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A Mutual Catastrophe Insurance Framework for Horizontal Collaboration in Prepositioning Strategic Reserves

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We develop a mutual catastrophe insurance framework for the prepositioning of strategic reserves to foster horizontal collaboration in preparedness against low-probability high-impact natural disasters. The framework consists of a risk-averse insurer pooling the risks of a portfolio of risk-averse policyholders. It encompasses the operational functions of planning the prepositioning network in preparedness for incoming insurance claims, in the form of units of strategic reserves, setting coverage deductibles and limits of policyholders, and providing insurance coverage to the claims in the emergency response phase. It also encompasses the financial functions of ensuring the insurer's solvency by efficiently managing its capital and allocating yearly premiums among policyholders. We model the framework as a very large-scale non-linear multi-stage stochastic program, and solve it through a Benders decomposition algorithm. We study the case of Caribbean countries establishing a horizontal collaboration for hurricane preparedness. Our results show that the collaboration is more effective when established over a longer planning horizon, and is more beneficial when outsourcing becomes expensive. Moreover, the correlation of policyholders affected simultaneously under the extreme realizations and the position of their claims in their global claims distribution directly affects which policyholders get deductibles and limits. This underlines the importance of pre-negotiating policyholders' indemnification policies at the onset of collaboration.

Key words: mutual catastrophe insurance; prepositioning; horizontal collaboration; multi-stage stochastic program; Benders decomposition.

1. Introduction

The economic losses from natural disasters have significantly increased in recent years, doubling between the 1980s and the early 2000s (Kunreuther and Michel-Kerjan 2013), mostly due to the climatic change altering the frequency, intensity, spatial diversity, and duration of catastrophic events, such as hurricanes. Resilience to low-probability high-impact disasters is a fundamental concern in catastrophe risk management, which aims to enhance the ability of communities to mitigate, prepare for, respond to, and recover from disasters (Kousky 2019). In particular, strengthening disaster preparedness for effective response is among the global priorities of the United Nations (UNDRR 2020). Prepositioning is a widely applied disaster preparedness policy by which strategic reserves for food, water, as well as medical, transportation, power, and rescue supplies are stored at selected locations to provide emergency response to disaster-affected areas (Sabbaghtorkan et al. 2020). While prepositioning is an important step to alleviate the impact of disasters after they occur, it can be substantially expensive (Balcik and Beamon 2008). How to reduce prepositioning costs poses important challenge to practitioners and researchers (Jahre et al. 2016). Hence, transformative approaches leading to higher benefits from the prepositioning investment are needed.

Stakeholders facing similar disaster risks (e.g., countries) can benefit from horizontal collaborative prepositioning by pooling their risks and sharing their resources. However, collaboration entails some challenges. It can be costly, both financially and in the time needed to create, operate, and manage it (Stephenson 2005), rendering only a few actors able to effectively provide it. One way to systematically address these challenges is to embed the operational and financial functions within an integrated catastrophic risk management framework (Dong and Tomlin 2012). In designing such a collaborative framework, it is essential to determine how to integrate these functions together, how to pool each stakeholder's individual risk jointly, and what are the most effective mechanisms for sharing risks, costs, and benefits.

Mutual catastrophe insurance provides an effective framework for establishing horizontal collaboration among stakeholders, which become policyholders when the risk is transferred from them to the insurer. Mutual insurance companies are non-profit and share the functions of the insurer and of the policyholders. They are owned by policyholders, and managed by an umbrella organization. Policyholders jointly hold the pool of insurance claims, and share the aggregate losses in a combination of prepaid premiums and paid-out dividends from the residual pool of capital after claims are covered (Doherty and Dionne 1993). The policyholder's premiums and indemnifications of claims are a function of their risk profile, individual losses, and the aggregate losses (Marshall 1974). Some mutual catastrophe insurers, such as the Caribbean Catastrophic Relief Insurance Facility (CCRIF) (CCRIF SPC 2019) and the African Risk Capacity (AFC) (African Risk Capacity 2020), provide participating countries under similar disaster risks with a set of catastrophe insurance schemes, but these only indemnify financial losses and do not integrate operational functions. We present a mutual catastrophe insurance framework that links disaster financing with the planning of disaster preparedness and emergency response operations through the design of multi-year insurance contracts and the consideration of the insurer's solvency.

1.1. Aim of this study

In contrast to what is observed in other lines of insurance (such as property insurance), catastrophic risks are difficult to insure since they are highly uncertain, unpredictable, spatially and temporally correlated (Kleindorfer and Klein 2003), and their distributions tend to be fat-tailed (Kousky and Cooke 2012). These features of catastrophes justify policyholders to seek catastrophe insurance by which one pays a premium to financially protect against uncertain low-probability high-impact future events (Zeckhauser 1995). Moreover, the specificities of catastrophic risks do not prevent the operation of a market for catastrophe insurance, as the risks are still sufficiently diversifiable and infrequent. In fact, the challenges faced by a catastrophe insurer are of a financial capital nature rather than of an insurance contracts nature, but the design of optimal insurance contracts still follows the principles applied in other lines of insurance (Jaffee and Russell 1997). The unpredictability, intensity, and spread of catastrophic risk result in a high variance of the insurer's cashflow. To be able to bear this risk, the insurer must have access to a large surplus capital pool, which allows it to cover the upper layers of risk, to financially recover, and to minimize the probability of insolvency (Kleffner and Doherty 1996). The latter is a very important factor for policyholders when choosing an insurer (Sommer 1996). It is therefore crucial for any catastrophe insurer to determine how to solve the intertemporal problem of matching the flow of premiums from policyholders to a highly non-smooth flow of losses indemnification, while congruently reducing its risk of insolvency (Jaffee and Russell 1997). Hence, the primary challenge of this paper is to integrate these financial insurance considerations with the preparedness and response operations over a multi-year planning horizon.

We design a mutual catastrophe insurance framework for the prepositioning of strategic reserves where a risk-averse insurer provides multi-year insurance contracts with coverage deductibles, limits, and yearly premiums to a portfolio of risk-averse policyholders. A deductible is the threshold below which no insurance coverage is provided, and a limit is the threshold above which no additional coverage is provided. The deductible and the limit are essential components of any insurance contract since they allow the reduction of total investments, and an effective premium allocation scheme yields a fair allocation of that investment among policyholders. Each policyholder's aim is to maximize its expected utility, and that of the insurer is to minimize its logistical costs (procuring, storing, and transporting supplies) while fulfilling the optimal insurance contracts offered in the portfolio. We coin the problem as the *Catastrophe Insurance Problem for the Prepositioning of Strategic Reserves (CIP-PSR)*, and model it as a multi-stage stochastic program (Birge and Louveaux 2011). The objective is to minimize over a planning horizon the expected insurance premiums of the portfolio, plus the expected cost of emergency outsourcing, which occurs due to the portion of the claims uncovered by the insurance contracts. The problem consists of four integrated decision making components: the insurance coverage design, the prepositioning network design, the management of surplus capital, and the premium allocation. In the first stage, the model determines the number of storage facilities and inventory level at each location in the prepositioning network, the coverage deductible and limit of each policyholder, as well as the surplus capital required to cover the worst-case catastrophes In each following stage, the model determines the premium to be paid by the policyholders at the beginning of the year, and whenever a disaster occurs in a year, the policyholders' insurance coverage from each location, as well as the replenishment amounts. We apply an elaborate Benders decomposition approach to solve the model and deal with its non-linear and large-scale complexities.

While our framework focuses primarily on hurricane catastrophes, it is generalizable to man-made disasters and other natural disasters, including pandemics. The COVID-19 pandemic has indeed shown that global and regional solidarity and collaboration are essential to support vulnerable populations and effectively prepare for, respond to, and recover from it. Efficient prepositioning of critical medical supplies and equipments, like in the Strategic National Stockpile, can strengthen the response to future pandemics (Gerstein 2020). Our framework is conceived so that better catastrophic risk management can be achieved through horizontal collaboration in such settings.

1.2. Positioning within the relevant literature

We position this study within two streams of literature: insurance schemes developed to support collaboration against disruptions, and collaborative prepositioning in humanitarian logistics.

1.2.1. Insurance schemes A conflict of interests among stakeholders in the supply chain makes non-collaborative strategies devoid of the benefits achievable through collaboration and risk pooling, and hence suboptimal (Bimpikis et al. 2018). Insurance schemes can be effective strategies to align stakeholders' interests and establish collaborations (Dong and Tomlin 2012). While there exists an abundance of research both in the field of insurance theory and catastrophe insurance, very few operations research papers apply these theories and methods to foster vertical or horizontal collaborations in operational settings.

Some papers apply insurance schemes for vertical collaboration to manage supply chain disruptions (e.g., production halts, increased lead time, etc.) (Kleindorfer and Saad 2005, Snyder et al. 2016). Other studies focus on the upstream of the supply chain and introduce insurance schemes to collaborate with suppliers (Lodree and Taskin 2008, Dong and Tomlin 2012, Serpa and Krishnan 2017, Dong et al. 2018, Qin et al. 2020). To a lesser extent, insurance is applied to manage disruptions downstream of the supply chain (Zhen et al. 2016, Wang et al. 2018).

While insurance has rarely been coupled with horizontal collaboration, the literature on horizontal collaboration in supply chains is abundant (Cruijssen et al. 2007, Basso et al. 2019), and is mostly concerned with agreements for coalition formation, and cost and benefit allocation, which are usually modeled and solved through game theory (Agarwal and Ergun 2010, Muggy and Heier Stamm 2014, Guajardo and Rönnqvist 2015, 2016, Karsten et al. 2015, Nagurney et al. 2016). Cooperative games are also applied in the related field of optimal allocation of risk capital among divisions of a firm (Boonen et al. 2020). These games consider all possible subcoalitions to determine the coalition that allocates the total gain among players using fairness allocation schemes such as the Shapley value (Guajardo and Rönnqvist 2016) or the Aumann-Shapley value (Boonen et al. 2020). The problem of finding such a coalition is computationally intractable even for relatively small-size instances since it requires enumerating all subsets of a set of players. Our proposed framework, which is based on a stochastic programming model, does not suffer from this problem and is applicable to much larger instances. It constitutes a practical approach for sharing risks, costs, and benefits in a cooperative agreement for disaster financing and preparedness.

1.2.2. Collaborative prepositioning The prepositioning problem has been widely studied in the humanitarian logistics literature (Sabbaghtorkan et al. 2020), and a rich variety of settings have been considered (Balcik and Beamon 2008, Rawls and Turnquist 2010, Campbell and Jones 2011, Duran et al. 2011, Tofighi et al. 2016, Dalal and Üster 2018). Most authors model prepositioning as a scenario-based two-stage stochastic program with recourse, since such models allow the consideration of different disaster scenarios, and the simultaneous incorporation of pre- and post-disaster events using first- and second-stage decision variables (Grass and Fischer 2016).

Whereas significant benefits can be derived from collaboration among stakeholders in disaster preparedness (Balcik et al. 2010, Jahre and Jensen 2010), few studies have explored collaborative prepositioning settings. Davis et al. (2013) consider location decisions coupled with the relocation of supplies between the prepositioning warehouses of local agencies, based on short-term hurricane forecasts. Similarly, Toyasaki et al. (2017) and Coskun et al. (2019) study horizontal collaboration between humanitarian organizations exchanging supplies in humanitarian depots. Rodríguez-Espíndola et al. (2018) present a multi-organisational collaborative model for prepositioning supplies against floods. The literature on horizontal collaboration and insurance is restricted to two papers that present a collaborative prepositioning network in a multi-partner setting to strengthen regional preparedness in the Caribbean. Balcik et al. (2019) consider two decisional components: prepositioning network design, and the allocation of the network costs among countries using a premium setting principle based on the countries' risk profiles, where premiums are calculated and the service level is fixed prior to the optimization model. Rodríguez-Pereira et al. (2021) also consider the countries' ability to pay in the cost sharing mechanism. Both papers solve their model using off-the-shelf commercial solvers.

Our paper differs from those of Balcik et al. (2019) and Rodríguez-Pereira et al. (2021) in the problem under consideration, the modelling, and the solution methodology. Unlike these studies that present a one-time cost sharing mechanism for the initial investment, we develop a full multi-year mutual catastrophe insurance framework that integrates the objectives of the insurer and of the policyholders.

1.2.3. Scientific contributions and organization of the paper Mutual catastrophe insurance constitutes an important mechanism for policyholders to jointly prepare for the consequences of disasters by pooling their physical and financial resources. Our proposed framework endogenously determines insurance coverage by imposing coverage deductibles and limits on the policyholders, and integrates financial solvency conditions for the insurer. It incorporates several concepts from insurance and risk theory into collaborative prepositioning planning, such as the design of optimal insurance contracts (Cummins and Mahul 2004), the top-down premium calculation method (Bühlmann 1985), the Cramér-Lundberg model in ruin theory (Schmidli 2017), and the standard deviation premium setting principle (Deelstra and Plantin 2014). We are not aware of any previous work that integrates these components for resilient preparedness logistics networks against uncertain disastrous events. In fact, we are the first to integrate these concepts together in one novel complex optimisation model. Moreover, the Cramér-Lundberg model is a process applied to continuous distributions. We show how such a process can be adapted and discretized to scenario-based multi-stage stochastic programs with non-anticipativity constraints. Finally, and most importantly, the solvency conditions we present are not standard conditions in the insurance literature, but have been tailored to horizontal collaboration in disaster management.

The proposed multi-year mutual catastrophe insurance framework which endogenously sets insurance parameters provides several distinctive advantages. First, the prepositioning network cost is reduced by including the network size as a decision variable so as to maximize its utilization by considering an outsourcing channel. Second, the outsourcing channel ensures that the needs of each policyholder are totally covered, while also reducing the total cost, an important aspect from a humanitarian perspective. Third, the coverage deductibles and limits variables allow the model to endogenously determine under which scenarios to outsource so as to optimize the network size. Fourth, by setting a deductible and limit in policyholders' contracts, the underlying mutual insurance principles enable a fair determination of the policyholders' yearly premiums according to their actual received coverage, as opposed to their requested coverage. Finally, the multi-year insurance is less taxing for the riskiest policyholders, as it allows the smoothing of the policyholders' contributions over time, and results in a smaller investment at the onset of the collaboration.

The introduction of the insurer's objective and its solvency considerations as well as the multi-year insurance contracts design add many layers of modeling complexity to this framework. In fact, the framework is modeled as a scenario-based multi-stage stochastic program which is non-linear and of very large-scale in the number of scenarios. Given that the proposed model cannot be solved efficiently by commercial solvers, we develop an efficient novel solution methodology based on Benders decomposition, which utilizes the analytical properties derived from the problem properties to effectively tackle its non-linearity and large-scale.

The remainder of the paper is structured as follows. Sec. 2 describes the proposed mutual catastrophe insurance framework, Sec. 3 presents the problem and its mathematical model, and Sec. 4 details the solution strategy. This is followed by a computational study in Sec. 5, and by conclusions and insights in Sec. 6.

2. The proposed mutual catastrophe insurance framework

Our mutual catastrophe insurance framework for the prepositioning of strategic reserves consists of a risk-averse mutual insurer offering multi-year insurance contracts to a portfolio of risk-averse policyholders with no moral hazard, i.e., the policyholders cannot engage in risky behaviour without financial consequences. The policyholders' preferences are represented by a von Neumann-Morgenstern utility function and their aim is to minimize their utility of the final wealth (Cummins and Mahul 2004). Risk-averse policyholders will always opt to insure against a risk when their expected final utility is higher under insurance than without it (Hillier 1997). We consider the policyholders to be risk-averse: given the choice between participating in the mutual insurance or fully outsourcing their needs in strategic reserves from an external supplier, policyholders will always choose the option that maximizes their final utility. This consists of minimizing their total expected costs of obtaining the needed strategic reserves, since the insurer does not give back yearly dividends to policyholders. The aim of the risk-averse insurer is to minimize its expected logistical costs associated with procuring, storing, and transporting the prepositioned reserves, while fulfilling the optimal insurance contracts offered in the portfolio. The insurer collects premiums that are as low as possible, but sufficient for the proper functioning of the prepositioning network and the insurance coverage. This allows the insurer to collect lower premiums in the subsequent years due to any surplus capital that may remain from the previous years.

2.1. Why mutual insurance?

The most efficient way to provide catastrophe insurance is to set up the insurer as a mutual insurer (Doherty and Dionne 1993). Traditional non-mutual catastrophe insurers face major hurdles preventing them from operating an efficient insurance (Marshall 1974). These are related to the high cashflow variance and solvency of the insurer, and hence to the access and holding of a large surplus capital. There may exist accounting requirements prohibiting the insurer from building a large surplus to pay for future catastrophic losses, and even when allowed, it is treated as a highly taxed earning (Zanjani 2002). Some insurers do not even build a surplus, counting on governmental intervention under major catastrophes (Kousky 2019). Even without regulatory constraints that may prevent the insurer from asking for higher premiums to build a surplus, too high premiums may deter individuals from insuring against catastrophes (Kousky and Cooke 2012). Finally, while insurers can choose to turn to reinsurance to cover the upper layers of large insurance claims instead of building a surplus, the reinsurance amount obtainable from the reinsurance market is capped at a limit that is far lower than the magnitude of high catastrophic losses (Jaffee and Russell 1997).

Mutual insurance is more suitable for catastrophes than non-mutual insurance where insurers prioritize profit maximization. Mutual insurers are better at absorbing the deviations in surplus capital by passing these effects onto policyholders through varying paid-out dividends, and more importantly and relevantly to our framework, by charging sufficiently high premiums to build the aggregate capital pool, hence reducing their insolvency risk (Doherty and Dionne 1993). Policyholders are willing to pay higher premiums to an insurer with a lower insolvency risk (Sommer 1996). Since the insurance is owned by the policyholders, these trust that the insurer has collected a sufficient capital pool for financial viability while at the same time charging the lowest possible premiums. Moreover, if the variability in the risk profiles among policyholders is high, a fair allocation of the aggregate capital pool among them is ensured in mutual insurance since they contribute to the aggregate pool proportionally to their risk profiles (Marshall 1974). Therefore, we opt to set up our framework as a mutual catastrophe insurance for all the aforementioned benefits it provides to the horizontal collaboration over traditional non-mutual catastrophe insurance.

2.2. Problem description of the framework

The proposed mutual catastrophe insurance framework comprises four integrated decision-making components: the insurance coverage design, the prepositioning network design, the management of surplus capital, and the premium allocation (see Fig. 1). With respect to Balcik et al. (2019) and Rodríguez-Pereira et al. (2021), the first, third, and fourth components are new, and so is the integration of all four components. The implication of this integration on the mathematical model and the solution approach is discussed in Section 3. Each component pertains to either the financial or the operational function, as well as to either the insurer or the policyholders. The insurance coverage design determines the optimal deductible and limit of policyholders that dictate their coverage. The prepositioning network design determines the inventory level and the portion of each insurance claim provided from each location in the collaborative network. These two components are operational functions, the former targeted at maximizing the utility of policyholders, and the latter targeted at minimizing the insurer's costs. The last two components are financial functions. The management of surplus capital ensures that the insurer remains solvent throughout the planning horizon by collecting a sufficiently high aggregate capital pool. The premium allocation ensures that the yearly capital pool is fairly apportioned between policyholders in accordance with their risk profiles. We now present how uncertainty is defined in our framework through catastrophe realizations and the indemnification policy (Sec. 2.2.1), and describe the four decision-making components of the framework (Sec. 2.2.2 to Sec. 2.2.5).

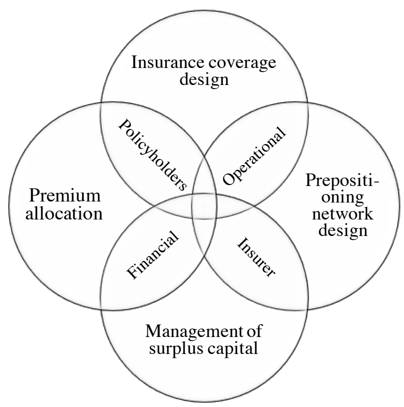

Figure 1 Relationships between the four components of the framework.

2.2.1. Catastrophe realizations and indemnification policy We define a catastrophe realization as a set of disastrous events occurring within a year. Each event is characterized by its severity level, and by the set of policyholders it affects. The severity level is a parameter that can be used to assess the losses incurred by each policyholder. Each realization is further divided into time periods (e.g., months), and a policyholder's quantified losses from all events occurring in a time period are aggregated into one insurance claim per period. We consider a multi-year planning horizon, and define a scenario as a sequence of realizations over the horizon.

An indemnification policy determines how much of its loss a policyholder can claim from the insurer when certain conditions are met, and the insurer is contractually responsible for indemnifying these claims. We study an ex ante parametric insurance indemnification policy, which is the most common policy type in catastrophe insurance (Kousky 2019), and is adopted by most catastrophe insurers such as the CCRIF (CCRIF SPC 2019) and AFC (African Risk Capacity 2020). Under such a policy, the policyholder and the insurer prenegotiate a claim value for which the policyholder will be indemnified for each possible severity level. The set of these negotiated claim values defines the loss schedule. When a policyholder negotiates a lower loss schedule, its claim values are lower but so are its contributed premiums, and conversely. The advantage of an ex ante parametric indemnification policy is that it allows the policyholder to negotiate an insurance contract in accordance with its financial means. Moreover, although the claim values in the loss schedule may deviate from the actual loss, the advantage is that it requires no assessment of the actual damages incurred, which can be a very lengthy, complex, and expensive process for the insurer. This ensures that the claims are rapidly covered within days since they are simply calculated from each event's severity level (CCRIF SPC 2019), as opposed to months or years under an indemnification policy that assesses the actual loss. We consider two basic parametric indemnification policies (Brettler and Gosnear 2020) that all policyholders in the portfolio opt for: the expected loss indemnification (EI) policy, and the maximal loss indemnification (MI) policy. Given all the possible loss values for each severity level, the loss schedule of the EI policy indemnifies the expected value over all possible losses. If the policyholders want a more conservative policy to counter the deviation of the EI policy from the actual loss, the MI policy provides it by setting the claims in the loss schedule to the maximal loss for each severity level among all possible values.

