclotron mode up to the correct frequency so that Kohn's theorem and the f-sum rule are properly satisfied. Setting the cyclotron energy $\omega_c = eB/m_b$ to infinity (i.e., taking the limit of the electron bare band mass m_b going to zero) then should remove any physics of this high energy scale from the problem. While this is not strictly equivalent to lowest Landau level projection, presumably much of the same physics is included.

We now briefly describe the calculation. More details are given in the Supplemental Material [53]. The MRPA scheme [9,67] accounts for mass renormalization via Landau Fermi theory. The polarization bubble for noninteracting fermions in zero effective field is calculated with an effective mass m^* which is set by the interaction scale. To preserve sum rules (stemming from Galilean invariance) we must include a Landau Fermi liquid interaction which amounts to an additional current-current interaction term $A\mathbf{j}\cdot\mathbf{j}$ with $A = (m_b - m^*)/(ne^2)$ with n the electron density and e the electron charge and $\mathbf{j}$ the current density. This current-current interaction term is then treated in RPA along with the Chern-Simons gauge interaction and the Coulomb interaction. The remainder of the Chern-Simons Eliashberg calculation follows that of Refs. [64,66] and is detailed in Supplemental Material [53] Sec. III. Using MRPA rather than RPA in CSRPAE theory we thus abbreviate as CSMRPAE. If we set $m^* = m_b$ in CSMRPAE we recover the CSRPAE results of Refs. [64,66].

We now consider CSMRPAE in the limit of m_b going to zero. This limit is meant to represent projection to a single Landau level, although as mentioned in Supplemental Material, Sec. VI, once one makes any sort of mean field approximation, some of the detailed structure of the lowest Landau level is lost, such as its particle-hole symmetry. Remarkably, in this limit we find that the divergences in coupling constants $\lambda_Z(\omega_m)$ and $\lambda_\phi^{(l)}(\omega_m)$ vanish proportional to m_b^n with $n \geq 1$ for any value of ω_m such that $0 < \omega_m < \mathcal{O}(\omega_c)$ (see Supplemental Material [53] Secs. IV and V) for both CE-CE pairing and for CE-CH pairing. By taking the $m_b \to 0$ limit, we push ω_c to infinity, removing all divergences at any finite frequency, and thus remove the need for an *ad hoc* q_c cutoff.

In this limit, we find that for CE-CE pairing $l = +1$ is now unambiguously the strongest pairing channel for all values of d and ω_m (see Fig. 1 and Supplemental Material [53] Sec. VII), and for CE-CH pairing $l = 0$ remains unambiguously the strongest pairing channel (see Fig. 1 and Supplemental Material [53] Sec. VIII). Further, without the divergences and cutoff dependencies we can now meaningfully compare CE-CE $l = +1$ pairing with CE-CH $l = 0$ pairing. To very high precision (roughly 1% level) we find that the CE-CE pairing and the CE-CH pairing are equivalently strong (see Fig. 1, inset). We also analyze this comparison analytically in Supplemental Material [53] Secs. VIII A–VIII C. The equivalence between the CE-CE and CE-CH pairing is rather surprising given that two very different integrals need to give almost precisely the same result. We show further (Supplemental Material [53] Sec. VIII D) what small modification of our MRPA approximation would make them exactly equal.

We note in passing that, as pointed out in Ref. [62] (see the supplementary material of that Ref.) both CE-CE and CE-CH pairing are equally able to remain paired in the presence of density imbalance between the layers, so long as the total filling remains $\nu_T = 1$, in agreement with experiment [68,69]. We reiterate this argument in

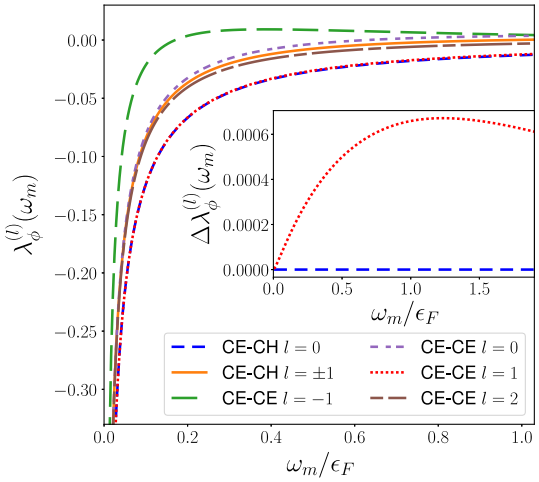

FIG. 1. The Eliashberg anomalous coupling constant $\lambda_\phi^l(\omega_m)$ calculated using CSMRPAE for interlayer spacing $d/l_B = 1$. The coupling constant is shown for different pairing channels l and for both composite-fermion-electron-composite-fermion-electron pairing (CE-CE) and for composite-fermion-electron-composite-fermion-hole pairing (CE-CH). We see that CE-CH pairing with $l = 0$ and CE-CE pairing with $l = 1$ are most attractive (most negative) and are very nearly equal to each other (the two lowest curves almost precisely overlap). The inset shows an enlargement of the difference between the two lowest curves ($l = 0$, CE-CH pairing and $l = 0$ CE-CE pairing). The difference is a factor of order 100 smaller than the difference to other pairing channels. Other values of d/ℓ_B, and larger range of ω_m are shown in the Supplemental Material [53] Sec. VII.

