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Security auctions with cash- and equity-bids: An experimental study*

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Abstract

We study the performance of cash- and equity-bid security auctions in an experiment using first- and second-price pricing rules. Theory predicts revenue equivalence between first- and second-price formats, equity auctions to generate more revenue than cash auctions, and for all formats to be efficient. We find that, on average, equity auctions produce larger revenue than cash auctions in absolute terms but not relative to equilibrium predictions. Important factors driving this result are substantial underbidding, large variance, and bidding functions being flatter in equity auctions. Furthermore, we find that first-price auctions produce larger revenues than second-price auctions, and cash auctions to be more efficient.

Keywords: equity auctions, security auctions, bidding behavior, revenue, efficiency.

JEL classification codes: C91, D44, G32.

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1 Introduction

Auctions are an integral part of many economic transactions, ranging from corporate takeovers (Dasgupta & Hansen, 2007) and selling collectibles (Ashenfelter, 1989) to mineral rights (Rothkopf & Engelbrecht-Wiggans, 1992). While such items are auctioned using very different formats they all share a feature: The value of the item being sold is uncertain or contingent upon future events, i.e. these are security auctions. In a seminal paper, DeMarzo et al. (2005) study the optimal design of such auctions and show that various auction formats generate different revenues for the auctioneer.

Interestingly, many security auctions have distinct features that allow sellers and bidders to better align their interests. For instance, in auctions for mineral rights, such as OCS Wildcat auctions (Li et al., 2000) bidders submit sealed cash bids, often for tracts with pre-defined royalty rates (Cramton, 2007). The royalty rates help lower the cash-price of tracts and introduces some risk-sharing between the seller and the bidder. Yet, as Cramton (2007) notes, in some jurisdictions there are also production-sharing contracts (PSCs): “With PSCs, it is common for bidding to be over the government’s highest profit share” (p. 127). In other words, in these auctions bidders bid with a share of future profits for an asset whose value is uncertain and where bids are contingent on a (later) realized value.

In this paper we design an experiment to investigate the performance of cash- and equity-bids in a security auction using first- and second-price pricing rules. We consider an auction in which bidders receive information about the distribution of the private value that they attach to the item being auctioned. Regardless of how bids are placed, in all auctions an asset is being sold, hence a ‘security auction’. In our design, bidders know the distribution of possible private values but not the ‘realized private value’. The realization of this value will be revealed only after all bids have been submitted.

Our design is different from many other security auctions in the sense that the realized value of the item is private rather than common to all bidders. One way to think about this choice, is that the item being auctioned is a scarce resource for which each bidder has a private technology to transform the resource into a revenue stream. The auctioneer does not observe the technology but only the bids from the bidders and selects the higher one as the winner of the auction.

We experimentally compare cash- and equity-bids in both a first- and second-price setting. We find that, in line with the theoretical prediction, equity auctions generate, on average, higher revenues than cash auctions in absolute terms. However, the difference is only significant under the first-price pricing rule. Comparing bids relative to equilibrium predictions, equity auctions generate less revenue than cash auctions under both pricing rules. Furthermore, we find that equity auctions are less efficient than cash auctions. Our results are in line with the theoretical predictions presented in Che and Kim (2010) on adverse selection in equity auctions – i.e. when the security being auctioned is steeper, the ‘wrong bidder’ wins more often. Other important factors driving low revenues relative to equilibrium predictions in the equity auctions are the substantial underbidding with respect to risk neutral Nash equilibrium bidding strategies as well as flatter equilibrium bidding functions.

We next compare the first- and second-price pricing rules. While theory predicts revenue to be equivalent between these two pricing rules for both the cash and the equity auctions,¹ we observe that first-price auctions generate higher revenues than second-price auctions in both cash and equity domains. In second-price auctions, on average, bidders bid lower with respect to equilibrium. Furthermore, due to order-statistics of the bids, the generated revenues in the second-price auctions are lower. This is not surprising, as higher revenues in first-price auctions are observed in several previous studies utilizing standard cash bids (Kagel & Levin, 1993; Cox et al., 1982; Kirchkamp et al., 2009). In accordance to theoretical prediction, we do not observe significant efficiency differences between pricing rules in the equity auctions.

Prior research on security auctions dates back to Hansen (1985) who shows that the revenue generated by second-price equity auctions is strictly higher than by cash auctions. This paper was later extended by Riley (1988) to first-price and common-value auctions. Since then, there have been several theoretical and empirical studies in the field such as Nachman and Noe (1994), DeMarzo and Duffie (1999), Giebe and Wolfstetter (2008), Hege et al. (2009), Gorbenko and Malenko (2011, 2014, 2018) and Abhishek et al. (2015). One of the most recent papers, Hernandez-Chanto and Fioriti (2019), studies a seller’s expected revenue in security auctions in the presence of negative externality. They find that without negative externality, the equity auction leads to the highest expected revenue, while with

¹Note that revenue equivalence is not satisfied for all types of securities; see DeMarzo et al. (2005).

negative externality the equity auction leads to the lowest.

Prior experimental research on security auctions is limited. Kogan and Morgan (2010) is an exception and closely related to our paper. In their paper the authors investigate, both theoretically and experimentally, the moral hazard problem in debt and equity auctions. The equity auction bears a high resemblance with the equity auction in our paper and similarly, the debt auction in their paper is similar to our cash auction. The authors implement a two-by-two (high/low returns to effort and debt/equity contracts) between-subjects design. The main results of their study show that debt auctions have higher efficiency compared to the equity auction and that there are smaller differences in revenues than predicted theoretically.

Our paper is different from Kogan and Morgan (2010). They study a very specific problem in their paper: An entrepreneur raising funds for a risky project. Thus, effort and moral hazard are key elements in their analysis. In our paper, the focus is exclusively on auction design and we do not model effort nor moral hazard. This is an important difference, as a risky asset is not always associated with a moral hazard problem and as a result the main design difference between Kogan and Morgan (2010) and this paper is that in our setup there is no effort stage. Instead of the effort stage, we utilize a strategy method to elicit bids. Employing a strategy method has the key advantage that it allows us to not only observe a single bid from each bidder but four different ones: one bid for each signal. Since each signal has a different risk and return trade-off associated with it, we are able to study how bids are placed relative to the underlying uncertainty of the asset under the hammer.

Synchronous to our study, Breig et al. (2022) also conducted security auction experiments. While similar in nature, our studies differ in important respects. One difference to our work is that they study debt auctions instead of cash auctions. Two attractive features of cash auctions are that, like in equity auctions, the final price to be paid is linear in the winning bid and revenues are equivalent between first- and second-price payment rules; features that are not present in debt auctions. Also the experimental implementations differ. Our study is a one-shot auction using a strategy method, while their study utilizes repeated decision making without the use of the strategy method. Furthermore, in our design the item being auctioned can realize multiple future values and signals are discrete, whereas their design includes only two possible future values and continuous signals. De-

spite these differences in design and procedure, as far as our studies can be compared, we report similar findings, including low performance of second-price equity auction and the minor role of risk attitude in explaining overbidding behavior.

The remainder of the paper is structured as follows. In Section 2, we introduce the experimental design and procedures. We next present our findings in Section 3 and search for underlying explanations in Section 4. Section 5 concludes.

2 Experiment

2.1 Setting

We consider the situation where two bidders² are bidding for an object for which the bidders’ private valuations are independent and uncertain. There are five possible values of the asset and these are commonly known to all bidders. The bidders receive private information about the probabilities by which each of these values may realize via one of four possible private signals. The private signals are drawn independently for the two bidders with each private signal having equal chance. Table 1 provides the parameters that we use in our experiment. There are two important properties associated with the values we chose. First, the values satisfy the strict monotone likelihood ratio property and second, the expected private valuations are linearly increasing over the four signals: They increase from 400 at the lowest signal to 1000 for the highest signal in constant steps of 200.

Private Value	(25%) Signal s_1	(25%) Signal s_2	(25%) Signal s_3	(25%) Signal s_4
200	70%	50%	30%	10%
600	15%	20%	25%	30%
1000	10%	15%	20%	25%
1400	5%	10%	15%	20%
1800	0%	5%	10%	15%

Table 1: Private valuations and signals.

We compare bidding behavior, revenues and efficiency between four different auction

²The choice of two bidders may appear restrictive but it is important to note that in many security auctions, the number of (potential) bidders is very low. We elaborate further on this in Section 5.

formats. These four auction formats have three properties in common. First, the winner is the bidder with the highest bid; and in case of equal bids, the bidder with the highest signal wins;³ and a coin-flip being decisive in case also signals are equal. Second, the losing bidder will have a zero payoff from the auction. Third, the benefit to the winner is their realized private valuation, which is drawn according to the probability distribution corresponding to their signal.