2.2.2. Prepositioning network design The collaborative prepositioning network must hold a sufficient inventory of emergency supplies to be able to provide coverage under any catastrophe realization. Each supplied unit is replenished after a lead time equal to a number of time periods. Therefore, the total inventory level required should cover the largest aggregate needed coverage within each replenishment period. The network consists of logistically advantageous candidate locations for storage facilities, in terms of infrastructure, connectivity, and low destruction likelihood. The costs associated with each location are those of operating a storage facility, and for each unit of supply, the procurement cost, the yearly holding cost, and the logistical coverage cost of transporting that unit to the policyholders. We consider that all storage facilities have the same capacity, that the inventory level at a location is unrestricted, and that there can be as many storage facilities as needed at any location. We assume that within the contractual obligations of the insurance contract, the transportation of strategic reserves is provided by the insurer to a single location pre-negotiated with the policyholder (e.g., an airport, a seaport, a warehouse). The policyholder is then responsible for the in-region distribution to the affected areas.

2.2.3. Insurance coverage design The insurer offers a multi-year contract to each policyholder with a coverage deductible, a coverage limit, and the set of yearly premiums. We assume that over the planning horizon, the set of policyholders is fixed, and the insurance contract remains unchanged. The aim of the insurance coverage design is to determine the optimal coverage deductibles and limits using optimal contract theory (Cummins and Mahul 2004). We assume that any claim falling between the deductible and the limit is fully covered. Any portion of a claim not covered by the insurance contract is procured by the insurer through emergency outsourcing during the time period in which the claim occurs, the policyholder bearing the cost. This ensures that the needed supplies are fully provided, which is an important concern in humanitarian contexts. We consider the unit emergency outsourcing cost to be higher than the cost of prepositioning and covering the unit from the network (Dong and Tomlin 2012), since in relief operations, it is driven by inflated market prices after a disaster (Balcik and Ak 2014, Moshtari et al. 2021).

Considering emergency outsourcing maximizes the benefits obtained through the collaboration. This is achieved by ensuring that the total inventory in the prepositioning network is at a level that is consistently, and not rarely, utilized, which otherwise would lead to a suboptimal, costlier, and underutilized network. The network size is determined by setting the deductibles and limits in insurance contracts so as to control the interaction between inventory level and the amount and correlation of claims (see Sec. 3.2.5). Limits control large claims by partially covering them, ensuring that they do not inflate the inventory so that its totality is infrequently utilized. Deductibles exclude small claims, ensuring that when aggregated with other claims, they do not result in the need for an extra quasi-empty storage facility holding only a portion of the small claims.

2.2.4. Management of surplus capital Along with determining the coverage of all policyholders from their deductibles and limits, the insurer must also compute the required surplus capital and manage its variation as it gets consumed in emergency response over the course of a realization, so as to remain solvent. At the beginning of each year, the insurer collects sufficient premiums in an aggregate capital pool to pay the prepositioning network expenses and to provide emergency response. We apply the Cramér-Lundberg model to set the insurer's solvency conditions, which is commonly used by insurers to compute the cash flow and surplus capital of an insurance portfolio (Schmidli 2017). Given a distribution of claims, this model computes at each point in time the surplus capital as the difference between the available capital plus the premiums collected, and the total claims paid out up to this point in time (see Fig. 2). It then calculates the probability of ruin, that is, the probability that the surplus capital drops below zero.

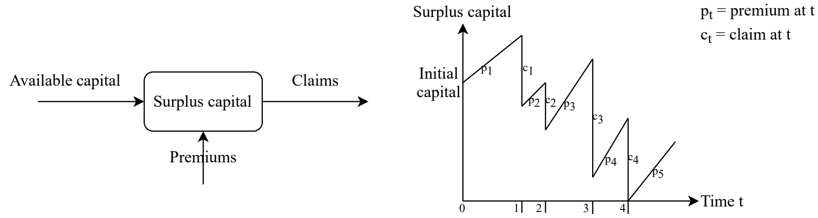

Figure 2 The Cramér-Lundberg model.

2.2.5. Premium allocation The yearly aggregate capital pools must be allocated in yearly premiums among the policyholders based on their risk profiles. A policyholder's risk profile is characterized by its expected insurance coverage and the standard deviation of its coverage, which themselves depend on their deductible and limit. We use a top-down premium calculation method (Bühlmann 1985) by first computing the aggregate capital pool, and then fairly allocating it among policyholders as a function of their risk profiles using the standard deviation premium setting principle. We use the standard deviation premium setting principle formulation of Deelstra and Plantin (2014) since the mathematical formulation used by Bühlmann (1985) cannot handle the case where the probability of ruin is zero.

3. Formulating the catastrophe insurance problem for the prepositioning of strategic reserves

We now present a multi-stage stochastic program to model the CIP-PSR. The notation is summarized in Tables 2–4 of Appendix A, while Appendix B provides an illustrative example that shows the computation of the specific functions of every component of the framework.

3.1. Modeling the uncertainty of catastrophes

We first show how we model the uncertainty of catastrophes using catastrophe realizations, a multi-year planning horizon, multi-stage scenarios, and a scenario tree representation.

The framework is operated for a portfolio C of policyholders over a planning horizon consisting of a set N of years. Let H be the set of catastrophe realizations (e.g., a hurricane season) that can occur in any year $n \in N$. Every realization $h \in H$ is divided into a set T of time periods. Let E_t^h be the set of disaster events that occur in period $t \in T$ of realization $h \in H$. Each event $e \in E_t^h$ is characterized by its severity level $\rho \in P$ and the set of policyholders $C^e \subseteq C$ it affects. Under an ex ante parametric indemnification policy, a loss value is attributed to each $e \in E_t^h$ according to its severity level. Let d_{ct}^h be the insurance claim of $c \in C$ received at the beginning of $t \in T$ for $h \in H$. The parameter d_{ct}^h is uncertain since it depends on h and is realized at the beginning of $t \in T$. It corresponds to the aggregate loss value of all $e \in E_t^h$ that affect $c \in C$.

We define a scenario $s \in S$ as a sequence of $|N|$ realizations $h \in H$. There are $|S| = |H|^{|N|}$ possible scenarios each with a probability p^s. We use a scenario tree representation for the set S with $|N|+1$ stages (see Fig. 3). Stage 0 is the investment stage where the prepositioning network is established, and the $|N|$ subsequent stages correspond to the years under which the framework is operational. Let L be the set of nodes in the scenario tree, and L_n those in stage n. Then $|L_0| = 1, |L_n| = |H|^n$, and $|L| = 1 + \sum_{n=1}^{|N|} |H|^n$. For a given $s \in S$, we write $\phi(s,n) = h$ to indicate that $h \in H$ occurs in year n. We denote by $\phi(s,j), j = 1,...,n$ the set of realizations that occur in s between years 1 and n. Finally, we use the notation $\theta(l,j)$ to indicate the predecessor node $v \in L_j$ in the tree of $l \in L_n, 0 \leq j \leq n \leq |N|$. Note that $\theta(l,n) = l \in L_n$. For ease of notation, we write $l \in L_n$ as also being the realization $h \in H$ occurring in the tree at node $l \in L_n$, and we write $\phi(s,n) = l$ when l corresponds to h. We also sometimes replace, where appropriate, the sn index in the parameters and variables by h when $\phi(s,n) = h$, and conversely.

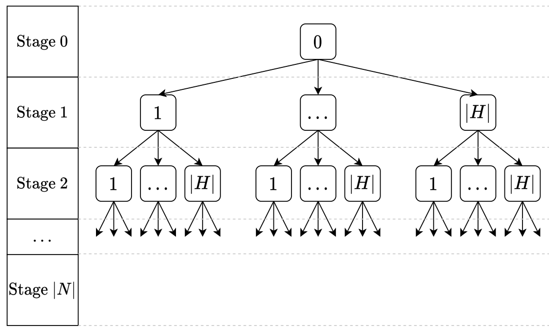

Figure 3 Scenario tree for $|N|+1$ stages and $|H|$ possible catastrophe realizations.

3.2. Modelling components

We now show how we model the framework's four components, and we explain how they interact.

3.2.1. Insurance coverage design Let V_c be the coverage deductible variable of $c \in C$, X_c its coverage limit variable, and G_{ct}^{sn} its claim coverage variable, $t \in T, n \in N, s \in S$. The values of V_c and X_c are determined at stage 0 and are unchanged over the planning horizon. We use a slightly altered version of the optimal insurance contract theory with a deductible and limit of Cummins and Mahul (2004) to design the insurance contracts. The optimal insurance contract contains three coverage states given by the non-decreasing piecewise linear function in Eq. (1). The contract provides no coverage when $d_{ct}^{sn} \leq V_c$, partial coverage when $d_{ct}^{sn} \geq X_c$, and full coverage when $d_{ct}^{sn} \in]V_c, X_c[$, unlike in Cummins and Mahul (2004) where V_c is subtracted from d_{ct}^{sn}. The portion of each claim that is not covered by the insurance corresponds to $d_{ct}^{sn} - G_{ct}^{sn}$, and is outsourced at a cost o_c.

$$G_{ct}^{sn} = \begin{cases} 0 & d_{ct}^{sn} \leq V_c \\ d_{ct}^{sn} & V_c < d_{ct}^{sn} < X_c \\ X_c & d_{ct}^{sn} \geq X_c. \end{cases} \tag{1}$$

We define two binary variables σ_{ct}^{sn} and δ_{ct}^{sn} for each G_{ct}^{sn} variable to linearize the piecewise Eq. (1): $\sigma_{ct}^{sn} = 1, \delta_{ct}^{sn} = 1$ correspond to the case $d_{ct}^{sn} \leq V_c$, $\sigma_{ct}^{sn} = 0, \delta_{ct}^{sn} = 0$ to $d_{ct}^{sn} \geq X_c$, and $\sigma_{ct}^{sn} = 0, \delta_{ct}^{sn} = 1$ to $V_c < d_{ct}^{sn} < X_c$. Consequently, the linearization constraints corresponding to Eq. (1) are as follows, where M is a large enough number:

$$V_c \leq X_c \quad c \in C \tag{2}$$

$$V_c - d_{ct}^{sn} \leq M\sigma_{ct}^{sn} \quad c \in C, t \in T, n \in N, s \in S \tag{3}$$

$$d_{ct}^{sn} - V_c + 1 \leq M\left(1 - \sigma_{ct}^{sn}\right) \quad c \in C, t \in T, n \in N, s \in S \tag{4}$$

$$G_{ct}^{sn} \leq d_{ct}^{sn}\left(1 - \sigma_{ct}^{sn}\right) \quad c \in C, t \in T, n \in N, s \in S \tag{5}$$

$$G_{ct}^{sn} \leq X_c \quad c \in C, t \in T, n \in N, s \in S \tag{6}$$

$$G_{ct}^{sn} \geq X_c - M\delta_{ct}^{sn} \quad c \in C, t \in T, n \in N, s \in S \tag{7}$$

$$G_{ct}^{sn} \geq d_{ct}^{sn}(1-\sigma_{ct}^{sn}) - M(1-\delta_{ct}^{sn}) \quad c \in C, t \in T, n \in N, s \in S. \tag{8}$$

3.2.2. Prepositioning network design The prepositioning network is designed based on the coverage given by the G_{ct}^{sn} variables. Let K be the set of potential strategic locations in the network where a number of storage facilities can be operated, a the capacity of a facility, and I_k and U_k the stage 0 investment variables corresponding to the inventory level, and number of storage facilities operated at $k \in K$, respectively. Moreover, let τ be the replenishment lead time, Q_{ctk}^{sn} the scenario-dependent variable indicating the amount of supply units sent from $k \in K$ to $c \in C$ in $t \in T, s \in S, n \in N$, A_{tk}^{sn} the beginning inventory variable at $t \in T$, and R_{tk}^{sn} the variable for the amount of units arriving in replenishment to the location at the beginning of $t \in T$. Then

$$G_{ct}^{sn} = \sum_{k \in K} Q_{ctk}^{sn} \quad c \in C, t \in T, n \in N, s \in S. \tag{9}$$

Constraints (9) link the prepositioning network component to the rest of the framework by specifying the amount of coverage provided to policyholders from each location in each scenario. In order to ensure that I_k at each location is sufficient to provide the coverage in any $s \in S, n \in N$, constraints (10) state that the starting inventory A_{1k}^{sn} is equal to I_k. Constraints (11)–(12) enforce the proper balance of the inventory between time periods, while constraints (13)–(14) ensure that what is used is replenished a lead time later, and that no replenishment takes place in the first τ periods. Finally, constraints (15) determine the number of storage facilities needed at each location:

$$A_{1k}^{sn} = I_k \quad k \in K, n \in N, s \in S \tag{10}$$

$$\sum_{c \in C} Q_{ctk}^{sn} \leq A_{tk}^{sn} \quad k \in K, n \in N, s \in S, t \in T \tag{11}$$

$$A_{t+1,k}^{sn} = A_{tk}^{sn} - \sum_{c \in C} Q_{ctk}^{sn} + R_{t+1,k}^{sn} \quad k \in K, n \in N, s \in S, t = 1, \ldots, |T|-1 \tag{12}$$

$$R_{tk}^{sn} = \sum_{c \in C} Q_{c,t-\tau-1,k}^{sn} \quad k \in K, n \in N, s \in S, t = \tau+2, ..., |T| \tag{13}$$

$$R_{tk}^{sn} = 0 \quad k \in K, n \in N, s \in S, t = 1, ..., \tau \tag{14}$$

$$I_k \leq aU_k \quad k \in K. \tag{15}$$

Constraints (10)–(15) extend the prepositioning model of Balcik et al. (2019) to a multi-year setting. Note that in the case of catastrophes where replenishment between events during a realization is not possible or is irrelevant to the operation, the model can easily be adapted by setting $\tau = \infty$. The current framework can also be modified to accommodate various settings, such as the possibility of damaged supplies, and varying emergency response levels.

3.2.3. Management of surplus capital The insurer must satisfy three financial solvency conditions to be able to provide insurance coverage to all policyholders under any scenario after having paid the prepositioning expenses. Prepositioning network expenses are scenario-independent and are paid upfront by the insurer at the beginning of each year $n \in N$. For each $k \in K$, f_k is the yearly fixed cost of operating a storage facility, r_k the procurement cost of a unit of supplies for prepositioning, and h_k the yearly cost of holding a unit in inventory. The total network expenses are $\sum_{k\in K}(f_kU_k + r_kI_k + h_kI_k)$ for $n = 1$, and $\sum_{k\in K}(f_kU_k + h_kI_k)$ for $n > 1$. Let g_{ck} be the logistical coverage cost of transporting one unit of supplies from $k \in K$ to $c \in C$. The total capital consumption of $c \in C$ in emergency response at any $n \in N, s \in S$ corresponds to $\sum_{k\in K}\sum_{t\in T}(g_{ck} + r_k)Q^{sn}_{ctk}$, which includes the coverage cost, and the replenishment cost of the coverage utilized.

Solvency Condition 1. The insurer should have sufficient surplus capital left in the aggregate capital pool to be able to cover the capital consumption in emergency response of the worst-case coverage realization.

The worst-case coverage realization is the realization with the largest total capital consumption in emergency response: $\max_{s\in S, n\in N}\{\sum_{k\in K}\sum_{t\in T}\sum_{c\in C}(r_k + g_{ck})Q^{sn}_{ctk}\}$. It is endogenously determined among the realizations by setting coverage deductibles and limits accordingly, which is not to be confused with the worst-case aggregate claim realization $\max_{s\in S, n\in N}\{\sum_{k\in K}\sum_{t\in T}\sum_{c\in C}(r_k + g_{ck})d^{sn}_{ct}\}$. Let B be the surplus capital variable required to cover this worst-case coverage realization. Then

$$B \geq \sum_{k\in K}\sum_{t\in T}\sum_{c\in C}(r_k + g_{ck})Q^{sn}_{ctk} \quad n \in N, s \in S. \tag{16}$$

Ensuring that the insurer remains solvent under any scenario is synonymous with a zero probability of ruin. Therefore, we use a similar cashflow balance equation as the Cramér-Lundberg model (Schmidli 2017) to compute the available capital for each $t \in T, n \in N, s \in S$ and ensure that it never drops below zero. For a given $s \in S, n \in N$, let Y^{sn}_c be the collected premium variable from $c \in C$ at the beginning of the year, BL^{sn} the surplus capital available at the beginning of the year, and EL^{sn} the capital remaining at the end of the year. These values can be calculated as follows for each $n \in N$. Note that $BL^{s1} = \sum_{c\in C}Y^{s1}_c - \sum_{k\in K}(r_kI_k + h_kI_k + f_kU_k)$ and $EL^{sn} = BL^{sn} - \sum_{k\in K}\sum_{t\in T}\sum_{c\in C}(r_k + g_{ck})Q^{sn}_{ctk}$. Moreover, we assume that $EL^{s,n-1}$ in year $n-1$ depreciates in value by a discount rate γ when carried to year n, with $BL^{sn} = EL^{s,n-1}/(1+\gamma) + \sum_{c\in C}Y^{sn}_c - \sum_{k\in K}(h_kI_k + f_kU_k)$:

$$\begin{aligned} BL^{sn} = &\sum_{j=1}^{n}\frac{\sum_{c\in C}Y^{sj}_c}{(1+\gamma)^{n-j}} - \sum_{j=1}^{n-1}\frac{\sum_{k\in K}\sum_{c\in C}\sum_{t\in T}(r_k + g_{ck})Q^{sj}_{ctk}}{(1+\gamma)^{n-j}} \\ &- \sum_{j=1}^{n}\frac{\sum_{k\in K}(h_kI_k + f_kU_k)}{(1+\gamma)^{n-j}} - \frac{\sum_{k\in K}r_kI_k}{(1+\gamma)^{n-1}} \qquad n \in N, s \in S \end{aligned} \tag{17}$$

$$\begin{aligned} EL^{sn} = & \sum_{j=1}^{n} \frac{\sum_{c\in C} Y_c^{sj}}{(1+\gamma)^{n-j}} - \sum_{j=1}^{n} \frac{\sum_{k\in K}\sum_{c\in C}\sum_{t\in T}(r_k+g_{ck})Q_{ctk}^{sj}}{(1+\gamma)^{n-j}} \\ & - \sum_{j=1}^{n} \frac{\sum_{k\in K}(h_k I_k + f_k U_k)}{(1+\gamma)^{n-j}} - \frac{\sum_{k\in K} r_k I_k}{(1+\gamma)^{n-1}} \qquad n\in N, s\in S. \end{aligned} \tag{18}$$

Solvency Condition 2. A zero probability of ruin is obtained by ensuring that the remaining surplus capital EL^{sn} at the end of each year $n\in N$ in each scenario $s\in S$ is non-negative. That is

$$EL^{sn} \geq 0 \quad n\in N, s\in S. \tag{19}$$

However, we cannot directly add constraints (19) to the multi-stage stochastic model since these breach its non-anticipativity conditions (Sec. 3.3). This is because, when collecting $\sum_{c\in C} Y_c^{sn}$ at the beginning of $n\in N$, the uncertain claims d_{ct}^{sn} have not yet been realized, and hence the capital consumption in emergency response $\sum_{k\in K}\sum_{t\in T}\sum_{c\in C}(r_k+g_{ck})Q_{ctk}^{sn}$ for that year is not yet known.

Solvency Condition 3. To respect the non-anticipativity conditions and meet the second condition, $BL^{sn}, n\in N, s\in S$ should be able to cover the worst-case coverage realization:

$$BL^{sn} \geq B \quad n\in N, s\in S. \tag{20}$$

3.2.4. Premium allocation The policyholders' contributions to the aggregate capital pool $\sum_{c\in C} Y_c^{sn}, n\in N, s\in S$ are in the form of a premium collected at the beginning of the year. We allocate the aggregate capital pool into individual premiums using a top-down premium calculation method (Bühlmann 1985) as follows. Let $\mathbb{E}[\mathbf{G}_c^n]$ be the expected insurance coverage of $c\in C$ in $n\in N$ over all scenarios, and $\mathbb{S}[\mathbf{G}_c^n]$ the standard deviation, $\mathbf{G}_c^n$ being the vector of variables G_{ct}^{sn}. The risk profile of $c\in C$ corresponds to the values of $\mathbb{E}[\mathbf{G}_c^n], \mathbb{S}[\mathbf{G}_c^n], n\in N$, and to its average logistical cost b_c of providing it with a unit of supplies in insurance coverage from the prepositioning network, where $b_c = \sum_{k\in K}(r_k+h_k+g_{ck})/|K|$. Moreover, let Z^{sn} be the safety margin variable of the standard deviation premium setting principle applied to $s\in S, n\in N$. The premiums Y_c^{sn} are calculated as in constraints (21) following the standard deviation premium setting principle (Deelstra and Plantin 2014). By this principle, the premium of each policyholder is calculated as a linear combination of the value of its expected coverage and of its standard deviation. The aggregate capital pool then corresponds to $\sum_{c\in C} Y_c^{sn}$. When $Y_c^{sn} = b_c\mathbb{E}[\mathbf{G}_c^n]$, we refer to the term as the pure premium:

$$Y_c^{sn} = b_c\mathbb{E}[\mathbf{G}_c^n] + b_c\mathbb{S}[\mathbf{G}_c^n] Z^{sn} \quad c\in C, n\in N, s\in S, \tag{21}$$

where $\mathbb{E}[\mathbf{G}_c^n] = \sum_{s\in S} p^s \sum_{t\in T} G_{ct}^{sn}$ and $\mathbb{S}[\mathbf{G}_c^n] = \sqrt{\mathbb{E}\left[(\mathbf{G}_c^n)^2\right] - (\mathbb{E}[\mathbf{G}_c^n])^2} \quad c\in C, n\in N.$ (22)

Note that even under the integration of the four components of the framework, the fairness in premium allocation, given by the Z^{sn} variables, still holds (see the proof of Proposition 3 in Appendix C). Also note that unlike in the model of Balcik et al. (2019) where insurance coverage is an input parameter, in our model it is a decision variable which has a major impact on the solution methodology since it renders the premium allocation constraints (21) non-linear due to the terms involving $\mathbb{S}[\mathbf{G}_c^n]$ and $\mathbb{S}[\mathbf{G}_c^n] Z^{sn}$. We will handle these non-linearities in the solution strategy.

