Improved Weighted Covariance-Based Detector for Spectrum Sensing in Rayleigh Fading Channel

AQ1 Huadong Lai, Mingxing Liu, *Member, IEEE*, Jinqiang Xu, Peng Luo, Changrun Chen, and Weichao Xu

***Abstract*—In this letter, we propose an improved weighted covariance based detector (IWCD) for spatially correlated time-varying Rayleigh fading channel. The proposed method uses adaptive weights that are tailored to the dynamic nature of the channels. These weights can be chosen manually to meet practical requirements or derived theoretically by optimizing some performance index, such that the IWCD outperforms traditional weighted covariance-based detectors (WCDs), which rely heavily on data-aided weights determined by the sample covariance matrix (SCM). Performance merits in terms of the probabilities of false alarm and detection are analyzed in the low signal-to-noise-ratio (SNR) regime. Besides, the optimal weights are derived via maximizing the modified deflection coefficient (MDC). A reasonable estimator of the optimal weights is also constructed armed with the available samples at hand. Theoretical analyses and experimental results demonstrate the superiority of our proposed method over existing works in various scenarios.**

***Index Terms*—Spectrum sensing, weighted covariance based detector, rayleigh fading channel, modified deflection coefficient.**

I. Introduction

COGNITIVE radio (CR), which allows the unlicensed secondary users to utilize the idle spectrum bands originally allocated to but not occupied by the licensed primary users, is recognized as a promising network architecture to improve the spectrum utilization efficiency and alleviate the problem of spectrum scarcity [1], [2], [3]. Spectrum sensing, as one of the most important functionalities of CR, aims at seeking the idle frequency band via continuously monitoring the activity state of PUs [4].

Traditional energy detection is widely utilized for spectrum sensing owing to its low computational complexity and simplicity of implementation. However, the performance of ED will degrade considerably in the presence of noise uncertainty [5]. To overcome it, a variety of robust spectrum sensing schemes have been addressed in the literature, such as the correlation-based detector (covariance absolute value (CAV) [6], volume based detection (VOL) [7], hadamard ratio test (HDM) [8]) and the machine learning-based schemes (CNN-LSTM [9] and CM-CNN [10]). By assuming the quasi-static fading channels, these approaches are capable of delivering desirable performance gain, but they may suffer from performance deterioration when the transmission channel is time-varying fading. To this end, several research efforts in the aspect of weighted covariance are addressed for time-varying fading channel, such as complex-valued WCD (CWCD) [11], real-valued WCD (RWCD) [12], generalization RWCD (GRWCD) [13] and modified GRWCD (MGRWCD) [13]. The pivotal idea is to construct the WCD-based statistic by employing the SCM-based weights within the principle of CAV. The performance of WCDs can be significantly enhanced by employing the SCM-aided weights to reduce the overlap between the distributions of test statistic with and without primary signals. However, the weights arising from the SCM are deterministic and fixed, and a heuristic method for achieving remarkable performance gain is to find the more flexible combined weights that are tailored to the time-varying channel.

Inspired by it, in this letter, an improved weighted covariance based detector (IWCD) is addressed for the time-varying correlated channels. Compared to the traditional data-aided WCDs, the proposed method exhibits the wider degree of flexibility because the utilized weights can be determined by manual selection for practical demands or by theoretical deduction via the optimization of some performance index. The analytic expressions of the false alarm probability and detection probability are derived in the scenarios where the SNR is low. Then, an optimization problem based on MDC is formulated, armed with which the optimal weights can be determined. In addition, the optimal weights are reasonably estimated after estimating the unknown parameters from the available samples. Numerical examples reveal that the proposed IWCD method is superior to other state-of-the-art detectors available in the literature.

Notation: The operators $\mathrm{tr}(\cdot)$, $|\cdot|$, $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ denote trace, modulus, conjugate, transpose and conjugate transpose, respectively. The symbols of $\mathbb{E}(x)$ and $\mathbb{V}(x)$ are utilized to represent the mean and variance of a random variable x. $\mathbf{x} \sim \mathcal{N}(\mu, \mathbf{\Sigma})(\mathcal{CN}(\mu, \mathbf{\Sigma}))$ means that $\mathbf{x}$ follows the real (complex) Gaussian distribution with mean μ and covariance matrix $\mathbf{\Sigma}$, whereas $\sim$ signifies "distributed as". The real and imaginary

AQ2 Manuscript received 9 January 2024; accepted 15 March 2024. This work was supported in part by the Program for Scientific Research Start-Up Funds of Guangdong Ocean University under Grant 060302112316, and in part by the National Natural Science Foundation of China under Grant 62171143 and Grant 62171141. The associate editor coordinating the review of this article and approving it for publication was J. Lee. *(Corresponding authors: Changrun Chen; Mingxing Liu.)*

Huadong Lai, Mingxing Liu, Jinqiang Xu, and Peng Luo are with the School of Electronics and Information Engineering, Guangdong Ocean University, Zhanjiang 524088, Guangdong, China (e-mail: hdlai@gdou.edu.cn; liumx@gdou.edu.cn; xujinqiang1@163.com; luopeng@gdou.edu.cn).