Differences between the four auction formats are along two design dimensions. The first dimension is the *bidding domain* on which bidders bid; the second dimension regards the *pricing rule*. On the bidding domain dimension, both bidders specify their bids either (1) in the *cash* domain or (2) in the *equity* domain (a percentage of future revenues). Furthermore, we have two variations on the pricing rule: the price to be paid by the winner is determined either (1) by the *winner's* bid or (2) by the *loser's* bid. Table 2 illustrates the four auction formats considered and introduces the labels used to refer to the four different treatments.

	Cash (C)	Equity (E)
First-price (FP)	FPC	FPE
Second-price (SP)	SPC	SPE

Table 2: The four auction formats (treatments) considered and compared.

Specifically, in the first-price cash auction (FPC), both bidders provide their bids in cash amounts and the price the winner pays is the amount equal to their winning bid. In the second-price cash auction (SPC), both bidders provide their bids in cash amounts and the price the winner pays is the amount equal to their rival bidder's bid. In the first-price equity auction (FPE), both bidders provide their bids as shares and the price the winner pays is a share of their realized private valuation in addition to a fixed amount of 200, where the share is equal to their winning bid. Finally, in the second-price equity auction (SPE), both bidders provide their bids as shares and the price the winner pays is a share of their realized private valuation in addition to a fixed amount of 200, where the share is

³Example 2 in Maskin and Riley (2000) shows that in first-price auctions with a finite type space a random tie-breaking rule may prevent the existence of the equilibrium. Wang et al. (2020) prove the existence of the equilibrium in this setting assuming that when there are multiple highest bids, the item will be allocated to the bidder with the highest value.

equal to their rival bidder’s bid.⁴

2.2 Equilibrium predictions and hypothesis

Appendix A specifies the equilibrium bids for each of the four auction formats, assuming symmetric bidding behavior and risk-neutrality. Notably, for both first-price auctions, equilibrium bidding involves randomization at the highest three signals. Based on these bidding functions, Table 3 presents the expected equilibrium bids at each signal in the four auction formats. In parenthesis the equivalent expected bid in the other bidding domain, using the transformation $\text{cash} = 200 + \frac{\text{share}}{100} \times \text{ExpPrivVal}$. The last column shows the resulting ex ante expected auction revenue for each of the four auction formats.

Auction format	Bids in cash (equivalence)				Bids in share (equivalence)				Revenue
	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4	
FPC	400.00	461.37	556.72	654.78	(50.00)	(43.56)	(44.59)	(45.58)	575.00
SPC	400.00	600.00	800.00	1000.00	(50.00)	(66.67)	(75.00)	(80.00)	575.00
FPE	(400.00)	(530.69)	(691.88)	(860.95)	50.00	55.11	61.48	66.10	718.75
SPE	(400.00)	(600.00)	(800.00)	(1000.00)	50.00	66.67	75.00	80.00	718.75

Table 3: Expected equilibrium bids at the different signals for each treatment (in parenthesis the equivalent expected bid on the other bidding domain), together with the resulting (ex ante) expected revenue.

Several observations can be made on basis of the information presented in Table 3. We use these observations to formulate the main hypotheses presented below. Our first series of hypotheses relate to bidding behavior, starting with bidding behavior within auction format.

Hypothesis 1 *For all four treatments, bids are increasing in the signal.*

⁴The fixed payment is needed to guarantee the existence of an equilibrium with increasing bidding functions. We have chosen for the value 200, in order to find sufficient variation in equilibrium bids across signals. Equilibrium bids range from 50% to 68% in FPE and from 50% to 80% in SPE. A lower value for the fixed amount would lower the variation between signals and make it hard to detect differences in the experiment, especially since we restrict bids to be integers. This fixed amount is not separately included in the cash auctions, since it is naturally embedded as part of the cash bid. Note that in our data, only 4% of the in total 496 reported cash bids are below 200. Furthermore, 88% of the 124 participants do not bid below 200 for any of the four signals.

For comparisons of bidding behavior across auction formats, we use the expected equilibrium cash bids for the cash auctions and for the equity auctions we use the cash equivalences of the expected equilibrium share bids.

Hypothesis 2 *Comparing bids across pricing-rules for both cash and equity auctions, bids are equal at the lowest signal and higher under the second-price pricing rule at the highest three signals, and hence also averaged over all four signals.*

Hypothesis 3 *Comparing bids between cash and equity auctions,*

- (a) under the first-price pricing rule, bids are equal at the lowest signal, and higher for the equity auction at the highest three signals, and hence also averaged over all four signals;*
- (b) under the second-price pricing rule, bids are equal at all signals, and hence also averaged over all four signals.*

The second series of hypotheses is related to market outcomes, starting with the seller's (expected) revenue from the auction.

Hypothesis 4 *For both bidding domains, the first-price and the second-price pricing rule generate equivalent revenues; and*

Hypothesis 5 *Under both pricing rules, the equity auction generates (in expectation) higher revenue than the cash auction.*

For all four auction formats expected equilibrium bids are increasing in signal. Given that the highest bidder wins the auction, the object will in all four auction formats be allocated to the bidder with the highest signal.⁵ This means that all auctions are 100 percent efficient from an ex ante perspective (i.e., a bidder with the higher signal wins) and this lets us state our last hypothesis as:

Hypothesis 6 *There is no difference in observed efficiency across the four treatments.*

⁵Note here that the intervals over which the bidders randomize in the first-price auctions are disjoint for each pair of consecutive signals.

2.3 Design and procedures

We conducted the experiment with the four formats as between-subjects treatment variations. Participants (18–65 yrs, fluent in English) were recruited via Prolific via which they were redirected to our oTree (Chen et al., 2016) server. After giving informed consent, the participants were presented with the instructions. In the instructions, they were given detailed information on the auction situation and procedures relevant to their treatment situation (see Appendix B). After reading the instructions, and before making their bidding decisions, each participant had to go through five pages of control questions in which we checked their understanding of the instructions. The procedure was implemented to ensure full understanding of the decision framework, which is very important given that decisions were made in a one-shot fashion.

For their bidding decisions, we asked participants to provide a bid for each possible private signal. On the decision making screen, the most important information – a table equivalent to Table 1 – was presented on the top of their screen, just above the four input fields in which they could enter their bids. Furthermore, at the bottom of the decision screen, subjects had at all times access to summarized instructions. All bidders were given an initial budget of 800, and we provided an unexpected gift of another 400 at the end. This was done to prevent extremely cautious bidding as well as to avoid a bankruptcy situation.⁶ At the end of the experiment, participants received a payment in British Pound Sterling (GBP) with an exchange rate of 500 points = 1 GBP.

After placing bids and before receiving feedback from the auction, the session continued with several additional tasks. In these tasks we elicited information relevant towards understanding bidding behavior. First, we elicited gender and age. Second, risk attitude, for which we used the dynamic version of the bomb risk elicitation task (BRET) developed by Crosetto and Filippin (2013) and implemented in oTree by Holzmeister and Pfurtscheller (2016) as well as the self-assessment question of Dohmen et al. (2011). Third, loss attitude, for which we used a BDM mechanism eliciting the maximum acceptable loss in a lottery. Fourth, we elicited their risk literacy using the Berlin numeracy test of Cokely et al. (2012).

⁶For all signals it is possible that the private value realizes as 200, while in the cash treatments bidders were expected to bid higher than that. Bankruptcies are less likely in the equity auctions, since a large part of the price concerns a share of the realized private value. Our precautions resulted in none of the participants to experience bankruptcy.

Apart from the demographic questions and their self-assessed risk attitude, all tasks were incentivized. Explanations of some of the key methods employed to elicit the individual characteristics in the experiment are provided in Appendix C.

Before running the experimental sessions, we were given ethical approval via University of Bath Social Science Research Ethics Review Board and Vrije Universiteit School of Business and Economics Research Ethics Review Board (SBE6/28/2021kwk350). Moreover, we pre-registered the experiment via the American Economic Association’s registry for randomized controlled trials (AEARCTR-0007950; <https://doi.org/10.1257/rct.7950-1.0>).

2.4 Data validity

Online experiments are now commonplace in economic research and have experienced an exploding growth since the global Covid-19 outbreak began. Several studies have shown that experiments conducted in brick-and-mortar laboratory environments replicate well in online environments (Casler et al., 2013; Crump et al., 2013; Arechar et al., 2018). For our online experiment, we recruit participants using Prolific, a platform that specializes in providing subjects for scientific studies and that has strict recruitment standards (Palan and Schitter, 2018). Moreover, the data quality produced by Prolific participants has been found to be of very high quality on multiple measures – including attention, reliability, reproducibility, naivety, dishonesty and comprehension (Peer et al., 2017; 2021).