3.2.5. Integrated relationship between the framework's components The purpose of this section is to show how the variables of the four framework components are directly related and affect each other.

The role of coverage deductibles and limits is to maximize the prepositioning network's utilization and its risk pooling benefits. Here we derive a direct relationship between the variables of the four framework components, by defining the policyholders' risk profiles (i.e., $\mathbb{E}[\mathbf{G}_c^n]$ and $\mathbb{S}[\mathbf{G}_c^n]$) as a function of their deductible and limit, and linking them with the prepositioning and surplus capital variables in the solvency constraints (20) through the premiums. Recall the three coverage states in Eq. (1) in Sec. 3.2.1. For a given $c \in C, h \in H$, we define ψ_{hc}^1, ψ_{hc}^2, and ψ_{hc}^3 as the set of all time periods corresponding to each coverage state, i.e., where $d_{ct}^h \leq V_c$, $V_c < d_{ct}^h < X_c$, and $X_c \leq d_{ct}^h$, respectively, such that $|\psi_{hc}^1| + |\psi_{hc}^2| + |\psi_{hc}^3| = |T|$. $|T| = |\psi_{hc}^2| + |\psi_{hc}^3|$ when $V_c = 0$, $|T| = |\psi_{hc}^1| + |\psi_{hc}^2|$ when $X_c = \max_{h \in H}\{d_{ct}^h\}$, and $|T| = |\psi_{hc}^2|$ when $V_c = 0$ and $X_c = \max_{h \in H}\{d_{ct}^h\}$. The total insurance coverage over all periods $t \in T$ of realization $h \in H$ is then

$$\sum_{t\in\psi_{hc}^1} G_{ct}^h + \sum_{t\in\psi_{hc}^2} G_{ct}^h + \sum_{t\in\psi_{hc}^3} G_{ct}^h = 0 + \sum_{t\in\psi_{hc}^2} d_{ct}^h + \sum_{t\in\psi_{hc}^3} X_c. \tag{23}$$

Similarly, we derive $\mathbb{E}[\mathbf{G}_c^n]$ and $\mathbb{S}[\mathbf{G}_c^n]$ as follows, using for simplicity the index $h \in H$ instead of $sn, s \in S, n \in N$ with $h = \phi(s,n)$:

$$\mathbb{E}[\mathbf{G}_c^n] = \sum_{h\in H} p^h \left(\sum_{t\in\psi_{hc}^1} G_{ct}^h + \sum_{t\in\psi_{hc}^2} G_{ct}^h + \sum_{t\in\psi_{hc}^3} G_{ct}^h \right) = \sum_{h\in H} p^h \left(\sum_{t\in\psi_{hc}^2} d_{ct}^h + \sum_{t\in\psi_{hc}^3} X_c \right) \tag{24}$$

$$\text{and } \mathbb{S}[\mathbf{G}_c^n] = \sqrt{\sum_{h\in H} p^h \left(\sum_{t\in\psi_{hc}^2} (d_{ct}^h)^2 + \sum_{t\in\psi_{hc}^3} (X_c)^2 \right) - \left(\sum_{h\in H} p^h \left(\sum_{t\in\psi_{hc}^2} d_{ct}^h + \sum_{t\in\psi_{hc}^3} X_c \right) \right)^2}. \tag{25}$$

Given that for any $s \in S, n \in N$, $\sum_{c\in C} Y_c^{sn} = \sum_{c\in C} b_c (\mathbb{E}[\mathbf{G}_c^n] + Z^{sn}\mathbb{S}[\mathbf{G}_c^n])$, replacing $\sum_{c\in C} Y_c^{sn}$ with $\sum_{c\in C} b_c (\mathbb{E}[\mathbf{G}_c^n] + Z^{sn}\mathbb{S}[\mathbf{G}_c^n])$ in constraints (20), and $\mathbb{E}[\mathbf{G}_c^n]$ and $\mathbb{S}[\mathbf{G}_c^n]$ by Eqs. (24) and (25), yields a direct relationship between the deductible V_c, the limit X_c, the prepositioning variables I_k and U_k, the worst-case surplus capital B, and the capital consumption variables Q_{ctk}^{sn}. If the insurer wants to alter the network size, the worst-case capital surplus, or the capital consumption in response, it does so by updating insurance contracts to provide altered coverage. The coverage G_{ct}^{sn} is altered by varying V_c and X_c so that the sets ψ_{hc}^1, ψ_{hc}^2, and ψ_{hc}^3 change sizes.

3.3. Multi-stage stochastic programming model

Given the four modeling components, we now show how they can be integrated together in a multi-stage stochastic program for the CIP-PSR . Fig. 4 presents the model's dynamics, the realization of uncertainty parameters, and the interaction between the variables. The objective of the model is to minimize the expected total cost, divided between premiums and outsourcing.

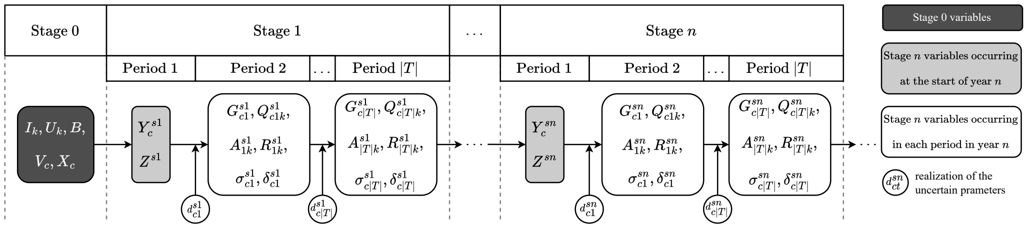

Figure 4 Model dynamics.

Our multi-stage stochastic model needs to respect the well-known non-anticipativity principle of stochastic programs (Rockafellar and Wets 1991), which ensures that the solution in each $l \in L_n$ at stage $n \in N$ in the scenario tree depends solely on variables and uncertain parameters that have already been realized, as applied for example in Solvency Condition 3 (Sec. 3.2.3). This principle also ensures that two scenarios $s, s' \in S$ that are undifferentiable in the tree up to stage $n \in N$ (i.e., sharing the same nodes $l \in L_j, j = 1, ..., n$) have the same solution for all their stage 0 to n variables.

Our model includes a different non-anticipativity condition for two subsets of the scenario-dependent variables (see Fig. 4). The first non-anticipativity condition (27) relates to the time period variables G^{sn}_{ct}, Q^{sn}_{ctk}, A^{sn}_{tk}, R^{sn}_{tk}, σ^{sn}_{ct} and δ^{sn}_{ct}. Two scenarios $s, s' \in S$ sharing the same nodes $\phi(s,j) = \phi(s',j), j = 1, ..., n$ in the tree are undifferentiable for these variables up to stage n. For example, $G^{sn}_{ct} = G^{s'n}_{ct}, c \in C, t \in T, n \in N, s, s' \in S, \phi(s,j) = \phi(s',j), j = 1, ..., n$. However, the premium allocation variables Y^{sn}_c and Z^{sn} are realized at the start of year $n \in N$ and depend solely on the variables and uncertain parameters realized in stages $0, ..., n-1$, but not on the information realized in stage n. Their non-anticipativity condition (28) for $s, s' \in S$ sharing the same nodes $\phi(s,j) = \phi(s',j), j = 0, ..., n$ is therefore only up to stage $n-1$ and not n, e.g. , $Y^{sn}_c = Y^{s'n}_c, c \in C, t \in T, n \in N, s, s' \in S, \phi(s,j) = \phi(s',j), j = 0, ..., n-1$. The complete model is as follows:

$$\text{minimize} \sum_{s\in S} p^s \sum_{n\in N} \left[\sum_{c\in C} Y^{sn}_c + \sum_{c\in C} o_c \sum_{t\in T} (d^{sn}_{ct} - G^{sn}_{ct}) \right] \tag{26}$$

subject to

$$(2)\text{–}(8),\ (9)\text{–}(15),\ (16),\ (20),\ (21)\text{–}(22)$$

$$\begin{aligned} &G^{sn}_{ct} = G^{s'n}_{ct},\ \sigma^{sn}_{ct} = \sigma^{s'n}_{ct},\ \delta^{sn}_{ct} = \delta^{s'n}_{ct}, \\ &Q^{sn}_{ctk} = Q^{s'n}_{ctk},\ A^{sn}_{tk} = A^{s'n}_{tk},\ R^{sn}_{tk} = R^{s'n}_{tk} \quad c \in C, t \in T, n \in N, s, s' \in S, \phi(s,j) = \phi(s',j), j = 1, ..., n, \end{aligned} \tag{27}$$

$$Y^{sn}_c = Y^{s'n}_c,\ Z^{sn} = Z^{s'n} \quad c \in C, n \in N \setminus \{1\}, s, s' \in S, \phi(s,j) = \phi(s',j), j = 1, ..., n-1. \tag{28}$$

$$\begin{aligned} &I_k, U_k \in \mathbb{Z}^+\ k \in K; \qquad G^{sn}_{ct}, Q^{sn}_{ctk} \in \mathbb{Z}^+\ n \in N, s \in S, t \in T, c \in C; \qquad B \geq 0; \\ &R^{sn}_{tk}, A^{sn}_{tk} \in \mathbb{Z}^+\ k \in K, n \in N, s \in S, t \in T; \qquad V_c, X_c \in \mathbb{Z}^+,\ 0 \leq V_c \leq X_c \leq \bar{X},\ c \in C; \end{aligned} \tag{29}$$

$$Y^{sn}_c \geq 0\ n \in N, s \in S, c \in C; \quad Z^{sn} \geq 0\ n \in N, s \in S. \tag{30}$$

The objective function (26) minimizes the expected cost of covering all claims over the planning horizon, divided between premiums and cost of outsourcing. Constraints (2)–(8) are the insurance coverage constraints, (9)–(15) the prepositioning network constraints, (16) and (20) the management of capital surplus constraints, (21)–(22) the premium allocation constraints, (27)–(28) the non-anticipativity constraints, while (29)–(30) define the domains of the variables.

3.3.1. Model properties Here we present some analytical properties of the multi-stage stochastic model for the CIP-PSR whose implications are beneficial for the implementation of our Benders decomposition algorithm. The proofs can be found in Appendix C.

PROPOSITION 1. *If* $\phi(s,n) = \phi(s',n'), n, n' \in N, s, s' \in S, s \neq s'$, *then* $G_{ct}^{sn} = G_{ct}^{s'n'}, c \in C, t \in T$.

Then, the non-anticipativity constraints (27) defined $\forall c \in C, t \in T, n \in N, s, s' \in S, \phi(s,j) = \phi(s',j), j = 1, ..., n$ can be redefined $\forall c \in C, t \in T, n, n' \in N, s, s' \in S, \phi(s,n) = \phi(s',n')$.

PROPOSITION 2. *For every* $s \in S, n \in N$, $BL^{sn} > B$ *when* $Z^{sn} = 0$ *and* $\sum_{c \in C} Y_c^{sn} = \sum_{c \in C} b_c \mathbb{E}[\boldsymbol{G}_c^n]$, *and* $BL^{sn} = B$ *when* $Z^{sn} > 0$ *and* $\sum_{c \in C} Y_c^{sn} > \sum_{c \in C} b_c \mathbb{E}[\boldsymbol{G}_c^n]$.

PROPOSITION 3. *The safety margin variables* Z^{sn} *ensure a fair allocation of the surplus capital pool among policyholders in each year* $n \in N$ *and each scenario* $s \in S$.

PROPOSITION 4. *Every scenario* $s \in S$ *is feasible in the scenario tree at every stage* $n = 1, ..., |N|$.

PROPOSITION 5. *The inventory level* I_k *at each location* $k \in K$ *has to respect the minimum required inventory level such that*

$$I_k \geq \max_{h \in H} \left\{ \max_{t \in T, t \geq \tau} \left\{ \sum_{c \in C} \sum_{i=t-\tau}^{t} Q_{cik}^h \right\} \right\}.$$

3.4. End of horizon

Our framework's contracts are binding over the $|N|$ years planning horizon. At its onset, it can be easily integrated in a rolling-horizon scheme to allow its renewal. At the end of the horizon, the remaining capital pool $EL^{s|N|}$ can be distributed among the policyholders using the premium setting principle ensuring that $EL^{s|N|} = \sum_{c \in C} b_c \mathbb{E}\left[\mathbf{G}_c^{|N|}\right] + \sum_{c \in C} b_c \mathbb{S}\left[\mathbf{G}_c^{|N|}\right] Z^{s|N|}$. At this point, the policyholders who may wish to quit the mutual insurance can do so, and new policyholders can be admitted. Moreover, the set of catastrophe realizations is updated with the new realizations realized during the $|N|$ years. As for the existing prepositioning network and the remaining strategic reserves at each location, these can be fixed in the model by adding the constraint $\overline{U_k} \leq U_k$ where $\overline{U_k}$ is the current number of warehouses open in location $k \in K$, and updating constraint (10) for year $n = 1$ to $A_{1k}^{s1} = I_k + \overline{I_k}$, where $\overline{I_k}$ is the remaining strategic reserves at each location.

4. Solution strategy

The multi-stage stochastic program presented in Sec. 3.3 is of very large-scale in the number of scenarios, and non-linear in some constraints, making it extremely hard to solve. The inherent challenge of solving multi-stage stochastic programs is the large size of the search space of integer variables coupled with the non-anticipativity constraints. To tackle these issues, we leverage the properties derived in Sec. 3.3.1 to develop an exact solution approach based on Benders decomposition.

We decompose the model into three phases (Fig. 5), where the operational components (insurance coverage design and prepositioning network design) are in the master problem and the financial components (management of surplus capital and premium allocation) are in the subproblem. First, the proposed decomposition is effective since the master problem phase contains all the integer variables in such a way that the non-anticipativity constraints are redundant for the constraints in the master problem. This transforms the master problem into a two-stage stochastic program of the order of the number of catastrophe realizations $|H|$ instead of the number of scenarios $|S| = |H|^{|N|}$. Second, the decomposition moves the non-anticipativity constraints to the subproblems phase, where only continuous variables exist. The challenge of the subproblem phase becomes restricted to solving a large-scale LP of the order of $|H|^{|N|-1}$ instead of $|H|^{|N|}$. To this end, different efficiency and speed-up strategies are employed. Instead of solving the full large-scale subproblem, a depth-first search of the scenario tree is employed where a subproblem is solved at each node of the tree, and an efficient index mapping is used to navigate the tree. Moreover, Proposition 4 guarantees that only optimality cuts and no feasibility cuts are generated by the subproblems, resulting in a faster convergence to the optimal solution. Since each optimality cut is prohibitively large in the number of variables it contains, an efficient memory management procedure is implemented to ensure the program does not run out of memory while computing the cut. Most importantly, given Proposition 2, a node reduction procedure is devised to ensure that the depth-first search only solves the LP subproblem at nodes whose aggregate premiums are larger than the pure premiums. Third, the intermediate phase between the master and subproblem phases ensures that the non-linearity is eliminated in the terms $\mathbb{S}[\mathbf{G}_c^n]$ and $\mathbb{S}[\mathbf{G}_c^n]Z^{sn}$ by first computing the G_{ct}^{sn} variables in the master problem, and then calculating $\mathbb{E}[\mathbf{G}_c^n]$ and $\mathbb{S}[\mathbf{G}_c^n]$, which are now constants. Finally, we note that the decomposition of the components of the framework where the operational components are in the master problem phase and the financial components in the subproblem phase makes our solution strategy easily generalizable to other disaster management contexts by simply adapting the prepositioning network design constraints in the master problem to the new context, and then changing the right-hand side of solvency constraints in the subproblem accordingly. However, since the right-hand side of the solvency conditions is a constant, the subproblem phase, which is the most challenging part of the solution strategy, with all its speed ups and node reduction procedure, remains directly applicable as is.

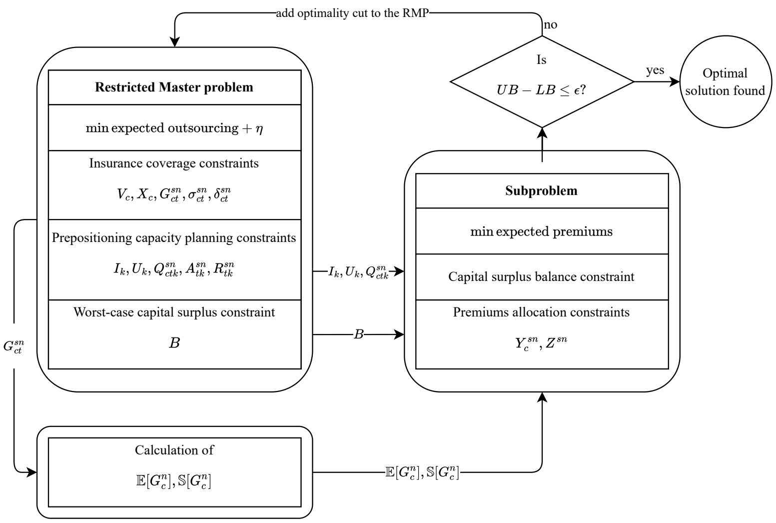

Figure 5 Flowchart of the Benders decomposition algorithm.

4.1. Restricted Master Problem phase

Let Λ be the set of optimality cuts generated at each iteration of the subproblem phase, and η the dummy variable associated with the cuts. The restricted master problem (RMP) is then

$$\text{minimize} \sum_{s\in S} p^s \sum_{n\in N}\sum_{c\in C} o_c \sum_{t\in T} (d^{sn}_{ct} - G^{sn}_{ct}) + \eta \tag{31}$$

subject to

$$(2)\text{–}(8),\ (9)\text{–}(15),\ (16),\ (27),\ (29)$$

$$\text{a set } \Lambda \text{ of optimality cuts.} \tag{32}$$

Proposition 6. *Based on the relaxed non-anticipativity constraints from Proposition 1, the RMP reduces to a two-stage stochastic program by reformulating the objective function and replacing the sn index with h in the objective, variables, and constraints.*

The non-anticipativity constraints (27) then become redundant and can be removed. The objective function is then

$$\text{minimize} \sum_{n\in N}\sum_{h\in H} p^{hn} \sum_{c\in C} o_c \sum_{t\in T} \left(d^h_{ct} - G^h_{ct}\right) + \eta \tag{33}$$

with $p^{hn} = \sum_{s\in S} p^{\phi(s,n)}$, $\phi(s,n) = h \in H, n \in N$. To speed up the reduced RMP, we eliminate the G^{sn}_{ct} variables and replace them with $\sum_{k\in K} Q^{sn}_{ctk}$ as given in constraints (9). Moreover, the valid inequalities (34)–(36) are added to the RMP. Constraints (34) ensure that if the inventory level is zero at a location, then no facilities are open, (35) correspond to Proposition 5, and (36) cap the total number of needed facilities by computing the maximal aggregate insurance claims within a replenishment period and dividing it by a facility's capacity. Finally, the first iteration of the RMP without any optimality cuts results in an optimal objective function value of 0 where $\sum_{k\in K} Q^h_{ctk} = d^h_{ct}$. We add the constraint $\sum_{k\in K} Q^h_{ctk} = d^h_{ct}, c\in C, t\in T, h\in H$ to speed up the first iteration of the RMP, and remove it in subsequent iterations:

$$U_k \le I_k \qquad k\in K \tag{34}$$

$$I_k \ge \max_{h\in H}\left\{\max_{t\in T, t\ge\tau}\left\{\sum_{c\in C}\sum_{i=t-\tau}^{t} Q^h_{cik}\right\}\right\} \qquad k\in K \tag{35}$$

$$\sum_{k\in K} U_k \le \max_{h\in H}\left\{\max_{t\in T, t\ge\tau}\left\{\left\lceil \frac{\sum_{c\in C}\sum_{i=t-\tau}^{t} d^h_{ci}}{a}\right\rceil\right\}\right\}. \tag{36}$$

4.2. Subproblem phase

Given the solution vector $(I_k, U_k, Q^{sn}_{ctk}, B, k\in K, c\in C, t\in T, n\in N, s\in S)$ obtained from the RMP, and the calculated values of $\mathbb{E}[\mathbf{G}_c]$ and $\mathbb{S}[\mathbf{G}_c]$, the subproblem is written in terms of the variables Y^{sn}_c and Z^{sn}:

$$\text{minimize} \sum_{s\in S} p^s \sum_{n\in N}\sum_{c\in C} Y^{sn}_c \tag{37}$$

subject to (20), (21)–(22), (28), (30).