Supplemental Material [53] Sec. IX. Imbalance will be studied in more depth in a forthcoming work.

To further demonstrate the usefulness of our approach, we now examine a number of extensions. First, we can consider other filling fractions, see Supplemental Material [53] Sec. VII B. The $\nu = 1/4 + 3/4$ case and the $\nu = 1/6 + 5/6$ cases described in the CE-CH picture are unambiguously found to be in the $l = 0$ pairing channel. At $\nu = 1/4 + 1/4$ in the CE-CE picture we find that the $l = 1$ channel is unambiguously favored. However, at $\nu = 1/6 + 1/6$, in the CE-CE picture $l = 0$ pairing is favored at low frequency (compared to the Fermi energy), but $l = 1$ is (only very slightly) favored at higher frequency. While this leaves a slight ambiguity in the result, it strongly suggests that $l = 0$ pairing is realized (this ambiguity was also seen by Ref. [64], although $l = 1$ is more strongly favored at high frequency in that case making the results more ambiguous). This suggests that $1/6 + 1/6$ would be interesting to examine further either in numerics or experiment. Such CE-CE $l = 0$ pairing would have zero quantized Hall drag at zero temperature [70–72] as compared to all of the other states considered here which have h/e^2 quantized Hall drag resistance.

We can further examine whether changing the inter-electron interaction might change our results, see Supplemental Material [53] Sec. VII A. Assuming that we start with two composite Fermi liquids before we turn on the interlayer interaction, we find that the favored pairing channel is remarkably insensitive to the details of the interelectron interaction within our CSMRPAE approximation. We have examined (i) $V(q) \sim 1/(q + q_0)$ which models interaction in the presence of nearby metal screening layers, (ii) $V(q) \sim e^{-q^2w^2}/q$ which models the effects of finite well width, (iii) longer-ranged potentials $V(q) \sim q^{-2+\eta}$ with $\eta \in (0, 1)$, (iv) Gaussian potentials $V(q) \sim \exp(-q^2w^2)$, and (v) inclusion of Landau level form factors $V(q) \sim [L_n(q^2\ell_B^2/2)]^2/q$ with L the Laguerre polynomials. We use the same form for inter- and intralayer interaction, although we reduce the strength of the interlayer compared to intralayer (see Supplemental Material [53] Sec. VII A for the full range of parameters that have been examined). For $\nu = 1/2 + 1/2$ in all cases, we find $l = 1$ favored for CE-CE pairing and $l = 0$ favored for CE-CH pairing with the two descriptions being very close to degenerate. This strongly suggests that very similar physics should occur in a wide range of two dimensional electron systems independent of details.

Finally, we turn to examine the robustness of our results to deformations of the spatial metric. Such deformations are of particular interest [73–79] because some physical two-dimensional electron systems have anisotropic effective mass. Even with isotropic effective mass, tilted magentic field can make the single-particle orbitals anisotropic. The simplest case to study is that of a Gaussian inter-electron interaction [case (iv) above]. As pointed out by Ref. [74], for a system projected to the lowest Landau level, the Gaussian interaction allows one to make a unitary transformation that implements an area preserving diffeomorphism without changing the spectrum—thus implying complete robustness against geometric deformation. We discuss geometric deformation further in Supplemental Material [53] Sec. IX C, where we argue that the pairing symmetry remains the same if one examines the system in rescaled coordinates, and we conjecture that the gap will always be robust to such deformation.

To conclude, we believe our approach of looking at the $m_b \to 0$ limit of CSMRPAE has satisfactorily tamed the divergences and ambiguities of CSRPAE theory which have been problematic for several decades. Our main results for $\nu = 1/2 + 1/2$ are: for CE-CE pairing the $l = +1$ pairing channel is unambiguously the strongest, and for CE-CH pairing the $l = 0$ pairing channel is unambiguously the strongest. To very high precision we also find that these two cases pair with the same strength in agreement with the results of prior trial wave function calculations [62]. This is rather satisfying since we believe that the two types of pairing are simply different descriptions of the same physics. In fact, it is perhaps a bit surprising that our two approximate approaches are so closely equivalent given that we have not enforced any sort of

symmetry between the two. One might think that this near equivalence is a result of particle-hole symmetry (which itself has been broken by the Chern-Simons calculational approach even in the $m_b \to 0$ limit). However, even given a perfect particle-hole symmetry, it is not obvious that binding CEs to CEs should be precisely equivalent to binding CEs to CHs. This should be interpreted as a nontrivial duality which is surprisingly accurately respected by the CSMRPAE approach. It is an open question whether the two apparently different types of pairing might look more equivalent within the Dirac CF theory, where particle-hole symmetry is manifest at least within each layer. We comment, however, that the system does not need to have particle-hole symmetry in order for the CSMRPAE to predict the (very near) degeneracy between CE-CE $l = +1$ pairing and CE-CH $l = 0$ pairing. See Supplemental Material [53] Sec. X for further elaboration of these issues.

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