We undertook several actions to ensure the integrity of the data. We made sure that subjects could only participate using either a desktop computer or tablet. This was a conscious choice as the instructions, control questions and decisions required several pieces of information that could be better presented on large screens to minimize the need for scrolling. We also provided a concise summary of the instructions at the bottom of the control question and decision making screens to give participants an easy reference to the rules of the auction if they needed to recall key elements of it. Lastly, before launching the study on Prolific, we ran several tests on the functionality and comprehension of experimental tasks among ourselves and with students to test the design and implementation of the experiment. This helped us ensure that subjects understood the experimental task and helped us corroborate that the data we collected is of high quality.

On average participants answered slightly more than 11 out of 19 control questions

correctly on the first try, and 90% of participants answered at least 8 of the control questions correctly on their first-try. If they answered incorrectly, a text explaining the correct answer and how to arrive at it would show up. Furthermore, response times in our experiment suggest that participants were actively engaged throughout the study. The mean reading time for the instructions and answering the control questions was 17 minutes and 35 seconds (median: 15 minutes and 6 seconds). The mean time needed to record their auction decision was 1 minute and 35 seconds (median: 1 minute and 3 seconds), and 95% of participants spent at least 28 seconds making the auction decision.

As a final check we asked participants to provide us with any comments or suggestions that they had at the very end of the experiment after payoffs had been finalized and communicated in the software. Overall, the sentiment was very positive, with one subject, for instance, saying that (first-price cash treatment),

I really really enjoyed this[,] it made me think a lot and really try and concentrate[.] [H]ope to have another survey like this[.]

As well as another mentioning (second-price equity treatment),

This study was very interesting and required fast thinking. The time limit was a bit short however, the exercise was very enjoyable and also challenging in a very good way. This was a very good survey.

Taken together, the measures we undertook together with the response behavior we observed, leads us to firmly believe that subjects took the experiment seriously and that the vast majority were giving it full attention.

3 Results

Our data collection started on 15 July 2021 with a pilot for the FPE treatment with 25 subjects participating. After this pilot we decided to not make any changes, except for increasing the fixed participation fee, because it took subjects a bit longer to complete the study than we expected upfront. Without any other changes we collected the remaining observations over four different sessions run on 23–24 August 2021, resulting in 65 observations for FPC, 59 for SPC, 63 for FPE (including the 25 from the pilot), and 61 for SPE.⁷

⁷For sessions with an odd number of participants, to resolve the payment for the last participant, we operated as the rival bidder for which we entered the bids provided by a random other participant.

Sessions took on average about 25–30 minutes. Participants received a fixed participation fee of £ 3.75 (this was £ 2.50 in the pilot) and on average a bonus payment of £ 2.76 for their performance in the auction and the additional tasks.

Treatment	N	Female	Age	Risk aversion		Loss	Risk
				BRET	Self rep.	aver.	lit.
FPC	65	55.38	26.83	29.31	6.09	24.92	1.14
SPC	59	63.79	26.93	28.83	6.42	26.08	1.19
FPE	63	50.79	27.13	32.14	6.65	30.00	1.22
SPE	61	58.33	26.08	30.25	6.11	26.46	1.26
All	248	56.91	26.74	30.15	6.32	26.87	1.20

Table 4: Participant characteristics.

We begin by tabulating the participant characteristics in the experiment. Table 4 summarizes these characteristics as elicited in the post-experimental questions and tasks. The column ‘Female’ presents the percentage of females conditional on the self-identified gender being male or female.⁸ The column ‘Age’ presents the average age conditional on the age being at least 18.⁹ The column ‘BRET’ presents the number of boxes opened in the bomb risk elicitation task – a number between 0 and 100.¹⁰ The column ‘Self rep.’ shows the average willingness to take risk as elicited on a scale from 0 until 10 by the self-assessment question. The column ‘Loss aver.’ presents the average loss attitude based on the BDM task eliciting certainty equivalences, ranging from 0 until 50.¹¹ Finally, ‘Risk lit.’ shows the average number of correctly answered questions in the risk literacy task and ranges from 0 until 4. Further details related to the individual characteristics are provided in Appendix C. Overall, we do not find systematic differences between the pools of participants across treatments.

⁸One participant revealed to be gender diverse and one participant preferred not to say; the other 246 participants identified as male or female.

⁹One participant revealed to be eight years old (possibly a typo), while we allowed only members of the Prolific participant pool between age 18 and 65 to participate.

¹⁰Here, we did not exclude the eight participants that opened all 100 boxes; possibly not paying sufficient attention.

¹¹Here, we did not exclude the participants that responded with one of the two extreme answers.

3.1 Bidding behavior

In this subsection we focus on the hypotheses related to bidding behavior: Hypotheses 1–3. The first hypothesis relates to bidding behavior across signals. Figure 1 presents cumulative distributions of the bids for the different signals (variation within plots) and the different auction formats (variation across plots).

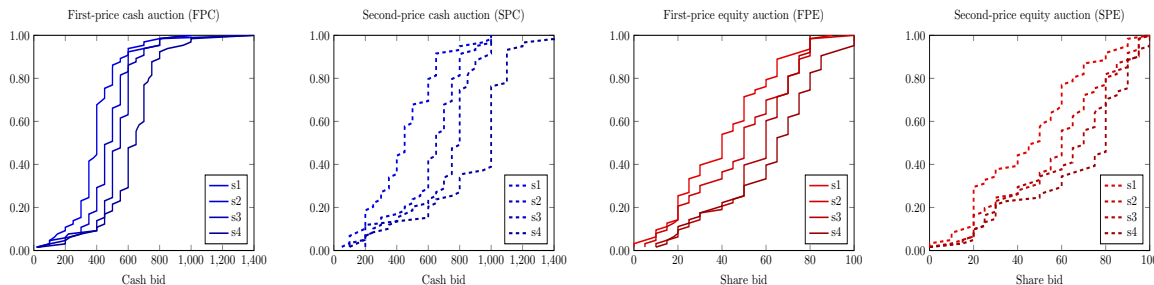


Figure 1: Cumulative distributions of bids at the different signals for the different treatments.

Table 5 presents for each auction format and each signal, the average bid, together with the equivalent bid in the other bidding domain in parentheses.¹² Visual inspection of Figure 1 and the numbers presented in the table, both, suggest that bids are increasing in the signal. Wilcoxon tests of all signal-pair combinations confirm this: for each of the four treatments, for every pair of signals, bids are larger at the higher signal (p -values from all tests are smaller than $p < .0389$).

Result 1 *For all four treatments, bids are increasing in the signal.*

Treatment	Bids in cash (equivalence)				Bids in share (equivalence)			
	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4
FPC	406.15	478.31	530.00	628.29	(51.54)	(46.38)	(41.25)	(42.83)
SPC	454.07	623.92	702.71	871.53	(63.52)	(70.65)	(62.84)	(67.15)
FPE	(364.76)	(490.29)	(642.54)	(818.73)	41.19	48.38	55.32	61.87
SPE	(381.57)	(542.49)	(691.93)	(871.48)	45.39	57.08	61.49	67.15

Table 5: Average (equivalent) bids for the four treatment and the four signals.

¹²Like in Table 3, we use the conversion $\text{cash} = 200 + \frac{\text{share}}{100} \times \text{ExpPrivVal}$.

Next, we turn to the comparison of bidding behavior across auction formats. To be able to perform comparisons between equity and cash formats, we use the cash bids as observed in the cash auctions and for the equity auctions we use the ‘cash equivalent’ bids (shown in parentheses in Table 5). Figure 2 replicates the cumulative distributions of Figure 1, but this time the plots are split by signals.

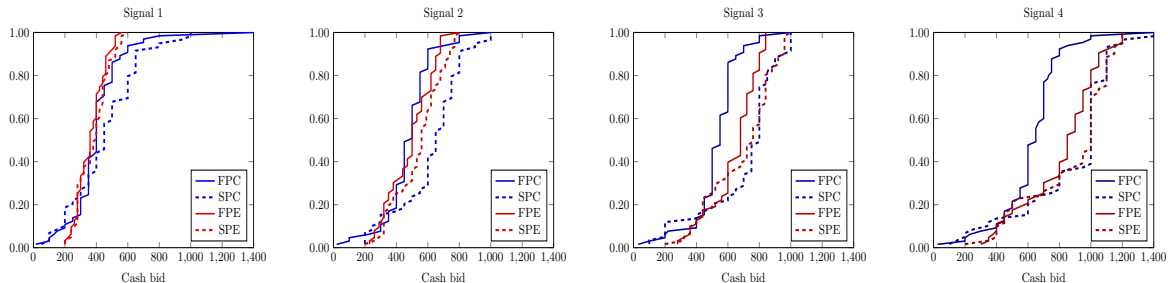


Figure 2: Cumulative distributions of cash (equivalent) bids for the different treatment at the different signals.