From Proposition 4, the subproblem is feasible $\forall n\in N, s\in S$, and is of very large-scale in the numbers $(|C|+1)|H|^{|N|}$ of variables and $(|C|+1)|N||H|^{|N|}$ of constraints. Solving it without decomposing it is computationally prohibitive. Recall that the non-anticipativity conditions (28) of Y^{sn}_c and Z^{sn} are up to stage $n-1$, i.e., there are $\sum_{n=0}^{|N|-1}|L_n| = \sum_{n=0}^{|N|-1}|H|^n$ distinct nodes in the scenario tree between stages 0 and $|N|-1$ where Y^{sn}_c and Z^{sn} are differentiable. Given that $\sum_{n=0}^{|N|-1}|H|^n < |H|^{|N|}$, a decomposition of the subproblem by nodes $l\in L_n, n=0,...,|N|-1$ instead of scenarios S results in solving $\sum_{n=0}^{|N|-1}|H|^n$ subproblems with $(|C|+1)$ variables and $(|C|+1)|N|$ constraints each, instead of $|H|^{|N|}$ subproblems. For example, with $|C|=10$, $|N|=4$ and $|H|=50$, $|S|=$6,250,000, while $\sum_{n=0}^{3}|H|^n=$127,550, i.e., only 2% of $|S|$. Model (38)–(42) corresponds to the subproblem at $l\in L_0,...,L_{|N|-1}$. Recall the use of the notation Y^l_c and Z^l to denote $Y^{sn}_c, Z^{sn}, n\in N, s\in S, \phi(s,n)=l\in L_{n-1}$, and $\theta(l,j), j<n-1$ to denote the predecessor of node $l\in L_{n-1}$ at stage j. The only variables included in (38)–(42) are Y^l_c and Z^l, with the values of $Y^{\theta(l,j)}_c, j<n-1$ being assumed as having been computed in the subproblem of the predecessors $\theta(l,j)$ of $l\in L_{n-1}$:

$$\text{minimize } p^l Y^l_c \tag{38}$$

subject to

$$\sum_{c\in C} Y^l_c \ge \overline{B} + \sum_{j=1}^{n}\frac{\sum_{k\in K}\left(f_k\overline{U_k}+h_k\overline{I_k}\right)}{(1+\gamma)^{n-j}} + \frac{\sum_{k\in K} r_k\overline{I_k}}{(1+\gamma)^{n-1}} + \sum_{j=1}^{n-1}\frac{\sum_{k\in K}\sum_{t\in T}\sum_{c\in C}(r_k+g_{ck})\overline{Q^{\theta(l,j)}_{ctk}}}{(1+\gamma)^{n-j}} - \sum_{j=0}^{n-2}\frac{\sum_{c\in C}\overline{Y^{\theta(l,j)}_c}}{(1+\gamma)^{n-j+1}} \tag{39}$$

$$b_c\overline{\mathbb{S}[\mathbf{G}^n_c]}Z^l - Y^l_c = -b_c\overline{\mathbb{E}[\mathbf{G}^n_c]} \qquad c\in C \tag{40}$$

$$Y^l_c \ge 0 \qquad c\in C \tag{41}$$

$$Z^l \ge 0. \tag{42}$$

4.2.1. Speed-ups for the subproblem phase In order to efficiently run through each node $l \in L_n, n = 0, ..., |N| - 1$, we use a depth-first search (DFS) of the tree, requiring at each $l \in L_n$ to only keep track of the current value of $\overline{Y_c^{\theta(l,j)}}, j = 0, ..., n-1$, i.e., up to $|C|(|N|-1)$ values. Moreover, since there are $|H|^n$ nodes at stage $n \in N$, to efficiently navigate through the tree, we use an index mapping given by the mathematical relation $\theta(l,j) = \lfloor l \mod |H|^j \rfloor$ to obtain the index of the predecessor node $\theta(l,j) \in L_j$, and the relation $\theta(l,j) = \left\lfloor l/|H|^{j-1} \mod |H| \right\rfloor$ to obtain the index of the catastrophe realization $h \in H$ occurring in node $\theta(l,j) \in L_j$. As a further speed-up, we implement a node reduction procedure using Proposition 2. Recall that (39) is satisfied as an equality when $\sum_{c \in C} Y_c^l > \sum_{c \in C} b_c \mathbb{E}[\mathbf{G}_c^n]$ and as an inequality when $\sum_{c \in C} Y_c^l = \sum_{c \in C} b_c \mathbb{E}[\mathbf{G}_c^n]$. In the former case, the Y_c^l variables can only be obtained by solving the subproblem to compute the value of the Z^l variable. However, in the latter case, we know that $Y_c^l = b_c \mathbb{E}[\mathbf{G}_c^n], c \in C$ and $Z^l = 0$ without solving the subproblem. Therefore, the rationale behind the node reduction procedure is to check for each $l \in L_n$ whether setting $Y_c^l = b_c \mathbb{E}[\mathbf{G}_c^n], c \in C$ in constraints (39) yields an inequality, and only solve the subproblem for l if not. Given that for each $l \in L_n, n = 0, ..., |N| - 2$ there are $|H|$ successor nodes, we rearrange constraint (39) as follows, and replace $\sum_{c \in C} Y_c^l$ with $\sum_{c \in C} b_c \mathbb{E}[\mathbf{G}_c^n]$:

$$\begin{aligned} &\sum_{c \in C} b_c \mathbb{E}[\mathbf{G}_c^n] - \overline{B} - \sum_{j=1}^{n-2} \frac{\sum_{k \in K} \sum_{t \in T} \sum_{c \in C} (r_k + g_{ck}) \overline{Q_{ctk}^{\theta(l,j)}}}{(1+\gamma)^{n-j}} - \sum_{j=1}^{n} \frac{\sum_{k \in K} \left(f_k \overline{U_k} + h_k \overline{I_k} \right)}{(1+\gamma)^{n-j}} \\ &- \frac{\sum_{k \in K} r_k \overline{I_k}}{(1+\gamma)^{n-1}} + \sum_{j=0}^{n-2} \frac{\sum_{c \in C} \overline{Y_c^{\theta(l,j)}}}{(1+\gamma)^{n-j+1}} > \frac{\sum_{k \in K} \sum_{t \in T} \sum_{c \in C} (r_k + g_{ck}) \overline{Q_{ctk}^h}}{(1+\gamma)}. \end{aligned} \tag{43}$$

For each $l \in L_n, n = 0, ..., |N| - 2$, (43) can be evaluated for each successor node $\theta(l, n+1) \in L_{n+1}$ by computing the left-hand side of (43) from the solution of l, and the right-hand side by computing the capital consumption $\sum_{k \in K} \sum_{t \in T} \sum_{c \in C} (r_k + g_{ck}) \overline{Q_{ctk}^h}$ of $h \in H$ occurring in the successor node $\theta(l, n+1)$. The capital consumption of each $h \in H$ can be precalculated from the RMP's solution, and the set H can be sorted in increasing order of the capital consumption. The DFS then works as follows. Starting from the successor node $\theta(l, n+1)$ of $l \in L_n, n = 0, ..., |N| - 2$ corresponding to the first catastrophe realization h in the sorted set H (i.e., the realization with the highest capital consumption), we check whether (43) is satisfied, and if not, we solve the subproblem for node $\theta(l, n+1)$, and then check the first successor node of $\theta(l, n+1)$ itself, and so on. However, if at any point in the DFS (43) is satisfied, then $Y_c^l = b_c \mathbb{E}[\mathbf{G}_c^n], c \in C, Z^l = 0$ for $h \in H$ occurring in $\theta(l, n+1)$. Given that the capital consumption of the remaining unsearched realizations in H is less than that of h, the remaining unsearched successor nodes of l all satisfy (43) and need not be checked. This procedure speeds up the subproblem phase by solving the subproblem at each node of the tree only if the node does not satisfy the inequality in (43).

4.2.2. Optimality cuts Let π_λ^l and ω_λ^{cl} be the dual variables of constraints (39) and (40). The set of optimality cuts Λ (constraints (32) in the RMP) is then

$$\eta \geq \sum_{n\in N}\sum_{l\in L_{n-1}} |H|^{|N|-n-1}\left(B+\sum_{j=1}^{n-1}\frac{\sum_{k\in K}\sum_{t\in T}\sum_{c\in C}(r_k+g_{ck})Q_{ctk}^{\theta(l,j)}}{(1+\gamma)^{n-j}}+\frac{\sum_{k\in K} r_k I_k}{(1+\gamma)^{n-1}}\right.$$
$$\left.+\sum_{j=1}^{n}\frac{\sum_{k\in K}(f_k U_k+h_k I_k)}{(1+\gamma)^{n-j}}\right)\overline{\pi_\lambda^l}-\sum_{n\in N}\sum_{l\in L_{n-1}}|H|^{|N|-n-1}\sum_{c\in C}\left(b_c\mathbb{E}\left[\mathbf{G}_c^n\right]\right)\overline{\omega_\lambda^{cl}} \qquad \lambda\in\Lambda. \tag{44}$$

The total number of dual variables π_λ^l and ω_λ^{cl} in each cut $\lambda\in\Lambda$ is $(|C|+1)\sum_{n=0}^{|N|-1}|H|^n$, which is a prohibitively large number of variables to store in one CPLEX expression for the current cut λ, making the cut in the form in (44) impractical to add to the RMP. To counter this, we devise a memory management procedure. From Proposition 2 and constraints (39)–(40), for a given $l\in L_n, n=0,...,|N|-1$, we obtain

$$\sum_{c\in C} Y_c^l=\sum_{c\in C} b_c\left(\mathbb{E}\left[\mathbf{G}_c^n\right]+\mathbb{S}\left[\mathbf{G}_c^n\right]Z^l\right)=\max\left\{\sum_{c\in C} b_c\mathbb{E}\left[\mathbf{G}_c^n\right]\quad,\quad B+\frac{\sum_{k\in K} r_k I_k}{(1+\gamma)^{n-1}}+\right.$$
$$\left.\sum_{j=1}^{n}\frac{\sum_{k\in K}(f_k U_k+h_k I_k)}{(1+\gamma)^{n-j}}+\sum_{j=1}^{n-1}\frac{\sum_{k\in K}\sum_{t\in T}\sum_{c\in C}(r_k+g_{ck})Q_{ctk}^{\theta(l,j)}}{(1+\gamma)^{n-j}}-\sum_{j=0}^{n-2}\frac{\sum_{c\in C} Y_c^{\theta(l,j)}}{(1+\gamma)^{n-j+1}}\right\}. \tag{45}$$

The maximum function in (45) is made up of a finite number of elements obtained from the RMP with a subset of those elements appearing for any $l\in L_n, n=0,...,|N|-1$. It also includes up to $n-2$ variables $Y_c^{\theta(l,j)}$, whose values can be obtained recursively from (45) for $\theta(l,j), j<n-1$. Let $\mathcal{P}=\sum_{c\in C} b_c\mathbb{E}\left[\mathbf{G}_c^n\right]$, $\mathcal{Q}^h=\sum_{k\in K}\sum_{t\in T}\sum_{c\in C}(r_k+g_{ck})Q_{ctk}^h$, and $\mathcal{C}^n=B+\sum_{k\in K} r_k I_k/(1+\gamma)^{n-1}+\sum_{j=1}^{n}\sum_{k\in K}(f_k U_k+h_k I_k)/(1+\gamma)^{n-j}$ be the finite elements. The memory management procedure is based on the rationale that, given that each cut only contains the $\mathcal{P}, \mathcal{Q}^h$, and $\mathcal{C}^n$ elements, at every $l\in L_n, n=0,...,|N|-1$, the global coefficient of each element $\mathcal{P}, \mathcal{Q}^h$, and $\mathcal{C}^n$ in the cut can be additively updated by the contribution of the subproblem at l through π_λ^l and ω_λ^{cl}, using Eq. (45) recursively. At the end of the subproblem phase, each element $\mathcal{P}, \mathcal{Q}^h$, and $\mathcal{C}^n$ is added to the cut with its global coefficient cumulatively obtained from all nodes. This way, only $|H|+|N|+1$ elements are added to the CPLEX expression of each $\lambda\in\Lambda$ instead of $(|C|+1)\sum_{n=0}^{|N|-1}|H|^n$, and up to $(|N|-1)(|H|+|N|+1)$ coefficients for each predecessor node need to be saved in memory at any point of the DFS to be able to execute the recursion in Eq. (45). Note that $\pi_\lambda^l=0$ and $\omega_\lambda^{cl}=1/|L_{n-1}|$ in the node reduction procedure when $Y_c^l=b_c\mathbb{E}\left[\mathbf{G}_c^n\right]$ for any $l\in L_n, n=0,...,|N|-1$, i.e., when the subproblem is not solved.

PROPOSITION 7. *Under the assumption of equiprobable scenarios ($p^s=1/|S|$) the model can be simplified further since $\mathbb{E}\left[\boldsymbol{G}_c^n\right]$ and $\mathbb{S}\left[\boldsymbol{G}_c^n\right]$ are the same for every year $n\in N, c\in C$. Consequently, the year index can be dropped and $\mathbb{E}\left[\boldsymbol{G}_c\right]$ and $\mathbb{S}\left[\boldsymbol{G}_c\right]$ can be used.*

5. Computational study

The aim of this section is to analyze the mutual catastrophe insurance framework through real-life data and provide managerial insights related to the operational and financial functions of the framework. We consider the case of a mutual insurer providing insurance contracts against hurricanes to a portfolio of 18 Caribbean countries.

5.1. Data description

The existing inter-governmental Caribbean Disaster and Emergency Management Agency (CDEMA) has the capacity and expertise to act as the insurer for the 18 Caribbean member states (see the map in Fig. 17 and the country acronyms in Table 7 in Appendix D). CDEMA currently manages a regional response mechanism in which emergency supplies are prepositioned in four subfocal countries. We consider the possibility of using $|K| = 10$ regional locations in the prepositioning network. The countries are under a constant threat of severe hurricanes during the yearly Atlantic hurricane season (June to November), most of which impact multiple countries.

We use historical hurricane tracks data from 67 past hurricane seasons between 1950 and 2017 as in Balcik et al. (2019). In this data set, each season is divided into $|T| = 16$ two-week time periods with a replenishment lead time of eight weeks ($\tau = 4$). Each season is characterized by a set of events occurring in each period and by the countries they affect. We consider that there are 188 events in total, spread over 67 equiprobable seasons, five of which having no hurricanes. These seasons are independent since what happens in a given year does not affect the future. The hurricane events can have one of three severity levels from an ordered set $P = \{\text{mild, severe, very severe}\}$, and each country $c \in C$ has a maximum demand Π_c for emergency supplies. Accordingly, mild hurricanes generate losses in $[0; 0.2\Pi_c]$, severe hurricanes in $]0.2\Pi_c; 0.5\Pi_c]$, and very severe hurricanes in $]0.5\Pi_c; \Pi_c]$.

Given the events and the countries they affect in each season, we extend the existent data set to test our methodology. First, we generate five distinct season sets by varying the severity level of each event in each set based on historical data. We consider in each set $|H| = 63$ catastrophe realizations (one of which being a zero-hurricane season with a probability five times higher than the rest), and vary the length $|N|$ of the planning horizon between two and five years. This implies that the number of multi-stage scenarios is $|S| = 3,969$ for $|N| = 2$, and $|S| = 992,436,543$ for $|N| = 5$. Moreover, based on Stumpf et al. (2017) who estimate that a dollar spent in disaster preparedness saves seven dollars that would be otherwise spent in emergency response, we consider the emergency outsourcing cost for each country o_c to be a multiple βb_c of its average prepositioning cost, and we set $\beta = 7$. We also consider $\beta = 4$ to study the effect of cheaper outsourcing channels. Finally, for each of the two parametric ex ante indemnification policies, the loss schedule for each country is generated as follows. For the EI policy, the schedule indemnifies $0.1\Pi_c$, $0.35\Pi_c$, and $0.75\Pi_c$ for each severity level, and it indemnifies $0.2\Pi_c$, $0.5\Pi_c$, and Π_c for the MI policy. The insurance claim d^h_{ct} for each country $c \in C$ affected by one or more events $e \in E^h_t, t \in T, h \in H$ is then obtained by matching the events' severity levels to the loss values in its loss schedule and aggregating these losses into a claim. We therefore obtain 80 distinct instances representing two indemnification policies (EI and MI), five season sets (from 1 to 5), four planning horizons (from 2 to 5), and two β values (4 and 7). Note that we set the discount rate γ to 4% and a facility's capacity a to 12,000. The remaining parameters can be found in Tables 8 and 9 in Appendix D.

While we use historical data to obtain our scenarios, our methodology can be easily extended to the generation of scenarios using rare event simulation techniques such as importance sampling, stratification, or splitting methods (Rubinstein and Kroese 2016). Such techniques are sampling-based and focus on refining the sampling process so as to sample scenarios representative of rare events in a computationally tractable time. To use such techniques in tandem with our methodology, one would first sample a representative catastrophe realization set using the rare event simulation technique. Then, if the number of realizations is computationally tractable, the algorithm can be applied directly to the sampled set. Otherwise, an approximation method such as sample average approximation (Verweij et al. 2003) can be used on the sampled set, which would consist of running our algorithm multiple times on different realization sets until the objective function of the approximation method converges. This would still be computational tractable for our case instances since our algorithm runs on average in one hour with a maximum of 5.5 hours.

5.2. Implementation of the algorithm

The Benders decomposition algorithm was implemented in C++ and the mathematical programs were solved by CPLEX 22.1.0 within a 24 hours time limit on an Intel E5-2683 v4 VMware virtual machine with eight cores Broadwell CPUs at 2.1Ghz, and 32 GB of vRAM on Gentoo Linux2.6 operating system. We used the following CPLEX settings, which proved to be most efficient during the tuning experiments. In the RMP phase, we used best-estimate search as the MIP node selection strategy, and zero cutting plane passes when solving the root node of the search tree. In the subproblem phase, we turned off presolve and used the dual simplex to solve linear programs. We also limited the runtime of the master problem to two hours, which has proven to be sufficient to identify an optimal integer solution in all instances, but not for the lower bound to converge to the upper bound in some instances. All instances ran within 5.5 hours, with an average runtime of 61 minutes and a minimum of 13 seconds. On average, one iteration of the subproblem phase ran in 0.11, 0.59, 41.15, and 3,507.11 seconds for a planning horizon of two to five years, and one iteration of the RMP phase ran in 12.16 minutes. The algorithm always converged to an optimal solution within very few iterations, with an average of 3.8 RMP iterations and 2.8 subproblem iterations, and a maximum of seven and six, respectively. Finally, the node reduction procedure eliminated on average 73% of the nodes. In the following sections, we analyze the solutions and discuss the results.

5.3. Results analysis

Our results suggest that the solutions yielded by our framework exhibit a high variation in the degree of resource pooling. A solution outsourcing claims exclusively corresponds to no-pooling, while a solution with prepositioning exclusively corresponds to full-pooling. For all combinations of policies, β, and n, the framework always found a degree of pooling between the two extremes of the pooling spectrum whose optimal cost is lower than the cost of both the no-pooling and full-pooling solutions. We define the degree of resource pooling by comparing the cost saving CS_n of the attained framework solution with that of the no-pooling solution (Fig. 6) and the cost saving CS_f of the attained solution compared with that of the full-pooling solution (Fig. 7). The degree of pooling also includes the proportion PO of the total cost used for outsourcing (Fig. 8), and the ratio IR of the inventory level in the framework to that of the inventory level under full-pooling (Fig. 9). Figs. 6–9 depict the average performance values over the five season sets #1–5 of CS_n, CS_f, PO, and IR for each policy, each β, and each $n \in N$. A larger degree of pooling is synonymous with a larger CS_n saving, a smaller CS_f saving, and a smaller outsourcing portion PO. Moreover, it is synonymous with an increase in the inventory level and the ratio IR, and in the worst-case surplus capital and its ratio to the full-pooling solution (which we do not depict as it follows a very similar trend as that of IR). Finally, it is also synonymous with an increase in the coverage of policyholders, given by a decrease in the number of policyholders with deductibles and limits, as well as a decrease in the value of these deductibles and an increase of the limits. We now analyze the degree of resource pooling with changes in the planning horizon, the indemnification policy, and the outsourcing factor β across the 80 instances, as well as the effect on policyholders' deductibles and limits. The electronic compendium EC.2 presents the detailed results for the 80 instances.

5.3.1. Effect of the planning horizon Horizontal collaboration is more attractive over a longer planning horizon. As observed in Figs. 6–9, the longer the planning horizon is, the higher CS_n is, the lower CS_f and PO are, and the more resource pooling benefits are obtained. For example, for EI and $\beta = 7$, CS_n is 25% for $n = 2$ and 52% for $n = 5$, while CS_f is 15% and 3%, PO is 31% and 4%, and IR is 55% and 91%, respectively. This is intuitive since fully outsourcing the claims for one more year results in a cost increase that exceeds that of operating the insurance for one more year with a larger network capacity. This benefits the policyholders whose aim is to establish a long-term collaboration. However, the degree of pooling is still significant enough to

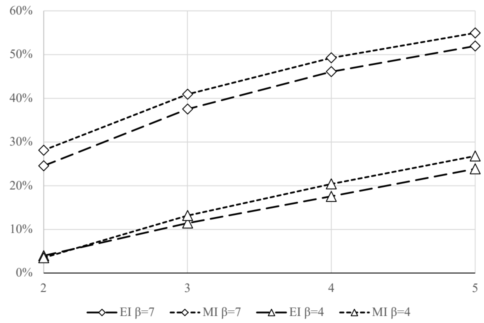

Figure 6 Cost saving CS_n compared with no-pooling.

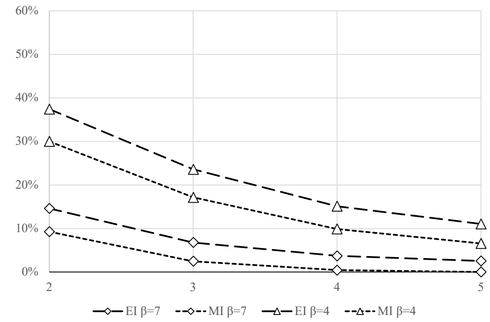

Figure 7 Cost saving CS_f compared with full-pooling.