Changrun Chen is with the School of Engineering, University of Kent, CT2 7NT Canterbury, U.K. (e-mail: c.chen@kent.ac.uk).

Weichao Xu is with the Department of Automatic Control, School of Automation, Guangdong University of Technology, Guangzhou 510006, Guangdong, China (e-mail: wcxu@gdut.edu.cn).

Digital Object Identifier 10.1109/LWC.2024.3379528

parts of x are denoted by Re(x) and Im(x), respectively. We utilize $\mathbf{0}_M$ and $\mathbf{I}_M$ to represent the $M \times 1$ zero vector and $M \times M$ identity matrix. $\Psi(x)$ and $\Gamma(\cdot)$ correspond to a special confluent hypergeometric function ${}_1\mathrm{F}_1(-1/2, 1, x)$ and Gamma function [14], respectively. The diag{x} stands for a diagonal matrix with diagonal elements consisting of x.

II. Preliminaries

A. Problem Formulation

Herein we consider the detection of primary signal for a CR system that composes of one PU and one SU equipped with M sensing antennas through time-varying Rayleigh fading channel. Denote the absence and presence of primary signal in a specific frequency band by $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively. Under the above binary hypothesis, the observation vector $\mathbf{x}(k)$ from $M-$antenna SU at time instant k can be expressed as [15]

$$\begin{cases} \mathcal{H}_0 : \mathbf{x}(k) = \mathbf{w}(k) \\ \mathcal{H}_1 : \mathbf{x}(k) = \mathbf{h}(k)s(k) + \mathbf{w}(k) \end{cases}, \; k = 1, 2, \ldots, K, \quad (1)$$

where $s(k) \sim \mathcal{CN}(0, \sigma_s^2(k))$, denotes the transmitted PU signal which is deterministic but unknown with instantaneous power $\sigma_s^2(k)$; $\mathbf{h}(k) \sim \mathcal{CN}(\mathbf{0}_M, \sigma_h^2 \mathbf{\Phi})$ represents the correlated Rayleigh fading channel with σ_h^2 and $\mathbf{\Phi}$ being the channel power and normalized correlation matrix, respectively; $\mathbf{w}(k) \sim \mathcal{CN}(\mathbf{0}_M, \mathbf{R}_w)$ is the additive background noise with unknown diagonal covariance matrix $\mathbf{R}_w = \mathrm{diag}\{\sigma_1^2, \ldots, \sigma_M^2\}$. Generally, it is assumed that $s(k)$, $\mathbf{h}(k)$ and $\mathbf{w}(k)$ are statistically independent with each other.

B. Channel Model

Due to its simplicity and excellent characterization of spatial correlation, the antenna correlation matrix $\mathbf{\Phi}$ is typically described by exponential correlation model [15], i.e.,

$$\mathbf{\Phi}_{mn} = \begin{cases} \rho^{n-m}, & m \le n \\ \mathbf{\Phi}_{nm}^*, & m > n \end{cases}, \; m, n = 1, 2, \ldots, M, \quad (2)$$

where $|\rho| \le 1$ is the complex-valued correlation coefficient between two neighboring antennas.

In such occasion, the channel vector $\mathbf{h}(k)$ is generated as

$$\mathbf{h}(k) = \mathbf{\Phi}^{\frac{1}{2}} \mathbf{g}(k), \; k = 1, 2, \ldots, N, \quad (3)$$

where $\mathbf{g}(k) \sim \mathcal{CN}(\mathbf{0}_M, \mathbf{I}_M)$, denotes the standard complex Gaussian distributed random vector.

III. Test Statistic and Performance Analysis

This section first briefly reviews weighted covariance-based sensing algorithms framework, and then elaborates the proposed test statistic. Besides, performance measures for the probabilities of false alarm probability and detection are studied in the low SNR regime with the assistance of central limit theorem (CLT) [16]. Finally, the optimal weights are computed via the optimization problem based on MDC [17], an estimate of which is also obtained with the available samples.