Table 6 presents for different pairs of auction formats and for each signal, the p -value of a Mann-Whitney U test testing equality of bids between two auction formats at a time. The last column tests equality of the average bid when the average is taken over the signals. First, regarding the comparison of bids between pricing-rules, the evidence is in support of Hypothesis 2 and we can state the following result:

Result 2 *Comparing bids across pricing-rules, for both cash and equity auctions, bids are not different at the lowest signal and larger under the second-price pricing rule at the highest three signals and averaged over all four signals.*¹³

Comparison	s_1	s_2	s_3	s_4	avg.
FPC vs. SPC	.0913	.0000	.0000	.0000	.0000
FPE vs. SPE	.3777	.0422	.0312	.0518	.0083
FPC vs. FPE	.1573	.4490	.0000	.0000	.0007
SPC vs. SPE	.0308	.0049	.7065	.9141	.0485

Table 6: Test results (p -values) for differences in bids across treatments at different signals and averaged over signals.

¹³At the highest signal, it is close to significant for the equity auction.

Second, we also find that under the first-price pricing rule the comparison of bids between cash and equity auction largely follows Hypothesis 3(a). Only at signal s_2 we do not find a significant difference. Under the second-price pricing rule we see that tests do not confirm the predictions reported in Hypothesis 3(b). Under this pricing rule there should not be a difference in bids between the cash and the equity auction. While test results confirm this at the highest two signals, at the lowest two signals and averaged over all four signals, we find bids to be larger in the cash auction. Accordingly, we state:

Result 3 *Comparing bids between cash and equity auctions,*

- (a) *under the first-price pricing rule, bids are not different at the lowest two signals, and larger for the equity auction at the highest two signals and averaged over all four signals;*
- (b) *under the second-price pricing rule, bids are not different at the highest two signals, and larger for the cash auction at the lowest two signals and averaged over all four signals.*

3.2 Revenue and efficiency

In this section we investigate the auction formats in terms of the revenue they produce and the efficiency they generate. For each possible combination of pairs of bidders (in the same auction format) we compute the expected revenue and the expected efficiency prior to the realization of signals and private values. Expected efficiency is the percentage of cases the bidder with the higher signal wins the auction. Figure 3 plots for each treatment the cumulative distribution of the expected revenue (left plot) and the expected efficiency (right plot) over all possible pairs. Respective summary statistics are presented in Table 7.

Treatment	Revenue		Efficiency	
	mean	(st.dev.)	mean	(st.dev.)
FPC	613.64	(115.92)	80.40	(12.86)
SPC	500.66	(158.78)	81.84	(15.34)
FPE	700.13	(95.01)	73.45	(12.25)
SPE	524.90	(157.67)	72.76	(12.41)

Table 7: Mean and standard deviation of expected revenue and efficiency.

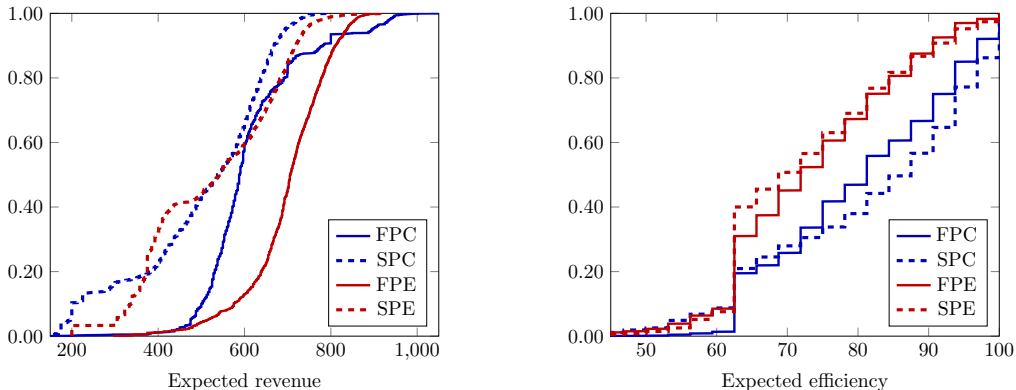


Figure 3: Cumulative distributions of ex ante expected revenue and efficiency over all possible pairs.

According to the theoretical predictions we should see the following ranking in terms of expected revenue: $FPE = SPE > FPC = SPC$. Visual inspection of Figure 3 shows that for both the cash and the equity auction, the distribution of the first-price pricing rule first-order stochastically dominates that of the second-price pricing rule. Though, the gap between the two appears larger for the equity auction format. Under the first-price pricing rule the equity auction appears to produce more revenue, while the relation between cash and equity auction revenues is less clear under the second-price pricing rule.

To test for statistical significances we use a statistical bootstrap technique. For each treatment we draw 29 pairs of participants.¹⁴ These pairs generate, for each treatment, 29 expected revenues and efficiency levels. Next, we apply two-sided Mann-Whitney tests over these 29 independent observations, and record the resulting p -value. We repeat this process for 10^6 draws of 29 pairs, resulting in 10^6 p -values for each treatment comparison. Table 8 presents the percentage of times the resulting p -values are below a given significance level.

For both the cash auction and the equity auction, a higher expected revenue is obtained under the first-price payment rule. The difference across pricing rules is statistically highly significant for the equity auction, where we can confidently reject revenue equivalence. For the cash auction the evidence is less conclusive. In slightly more than two-thirds of the 10^6 bootstraps revenue equivalence is rejected at the five percent significance level. This number increases to almost 90% of the bootstraps when considering ten percent significance

¹⁴This is the largest number of pairs we can draw in all treatments, given that SPC has with 59 participants the lowest number among the four treatments.

Comparison	Revenue			Efficiency		
	1% sign.	5% sign.	10% sign.	1% sign.	5% sign.	10% sign.
FPC vs. SPC	14.14	68.29	89.30	0.01	0.43	2.09
FPE vs. SPE	100.00	100.00	100.00	0.02	0.51	2.00
FPC vs. FPE	98.29	99.99	100.00	12.09	47.88	69.67
SPC vs. SPE	0.00	0.01	0.15	35.82	81.78	94.05

Table 8: Comparing revenue and efficiency across treatments. Percentage of times the resulting p -value is below the stated significance level.

level. Although with less confidence than for the equity auction, we conclude that also for the cash auction we do not observe revenue equivalence.

Result 4 *For both the cash and the equity auction, the first-price pricing rule generates a higher revenue than the second-price pricing rule.*

Further, the equity auction generates on average more revenues than the cash auction under both pricing rules. For the first-price pricing rule the observed revenue difference is highly significant. However, for the second-price pricing rule we are not able to reject revenues being different, possibly due to the larger standard deviation in both the cash and equity auction formats that this pricing rule produces.

Result 5 *Under both pricing rules, the equity auction generates on average a higher revenue than the cash auction. Though, the difference is only significant under the first-price pricing rule.*

In terms of the efficiency of the auctions, we find that for both the cash and the equity auction only 2% of the 10^6 bootstraps reports a significant difference between the two pricing rules when adopting a very generous ten percent significance level. This drops to about 0.5% if we adopt the more conventional five percent significance level. It is visible in Figure 3 and in Table 7 that there is a difference between cash and equity auctions, with cash auctions yielding higher efficiency than the equity auction under both pricing rules. Statistically the significance of the difference is higher under the second-price pricing rule (see Table 8).

Result 6 *The pricing rule has no significant impact on the efficiency of the auction. Cash auctions yield higher levels of efficiency than equity auctions.*

Based on the observed differences in revenue and efficiency, we can rank the treatments as $SPE = SPC < FPC < FPE$ for revenue and as $SPE = FPE < FPC = SPC$. The first-price equity auction maximizes auction revenues, but with a slight reduction in efficiency. The second-price cash auction is the opposite: low revenues and high efficiency. The first-price cash auction operates fairly well on both dimensions, while the second-price equity auction performs poorly on both dimensions.

4 Discussion

The results of the study show that there are significant performance differences between the auction formats. Note, however, that the risk-neutral Nash equilibrium does predict that expected revenues are higher in equity auctions than in cash auctions. Once we put the observed revenues in relation to the expected equilibrium revenues we find: +6.78% for FPC, -12.87% for SPC, -2.64% for FPE, and -26.98% for SPE. These numbers indicate that, relative to equilibrium, equity auctions perform worse than cash auctions.