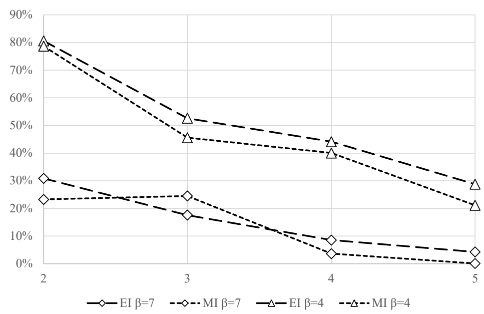

Figure 8 Portion of outsourcing costs PO.

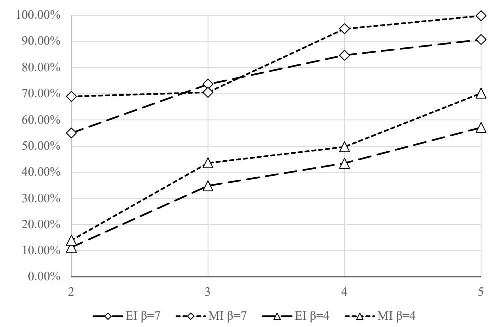

Figure 9 Inventory level ratio IR to that of full-pooling.

motivate the establishment of the collaboration, albeit with a smaller level of resource pooling, even for a two-year horizon. When extending the planning horizon by one year, since the framework already pools resources for a smaller n, we observe a certain increase in the resource pooling level. This results in a cost increase spread over the yearly premiums, which is still lower than the cost increase of outsourcing these claims for an extra year. The discussed trends of the CS_n, CS_f, PO, and IR curves with the planning horizon are in line with this observation.

5.3.2. Effect of the outsourcing factor Even if some emergency outsourcing channels could provide outsourcing at a lower price ($\beta = 4$), there will always be a certain benefit derived from the collaboration, as there still will be a reduced yet positive degree of pooling. An insurer wanting to reduce the amount of inventory and surplus capital it collects can seek stronger relationships and agreements with suppliers in order to reduce the outsourcing costs, this without voiding the collaboration from its resource pooling benefits. This is intuitive since a lower β value makes the no-pooling solution cheaper, while the cost of the full-pooling solution remains unchanged. As seen in Figs. 6–9, this results in less resource pooling IR, a larger outsourcing portion PO, less savings CS_n, and more savings CS_f. For example, for the EI policy and $n = 4$, IR drops from 85% ($\beta = 7$) to 43% ($\beta = 4$), PO increases from 9% to 44%, CS_n drops from 46% to 18%, and CS_f increases from 4% to 15%. However, with a longer planning horizon, the effect of the outsourcing factor β is dampened since the degree of pooling seems to converge irrespective of its value. This is evident from the smaller gap between the CS_n, CS_f, PO, and IR curves of $\beta=7$ and $\beta=4$ when $n=5$. The intuition behind this observation is the same as that derived in Sec. 5.3.1.

5.3.3. Analysis of the coverage deductibles and limits Our results show that the deductibles and limits set by the framework to reach the desired degree of pooling depend on two factors. The first is driven by the subset $\overline{C} \subseteq C$ of all policyholders affected by catastrophes together in the same time periods in the two extreme catastrophe realizations: one that dictates the total inventory level $\sum_{k\in K} I_k$, and the other that dictates the worst-case surplus capital B. Indeed, among all realizations $h \in H$, there exists one whose total insurance coverage for all policyholders is the largest and hence dictates the value of $\sum_{k\in K} I_k$, and one whose total emergency response cost for all policyholders is the largest and hence dictates B. These extreme realizations cannot be determined a priori (see Eq. 16 and Proposition 5), and are the same independently of the indemnification policy, since the difference in the total claims for all policyholders in each realization is proportional to the difference in the value of the loss schedules of the EI and MI policies.

The second factor is, for all the insurance claims of each policyholder in the subset $\overline{C}$ of policyholders simultaneously affected under the extreme realizations, the position of these claims in the global distribution of claims over all realizations. If the claim of a policyholder falls at the lower end of the distribution, it makes for a good candidate to set it as the deductible. If it falls at the upper end of the distribution, it makes for a good candidate to set it as the limit (recall Fig. **??** of Section 3.2.5). We observe that in order to optimize the cost and reach the optimal resource pooling level, the framework almost always sets deductibles and limits only to the policyholders that are in $\overline{C}$. This can be seen in Fig. 10 showing, for each of the five season sets, which countries are in $\overline{C}$, and which countries got deductibles and limits in the EI and MI policies. The figure shows the results for $\beta=7$ aggregated over the planning horizon.

Among the policyholders in $\overline{C}$, and given their claims for all the events in the extreme realizations, the framework determines for which policyholders it should set deductibles and limits, based on the position of their claims in their global distribution of claims over all realizations. For example, for the EI policy in season sets #1 and #2, the maximal claim of HTI and JAM occurs in the extreme realizations, and therefore the framework sets a limit to these countries to reduce the inflating effect that their full coverage would have on the inventory level and on the surplus capital. Moreover, for season set #5, the framework sets a deductible to the Turks and Caicos Islands (TCA), the Cayman Islands (CYM), and JAM, each of which had one claim or more in the extreme realizations. This is because TCA's single claim was its lowest global claim, making it a perfect candidate for a deductible, while CYM's lowest out of two claims resulted in only 21% of its claims

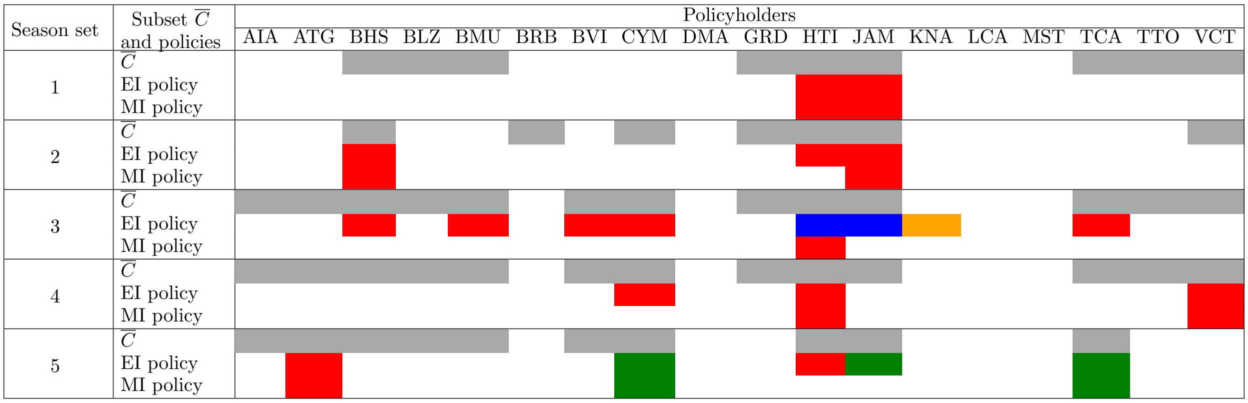

Figure 10 Deductibles and limits of policyholders for the EI and MI policies. The gray cells are the countries that form $\overline{C}$, the green cells are those in $\overline{C}$ that got a deductible ($V_c > 0$), the red cells are those in $\overline{C}$ that got a limit ($X_c > 0$), the blue cells are those in $\overline{C}$ that got both a deductible and limit ($V_c, X_c > 0$), and the orange cell is the one that is not in $\overline{C}$ and got a deductible or a limit ($V_c > 0$ or $X_c > 0$).

not being covered due to deductibles. On the other hand, neither Antigua & Barbuda (ATG) nor the Bahamas (BHS) get a deductible. This is because their lowest claim in the extreme realizations falls at the upper end of their global distribution of claims, which would have resulted in 91% and 51% of their respective claims not being covered due to deductibles, making them poor candidates for a deductible. Finally, in this case, JAM's claim in the extreme realizations does not correspond to its maximal claim, and therefore made it a better candidate for a deductible than for a limit.

5.3.4. Effect of the indemnification policy The degree of pooling in the framework increases when changing the policy from EI to MI, as the resource pooling benefits become larger. For example, we observe in Figs. 6–9 that if the policy changes from EI to MI for $\beta = 7$ and $n = 4$, then CS_n increases from 46% to 49%, CS_f drops from 4% to 0.4%, PO drops from 9% to 4%, and IR increases from 85% to 95%. However, this change in the degree of pooling with the policy is not as prominent as a change in the planning horizon or in the outsourcing factor. In fact, the difference in the degree of pooling between EI and MI is more apparent when β and n increase. Moreover, this difference is also observed in the reduced number of policyholders with deductibles and limits in the season set, as seen in Fig. 10.

Given that the hurricane events and their severity levels are the same under the EI and MI policies, the extreme realizations are unchanged under these policies. The larger values of the claims in the loss schedule of the MI policy, compared with those of the EI policy, result in larger total claims under the extreme realizations. When the total claims amount increases, there is more resource pooling, which results in larger yearly premiums, but this increase is still lower than the cost increase resulting from outsourcing these extra claims. The framework therefore opts to increase the degree of pooling and the resource pooling to a certain level so as to absorb in premiums what would otherwise result in higher outsourcing costs. For example, for $\beta = 7$ and

$n = 4$, the inventory level for the MI policy was 1.7 times that of the EI policy, and the surplus capital 1.7 times, while the cost difference was only 1.5 times.

Finally, it is interesting to note that if the insurer allows each policyholder to negotiate a different policy (e.g., 30% of policyholders opt for an EI policy and 70% opt for an MI policy), it is the claim values of the policyholders affected under extreme realizations that dictate the degree of pooling. Negotiating smaller loss schedules in the policies of these policyholders would be a consequential exercise for the insurer to maintain a similar degree of pooling while simultaneously reducing the network's capacity and the surplus capital.

5.3.5. Detailed analysis of a framework's solution We now analyze in more detail one of the framework's solutions for season set #1 under the base-case parameters (i.e., with $\beta = 7$ and $|N| = 5$) and for the two indemnification policies EI and MI. The results for these two instances are presented in Table 1. Respectively for EI and MI, the CS_n saving with no-pooling is 52% and 55%, and the CS_f saving with full-pooling is 0.6% and 0.01%, the proportion PO of outsourcing from the total cost is 2.7% and 0.2%, and the inventory level ratio IR is 95.7% and 99.8%. These observations suggest that the degree of resource pooling in the base-case is very close to full-pooling. Moreover, we note that the total value of claims under the MI policy is 1.5 times the total value of claims under the EI policy. This proportion is also observed in the total cost, the total premiums, the inventory level, and the worst-case surplus capital.

Table 1 Results for the indemnification policies for season set #1, $\beta = 7$, and $n = 5$.

Indemnification policy	Total cost	Total premiums	Total outsourcing cost	Inventory level	Worst-case surplus capital
EI	939,281,436	914,275,324	25,006,112	1,260,000	384,138,736
MI	1,427,179,290	1,424,980,026	2,199,264	1,908,000	598,864,143

The following values are common to both policies. The aggregate capital pool in the first year is broken down into 65% surplus capital, 31% procurement costs, 2% holding costs, and 2% facility operating costs. While the first year capital pool is large due to the investment needed to build the network and store the worst-case surplus capital, the yearly premiums of policyholders beyond year one are consistently around 1.5 times the pure premium (i.e., the minimum premium policyholders would pay). This is due to the fact that the expected capital consumption in emergency response is only 11% of the aggregate capital pool in the first year, and recouping that consumed capital in an aggregate pool results in a much lower capital pool than in the first year. This implies that donor support may be necessary to cover the initial investments. Moreover, the yearly contribution of each policyholder to the capital pool is consistent with its risk profile. For example, the premiums of the most risky policyholders such as Haiti (HTI) and Jamaica (JAM) with the highest and second highest maximal claim correspond to the largest portion of the aggregate capital pool with 68%

and 15%, respectively. These factors are also similar in the other instances, and are consistent with what is generally observed in mutual catastrophe insurance (Kousky and Cooke 2012). Finally, we concentrate on the policyholders for which the framework sets a coverage deductible or limit. In the base-case instances of season set #1, for the EI policy, the framework only sets a coverage limit to HTI which partially covers 11% of all its claims, since it has the largest outlier claims that would inflate the inventory level if included fully. For the MI policy, it sets a coverage limit to both HTI and JAM which partially covers 11% and 22% of all their claims, respectively.

6. Conclusions and managerial insights

We have developed a novel mutual catastrophe insurance framework for the establishment of a horizontal collaboration between policyholders to jointly preposition strategic reserves. This framework consists of a risk-averse mutual insurer providing multi-year insurance contracts with coverage deductibles and limits to a portfolio of risk-averse policyholders. It integrates the operational and financial functions of covering the policyholders' insurance claims in the form of emergency supplies, and establishing and operating the prepositioning network from which coverage is provided. The underlying problem, the CIP-PSR, includes four integrated subproblems: determining insurance coverage, prepositioning network capacity, management of the surplus capital, and premium allocation. The problem was modeled as a very large-scale non-linear multi-stage stochastic program. Exploiting the model's properties, we solved it by applying Benders decomposition so as to eliminate the non-linearities and move the large-scale aspect to the subproblem phase, where a problem-specific node reduction procedure is applied as a substantial speed-up mechanism.

We have tested the framework on real data from 18 Caribbean countries collaborating for hurricane preparedness, and we have studied the degree of resource pooling in the framework as a function of changes in the planning horizon, the cost of outsourcing, and the loss schedule in the indemnification policies. A larger degree of pooling is synonymous with less outsourcing, more coverage, smaller deductibles and larger limits. The results showed that the least-cost framework solution was always better than the no-pooling and full-pooling solutions. Our main insights from the Caribbean case study can be summarized in the following key messages.

Horizontal collaboration is more effective when established for the long-term. Our analysis demonstrates that the degree of pooling highly increases when the collaboration is established over a larger planning horizon. With a five-year planning horizon, the degree of pooling is close to full-pooling. Moreover, even for a two-year planning horizon, there exist some positive, albeit reduced, benefits resulting from the collaboration.

Horizontal collaboration is still beneficial in the presence of cheaper outsourcing channels. Our results show that even though the presence of cheaper outsourcing channels lowers the degree of pooling, there are still resource pooling benefits that justify the continuation of the collaboration. Therefore, if the insurer wants to reduce its surplus capital and the inventory level of the network while also reducing the total framework cost, it needs to pursue stronger relationships with suppliers and make agreements to reduce the cost of outsourcing.

The setting of deductibles and limits needed to reach a desired degree of pooling depends on the degree of correlation of policyholders simultaneously affected by disasters. The framework sets deductibles and limits to the subset of policyholders simultaneously affected by disasters in the two extreme realizations dictating the total inventory level and the worst-case surplus capital. The good candidates for a deductible or limit setting among these policyholders are those whose claims in the extreme realizations fall at the end of their global claims distribution. The larger is the number of policyholders simultaneously affected under the extreme realizations, the larger is the total amount of claims that need to be covered. This implies that more inventory needs to be held in the network and more surplus capital needs to be collected from policyholders in premiums. Yet this provides the framework with more options to set deductibles and limits in order to reduce that inventory level and surplus capital.

A careful pre-negotiation of policyholders' indemnification policies can strengthen the collaboration. We observed in our analysis that adopting the more conservative parametric ex ante indemnification policy MI leads to a cost increase proportional to the increase in the total amount of claims, which is not the case for the EI policy. Moreover, the degree of pooling is larger under an MI policy since the resource pooling benefits increase with the total amount of claims. In fact, in order to strengthen the collaboration by maintaining it while maximizing the pooling benefits and reducing the total inventory level and worst-case surplus capital, the insurer should negotiate at the onset of the collaboration the claim values in the loss schedule of each policyholder so as to reduce the total amount of claims that need to be covered under the extreme realizations. Alternatively, it could alter the parametrization of the policies by considering a larger number of severity levels in the loss schedules, resulting in a more populated global distribution of claims for policyholders, and hence allow a larger flexibility for the framework to set deductibles and limits.

The framework provides a transparent disaster financing plan to potential donors. Our results show that while the initial investment to build the surplus capital and the prepositioning network's capacity is substantial, the subsequent years' premiums for the Caribbean case are consistently around 1.5 times the pure premium for each policyholder. Moreover, the more risky countries (such as HTI and JAM) benefit the most from the collaboration. Therefore, vulnerable regions and countries that want to strengthen their capacity but do not have the means to finance the initial investments themselves can use the framework's solution as a proof of concept and transparent disaster financing plan in front of potential donors.

Considering in our framework a zero probability of ruin to ensure the solvency of the insurer under any scenario may be too conservative, but it allows us to fully cover the needs of the policyholders, be it from the insurance framework or from outsourcing. This is an important consideration to take into account in a humanitarian setting. Moreover, the imposition of a non-zero probability of ruin would result in a chance-constrained model, and is non-trivial from a contractual perspective, since it is not clear which type of contracts should be designed between the different stakeholders (i.e., policyholders, insurer, and suppliers). This would result in an added modelling complexity, which would deserve future studies.

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Appendix A: Notation

Table 2 presents the sets used in the mathematical model, Table 3 the parameters, and Table 4 the variables.

Table 2 Model sets.

C	Set of policyholders
N	Set of years in the planning horizon
T	Set of time periods
K	Set of storage locations in the prepositioning network
P	Set of catastrophe severities
H	Set of catastrophe realizations
E_t^h	Set of events occurring in $t \in T$ of $h \in H$
S	Set of scenarios
C^e	Set of policyholders affected by catastrophic event $e \in E_t^h, t \in T, h \in H$
L	Set of nodes in the scenario tree
L_n	Set of nodes in stage $n \in N$
L_0	Set of nodes in stage 0

Table 3 Model parameters.

p^s	Probability of $s \in S$
p^h	Probability of $h \in H$
τ	Inventory replenishment time
a	Capacity of a facility
γ	Discount rate
d_{ct}^{sn}	Insurance claim of $c \in C$ in $t \in T$ of year $n \in N$ of $s \in S$
f_k	Yearly fixed cost of opening and operating a storage facility at $k \in K$
r_k	Procurement cost of a unit of strategic reserve supplies for prepositioning at $k \in K$
h_k	Yearly cost of holding a unit in inventory at $k \in K$
g_{ck}	Logistical coverage cost of transporting one unit of supplies from $k \in K$ to $c \in C$
o_c	Cost of emergency outsourcing a unit of supplies for $c \in C$
b_c	Average logistical cost of providing a unit of supplies in insurance coverage to $c \in C$ $b_c = \sum_{k \in K} (r_k + h_k + g_{ck}) / \lvert K \rvert$
$\phi(s, j)$	Catastrophe realization $h \in H$ occurring in scenario $s \in S$ at year $j \in N$
$\theta(l, j)$	Predecessor node at stage $j \in L_j$ of node $l \in L_n$ in the scenario tree

Table 4 Model variables.

V_c	Coverage deductible of $c \in C$
X_c	Coverage limit of $c \in C$
I_k	Inventory level at $k \in K$
U_k	Number of operational facilities at $k \in K$
B	Capital surplus needed to cover this worst-case coverage realization
G_{ct}^{sn}	Insurance coverage provided to $c \in C$ in $t \in T$ of year $n \in N$ of $s \in S$
Q_{ctk}^{sn}	Insurance coverage provided to $c \in C$ from $k \in K$ in $t \in T$ of year $n \in N$ of $s \in S$
$\sigma_{ct}^{sn}, \delta_{ct}^{sn}$	Insurance coverage linearization variables for $c \in C$ in $t \in T$ of year $n \in N$ of $s \in S$
A_{tk}^{sn}	Inventory level at $k \in K$ at the start of $t \in T$ of year $n \in N$ of $s \in S$
R_{tk}^{sn}	Replenishment amount arriving to $k \in K$ at the start of $t \in T$ of year $n \in N$ of $s \in S$
Y_c^{sn}	Premium of $c \in C$ in year $n \in N$ of $s \in S$
Z^{sn}	Safety margin variable of the standard deviation premium setting principle in year $n \in N$ of $s \in S$
$\mathbb{E}[\mathbf{G}_c^n]$	Expected insurance coverage of $c \in C$ in year $n \in N$
$\mathbb{S}[\mathbf{G}_c^n]$	Standard deviation of the insurance coverage of $c \in C$ in year $n \in N$

Appendix B: Illustrative example of the functioning of the framework's components

We present here a small illustrative example that shows how each of the four components of the framework functions. This example is composed of two policyholders $|C| = 2$, three time periods $|T| = 3$, two possible locations in the prepositioning network $|K| = 2$, three catastrophe realizations $|H| = 2$, and a planning horizon of two years $|N| = 2$, giving $|S| = 9$ scenarios. We assume that the catastrophe realizations and therefore the scenarios are equiprobable. Let the inventory replenishment time $\tau = 2$, a facility's capacity $a = 300$, and the discount rate $\gamma = 0\%$ for simplicity. For each of the two prepositioning locations, the yearly fixed cost is $f_1 = 100{,}000$ and $f_2 = 50{,}000$, the procurement cost is $r_1 = 100$ and $r_2 = 250$, and the holding cost is $h_1 = 15$ and $h_2 = 30$. Finally, for the two policyholders and the two locations, the logistical coverage cost is $g_{11} = 125$, $g_{12} = 75$, $g_{21} = 75$, and $g_{22} = 125$. Based on these costs, the average logistical costs b_1 and b_2 over the two locations can be calculated as $b_1 = b_2 = 297.5$, and the outsourcing costs o_1 and o_2 are assumed to be thrice as large as b_1 and b_2, respectively. Finally, Fig. 11 presents, for each catastrophe realization $h \in H$ and each time period $t = 1, ..., 3$, the claims d_{ct}^h for policyholder c_1, and Fig. 12 presents the claims for policyholder c_2. We note that h_1 contains three events $e \in E_t^1, t = 1, ..., 3$, h_2 three events $e \in E_t^2, t = 1, ..., 3$, and h_3 two events $e \in E_t^3, t = 1, 2$.