A. Improved Weighted Covariance Based Detection

It is stated in [11], [12], [13] that the test statistics of WCDs are constructed by means of applying different weights to the entries of normalized SCM, i.e., $T_{\mathrm{WCD}} \triangleq \sum_{i=1}^{M-1} \omega_i \sum_{n-m=i} |r'_{mn}|$ where $r'_{mn} = r_{mn}/\hat{\sigma}^2$ with $\hat{\sigma}^2 = \sum_{m=1}^{M} r_{mm}/M$ and r_{mn} being the (m, n) entries of SCM defined as $\mathbf{R} = \frac{1}{K}\sum_{i=1}^{K} \mathbf{x}(k)\mathbf{x}^H(k)$, and ω_i is the weight obtained from the SCM. The data-aided weights can reduce the overlap between the distributions of detection statistic with and without the primary signal, thereby improving the detection power. However, the weights from the SCM are deemed to be fixed and unalterable, and a natural idea to achieve the better performance is to adopt the more flexible strategy with alterable weights, leading to our proposed method, as

$$T_{\mathrm{IWCD}} \triangleq \sum_{i=0}^{M-1} \omega_i \sum_{n-m=i} |r_{mn}| \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda, \quad (4)$$

where λ is the decision threshold for a given false alarm probability, $\{\omega_i\}_{i=0}^{M-1}$ are the weights, which play a pivotal role in improving the performance of our detection scheme. The weights can be manually prescribed according to practical requirements or theoretically determined by optimizing some performance index. Note that when $\omega_0 = 0$, IWCD reduces to CAV for $\omega_i = 1/M$ $(i = 1, \ldots, M-1)$, or reduces to CWCD for $\omega_i = 4\sum_{n-m=i} \mathrm{Re}(r_{mn})/\hat{\sigma}^4$ $(i = 1, \ldots, M-1)$.

B. False Alarm Probability

We first establish the closed-from expression for the false alarm probability by following along the line in [18]. Specifically, in the scenario of low SNR and large K, $\{r_{mm}\}_{m=1}^{M}$ and $\{r_{mn}\}_{n-m=1}^{M-1}$ are statistically independent, with PDFs:

$$r_{mm}|\mathcal{H}_0 \sim \mathcal{N}\left(\sigma_m^2, \frac{1}{K}\sigma_m^4\right), \quad (5)$$

$$r_{mn}|\mathcal{H}_0 \sim \mathcal{CN}\left(0, \frac{1}{K}\sigma_m^2\sigma_n^2\right), \quad (6)$$

where σ_m^2 and σ_n^2 are m-th and n-th diagonal elements in $\mathbf{R}_w$.

Denote $T_i \triangleq \sum_{n-m=i} |r_{mn}|$, it is very easy to obtain

$$\mathbb{E}(T_0|\mathcal{H}_0) = \sum_{m=1}^{M} \mathbb{E}[|r_{mm}|] = \sum_{m=1}^{M} \sigma_m^2, \quad (7)$$

$$\mathbb{V}(T_0|\mathcal{H}_0) = \sum_{m=1}^{M} \mathbb{V}[|r_{mm}|] = \frac{1}{K}\sum_{m=1}^{M} \sigma_m^4. \quad (8)$$

The amplitude $|r_{mn}|$ for $n > m$, follows the Rayleigh distribution with scale parameter $\tilde{\sigma} = \frac{\sigma_m \sigma_n}{\sqrt{2K}}$, whose first few raw moments are $\mathbb{E}(|r_{mn}|^j) = \tilde{\sigma}^j 2^{\frac{j}{2}} \Gamma(1 + \frac{j}{2})$ [19].

We then obtain T_i for $i = 1, \ldots, M-1$,

$$\mathbb{E}[T_i|\mathcal{H}_0] = \sum_{n-m=i} \mathbb{E}[|r_{mn}|] = \sqrt{\frac{\pi}{4K}} \sum_{m=1}^{M-i} \sigma_m \sigma_{m+i}, \quad (9)$$

$$\mathbb{V}[T_i|\mathcal{H}_0] = \sum_{n-m=i} \mathbb{V}[|r_{mm}|] = \frac{4-\pi}{4K} \sum_{m=1}^{M-i} \sigma_m^2 \sigma_{m+i}^2. \quad (10)$$

The mean (denoted by μ_0) and variance (denoted by σ_0^2) of T_{IWCD} under $\mathcal{H}_0$ can be respectively computed as

$$\begin{aligned}\mu_0 &= \sum_{i=0}^{M-1} \mathbb{E}[\omega_i T_i|\mathcal{H}_0] \\ &= \omega_0 \sum_{m=1}^{M} \sigma_m^2 + \sqrt{\frac{\pi}{4K}} \sum_{i=1}^{M-1} \omega_i \sum_{m=1}^{M-i} \sigma_m \sigma_{m+i}, \quad (11)\end{aligned}$$

$$\begin{aligned}\sigma_0^2 &= \sum_{i=0}^{M-1} \mathbb{V}[\omega_i T_i|\mathcal{H}_0] \\ &= \frac{\omega_0^2}{K} \sum_{m=1}^{M} \sigma_m^4 + \frac{4-\pi}{4K} \sum_{i=1}^{M-1} \omega_i^2 \sum_{m=1}^{M-i} \sigma_m^2 \sigma_{m+i}^2. \quad (12)\end{aligned}$$