One potential cause for the observed discrepancies is the difference between auction formats in terms of their complexity. However, we argue that our observations so far cannot be attributed to this difference for two reasons. First, there is no strong evidence for there being major differences in participants' understanding of the functioning of the auction mechanisms. A Kruskal-Wallis test of the time spent on the instructions and the control questions does not show any significant differences across auction formats ($p = 0.290$). We do find significant differences across the treatments when it comes to the fraction of correctly answered control questions on the first try ($p < 0.001$), but the proportion of correctly answered control questions is only marginally higher in FPC (64%) compared to the other treatments (SPC: 57%; FPE: 56%; SPE: 60%). Thus, at best, it could explain why FPC does better relative to SPC but not why FPE outperforms SPE relative to expected equilibrium predictions. Second, we also do not find strong evidence of potential complexity differences in the bidding process. A Kruskal-Wallis test shows insignificant differences in the time spent on the decision screen ($p = 0.605$) in the experiment. Given this evidence, we can exclude complexity as the main factor driving the observed differences between auction formats. We next take a closer look at individual bidding behavior within the auctions.

To facilitate this analysis, we define bidder i 's relative bid at signal s as $r_s^i = \frac{b_s^i - b_s^*}{b_s^*} \times 100$, where b_s^i denotes bidder i 's bid at signal s and b_s^* the expected equilibrium bid at signal s . Hence, r_s^i captures how bidder i bids relative to equilibrium at signal s in percentage terms. Next, we define bidder i 's average relative bid as $r^i = \frac{1}{4} \sum_s r_s^i$. Table 9 presents, for each auction format, the mean and standard deviation of r^i . Overall, we see the lowest deviations from equilibrium in FPC and SPC and significantly higher deviations in the two equity auction treatments. Testing for significance of the values reported in the table suggests that we find significant underbidding in the two equity auction formats (two-sided p -values from t-test, FPE: 0.004; SPE: 0.001) but not in the two cash auction formats (two-sided p -values from t-test: FPC: 0.803; SPC: 0.595).

Treatment	avg.	st.dev.
FPC	-0.9	29.2
SPC	-1.9	27.0
FPE	-11.6	31.1
SPE	-14.4	33.6

Table 9: Mean and standard deviation of relative bids by averaged over signals (r_i).

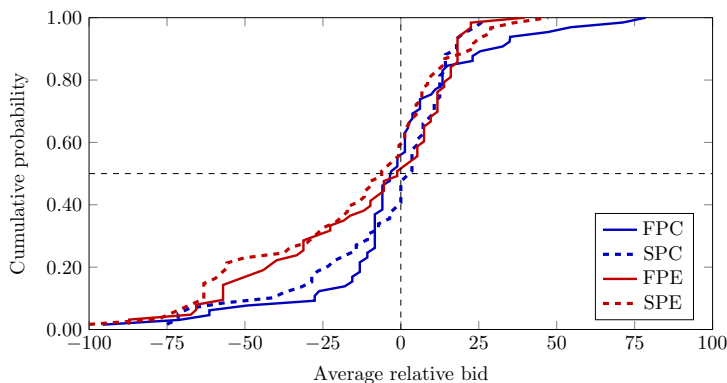


Figure 4: Cumulative distributions of the average relative bids.

To better understand bidding behavior, we present, for each auction format, the cumulative distribution over average relative bids in Figure 4. Bidders with a positive (negative) value, on average, overbid (underbid) relative to equilibrium. For the cash auctions (blue lines in the figure), the distributions between the two pricing rules look quite similar around the equilibrium bid. One notable difference however, is in the upper tail that extends much

further in the first-price auction than in the second-price auction. Such (extreme) overbidding has been observed previously in other first-price auctions settings such as Cox et al. (1982), Cox et al. (1985), Goeree et al. (2002). Furthermore, although we find the fraction of participants underbidding to be larger in the first-price auction compared to the second-price auction, prices are likely to be determined by the bids in the lower quantiles of the distribution more frequently in the second-price auction due to the small number of bidders per auction in the experiment.¹⁵ Taken together these two effects lead to higher relative revenues to be realized in FPC than in SPC.

In the equity auctions (red lines), we see a smaller difference between the distributions of the two pricing rules. For both pricing rules we neither find substantial overbidding nor underbidding (a fat lower tail). However, similarly to the cash auctions, the negative impact of prices being set by the lower bid is detrimental to the performance of the second-price auction and can explain the poor performance of SPE relative to FPE. Comparing cash to equity auctions (comparing the solid blue and red lines as well as the dashed red and blue lines separately), the most substantial difference is the large amount of underbidding (a fatter lower tail) in the equity auctions. This underbidding can explain why equity auctions perform worse than cash auctions relative to equilibrium.

An additional mechanical effect that also may affect the observed bidding behavior relates to the steepness of the bidding functions. Equilibrium bids across signals are much steeper in cash auctions than in equity auctions. Comparing the bid at the highest signal relative to the bid at the lowest signal we find +64% for FPC, +150% for SPC, +33% for FPE, and +60% for SPE (see Table 3). This leads to a situation where there is a large negative effect for auctions with flatter bidding functions due to the fact that the probability that a bid for a lower signal may dominate that for a higher signal is higher in the presence of over- and under-bidding. With this in mind, equity auctions are expected to generate less efficiency when bidders underbid compared to cash auctions.

We next attempt to understand what drives the extreme overbidding in the first-price

¹⁵Kagel and Levin (1993) study first-, second-, and third-price auctions in an experiment with 5 and 10 bidders. Our finding is in-line with their results for 5 bidders, where they find that first-price auction generates the highest average revenue and third-price auction the lowest, consistent with the theory predictions for risk averse bidders. When they increase the number of bidders to 10, they observe that in all the three auction formats the average revenue increases, specifically, they find no significant differences in average revenue under the three pricing rules, which is consistent with the revenue equivalence theorem under risk neutrality.

cash auction and the fat lower tails in the equity auctions. To do so, we regress the average relative bids on the elicited individual characteristics. Of particular interest we consider risk and loss attitude. Usually, risk attitude plays a more important role in first-price auctions where bidders trade off the potential surplus from winning against the probability of winning. In our setting, with the private value being determined by a lottery, risk attitude may also play a role in the second-price auctions. We consider loss attitude of interest because it is possible to make losses in the auction and the likelihood of this happening varies across the auction formats. Especially in the cash auctions it is possible to make a loss since the realized value of the asset may be less than the amount paid. Compare this to the equity auction where the price to be paid is linked to the realized private value and therefore, even if one wins the auction by committing to give away a substantial fraction of the revenues.

	Cash	Equity
Constant	5.796 (18.103)	10.770 (20.102)
Second-price (SP)	-1.316 (5.089)	-2.300 (6.014)
Risk tolerance (R)	9.531** (3.947)	3.077 (4.280)
SP \times R	-7.295 (6.040)	4.972 (6.287)
Loss tolerance	-0.128 (0.198)	-0.119 (0.212)
Risk literacy	1.406 (3.272)	-1.968 (3.054)
Gender	3.235 (5.595)	-4.646 (6.581)
Age	-0.342 (0.320)	-0.364 (0.456)
R + SP \times R = 0 (<i>p</i> -value of F-test)	0.649	0.078
Observations	122	123
R-squared	0.092	0.044

Table 10: Relation between relative bid and individual characteristics. Robust standard errors in parentheses. *, **, and *** indicate significance at the 0.10, 0.05, and 0.01 levels, respectively.

Table 10 presents the results of the regression. Regressions are separated for cash and

equity auctions due to the difference in bidding domains. For risk tolerance we use the first principal component of the decisions in the two risk elicitation tasks; see Appendix C for further details. For both the cash and the equity auctions, we observe that relative bids are lower in the second-price auctions; yet, this effect is insignificant. We find no significant relation of loss attitude on relative bids in either of the two models. This is, as explained above, not too surprising for the equity auction, but one could have expected an effect for the cash auctions.

For the cash auctions, we find that risk attitude has no significant effect in the second-price auction, which may be due to not having to make the trade-off between surplus and winning probability. In contrast, in the first-price auction, we do find a significant positive effect: more risk tolerant individuals bid higher. This is consistent with higher bids being more risky due to the higher likelihood of generating a loss conditional on winning the auction. In particular, the observed extreme overbidding appears to be due to the more risk-tolerant bidders.