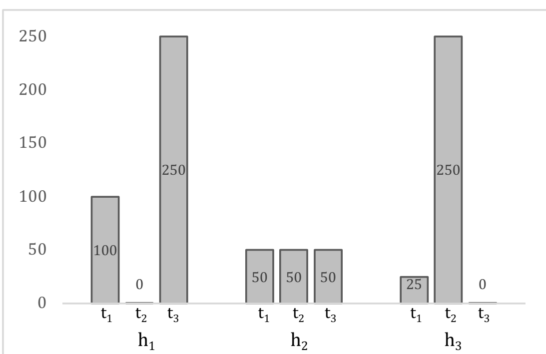

Figure 11 Claims d_{ct}^h for c_1.

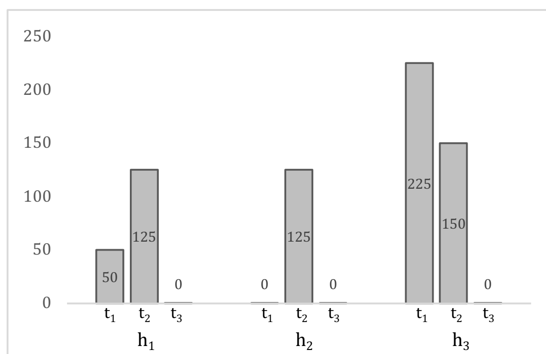

Figure 12 Claims d_{ct}^h for c_2.

The total value of claims $\sum_{c \in C} \sum_{t \in T} d_{ct}^h$ for catastrophe realizations h_1, h_2 and h_3 is 525, 275, and 650, respectively. The total cost of covering these claims from outsourcing without any insurance framework (i.e., under no-pooling of resources) is, respectively for each $h \in H$, 468,562.50, 245,437.50, and 580,125. This gives an expected yearly outsourcing cost over the three realizations of 431,375, and a total expected cost over the planning horizon of two years of 862,750.

To cover all claims from the insurance framework without any outsourcing (i.e., full-pooling of resources), the total inventory level needed is 650 units, given by the total value of claims in h_3. In fact, h_3 is the worst-case coverage realization. This leads to requiring $\lceil 650/300 \rceil = \lceil 2.17 \rceil$, i.e., three facilities. The optimal prepositioning network in this case without outsourcing is two facilities at k_1 with $I_1 = 600$, and one facility at k_2 with $I_2 = 50$ units. This leads to a total expected cost of

829,653.85. This solution is not very attractive since it requires the opening two full facilities and a third facility that is only 17% full. The yearly fixed cost of that facility is too high compared with the small amount of inventory it holds.

One way to remedy this is to allow outsourcing in order to obtain a more attractive solution with a lower total cost. Such a solution consists of opening only two full facilities at k_1 with a total inventory level of 600. This would be sufficient to cover the total claims of 525 in h_1 and 275 in h_2, but insufficient to cover the 650 units needed in h_3, leading to the need to outsource the remaining 50 units. In the following, we present two possible solutions that would lead to this outcome, we exemplify in detail each of the four modelling components of the framework for these solutions, and we discuss which of the two solutions is better and why.

B.1. Insurance coverage design

In order to outsource 50 units in h_3, we need to set a coverage deductible and a limit to policyholders 1 and 2 in a way that ensures that 50 units in total are not covered by the insurance contracts. One possible solution (S_1) is to set a coverage deductible $V_1 = 25$ and a limit $X_2 = 200$, as shown in Figs. 13 and 14. The grey columns correspond to the part of the claims that are covered by the insurance contract (G^h_{ct} variables), while the white columns correspond to the amount of the claims not covered by it. With $V_1 = 25$, the 25 units claimed by c_1 in t_1 are not covered, while the full claim of 250 units in t_2 is. With $X_2 = 200$, the 225 units claimed by c_1 in t_1 are partially covered up to $X_2 = 200$, with the remaining 25 units not covered by the contract. The 150 units in t_2 are fully covered. This leads to a total of 50 units in h_3 that need outsourcing.

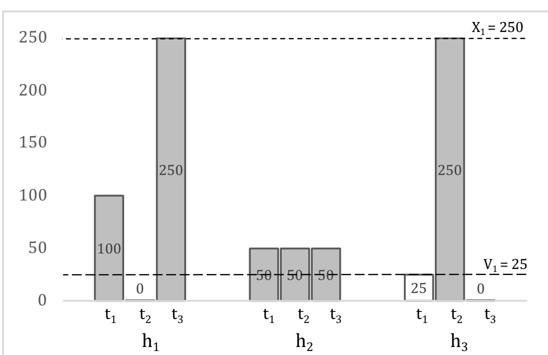

Figure 13 Coverage G^h_{ct} for c_1 in S_1.

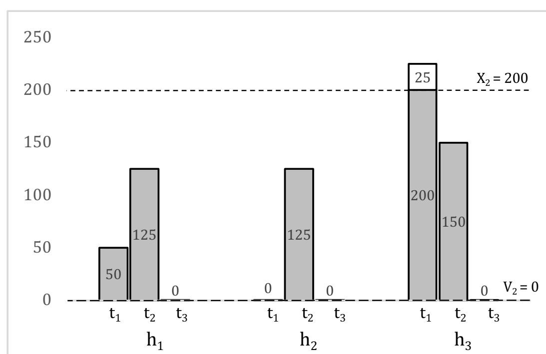

Figure 14 Coverage G^h_{ct} for c_2 in S_1.

Another possible solution (S_2) is to set no coverage deductible or limit to c_2, and set $X_1 = 200$, as show in Figs. 15 and 16. This leads to the 25 units claimed by c_1 in t_1 to be covered, while the 250 units in t_2 are partially covered up to X_1, and the remaining 50 units are not covered by the insurance contract. However, since c_1 has a claim in t_3 of h_1 that is larger than $X_1 = 200$, this claim of 250 is also partially covered up to 200, with the remaining 50 units being outsourced. This ensures that a total of 50 units are outsourced in h_3, but also results in 50 units in h_1.

Note that since the demands for both countries under catastrophe realization h_2 are neither too high nor too low (i.e., are not good candidates for either coverage deductibles or limits), both countries get full coverage for that realization, be it under solution S_1 or S_2.

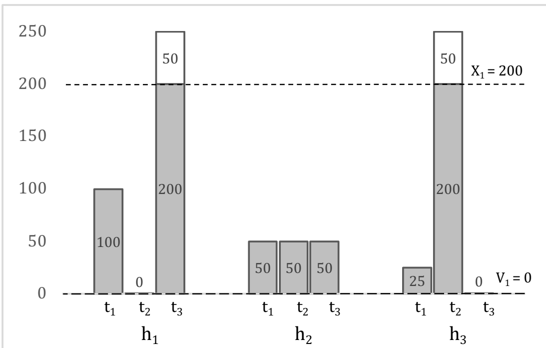

Figure 15 Coverage G^h_{ct} for c_1 in S_2.

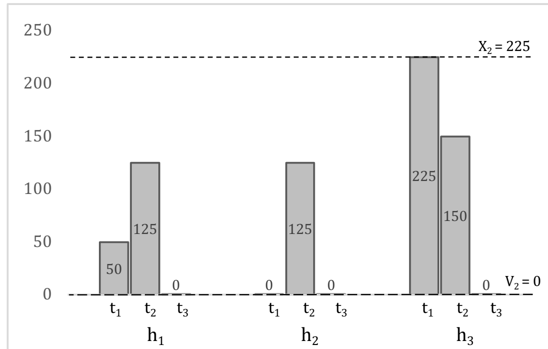

Figure 16 Coverage G^h_{ct} for c_2 in S_2.

B.2. Prepositioning network design

In both of the presented solutions, the optimal prepositioning network consists of opening two facilities at k_1 with a total inventory $I_1 = 600$, and no facility at k_2. Table 5 presents, for each of the two solutions, the optimal value of the prepositioning variables for location k_1, the value of these variables for k_2 being 0.

Table 5 Prepositioning network design variables for location k_1 with $U_1 = 2$ and $I_1 = 600$.

		A^h_{tk}			R^h_{tk}			Q^h_{1tk}			Q^h_{2tk}		
		t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3
Solution 1	h_1	600	450	475	0	0	150	100	0	250	50	125	0
	h_2	600	550	425	0	0	50	50	50	50	0	125	0
	h_3	600	400	200	0	0	200	0	250	0	200	150	0
Solution 2	h_1	600	450	475	0	0	150	100	0	200	50	125	0
	h_2	600	550	425	0	0	50	50	50	50	0	125	0
	h_3	600	350	250	0	0	250	25	200	0	225	150	0

B.3. Management of surplus capital

Given the optimal prepositioning network in Sec. B.2 for both solutions the total initial procurement costs $\sum_{k\in K}(r_k I_k) = 60{,}000$ and the total yearly holding plus facility costs $\sum_{k\in K}(f_k U_k + h_k I_k) = 209{,}000$. The total capital consumption in emergency response $\sum_{c\in C}\sum_{k\in K}\sum_{t\in T}(g_{ck} + r_k)Q^h_{ctk}$ for realizations h_1, h_2 and h_3 is 109,375, 55,625, and 117,500, respectively for S_1, and 98,125, 55,625, and 116,250 for S_2. This means that the capital surplus variable B required to cover the worst-case realization is 117,500 in S_1 and 116,250 in S_2. Finally, the total outsourcing $\sum_{c\in C} o_c \sum_{t\in T}(d^h_{ct} - G^h_{ct})$ for each realization is 0, 0, and 44,625, respectively for S_1 and 44,625, 0, and 44,625 for S_2. For illustrative purposes, Table 6 presents in solution S_1 for each scenario and each year, the aggregate capital pool (i.e., total premiums), the beginning surplus capital BL^{sn} and the remaining surplus capital EL^{sn}.

Table 6 Management of surplus capital variables for solution S_1.

$s \in S$	h in $n=1$	h in $n=2$	Aggregate capital pool $n=1$	BL^{s1}	EL^{s1}	Aggregate capital pool $n=2$	BL^{s2}	EL^{s2}
1	1	1	386,500	117,500	8,125	318,375	117,500	8,125
2	2	2	386,500	117,500	61,875	264,625	117,500	61,875
3	3	3	386,500	117,500	0	326,500	117,500	0
4	1	2	386,500	117,500	8,125	318,375	117,500	61,875
5	2	1	386,500	117,500	61,875	264,625	117,500	8,125
6	1	3	386,500	117,500	8,125	318,375	117,500	0
7	3	1	386,500	117,500	0	326,500	117,500	8,125
8	2	3	386,500	117,500	61,875	264,625	117,500	-
9	3	2	386,500	117,500	0	326,500	117,500	61,875

Adding up the total expected premiums and the total expected outsourcing over the two years, we obtain that the total expected cost is 719,416.67 for S_1 and 743,750.00 for S_2, making S_1 an optimal solution, both solutions being much cheaper than the no-pooling solution (862,750) and the full-pooling solution (829,635.85). Even though the difference in expected capital consumption in emergency response is 4,166 between S_1 and S_2 due to the higher coverage provided given in S_1, the difference in the expected outsourcing cost is larger ($-29{,}750$), making S_1 with a deductible $V_1 = 25$ and limit $X_2 = 200$ a cheaper solution than S_2 with a limit $X_1 = 200$.

B.4. Premium allocation

Given the value of the G_{ct}^{h} variables in S_1, the expected coverage $\mathbb{E}[\mathbf{G}_c^n]$ and standard deviation of coverage $\mathbb{S}[\mathbf{G}_c^n]$ for c_1 in year 1 are $\mathbb{E}\left[\mathbf{G}_1^1\right] = 250$ and $\mathbb{S}\left[\mathbf{G}_1^1\right] = 81.65$, and $\mathbb{E}\left[\mathbf{G}_2^1\right] = 216.67$ and $\mathbb{S}\left[\mathbf{G}_2^1\right] = 96.47$ for c_2. Using the $\mathbb{E}[\mathbf{G}_c^n]$ and $\mathbb{S}[\mathbf{G}_c^n]$ variables, the aggregate capital pool in each year and each scenario is allocated among the two policyholders using the standard deviation premium setting principle. We illustrate this with an example for scenario 1 and year 1. This gives that $\sum_{c \in C} Y_c^{11} = b_1\mathbb{E}[\mathbf{G}_1^n] + b_2\mathbb{E}[\mathbf{G}_2^n] + (b_1\mathbb{S}[\mathbf{G}_1^n] + b_2\mathbb{S}[\mathbf{G}_2^n])\, Z^{11}$. Solving for Z^{11} we obtain $Z^{11} = 4.67$, $Y_1^{11} = 187{,}907.84$, and $Y_2^{11} = 161{,}865.57$.

Appendix C: Proofs of the propositions

PROPOSITION 1. *If $\phi(s,n) = \phi(s',n'), n, n' \in N, s, s' \in S, s \neq s'$, then $G_{ct}^{sn} = G_{ct}^{s'n'}, c \in C, t \in T$.*

Proof of Proposition 1 From Eq. (1), the insurance coverage G_{ct}^{sn} is dependent on three values: V_c, X_c, and d_{ct}^{sn}, while $G_{ct}^{s'n'}$ on V_c, X_c, and $d_{ct}^{s'n'}$. If $\phi(s,n) = \phi(s',n') = h \in H$ then $d_{ct}^{sn} = d_{ct}^{s'n'}, c \in C, t \in T$, and therefore $G_{ct}^{sn} = G_{ct}^{s'n'}, c \in C, t \in T$. □

PROPOSITION 2. *For every* $s \in S, n \in N$*,* $BL^{sn} > B$ *when* $Z^{sn} = 0$ *and* $\sum_{c\in C} Y_c^{sn} = \sum_{c\in C} b_c \mathbb{E}[\boldsymbol{G}_c^n]$*, and* $BL^{sn} = B$ *when* $Z^{sn} > 0$ *and* $\sum_{c\in C} Y_c^{sn} > \sum_{c\in C} b_c \mathbb{E}[\boldsymbol{G}_c^n]$*.*

Proof of Proposition 2 Using the result of Proposition 1, and summing constraints (21) over all policyholders $c \in C$ and isolating Z^{sn}, we obtain

$$Z^{sn} = \frac{\sum_{c\in C} Y_c^{sn} - \sum_{c\in C} b_c \mathbb{E}[\mathbf{G}_c^n]}{\sum_{c\in C} b_c \mathbb{S}[\mathbf{G}_c^n]} \quad n \in N, s \in S. \tag{46}$$

Rearranging constraint (20), yields

$$\begin{aligned}\sum_{c\in C} Y_c^{sn} \geq & B + \sum_{j=1}^{n} \frac{\sum_{k\in K}(f_k U_k + h_k I_k)}{(1+\gamma)^{n-j}} + \frac{\sum_{k\in K} r_k I_k}{(1+\gamma)^{n-1}} \\ & + \sum_{j=1}^{n-1} \frac{\sum_{k\in K}\sum_{t\in T}\sum_{c\in C}(r_k + g_{ck}) Q_{ctk}^{sj}}{(1+\gamma)^{n-j}} - \sum_{j=1}^{n-1} \frac{\sum_{c\in C} Y_c^{sj}}{(1+\gamma)^{n-j}} \quad n \in N, s \in S.\end{aligned} \tag{47}$$

When the left-hand side of (47) is larger than its right-hand side, the capital needed to be collected in year $n \in N$ of scenario $s \in S$ is smaller than the aggregate pool of capital $\sum_{c\in C} Y_c^{sn}$ to be collected. In order to ensure that $\sum_{c\in C} Y_c^{sn}$ is the minimum possible, Z^{sn} is set to 0 in Eq. (46), and hence the aggregate capital pool is equal to the sum of pure premiums $\sum_{c\in C} Y_c^{sn} = \sum_{c\in C} b_c \mathbb{E}[\mathbf{G}_c^n]$. On the other hand, if (47) is satisfied as an equality, the capital to be collected is larger than the sum of pure premiums. In this case, Z^{sn} takes the least possible non-negative value that will ensure that $\sum_{c\in C} b_c \mathbb{E}[\mathbf{G}_c^n] + Z^{sn} \sum_{c\in C} b_c \mathbb{S}[\mathbf{G}_c^n]$ is equal to the right-hand side of (47). □

The following proposition shows that even under the integration of the four components of the framework, the fairness of premium allocation still holds. This is not trivially deducible when integrating insurance contract design and solvency conditions with premium allocation, as opposed to covering all the demand as in Balcik et al. (2019). The proof of Proposition 1 in Balcik et al. (2019) and the proof of Proposition 3 in this paper consider two different cases: the former describes a cost allocation mechanism for sharing a one-time initial investment, while the latter describes a multi-year mutual catastrophe insurance framework in which its four components are integrated. We note that this proposition can be used to model other collaborative disaster preparedness frameworks that integrate operational and financial planning and can be generalized by substituting the relevant operational design component and its related terms in the solvency conditions with the terms and constraints of the application under study.

PROPOSITION 3. *The safety margin variables* Z^{sn} *ensure a fair allocation of the surplus capital pool among policyholders in each year* $n \in N$ *and each scenario* $s \in S$*.*

Proof of Proposition 3 To ensure fairness in premium allocation in each year and each scenario, the safety margin variables Z^{sn} need to be minimized so as to ensure they take on the least possible non-negative value (Balcik et al. 2019). Our model ensures this implicitly as demonstrated below.

As mentioned in Proposition 2, when $BL^{sn} > B$, there is enough capital remaining from the previous year to cover the expenses of year $n \in N$, which means that the insurer needs only to collect the minimum premium from each policyholder, i.e., the pure premium $Y_c^{sn} = b_c\mathbb{E}[\mathbf{G}_c^n]$. Since the Y_c^{sn} variables are minimized in the objective function, the model implicitly ensures that Z^{sn} is set to zero in Eq. (21) sand therefore $Y_c^{sn} = b_c\mathbb{E}[\mathbf{G}_c^n]$.

On the other hand, when $BL^{sn} = B$, there is not enough capital remaining from the previous year to cover the expenses of year $n \in N$. This means that the insurer needs to collect a premium from each policyholder that is larger than their pure premium, i.e., $Z^{sn} \geq 0$. Replacing Y_c^{sn} in Eq. (47) with the RHS of Eq. (21), we get that when $BL^{sn} = B$:

$$\begin{aligned}\sum_{c\in C} b_c\mathbb{E}[\mathbf{G}_c^n] + \sum_{c\in C} b_c\mathbb{S}[\mathbf{G}_c^n]\, Z^{sn} =& B + \sum_{j=1}^{n} \frac{\sum_{k\in K}(f_kU_k + h_kI_k)}{(1+\gamma)^{n-j}} + \frac{\sum_{k\in K} r_kI_k}{(1+\gamma)^{n-1}} \\ &+ \sum_{j=1}^{n-1} \frac{\sum_{k\in K}\sum_{c\in C}\sum_{t\in T}(r_k + g_{ck})Q_{ctk}^{sj}}{(1+\gamma)^{n-j}} + \sum_{j=1}^{n-1} \frac{\sum_{c\in C} Y_c^{sj}}{(1+\gamma)^{n-j}}.\end{aligned}$$

From the above equation, we obtain:

$$Z^{sn} = \frac{B + \sum_{j=1}^{n} \frac{\sum_{k\in K}(f_kU_k+h_kI_k)}{(1+\gamma)^{n-j}} + \frac{\sum_{k\in K} r_kI_k}{(1+\gamma)^{n-1}} + \sum_{j=1}^{n-1} \frac{\sum_{k\in K}\sum_{c\in C}\sum_{t\in T}(r_k+g_{ck})Q_{ctk}^{sj}}{(1+\gamma)^{n-j}} + \sum_{j=1}^{n-1} \frac{\sum_{c\in C} Y_c^{sj}}{(1+\gamma)^{n-j}} - \sum_{c\in C} b_c\mathbb{E}[\mathbf{G}_c^n]}{\sum_{c\in C} b_c\mathbb{S}[\mathbf{G}_c^n]}.$$

This ensures that the model sets the variable Z^{sn} to take the least possible non-negative value that will ensure that $BL^{sn} = B$. Since the model implicitly ensures that the Z^{sn} variable takes the least possible value in both cases when $BL^{sn} > B$ or $BL^{sn} = B$, the fairness of allocation property given by the safety margin of the standard deviation premium setting principle is achieved. □

PROPOSITION 4. *Every scenario $s \in S$ is feasible in the scenario tree at every stage $n = 1, ..., |N|$.*

Proof of Proposition 4 Given Proposition 2, for every $s \in S, n \in N$, in the case where $\sum_{c\in C} Y_c^{sn} > \sum_{c\in C} b_c\mathbb{E}[\mathbf{G}_c^n]$ and constraint (20) needs to be satisfied as equality, constraint (21) ensures that the premiums Y_c^{sn} are sufficiently large to satisfy constraint (20) by setting the unbounded variable Z^{sn} to a suitably large value. □

PROPOSITION 5. *The inventory level I_k at each location $k \in K$ has to respect the minimum required inventory level such that*

$$I_k \geq \max_{h\in H}\left\{\max_{t\in T, t\geq\tau}\left\{\sum_{c\in C}\sum_{i=t-\tau}^{t} Q_{cik}^h\right\}\right\}.$$

Proof of Proposition 5 Given a time period $t \in T$ in a catastrophe realization $h \in H$, the amount of insurance coverage provided to the whole portfolio from location $k \in K$ in the period between a lead time before $t-\tau$ and t is given by $\sum_{i=t-\tau}^{t} Q_{cik}^{h}$. This value corresponds to the minimum amount that needs to be in inventory to be able to provide the insurance coverage without defaulting. Therefore, the minimum inventory level needed to insure the full functioning of the insurance coverage from location k under any catastrophe realization and any time period is the maximum insurance coverage given within any period $t-\tau$ and t of any realization h. □

PROPOSITION 6. *Based on the relaxed non-anticipativity constraints from Proposition 1, the RMP reduces to a two-stage stochastic program by reformulating the objective function and replacing the sn index with h in the objective, variables, and constraints.*

Proof of Proposition 6 The variables included in the RMP are V_c, X_c, I_k, U_k and B (stage-0 variables) and $G_{ct}^{sn}, \sigma_{ct}^{sn}, \delta_{ct}^{sn}, Q_{ctk}^{sn}, A_{tk}^{sn}$ and R_{tk}^{sn} (stage-n variables, $n = 1, ..., |N|$). From Proposition 1, $G_{ct}^{sn} = G_{ct}^{s'n'}$ for any $s \in S, n \in N$ and $s' \in S, n' \in N$ such that $\phi(s,n) = \phi(s',n') = h \in H$. Since the linearization of the variables σ_{ct}^{sn} and δ_{ct}^{sn} depends on V_c, X_c, d_{ct}^{sn} and G_{ct}^{sn}, it follows that $\sigma_{ct}^{sn} = \sigma_{ct}^{s'n'}$ and $\delta_{ct}^{sn} = \delta_{ct}^{s'n'}$ as in Proposition 1. Similarly, we know from constraint (9) that $G_{ct}^{sn} = \sum_{k \in K} Q_{ctk}^{sn}$, which results in $Q_{ctk}^{sn} = Q_{ctk}^{s'n'}$. Finally, the prepositioning variables A_{tk}^{sn} and R_{tk}^{sn} depend on the stage 0 variables I_k and U_k as well as Q_{ctk}^{sn}, which results in $A_{tk}^{sn} = A_{tk}^{s'n'}$ and $R_{tk}^{sn} = R_{tk}^{s'n'}$. Therefore, for any $s \in S, n \in N$ and $s' \in S, n' \in N$ such that $\phi(s,n) = \phi(s',n') = h \in H$, the stage 1 to N variables are equal, which renders the non-anticipativity constraints for these variables redundant, and hence all non-anticipativity constraints from the RMP can be removed. Rewriting the objective function as in Eq. (33) and solving the model over the set of catastrophe realizations H leads to the same optimal solution for the RMP as solving it over all scenarios and all years $S \times N$. This allows to replace the index sn in the variables, objective function, and constraints by the index h, which reduces the RMP to be an equivalent two-stage stochastic model over the set H. □

PROPOSITION 7. *Under the assumption of equiprobable scenarios ($p^s = 1/|S|$) the model can be simplified further since $\mathbb{E}[\boldsymbol{G}_c^n]$ and $\mathbb{S}[\boldsymbol{G}_c^n]$ are the same for every year $n \in N, c \in C$. Consequently, the year index can be dropped and $\mathbb{E}[\boldsymbol{G}_c]$ and $\mathbb{S}[\boldsymbol{G}_c]$ can be used.*

Proof of Proposition 7 Proposition 1, implies that $\sum_{t \in T} G_{ct}^{sn} = \sum_{t \in T} G_{ct}^{s'n'}, c \in C$. Moreover, at every stage $n \in N$, each catastrophe realization $h \in H$ appears $|H|^{n-1}$ times in the nodes of the scenario tree, which yields

$$\mathbb{E}[\mathbf{G}_c^n] = \sum_{s \in S} p^s \sum_{t \in T} G_{ct}^{sn} = \frac{\sum_{s \in S} \sum_{t \in T} G_{ct}^{sn}}{|S|} = \sum_{h \in H} \frac{|H|^{n-1}}{|H|^n} \sum_{t \in T} G_{ct}^{h} = \sum_{h \in H} \frac{1}{|H|} \sum_{t \in T} G_{ct}^{h} \quad n \in N.$$

A similar analysis can be derived for $\mathbb{S}[\mathbf{G}_c^n]$ since it is based on $\mathbb{E}[\mathbf{G}_c^n]$ and G_{ct}^{sn}. □

Appendix D: Data description

Fig. 17 depicts the 18 Caribbean countries in the CDEMA portfolio. Table 7 contains the abbreviations of the countries used throughout the paper. Table 8 presents the cost parameters corresponding to each location $k \in K$, and Table 9 the cost parameter g_{ck} for each location $k \in K$ and country $c \in C$.

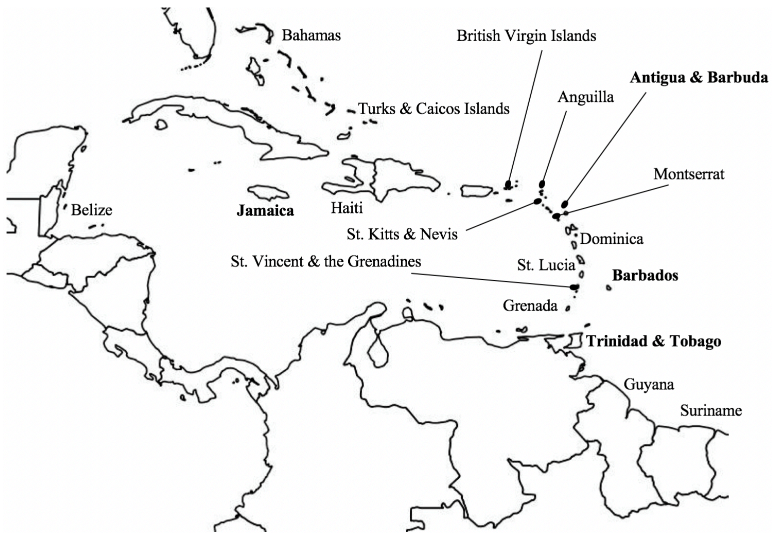

Figure 17 Map of the 18 Caribbean CDEMA member countries (Source: Balcik et al. (2019))

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Table 7 Abbreviation for each country's name.

AIA	Anguilla	GUY	Guyana
ATG	Antigua & Barbuda	HTI	Haiti
BHS	Bahamas	JAM	Jamaica
BLZ	Belize	KNA	St. Kitts & Nevis
BMU	Bermuda	LCA	St. Lucia
BRB	Barbados	MST	Montserrat
BVI	British Virgin Islands	SUR	Suriname
CYM	Cayman Islands	TCA	Turks & Caicos Islands
DMA	Dominica	TTO	Trinidad & Tobago
GRD	Grenada	VCT	St. Vincent & the Grenadines

Table 8 Parameters for each location $k \in K$.

Cost	Candidate warehouse locations									
parameters	ATG	BHS	BLZ	BRB	DMA	GRD	GUY	JAM	SUR	TTO
f_k	209,067	288,131	104,452	149,741	96,754	110,987	105,020	119,307	92,899	139,930
r_k	149.08	149.16	149.03	149.08	149.03	149.06	149.01	149.02	149.03	149.05
h_k	8.94	8.95	8.94	8.95	8.94	8.95	8.94	8.94	8.94	8.95

Table 9 g_{ck} parameter for each country $c \in C$ and warehouse location $k \in K$.

g_{ck} parameter	AIA	ATG	BHS	BLZ	BMU	BRB	BVI	CYM	DMA	GRD	HTI	JAM	KNA	LCA	MST	TCA	TTO	VCT
g_{c1}	139.47	0.00	93.31	91.75	92.98	95.75	139.97	100.36	155.05	94.78	113.27	88.14	112.80	90.84	143.29	93.75	81.53	88.81
g_{c2}	139.47	82.74	0.00	91.75	92.98	97.56	140.63	100.36	152.16	96.57	112.15	87.72	111.12	92.55	139.47	93.31	81.53	87.18
g_{c3}	113.71	60.61	79.16	74.57	75.80	0.00	118.75	83.18	130.51	55.79	96.09	66.63	96.44	54.78	118.75	75.80	47.75	53.98
g_{c4}	139.47	124.12	140.63	137.62	139.47	146.34	0.00	150.54	150.71	141.12	169.04	132.84	165.23	136.26	143.29	140.63	119.82	128.90
g_{c5}	143.29	127.03	140.63	137.62	139.47	146.34	139.97	150.54	0.00	141.12	172.27	135.33	166.02	138.83	143.29	143.29	119.82	128.90
g_{c6}	117.40	59.86	81.51	77.03	78.27	57.20	117.40	85.64	129.76	0.00	98.56	75.85	96.41	58.22	117.40	78.27	49.96	63.55
g_{c7}	150.47	88.94	100.12	99.08	100.32	118.49	153.76	113.57	141.84	113.98	114.85	0.00	111.12	122.18	150.47	102.51	107.83	87.74
g_{c8}	139.47	84.69	95.53	91.75	92.98	95.75	143.29	100.36	155.05	96.57	113.27	90.22	111.12	0.00	143.29	95.53	81.53	88.81
g_{c9}	139.47	84.69	95.53	91.75	92.98	97.56	139.47	100.36	151.83	96.57	113.27	90.22	111.12	92.55	139.47	92.98	81.53	85.94
g_{c10}	139.47	82.74	93.31	91.75	92.98	95.75	143.29	100.36	155.05	94.78	114.85	88.56	111.12	90.84	139.47	92.98	0.00	87.18

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Data description and results tables

Appendix EC.1: Data description tables

Tables EC.1 to EC.4 presents the profile of each country for the five season sets and the two indemnification policies, i.e., their minimum, average, and maximum insurance claim, as well as the standard deviation of the claim.

Table EC.1 Minimum claim data.

Country		EI1	EI2	EI3	EI4	EI5	MI1	MI2	MI3	MI4	MI5
Min claim	AIA	271	271	271	271	271	542	542	542	542	542
	ATG	1,030	1,030	1,030	1,030	1,030	2,060	2,060	2,060	2,060	2,060
	BHS	3,990	3,990	3,990	3,990	3,990	7,979	7,979	7,979	7,979	7,979
	BLZ	4,047	4,047	4,047	4,047	4,047	8,093	8,093	8,093	8,093	8,093
	BMU	611	611	611	611	611	1,222	1,222	1,222	1,222	1,222
	BRB	2,864	2,864	2,864	2,864	2,864	5,727	5,727	5,727	5,727	5,727
	BVI	571	571	571	571	571	1,141	1,141	1,141	1,141	1,141
	CYM	922	922	922	922	922	1,844	1,844	1,844	1,844	1,844
	DMA	1,427	1,427	1,427	1,427	1,427	2,853	2,853	2,853	2,853	2,853
	GRD	1,257	1,257	1,257	1,257	1,257	2,513	2,513	2,513	2,513	2,513
	HTI	111,012	111,012	111,012	111,012	111,012	222,023	222,023	222,023	222,023	222,023
	JAM	28,980	28,980	28,980	28,980	28,980	57,959	57,959	57,959	57,959	57,959
	KNA	559	559	559	559	559	1,117	1,117	1,117	1,117	1,117
	LCA	3,233	3,233	3,233	3,233	3,233	6,466	6,466	6,466	6,466	6,466
	MST	94	94	94	94	94	188	188	188	188	188
	TCA	360	360	360	360	360	719	719	719	719	719
	TTO	13,723	13,723	13,723	13,723	13,723	27,446	27,446	27,446	27,446	27,446
	VCT	1,102	1,102	1,102	1,102	1,102	2,204	2,204	2,204	2,204	2,204

Table EC.2 Expected claim data.

Country		EI1	EI2	EI3	EI4	EI5	MI1	MI2	MI3	MI4	MI5
Expected claim	AIA	807	733	750	795	705	1,196	1,105	1,128	1,184	1,072
	ATG	3,327	3,129	3,267	2,198	2,495	4,871	4,634	4,792	3,485	3,842
	BHS	12,442	10,819	10,041	12,577	12,070	18,189	16,228	15,214	18,527	17,986
	BLZ	12,416	16,002	13,611	14,807	16,002	18,393	22,808	19,865	21,336	22,808
	BMU	1,082	1,082	764	1,082	764	1,808	1,808	1,426	1,808	1,426
	BRB	5,467	4,165	3,515	3,840	4,491	8,851	7,289	6,508	6,898	7,679
	BVI	1,937	2,720	2,435	2,507	2,435	2,827	3,754	3,445	3,445	3,350
	CYM	2,912	3,729	3,184	3,184	5,091	4,190	5,195	4,609	4,525	6,955
	DMA	4,332	3,909	4,754	3,857	3,857	6,339	5,916	6,867	5,863	5,863
	GRD	5,025	3,377	2,474	4,280	2,435	7,145	5,104	4,005	6,203	3,926
	HTI	240,525	400,875	326,867	411,668	380,831	391,624	582,810	468,715	570,475	551,973
	JAM	78,749	68,669	110,878	80,639	110,878	114,658	104,578	153,717	123,477	161,276
	KNA	1,160	1,847	2,018	1,600	1,493	1,825	2,705	2,877	2,405	2,276
	LCA	12,784	6,098	4,188	5,143	8,964	18,221	9,993	7,642	8,817	13,519
	MST	314	308	228	243	277	461	444	347	371	416
	TCA	1,398	1,055	1,234	852	914	1,984	1,562	1,718	1,344	1,422
	TTO	21,640	19,001	19,001	21,640	21,640	36,947	33,780	33,780	36,947	36,947
	VCT	1,682	1,537	1,827	1,537	1,682	2,900	2,726	3,074	2,726	2,900

Table EC.3 Standard deviation of claim data.

Country		EI1	EI2	EI3	EI4	EI5	MI1	MI2	MI3	MI4	MI5
Standard deviation of claims	AIA	687	645	701	746	650	844	792	861	917	798
	ATG	2,838	2,901	2,667	2,196	2,192	3,492	3,569	3,280	2,695	2,687
	BHS	10,570	10,479	9,758	11,498	9,650	12,685	12,527	11,437	14,067	11,920
	BLZ	12,251	13,097	12,652	12,932	13,097	15,078	16,119	15,572	15,916	16,119
	BMU	689	689	432	908	432	836	836	548	1,163	548
	BRB	3,443	2,761	2,058	2,456	3,000	4,132	3,313	2,469	2,948	3,600
	BVI	1,497	1,782	1,637	1,823	1,670	1,851	2,164	2,005	2,142	1,942
	CYM	2,923	3,085	2,918	3,002	2,770	3,434	3,671	3,504	3,547	3,331
	DMA	3,812	3,226	4,162	3,580	3,580	4,565	3,952	5,016	4,431	4,431
	GRD	3,947	3,139	2,733	3,258	1,520	4,859	3,858	3,359	4,009	1,824
	HTI	229,602	312,926	331,173	311,902	317,552	289,584	376,799	359,824	348,261	379,615
	JAM	86,181	72,286	86,158	56,333	83,795	94,573	78,199	96,468	70,581	103,194
	KNA	1,245	1,410	1,592	1,403	1,425	1,456	1,728	1,919	1,736	1,764
	LCA	10,463	7,211	4,377	6,041	9,358	12,877	8,875	5,387	7,435	11,518
	MST	272	261	218	184	261	336	313	255	229	321
	TCA	1,051	962	998	805	886	1,270	1,144	1,095	1,000	1,098
	TTO	14,455	12,378	12,378	14,455	14,455	17,346	14,854	14,854	17,346	17,346
	VCT	1,123	1,005	1,213	1,005	1,123	1,347	1,205	1,455	1,205	1,347

Table EC.4 Maximum claim data.

Country		EI1	EI2	EI3	EI4	EI5	MI1	MI2	MI3	MI4	MI5
Max claim	AIA	2,030	2,030	2,030	2,030	2,030	2,706	2,706	2,706	2,706	2,706
	ATG	7,722	7,722	7,722	7,722	7,722	10,296	10,296	10,296	10,296	10,296
	BHS	37,900	39,894	39,894	31,916	29,920	39,894	39,894	39,894	39,894	39,894
	BLZ	30,349	30,349	30,349	30,349	30,349	40,465	40,465	40,465	40,465	40,465
	BMU	2,139	2,139	2,139	4,278	2,139	3,055	3,055	3,055	6,110	3,055
	BRB	10,022	10,022	10,022	10,022	10,022	14,317	14,317	14,317	14,317	14,317
	BVI	4,276	4,847	4,276	5,701	5,701	5,701	5,701	5,701	5,701	5,701
	CYM	7,835	7,835	7,835	7,835	7,835	9,217	9,217	9,217	9,217	9,217
	DMA	12,123	10,696	12,123	10,696	10,696	14,261	14,261	14,261	14,261	14,261
	GRD	9,422	9,422	9,422	9,422	4,397	12,562	12,562	12,562	12,562	12,562
	HTI	832,586	943,598	1,110,114	1,110,114	943,598	1,110,114	1,110,114	1,110,114	1,110,114	1,110,114
	JAM	289,793	289,793	289,793	217,345	217,345	289,793	289,793	289,793	289,793	289,793
	KNA	4,745	4,186	4,745	4,186	4,186	5,581	5,581	5,581	5,581	5,581
	LCA	24,246	24,246	24,246	24,246	24,246	32,327	32,327	32,327	32,327	32,327
	MST	703	797	797	703	703	937	937	937	937	937
	TCA	3,054	3,054	3,592	2,694	2,694	3,592	3,592	3,592	3,592	3,592
	TTO	48,031	48,031	48,031	48,031	48,031	68,615	68,615	68,615	68,615	68,615
	VCT	3,857	3,857	3,857	3,857	3,857	5,509	5,509	5,509	5,509	5,509

Appendix EC.2: Detailed results tables

This section presents the detailed results from the framework's solutions for the 80 instances. We consider a set of aggregate metrics over the planning horizon and a set of yearly metrics. The aggregate metrics are the total expected cost, the total expected premiums, the total expected emergency outsourcing cost, the worst-case capital surplus, and the total inventory level. For comparison purposes, we also compute the total expected cost of the no-pooling and full-pooling solutions. The yearly financial metrics are the prepositioning network expenses for year 1 and those of every subsequent year, the expected yearly capital consumption in emergency response, the expected yearly outsourcing, the expected yearly aggregate premiums, and the aggregate pure premiums. The values of these metrics for the 80 instances are presented in Tables EC.5 and EC.6. The runtime of the algorithm and the number of RMP and subproblem phases in each instance are given in Table EC.7. We denote each instance by `WX_Y_Z`, where `W` is the indemnification policy (EI for expected loss, and MI for maximal loss), `X` the season set (from 1 to 5), `Y` the planning horizon (from 2 to 5), and `Z` the β value (4 and 7).

Table EC.5 Results for the aggregate metrics.