For the equity auctions, the risk relationship is reversed. In the first-price auction, there is a positive but insignificant effect of risk tolerance. In the second-price auction, the magnitude is about three times higher; yet, it is only significant at ten percent significance level. That risk attitudes play a lesser role in the equity auction is not unsurprising given that the possibility of making a loss is suppressed. While we earlier were able to rule out complexity of the auction formats playing a dominant role, participants in the equity auction may have found difficulties in forming accurate beliefs about their rival's bidding strategy. This can result in a higher variance in beliefs, and subsequently relative bids, and causing the fatter lower tail. Since we did not elicit participants' beliefs about their rival's bidding strategy, we are unfortunately not able to explore this argument further.

5 Conclusion

This study produces several findings. First, for neither the cash nor the equity auction do we find revenue equivalence: first-price auctions generate higher revenue than second-price auctions. For the cash auction this is partially explained by risk tolerant bidders placing very high bids under the first-price pricing rule; for the equity auction this can be explained by substantial amount of underbidding under both pricing rules and this being

more effective under the second-price pricing rule due to an order-statistics effect. Second, although in absolute terms equity auctions generate larger revenue than cash auctions, relative to equilibrium they do worse. Our main explanation is related to bidding functions being steeper in cash auctions, which results in an increased tendency to produce inefficient allocations in the equity auctions. This effect is amplified by the higher variance in bidding behavior that we observe in equity auctions. Third, equity auctions are less efficient, but there is no efficiency difference between pricing rules.

For an auctioneer seeking to auction an asset with a random value, it thus appears as if the choice is straight-forward. If one is concerned about both revenue and efficiency, there is no reason to use one of the second-price auctions. If one is primarily concerned about efficiency, the first-price cash auction is the best choice. In contrast, if one is more concerned about revenues, the first-price equity auction may be more suitable, but only because equilibrium revenues are much higher which makes up for the loss from the observed underbidding.

In the bigger picture, our study contributes to the understanding of the functioning of security auctions. For the experiment reported here, we focus on two bidders. Our restriction to two bidders was for the practical reason, to be able to refer to ‘the second-highest bid’ as ‘the other bidder’s bid’ in the instructions of our experiment. This significantly simplifies the communication of the auction rules to the participants in the experiment. Yet, it is not uncommon for auctions of assets with uncertain values to only feature a few bidders, making our choice align with real-life auctions in this space. For instance, Boone and Mulherin (2007) report that the average number of bidders that submit a private written offer is 1.29 and the average that submit a public written offer is 1.13 in a sample of 400 firms that are sold. Thus, the total average is less than 2.5 bids per sale, closely matching our setting. We expect that with more bidders, the observed gap in revenues between pricing rules would shrink. However, there are several effects that are unrelated to the number of bidders that we do expect to persist: First, the higher variance in relative bids in the equity auctions is unlikely to change with more bidders, contributing to high levels of inefficiencies in this format. Second, bidding functions are flatter in the equity auction and therefore our findings on revenues relative to equilibrium are also expected to persist when comparing within the first-price and second-price auction rules.

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Online appendix

corresponding to

Bajoori E, R Peeters & L Wolk (2022).

Security auctions with cash- and equity-bids: An experimental study.

A Equilibrium bids

A.1 First-price cash auction (FPC)

In the symmetric risk-neutral Nash equilibrium, both bidders bid as follows:

$$s_1 : 400$$

$$s_2 : \text{randomize over } [400, 500] \text{ with density function } f(b) = \frac{200}{(600-b)^2}$$

$$s_3 : \text{randomize over } [500, 600] \text{ with density function } f(b) = \frac{600}{(800-b)^2}$$

$$s_4 : \text{randomize over } [600, 700] \text{ with density function } f(b) = \frac{1200}{(1000-b)^2}$$

A.2 Second-price cash auction (SPC)

In the symmetric risk-neutral Nash equilibrium, both bidders bid as follows:

$$s_1 : 400$$

$$s_2 : 600$$

$$s_3 : 800$$

$$s_4 : 1000$$

A.3 First-price equity auction (FPE)

In the symmetric risk-neutral Nash equilibrium, both bidders bid as follows:

$$s_1 : \frac{1}{2}$$

$$s_3 : \text{randomize over } \left[\frac{1}{2}, \frac{7}{12}\right] \text{ with density function } f(b) = \frac{3}{2(2-3b)^2}$$

$$s_3 : \text{randomize over } \left[\frac{7}{12}, \frac{23}{36}\right] \text{ with density function } f(b) = \frac{16}{3(3-4b)^2}$$

$$s_4 : \text{randomize over } \left[\frac{23}{36}, \frac{163}{240}\right] \text{ with density function } f(b) = \frac{145}{12(4-5b)^2}$$

A.4 Second-price equity auction (SPE)

In the symmetric risk-neutral Nash equilibrium, both bidders bid as follows:

$$s_1 : \frac{1}{2}$$

$$s_2 : \frac{2}{3}$$

$$s_3 : \frac{3}{4}$$

$$s_4 : \frac{4}{5}$$

B Screenshots

Instructions

Read the instructions below carefully before proceeding.

Your decision

The owner of a company is retiring, and they have decided to sell the company by means of an auction with two candidates bidding in the auction. You are one of the two candidates and the winner of the auction will take over the company. The information below makes clear the precise details regarding this auction. You are advised to read the information very carefully since the outcome of the auction, and hence your final earnings depend on your decisions.

1. The other candidate who potentially can take over the company is another participant in this experiment, who receives the exact same instructions as you.
2. Each of you are given an initial budget of 800 points.
3. The future profit of the company is uncertain, but will be either 200, 600, 1000, 1400 or 1800 points.
4. There are four possible situations regarding the future profit and the table below explains for each situation the chances to result in the possible future profit levels.

For example, in Situation 3, the probability that the future profit equals 1000 is 20%. Note that situations with a higher number have higher chance to result in a higher future profit. The future profit will be realized by the central computer according to the probabilities below, after you and the other candidate have submitted the bids (see point 10a below).

Future profit	Situation 1 (25%)	Situation 2 (25%)	Situation 3 (25%)	Situation 4 (25%)
200	70%	50%	30%	10%
600	15%	20%	25%	30%
1000	10%	15%	20%	25%
1400	5%	10%	15%	20%
1800	0%	5%	10%	15%

Note: While you are advised to take a very close look at the table, this table will be presented on all screens that follow where the information in the table is relevant for you.

5. For each of the four possible situations, you are asked to specify your bid. For each situation this is to be a number between 0% and 100%. The other candidate is given the exact same task.

What happens after bidding

6. The central computer selects at random one of the four situations for you - each has equal chance to be chosen. Also for the other candidate one of the situations is selected, and also each with equal chance, but note that a different situation can be selected for the other candidate than for you.
7. The bid that you placed in the situation that is selected for you is your relevant bid. Likewise, the bid that the other candidate placed in the situation that is selected for them is their relevant bid.
8. The candidate with the higher relevant bid is the winner of the auction. In case of equal relevant bids, the bidder with the higher numbered situation is the winner. If also the situation number is equal, the winner will be decided at random (each having equal chance to win).
9. In case you are not the winner, you neither incur any costs nor make any earnings in addition to your initial budget of 800 points.
10. In case you are the winner of the auction, the earnings you make in addition to your initial budget of 800 points depends on the realized future profit and the price you have to pay.
 - a. The realized future profit will be decided in accordance to the probabilities for the situation that is chosen for you (see the table above).
 - b. The price that you have to pay is 200 plus the share of the realized future profit equal to the other candidate's relevant bid.
11. At the end of the auction, you will be given feedback on all details that are relevant for your final earnings: (1) the situations selected for you and the other candidate, (2) your and the other candidate's relevant bids, (3) the winner of the auction, and (4) the realized future profit and the price you have to pay if you are the winner.
12. The exchange rate is such that 500 points equal 1 GBP.

The instructions will be summarized on the decision page, identical to the summary box below.

I have finished reading the instructions.

Next

Short summary

- You and another candidate are bidding to take over a firm.
- There are five possible future profit levels and the likelihood of each one of these realizing depends on the situation selected for you.
- There are four situations and each is equally likely to be selected.
- You do not know the situation that is selected for you at the moment you specify your bids.
- Your relevant bid is the bid you specified for the situation that is ultimately selected for you.
- The same applies to the other candidate, but another situation may be selected for them.
- The candidate with the highest relevant bid wins the auction.
- The loser of the auction earns nothing apart from the initial budget.
- The additional earnings to the winner of the auction is the realized future profit minus the price. The realized profit is determined in accordance to the situation of the winner. The price to be paid is 200 plus the share of the realized future profit, where this share equals the relevant bid of the loser.

Figure 5: Instructions

Control questions (1 of 5)

The table below repeats the different profit levels and their likelihood for each possible situation separately.