Instance	Total cost	Total outsourcing cost	Total premiums	Full-pooling cost	No-pooling cost	Total inventory level	
EI0_2_7	527,507,957	328,447,705	199,060,253	701,501,581	704,678,109	302,337	26
EI0_3_7	708,749,087	244,244,253	464,504,834	783,595,028	1,057,017,164	691,262	58
EI0_4_7	837,393,881	218,572,469	618,821,412	864,694,161	1,409,356,218	867,702	73
EI0_5_7	939,281,436	25,006,112	914,275,324	945,191,651	1,761,695,273	1,260,000	105
EI1_2_7	718,846,021	102,757,191	616,088,830	751,040,610	1,013,943,441	848,291	72
EI1_3_7	849,483,648	40,547,544	808,936,105	862,003,104	1,520,915,161	963,934	81
EI1_4_7	964,863,278	5,657,668	959,205,610	969,865,073	2,027,886,882	1,079,851	90
EI1_5_7	1,071,693,036	7,072,085	1,064,620,950	1,075,941,183	2,534,858,602	1,079,851	90
EI2_2_7	708,599,476	270,501,883	438,097,593	932,117,630	901,291,252	664,814	56
EI2_3_7	896,279,450	369,976,439	489,898,571	1,036,725,734	1,351,936,877	691,719	58
EI2_4_7	1,051,617,157	77,976,158	973,640,998	1,139,889,019	1,802,582,503	1,135,765	95
EI2_5_7	1,164,478,759	97,470,198	1,067,008,561	1,242,296,720	2,253,228,129	1,135,765	95
EI3_2_7	860,359,320	276,230,805	584,128,515	1,011,337,817	1,052,162,454	931,217	78
EI3_3_7	1,070,561,836	54,334,899	1,016,226,938	1,135,334,741	1,578,243,681	1,816,441	152
EI3_4_7	1,203,632,437	71,384,203	1,132,248,235	1,257,853,230	2,104,324,909	1,817,363	152
EI3_5_7	1,335,448,136	89,230,253	1,246,217,883	1,379,407,359	2,630,406,136	1,817,363	152
EI4_2_7	731,272,466	55,852,390	675,420,077	770,667,114	1,028,594,951	1,364,054	114
EI4_3_7	852,878,279	20,916,758	831,961,521	884,734,228	1,542,892,426	1,494,462	125
EI4_4_7	968,263,931	26,779,539	941,484,392	996,349,829	2,057,189,901	1,496,440	125
EI4_5_7	1,081,498,080	32,447,471	1,049,050,608	1,106,058,647	2,571,487,377	1,498,284	125
MI0_2_7	799,179,548	422,438,597	376,740,952	1,050,570,541	1,117,183,472	572,039	48
MI0_3_7	1,076,087,716	621,013,264	455,074,452	1,177,846,584	1,675,775,208	595,697	50
MI0_4_7	1,276,011,914	230,033,924	1,045,977,990	1,303,081,032	2,234,366,945	1,432,777	120
MI0_5_7	1,427,179,290	2,199,264	1,424,980,026	1,427,330,018	2,792,958,681	1,908,000	159
MI1_2_7	996,933,331	24,359,110	972,574,222	1,001,265,531	1,488,078,434	1,420,418	119
MI1_3_7	1,157,838,460	3,247,506	1,154,590,954	1,159,366,157	2,232,117,651	1,428,000	119
MI1_4_7	1,311,982,277	3,690,306	1,308,291,971	1,312,804,260	2,976,156,868	1,428,000	119
MI1_5_7	1,463,231,267	4,612,883	1,458,618,384	1,463,413,699	3,720,196,085	1,428,000	119
MI2_2_7	964,417,384	330,408,625	634,008,759	1,108,598,716	1,298,266,820	1,052,592	88
MI2_3_7	1,205,797,132	389,367,550	816,429,582	1,251,046,559	1,947,400,230	1,153,563	97
MI2_4_7	1,390,676,217	0	1,390,676,217	1,390,676,217	2,596,533,640	1,708,620	143
MI2_5_7	1,528,578,042	0	1,528,578,042	1,528,578,042	3,245,667,050	1,708,620	143
MI3_2_7	1,154,031,536	305,518,600	848,512,936	1,264,526,959	1,487,231,717	1,349,358	113
MI3_3_7	1,432,155,318	458,277,900	973,877,418	1,432,206,888	2,230,847,576	1,349,358	113
MI3_4_7	1,596,766,948	(0)	1,596,766,948	1,596,772,310	2,974,463,434	2,459,472	205
MI3_5_7	1,759,373,919	(0)	1,759,373,919	1,759,384,139	3,718,079,293	2,459,472	205
MI4_2_7	1,034,916,332	2,579,775	1,032,336,557	1,036,118,430	1,499,497,116	2,137,923	179
MI4_3_7	1,197,588,910	1,232,342	1,196,356,568	1,197,974,325	2,249,245,673	2,141,157	179
MI4_4_7	1,355,734,830	1,643,123	1,354,091,706	1,355,822,315	2,998,994,231	2,141,157	179
MI4_5_7	1,510,988,914	-	1,510,988,914	1,510,988,914	3,748,742,789	2,144,845	179
EI0_2_4	380,655,624	275,406,382	105,249,242	701,501,581	402,673,205	162,146	14
EI0_3_4	515,912,642	289,081,526	226,831,116	783,595,028	604,009,808	286,696	24
EI0_4_4	646,006,621	376,853,481	269,153,141	864,694,161	805,346,410	299,999	25
EI0_5_4	774,343,292	466,473,573	307,869,720	945,191,651	1,006,683,013	307,716	26
EI1_2_4	559,762,578	456,373,869	103,388,708	751,040,610	579,396,252	188,885	16
EI1_3_4	774,447,759	222,325,389	552,122,369	862,003,104	869,094,378	792,000	66
EI1_4_4	924,608,911	291,718,851	632,890,061	969,865,073	1,158,792,504	792,000	66
EI1_5_4	1,048,315,247	146,795,987	901,519,259	1,075,941,183	1,448,490,630	848,291	71
EI2_2_4	483,667,098	406,428,716	170,088,161	932,117,630	515,023,572	144,000	12
EI2_3_4	693,227,563	457,791,228	235,436,335	1,036,725,734	772,535,358	303,741	26
EI2_4_4	874,827,418	350,720,140	524,107,278	1,139,889,019	1,030,047,145	619,187	52
EI2_5_4	996,444,519	352,358,513	622,979,531	1,242,296,720	1,287,558,931	691,719	58
EI3_2_4	587,349,291	496,293,650	91,055,640	1,011,337,817	601,235,688	141,910	12
EI3_3_4	827,401,653	600,760,122	226,641,531	1,135,334,741	901,853,532	286,030	24
EI3_4_4	1,057,160,685	515,497,290	541,663,395	1,257,853,230	1,202,471,376	659,497	55
EI3_5_4	1,224,708,321	394,615,436	830,092,885	1,379,407,000	1,503,089,220	931,217	78
EI4_2_4	572,085,013	461,843,633	110,241,380	770,667,114	587,768,543	168,000	14
EI4_3_4	765,898,798	304,342,354	461,556,444	884,734,228	881,652,815	754,116	63
EI4_4_4	933,267,778	392,340,153	540,927,625	996,349,829	1,175,537,086	779,034	65
EI4_5_4	1,066,933,423	20,796,717	1,046,136,706	1,106,058,647	1,469,421,358	1,494,462	125
MI0_2_4	605,725,106	446,707,317	159,017,789	1,050,570,541	638,390,556	241,867	21
MI0_3_4	805,279,858	371,288,658	433,991,200	1,177,849,436	957,585,833	552,000	46
MI0_4_4	992,684,622	483,037,089	509,647,533	1,303,074,777	1,276,781,111	571,697	48
MI0_5_4	1,177,706,698	601,473,371	576,233,327	1,427,330,018	1,595,976,389	575,984	48
MI1_2_4	821,368,837	620,032,628	201,336,209	1,001,265,531	850,330,534	351,583	30
MI1_3_4	1,120,338,494	206,198,801	914,139,693	1,159,366,157	1,275,495,800	1,390,553	116
MI1_4_4	1,306,408,060	27,838,983	1,278,569,077	1,312,804,260	1,700,661,067	1,420,418	119
MI1_5_4	1,460,478,978	18,796,410	1,441,682,568	1,463,413,699	2,125,826,334	1,428,000	119
MI2_2_4	718,659,132	601,912,874	116,746,258	1,108,598,716	741,866,754	177,200	15
MI2_3_4	964,444,000	505,311,063	459,132,937	1,251,046,559	1,112,800,131	598,506	50
MI2_4_4	1,198,442,338	642,442,753	555,999,585	1,390,691,714	1,483,733,509	660,000	55
MI2_5_4	1,394,210,524	372,392,567	1,021,817,956	1,528,606,875	1,854,666,886	1,152,000	96
MI3_2_4	826,317,628	699,542,353	126,775,275	1,264,526,959	849,846,696	191,998	16
MI3_3_4	1,138,806,148	706,791,822	432,014,326	1,432,206,888	1,274,770,043	549,259	46
MI3_4_4	1,430,777,529	859,526,910	571,250,618	1,596,772,310	1,699,693,391	666,338	56
MI3_5_4	1,651,422,121	436,455,142	1,214,966,979	1,759,384,139	2,124,616,739	1,349,358	113
MI4_2_4	830,087,069	628,727,660	201,359,410	1,036,118,430	856,855,495	310,748	26
MI4_3_4	1,109,076,927	542,596,002	566,480,926	1,197,974,325	1,285,283,242	749,989	63
MI4_4_4	1,344,410,402	481,912,293	862,498,110	1,355,822,315	1,713,710,989	1,163,999	97
MI4_5_4	1,510,050,906	2,590,293	1,507,460,613	1,510,988,914	2,142,138,736	2,137,923	179

Table EC.6 Results for the yearly metrics.

Instance	Year 1 expenses	Year > 1 expenses	Yearly emergency response	Yearly outsourcing cost	Yearly premiums					Pure premiums
					1	2	3	4	5	
EI0_2_7	51,169,194	6,107,181	22,416,658	164,223,852	159,834,528	39,225,724				27,685,441
EI0_3_7	116,041,546	13,017,204	32,911,971	81,414,751	345,300,053	61,076,724	58,128,057			39,515,313
EI0_4_7	145,497,747	16,179,651	35,697,711	54,643,117	421,713,057	67,983,701	65,122,728	64,001,926		43,339,832
EI0_5_7	210,644,948	22,867,148	41,200,638	5,001,222	594,783,684	82,167,775	79,940,352	78,913,634	78,469,879	50,431,531
EI1_2_7	142,083,383	15,660,714	56,839,444	51,378,596	513,139,549	102,949,281				66,252,870
EI1_3_7	161,292,217	17,634,676	61,099,354	13,515,848	589,190,547	112,490,813	107,254,744			71,661,834
EI1_4_7	180,368,374	19,439,023	62,302,974	1,414,417	625,988,845	115,638,584	110,342,869	107,235,311		73,390,610
EI1_5_7	180,368,374	19,439,023	62,302,974	1,414,417	625,988,845	115,638,584	110,342,869	107,235,311	105,415,341	73,390,610
EI2_2_7	111,288,562	12,212,752	37,965,065	135,250,942	368,726,815	69,370,778				46,094,737
EI2_3_7	115,749,611	12,663,963	38,951,365	123,325,480	354,708,725	69,500,037	65,689,809			47,798,375
EI2_4_7	189,752,557	20,490,760	51,733,130	19,494,040	683,059,005	99,968,465	96,180,051	94,433,476		62,631,438
EI2_5_7	189,567,198	20,306,961	51,886,831	19,494,040	683,134,781	99,948,176	96,154,997	94,410,298	93,360,309	62,631,438
EI3_2_7	155,853,839	17,072,202	47,485,311	138,115,402	496,151,095	87,977,421				56,635,858
EI3_3_7	303,236,866	32,531,824	61,740,322	18,111,633	778,474,989	120,699,800	117,052,148			73,779,254
EI3_4_7	303,465,681	32,624,913	61,690,267	17,846,051	779,263,099	120,721,648	117,078,077	115,185,411		73,817,194
EI3_5_7	303,511,370	32,670,269	61,593,792	17,846,051	779,328,444	120,707,228	117,056,729	115,157,775	113,967,707	73,817,194
EI4_2_7	227,942,663	24,653,528	61,472,446	27,926,195	562,871,333	112,548,744				70,666,629
EI4_3_7	249,801,172	27,079,238	61,944,516	6,972,253	604,763,932	116,043,615	111,153,974			73,660,049
EI4_4_7	250,120,879	27,103,589	61,938,971	6,694,885	605,595,679	116,093,432	111,198,828	108,596,453		73,699,673
EI4_5_7	250,414,549	27,124,050	61,320,097	6,489,494	607,917,627	115,538,134	110,740,372	108,213,537	106,640,938	73,729,015
MI0_2_7	96,344,522	11,087,273	41,759,651	211,219,298	303,464,225	73,276,726				50,911,712
MI0_3_7	100,339,464	11,555,565	42,148,830	207,004,421	311,888,081	74,400,330	68,786,041			51,513,837
MI0_4_7	239,936,799	26,403,964	60,215,291	57,508,481	714,415,028	114,613,832	109,587,718	107,361,412		72,870,400
MI0_5_7	319,064,471	34,712,231	66,529,553	439,853	917,928,614	131,076,845	126,924,876	124,954,318	124,095,374	81,023,061
MI1_2_7	237,341,400	25,654,706	90,898,522	12,179,555	808,532,876	164,041,346				106,265,756
MI1_3_7	238,581,584	25,767,224	91,295,682	1,082,502	830,213,308	166,590,409	157,787,237			107,851,050
MI1_4_7	238,581,584	25,767,224	91,316,495	922,577	830,670,330	166,631,382	157,824,046	153,166,213		107,873,896
MI1_5_7	238,581,584	25,767,224	91,316,495	922,577	830,670,330	166,631,382	157,824,046	153,166,213	150,326,413	107,873,896
MI2_2_7	175,926,879	19,060,995	58,322,973	165,204,313	528,294,352	105,714,407				70,628,427
MI2_3_7	192,810,306	20,896,648	61,812,187	129,789,183	594,800,826	114,321,125	107,307,632			75,687,732
MI2_4_7	285,297,396	30,663,650	78,047,679	0	959,455,797	149,208,293	142,382,469	139,629,657		94,229,043
MI2_5_7	285,312,076	30,678,505	77,926,509	0	959,676,663	149,142,322	142,312,491	139,560,238	137,886,328	94,229,043
MI3_2_7	225,822,544	24,724,583	72,658,754	152,759,300	715,945,130	132,567,806				86,121,480
MI3_3_7	225,779,169	24,681,340	71,402,249	152,759,300	716,076,425	132,476,831	125,324,163			86,121,480
MI3_4_7	410,686,831	44,152,559	89,810,271	0	1,090,316,899	174,248,688	167,648,870	164,552,490		107,944,238
MI3_5_7	410,686,831	44,152,559	89,810,271	0	1,090,316,899	174,248,688	167,648,870	164,552,490	162,606,972	107,944,238
MI4_2_7	357,276,494	38,659,405	92,080,878	1,289,888	862,769,567	169,566,990				108,650,198
MI4_3_7	357,794,486	38,694,934	91,134,906	410,781	864,889,610	169,709,882	161,757,076			108,775,785
MI4_4_7	357,809,403	38,709,827	91,013,038	410,781	865,055,577	169,643,770	161,691,122	157,701,238		108,775,785
MI4_5_7	358,394,256	38,746,774	90,399,046	0	868,133,824	169,195,781	161,272,849	157,345,796	155,040,663	108,834,468
EI0_2_4	27,854,550	3,684,569	13,292,308	137,703,191	82,606,121	22,643,120				16,720,194
EI0_3_4	48,453,739	5,722,874	21,701,915	96,360,509	153,153,141	38,313,005	35,364,970			27,055,865
EI0_4_4	50,695,504	5,981,853	22,108,721	94,213,370	158,820,245	39,068,325	36,139,476	35,125,095		27,592,649
EI0_5_4	51,947,663	6,084,667	22,282,977	93,294,715	161,781,498	39,371,358	36,455,824	35,434,147	34,826,893	27,822,313
EI1_2_4	31,740,042	3,590,435	13,128,982	228,186,935	81,451,700	21,937,008				16,545,935
EI1_3_4	132,329,072	14,298,752	45,873,974	74,108,463	389,210,638	83,782,603	79,129,128			55,065,553
EI1_4_4	132,432,912	14,402,112	46,070,373	72,929,713	391,916,071	84,295,667	79,650,883	77,027,439		55,360,241
EI1_5_4	141,801,380	15,379,179	56,392,230	29,359,197	513,430,605	102,872,130	97,752,069	94,645,962	92,818,494	66,252,870
EI2_2_4	24,372,478	2,909,518	11,963,530	203,214,358	133,192,986	36,895,174				14,612,711
EI2_3_4	51,076,591	5,808,793	22,315,015	152,597,076	160,457,416	38,915,331	36,063,587			27,267,032
EI2_4_4	103,655,760	11,379,378	35,135,682	87,680,035	338,679,742	64,889,233	61,128,828	59,409,475		43,496,292
EI2_5_4	115,556,528	12,472,466	38,890,565	70,471,703	358,863,960	70,233,291	66,261,300	64,385,096	63,235,884	47,798,375
EI3_2_4	24,227,387	3,074,612	11,546,393	248,146,825	72,054,669	19,000,972				14,329,923
EI3_3_4	48,290,645	5,659,291	21,443,278	200,253,374	154,625,900	37,370,328	34,645,303			26,303,286
EI3_4_4	110,528,069	12,240,656	36,269,567	128,874,323	350,976,047	66,564,645	62,980,833	61,141,870		44,148,049
EI3_5_4	155,853,839	17,072,202	47,211,972	78,923,087	496,151,095	87,977,421	83,982,838	81,713,671	80,267,860	56,635,858
EI4_2_4	28,409,484	3,370,284	13,448,387	230,921,816	87,833,722	22,407,658				16,925,631
EI4_3_4	126,541,248	14,152,701	40,444,012	101,447,451	318,578,847	73,545,364	69,432,233			49,294,222
EI4_4_4	130,458,080	14,357,808	41,504,815	98,085,038	326,331,409	75,048,167	70,895,835	68,652,213		50,134,826
EI4_5_4	249,720,135	27,000,100	61,201,703	4,159,343	605,887,865	115,325,252	110,521,450	107,990,732	106,411,407	73,616,249
MI0_2_4	41,393,348	5,340,671	20,218,567	223,353,659	124,993,903	34,023,886				25,247,483
MI0_3_4	92,934,182	10,663,622	40,630,408	123,762,886	295,397,626	72,176,637	66,416,937			50,145,176
MI0_4_4	96,290,496	11,084,215	41,217,946	120,759,272	303,284,521	73,251,654	67,532,177	65,579,181		50,896,079
MI0_5_4	96,896,329	11,051,913	41,302,767	120,294,674	304,924,624	73,383,821	67,654,319	65,697,612	64,572,950	51,012,229
MI1_2_4	59,125,290	6,727,798	24,614,450	310,016,314	160,011,676	41,324,533				30,501,614
MI1_3_4	232,143,648	24,910,926	75,668,453	68,732,934	645,034,729	138,602,353	130,502,611			90,822,459
MI1_4_4	237,505,036	25,817,471	89,440,785	6,959,746	808,696,512	163,909,850	155,314,262	150,648,454		106,265,756
MI1_5_4	238,704,338	25,887,578	90,000,863	3,759,282	819,567,838	165,117,734	156,396,616	151,727,973	148,872,407	107,065,872
MI2_2_4	29,930,438	3,519,682	15,544,717	300,956,437	91,327,741	25,418,518				18,989,934
MI2_3_4	100,178,357	10,982,198	42,353,833	168,437,021	314,901,868	75,037,603	69,193,466			52,119,788
MI2_4_4	110,450,707	12,089,827	43,836,626	160,610,688	335,793,654	78,027,949	72,090,591	70,087,391		54,076,371
MI2_5_4	192,473,478	20,793,078	61,603,391	74,478,513	594,084,415	114,067,189	107,046,116	104,170,966	102,449,271	75,609,415
MI3_2_4	32,750,591	4,131,249	16,482,167	349,771,177	99,927,411	26,847,864				20,501,443
MI3_3_4	92,392,985	10,531,053	40,320,361	235,597,274	295,817,958	70,815,410	65,380,957			49,044,919
MI3_4_4	111,949,889	12,641,658	43,758,551	214,881,728	349,862,282	78,176,961	72,604,821	70,606,554		54,223,806
MI3_5_4	225,793,956	24,696,195	71,232,358	87,291,028	716,305,899	132,395,797	125,232,353	121,590,166	119,442,763	86,121,480
MI4_2_4	52,435,672	6,122,112	23,880,715	314,363,830	161,102,880	40,256,529				30,243,511
MI4_3_4	126,061,548	14,287,588	52,250,240	180,865,334	387,724,808	92,774,581	85,981,538			63,618,135
MI4_4_4	195,125,743	21,651,492	64,869,078	120,478,073	526,614,578	117,947,299	110,773,618	107,162,615		78,714,950
MI4_5_4	357,293,133	38,677,195	90,142,451	518,059	865,704,659	168,938,465	160,999,637	157,065,808	154,752,044	108,704,953

Table EC.7 Runtime results.

Instance	Run time (hours)	Num. RMP iterations	Num. Optimality cuts
EI0_2_7	0.11	4	3
EI0_3_7	0.00	3	2
EI0_4_7	0.06	4	3
EI0_5_7	2.99	3	2
EI1_2_7	0.01	3	2
EI1_3_7	0.01	3	2
EI1_4_7	0.04	4	3
EI1_5_7	1.68	3	2
EI2_2_7	0.01	3	2
EI2_3_7	0.02	5	4
EI2_4_7	0.05	4	3
EI2_5_7	3.35	4	3
EI3_2_7	0.01	3	2
EI3_3_7	0.01	3	2
EI3_4_7	0.06	4	3
EI3_5_7	3.16	4	3
EI4_2_7	0.01	3	2
EI4_3_7	0.01	3	2
EI4_4_7	0.04	4	3
EI4_5_7	2.68	4	3
MI0_2_7	0.25	3	2
MI0_3_7	0.01	3	2
MI0_4_7	0.06	5	4
MI0_5_7	3.84	4	3
MI1_2_7	0.01	3	2
MI1_3_7	0.02	3	2
MI1_4_7	0.03	3	2
MI1_5_7	1.67	3	2
MI2_2_7	0.02	3	2
MI2_3_7	0.01	3	2
MI2_4_7	0.19	4	3
MI2_5_7	3.07	4	3
MI3_2_7	0.01	3	2
MI3_3_7	0.02	4	3
MI3_4_7	0.10	4	3
MI3_5_7	2.74	4	3
MI4_2_7	0.00	2	1
MI4_3_7	0.01	3	2
MI4_4_7	0.04	4	3
MI4_5_7	2.42	4	3
EI0_2_4	5.50	4	3
EI0_3_4	0.12	3	2
EI0_4_4	0.20	3	2
EI0_5_4	3.50	4	3
EI1_2_4	1.30	7	6
EI1_3_4	0.04	3	2
EI1_4_4	0.07	5	4
EI1_5_4	2.40	4	3
EI2_2_4	2.57	6	5
EI2_3_4	2.15	6	5
EI2_4_4	2.06	4	3
EI2_5_4	2.29	3	2
EI3_2_4	0.31	7	6
EI3_3_4	0.01	3	2
EI3_4_4	0.05	4	3
EI3_5_4	1.48	3	2
EI4_2_4	2.28	4	3
EI4_3_4	0.36	4	3
EI4_4_4	0.04	4	3
EI4_5_4	1.87	3	2
MI0_2_4	3.11	4	3
MI0_3_4	1.22	3	2
MI0_4_4	2.34	4	3
MI0_5_4	4.47	4	3
MI1_2_4	0.13	4	3
MI1_3_4	0.04	3	2
MI1_4_4	0.04	4	3
MI1_5_4	1.71	3	2
MI2_2_4	0.32	6	5
MI2_3_4	0.03	3	2
MI2_4_4	0.24	5	4
MI2_5_4	2.10	3	2
MI3_2_4	0.13	5	4
MI3_3_4	0.01	3	2
MI3_4_4	0.03	3	2
MI3_5_4	2.13	3	2
MI4_2_4	1.60	7	6
MI4_3_4	0.20	3	2
MI4_4_4	2.08	4	3
MI4_5_4	1.92	3	2