Future profit	Situation 1 (25%)	Situation 2 (25%)	Situation 3 (25%)	Situation 4 (25%)
200	70%	50%	30%	10%
600	15%	20%	25%	30%
1000	10%	15%	20%	25%
1400	5%	10%	15%	20%
1800	0%	5%	10%	15%

Please answer the following control questions.

The situation that is selected for me, is also selected for the other candidate.

False True

Situations with a higher number are more favorable.

False True

The probability that the future profit is 1000 is always 20%.

False True

If I win the auction, the amount that I pay to the current owner depends only on my relevant bid.

False True

Next

Short summary

- You and another candidate are bidding to take over a firm.
- There are five possible future profit levels and the likelihood of each one of these realizing depends on the situation selected for you.
- There are four situations and each is equally likely to be selected.
- You do not know the situation that is selected for you at the moment you specify your bids.
- Your relevant bid is the bid you specified for the situation that is ultimately selected for you.
- The same applies to the other candidate, but another situation may be selected for them.
- The candidate with the highest relevant bid wins the auction.
- The loser of the auction earns nothing apart from the initial budget.
- The additional earnings to the winner of the auction is the realized future profit minus the price. The realized profit is determined in accordance to the situation of the winner. The price to be paid is 200 plus the share of the realized future profit, where this share equals the relevant bid of the loser.

Figure 6: Control questions 1

Control questions (2 of 5)

In the control questions below we test your understanding of some of the details regarding the auction. Let the last two rows of table below be your and the other candidate's bids. Note that the numbers are just chosen as an example and should not be perceived as a suggestion for how you should make your decisions later on.

Future profit	Situation 1 (25%)	Situation 2 (25%)	Situation 3 (25%)	Situation 4 (25%)
200	70%	50%	30%	10%
600	15%	20%	25%	30%
1000	10%	15%	20%	25%
1400	5%	10%	15%	20%
1800	0%	5%	10%	15%
Your bids:	40%	95%	65%	80%
The other candidate's bids:	55%	60%	70%	90%

Suppose Situation 2 is selected for you and Situation 3 for the other candidate.

- A. What is your relevant bid? 40% 65% 80% 95%
-
- B. What is the other candidate's relevant bid? 55% 60% 70% 90%
-
- C. Who is the winner of this auction? I am winning
 The other candidate is winning
-
- D. What is the chance that the realized future profit is 1400? 5% 10% 15% 20%
-
- E. What realized future profit does this situation result in on average? 400 600 800 1000
-
- F. Which price needs to be paid if the realized future profit is 1400?
 $0.7 \times 1400 = 980$
 $200 + 0.7 \times 1400 = 1180$
 $0.95 \times 1400 = 1330$
 $200 + 0.95 \times 1400 = 1530$
-
- G. What will your payoff be (including the initial budget) if the realized future profit is 1400?
 $1400 - (200 + 0.7 \times 1400) = 220$
 $800 + 1400 - (200 + 0.95 \times 1400) = 670$
 $800 + 1400 - (200 + 0.7 \times 1400) = 1020$

Next

Short summary

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- The same applies to the other candidate, but another situation may be selected for them.
- The candidate with the highest relevant bid wins the auction.
- The loser of the auction earns nothing apart from the initial budget.
- The additional earnings to the winner of the auction is the realized future profit minus the price. The realized profit is determined in accordance to the situation of the winner. The price to be paid is 200 plus the share of the realized future profit, where this share equals the relevant bid of the loser.

Figure 7: Control questions 2

Control questions (3 of 5)

Future profit	Situation 1 (25%)	Situation 2 (25%)	Situation 3 (25%)	Situation 4 (25%)
200	70%	50%	30%	10%
600	15%	20%	25%	30%
1000	10%	15%	20%	25%
1400	5%	10%	15%	20%
1800	0%	5%	10%	15%
Your bids:	40%	95%	65%	80%
The other candidate's bids:	55%	60%	70%	90%

Suppose again that Situation 2 is selected for you and Situation 3 for the other candidate.

A. What is the chance that the realized future profit is 600? 15% 20% 25% 30%

B. Which price needs to be paid if the realized future profit is 600? $0.7 \times 600 = 420$
 $0.95 \times 600 = 570$
 $200 + 0.7 \times 600 = 620$
 $200 + 0.95 \times 600 = 770$

C. What will your payoff be (including the initial budget) if the realized future profit is 600? $600 - (200 + 0.7 \times 600) = -20$
 $800 + 600 - (200 + 0.95 \times 600) = 630$
 $800 + 600 - (200 + 0.7 \times 600) = 780$

Next

Short summary

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- There are five possible future profit levels and the likelihood of each one of these realizing depends on the situation selected for you.
- There are four situations and each is equally likely to be selected.
- You do not know the situation that is selected for you at the moment you specify your bids.
- Your relevant bid is the bid you specified for the situation that is ultimately selected for you.
- The same applies to the other candidate, but another situation may be selected for them.
- The candidate with the highest relevant bid wins the auction.
- The loser of the auction earns nothing apart from the initial budget.
- The additional earnings to the winner of the auction is the realized future profit minus the price. The realized profit is determined in accordance to the situation of the winner. The price to be paid is 200 plus the share of the realized future profit, where this share equals the relevant bid of the loser.

Figure 8: Control questions 3

Control questions (4 of 5)

Future profit	Situation 1 (25%)	Situation 2 (25%)	Situation 3 (25%)	Situation 4 (25%)
200	70%	50%	30%	10%
600	15%	20%	25%	30%
1000	10%	15%	20%	25%
1400	5%	10%	15%	20%
1800	0%	5%	10%	15%
Your bids:	40%	95%	65%	80%
The other candidate's bids:	55%	60%	70%	90%

Suppose now that Situation 3 is selected for both of you.

- A. Who is the winner of this auction?
- I am winning
- The other candidate is winning

- B. What will your payoff be (including the initial budget) if the realized future profit is 1400?
- 800
- $800+1400-(200+0.7 \times 1400) = 1020$
- $800+1400-(200+0.65 \times 1400) = 1090$

- C. What will your payoff be (including the initial budget) if the realized future profit is 600?
- $800+600-(200+0.7 \times 600) = 780$
- 800
- $800+600-(200+0.65 \times 600) = 810$

- D. Which price needs to be paid if the realized future profit is 1000?
- $200+0.65 \times 1000 = 850$
- $200+0.7 \times 1000 = 900$
- 1000

Next

Short summary

- You and another candidate are bidding to take over a firm.
- There are five possible future profit levels and the likelihood of each one of these realizing depends on the situation selected for you.
- There are four situations and each is equally likely to be selected.
- You do not know the situation that is selected for you at the moment you specify your bids.
- You relevant bid is the bid you specified for the situation that is ultimately selected for you.
- The same applies to the other candidate, but another situation may be selected for them.
- The candidate with the highest relevant bid wins the auction.
- The loser of the auction earns nothing apart from the initial budget.
- The additional earnings to the winner of the auction is the realized future profit minus the price. The realized profit is determined in accordance to the situation of the winner. The price to be paid is 200 plus the share of the realized future profit, where this share equals the relevant bid of the loser.

Figure 9: Control questions 4

Control questions (5 of 5)

Future profit	Situation 1 (25%)	Situation 2 (25%)	Situation 3 (25%)	Situation 4 (25%)
200	70%	50%	30%	10%
600	15%	20%	25%	30%
1000	10%	15%	20%	25%
1400	5%	10%	15%	20%
1800	0%	5%	10%	15%
Your bids:	40%	95%	65%	80%
The other candidate's bids:	55%	60%	70%	90%

Suppose Situation 4 is selected for you.

- A. What is, given the bids above, the chance you win the auction?
- 0%
 - 25%
 - 50%
 - 75%
 - 100%
 - Other

Next

Short summary

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- There are four situations and each is equally likely to be selected.
- You do not know the situation that is selected for you at the moment you specify your bids.
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- The same applies to the other candidate, but another situation may be selected for them.
- The candidate with the highest relevant bid wins the auction.
- The loser of the auction earns nothing apart from the initial budget.
- The additional earnings to the winner of the auction is the realized future profit minus the price. The realized profit is determined in accordance to the situation of the winner. The price to be paid is 200 plus the share of the realized future profit, where this share equals the relevant bid of the loser.

Figure 10: Control questions 5

Task

Please enter your bid for each of the four possible situations in the last row in the table below:

Future profit	Situation 1 (25%)	Situation 2 (25%)	Situation 3 (25%)	Situation 4 (25%)
200	70%	50%	30%	10%
600	15%	20%	25%	30%
1000	10%	15%	20%	25%
1400	5%	10%	15%	20%
1800	0%	5%	10%	15%
Your bid	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

Note: Enter a bid of e.g. 20% as '20' in the fields above. You should not add the % sign to your entry.

I confirm my bids.

[Next](#)

Short summary

- You and another candidate are bidding to take over a firm.
- There are five possible future profit levels and the likelihood of each one of these realizing depends on the situation selected for you.
- There are four situations and each is equally likely to be selected.
- You do not know the situation that is selected for you at the moment you specify your bids.
- Your relevant bid is the bid you specified for the situation that is ultimately selected for you.
- The same applies to the other candidate, but another situation may be selected for them.
- The candidate with the highest relevant bid wins the auction.
- The loser of the auction earns nothing apart from the initial budget.
- The additional earnings to the winner of the auction is the realized future profit minus the price. The realized profit is determined in accordance to the situation of the winner. The price to be paid is 200 plus the share of the realized future profit, where this share equals the relevant bid of the loser.

Figure 11: Decision screen

C Individual characteristics

C.1 Risk attitude

For our first measure of risk attitude we use the dynamic Bomb Risk Elicitation Task of Crosetto and Filippin (2013). Starting from zero, participants collect randomly picked boxes from a 10-by-10 field with 100 boxes, until they stop the collection process. Every time a box is collected their provisional payoff increases by one point. However, one of the boxes is hiding a bomb. If the bomb is not among the collected boxes, the payoff for this task equals the provisional payoff at the time the collection process was terminated (i.e., they receive one point for each box they collected); otherwise, if the bomb is in one of the collected boxes, they conclude with a zero payoff. A participant’s risk attitude is defined by the number of boxes they collect (with collecting 50 boxes being the expected payoff maximizing decision). Figure 12 plots, for each treatment, the cumulative distribution over the number of boxes that participants collected. The figure does not show major differences in distributions across treatments, which is confirmed via Mann-Whitney tests ($p > .182$).

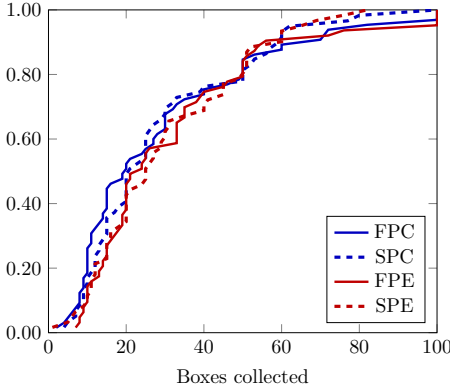


Figure 12: Cumulative distributions over the number of boxes opened.

For our second measure of risk attitude, following Dohmen et al. (2011), we use answers to the question: How do you see yourself: Are you generally a person who is fully prepared to take risks or do you try to avoid taking risks? Please tick a box on the scale, where the value 0 means: “not at all willing to take risks” and the value 10 means: “very willing to take risks”. Figure 13 shows, for each treatment, the cumulative distribution of the answers provided. Mann-Whitney tests do not reject equality in any of the four relevant

pairwise comparisons ($p > .112$).

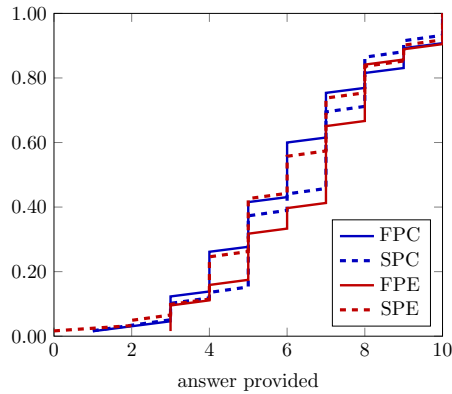


Figure 13: Cumulative distributions over answers in the self-assessed risk attitude question.

The first scatter plot in Figure 14 plots for each of the participants the pair of answers to the two risk attitude tasks. The two measures of risk attitude are positively correlated, significant at 1% level, but the relation is not very strong ($\rho = 0.1874$). Using principal-component analysis we create a new measure. The latter two scatter plots in the figure plot for each participant the new measure against the elicited measures. The positive correlation of this new measure with each of the elicited measures is significant at the 1% level and, more importantly, very high ($\rho = 0.7705$ for both measures). In our analysis we incorporate this newly created measure of risk attitude, with values ranging from -2.55 to 3.41 , as a linear variable.

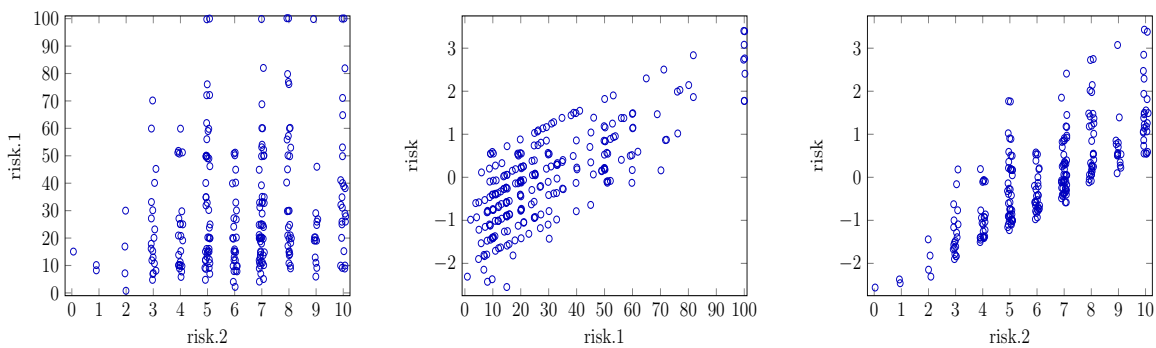


Figure 14: Scatter plots relating the three measures of risk attitude.

C.2 Loss attitude

Participants are offered a lottery that yields a payoff of +25 with 50% chance and a negative payoff of $-X$ with the complementary 50% chance. The value of X will be a number uniformly randomly drawn out of the numbers from 0 to 50. Participants are asked to provide the maximum value of this number X for which they are willing to accept the lottery. Following the Becker-DeGroot-Marschak mechanism (Becker et al., 1964), the lottery is accepted and played if the number X drawn is at most the number stated by the participant; otherwise, the lottery is declined and not played. Figure 15 shows, for each treatment, the cumulative distribution of the answers provided. Mann-Whitney tests reject equality for the comparison between FPC and FPE ($p = .0353$), but not between any other pair of treatments ($p > .264$). However, the significance of the difference between FPC and FPE disappears when ignoring individuals that reported 0 or 50 ($p = .1143$).

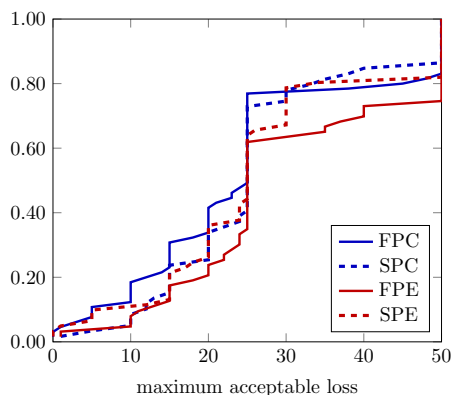


Figure 15: Cumulative distributions over answers in the loss attitude question.

C.3 Risk literacy

We use the four multiple-choice questions from the Berlin Numeracy Test of Cokely et al. (2012) to elicit participants' risk literacy. Participants are given 150 seconds to answer the four questions. Table 11 shows for each treatment separately, and all treatments combined, the percentage of participants that answered each of the four question correctly and the distribution over the number of correctly answered questions.¹⁶ In our analysis

¹⁶For the last two questions, the percentage of participants that answered each of them correctly is below 25%, which would be the chance a perfect randomization mechanism would answer correctly. Since there

we incorporate risk literacy as a linear variable encoded via the number of correct answers given.

Treatment	Question correct				Number of questions correct				
	Q1	Q2	Q3	Q4	0	1	2	3	4
FPC	49.23	41.54	13.85	9.23	26.15	38.46	33.77	4.62	0.00
SPC	52.54	35.59	20.34	10.17	16.95	52.54	25.42	5.08	0.00
FPE	44.44	42.86	22.22	12.70	23.81	41.27	25.40	7.94	1.59
SPE	49.18	42.62	19.67	14.75	24.59	31.15	37.70	6.56	0.00
All	48.79	40.73	18.95	11.69	22.98	40.73	29.84	6.05	0.40

Table 11: Details on the risk literacy task.

was time pressure, not all participants managed to answer all questions. Among those that provided an answer, the overall percentages are 49.79%, 42.98%, 22.17% and 14.95%; still, worse than a randomization device for the last two questions.