Based on CLT, The distribution of IWCD can be computed as

$$T_{\text{IWCD}}|\mathcal{H}_0 \sim \mathcal{N}\left(\mu_0, \sigma_0^2\right). \quad (13)$$

When the threshold λ is pre-given, the false alarm probability is computed as

$$P_f = \Pr(T_{\text{IWCD}} > \lambda) = \mathrm{Q}\left(\frac{\lambda - \mu_0}{\sigma_0}\right), \quad (14)$$

where $\mathrm{Q}(x) \triangleq \frac{1}{2\pi} \int_x^{+\infty} \mathrm{e}^{-\frac{u^2}{2}} \,\mathrm{d}\, u$ is the Gaussian-Q function.

Denote by $\mathrm{Q}^{-1}(\cdot)$ the inverse function of $\mathrm{Q}(\cdot)$. The decision threshold can be evaluated with a prescribed P_f, as

$$\lambda = \sigma_0 \,\mathrm{Q}^{-1}(P_f) + \mu_0. \quad (15)$$

C. Detection Probability

The analytic form for the probability of detection are investigated in this subsection. To continue, the lemma below is required to establish the distribution of r_{mn} under $\mathcal{H}_1$.

Lemma 1: Let $\rho_i \triangleq \rho^{n-m}$ for $n \geq m$, $\sigma_{sh}^2(k) \triangleq \sigma_s^2(k)\sigma_h^2$, $\sigma_{sh}^2 \triangleq \frac{1}{K}\sum_{k=1}^{K} \sigma_{sh}^2(k)$, $\tilde{\sigma}_m^2 \triangleq \sigma_{sh}^2 + \sigma_m^2$ and $\tilde{\sigma}_n^2 \triangleq \sigma_{sh}^2 + \sigma_n^2$. The SNR is defined as $\text{SNR} = 10\log_{10}\left(\frac{\sigma_{sh}^2}{\frac{1}{M}\operatorname{tr}(\mathbf{R}_w)}\right)$. Then, when $\mathcal{H}_1$ holds, $\{r_{mm}\}_{m=1}^{M}$ and $\{r_{mn}\}_{n-m=1}^{M-1}$ are mutually independent in the low SNR regime, whose PDFs are respectively given by

$$r_{mm}|\mathcal{H}_1 \sim \mathcal{N}\left[\tilde{\sigma}_m^2, \frac{1}{K}\tilde{\sigma}_m^4\right], \quad (16)$$

$$r_{mn}|\mathcal{H}_1 \sim \mathcal{CN}\left[\rho_i \sigma_{sh}^2, \frac{1}{K}\tilde{\sigma}_m^2 \tilde{\sigma}_n^2\right]. \quad (17)$$

Proof: Due to the space limitation, the proof is integrated in the supplementary material. ■

AQ3 According to (16), we have

$$\mathbb{E}[T_0|\mathcal{H}_1] = \sum_{m=1}^{M} \mathbb{E}[T_{mm}|\mathcal{H}_1] = \sum_{m=1}^{M} \tilde{\sigma}_m^2, \quad (18)$$

$$\mathbb{V}[T_0|\mathcal{H}_1] = \sum_{m=1}^{M} \mathbb{V}[T_{mm}|\mathcal{H}_1] = \frac{1}{K}\sum_{m=1}^{M} \tilde{\sigma}_m^4. \quad (19)$$

It is obvious that $\mathrm{Re}(r_{mn})$ and $\mathrm{Im}(r_{mn})$ is independent of each other when the SNR is low. Thus, the amplitude $|r_{mn}|$ follows the Rician distribution $\mathcal{R}(\nu, \mathcal{V})$ with $\nu = |\rho_i|\sigma_{sh}^2$ and $\mathcal{V} = \sqrt{\frac{1}{2K}\tilde{\sigma}_m^2 \tilde{\sigma}_n^2}$. We then have

$$\mathbb{E}(|r_{mn}|) = \mathcal{V}\sqrt{\frac{\pi}{2}}\Psi\left(-\frac{\nu^2}{2\mathcal{V}^2}\right), \quad (20)$$

$$\mathbb{E}\left(|r_{mn}|^2\right) = 2\mathcal{V}^2 + \nu^2, \quad (21)$$

which produces T_i for $i = 1, \ldots, M-1$,

$$\mathbb{E}[T_i|\mathcal{H}_1] = \sum_{m=1}^{M-i} \sqrt{\frac{\pi}{4K}}\tilde{\sigma}_m \tilde{\sigma}_{m+i} \Psi\left(-\frac{K\sigma_{sh}^4 |\rho_i|^2}{\tilde{\sigma}_m^2 \tilde{\sigma}_{m+i}^2}\right), \quad (22)$$

$$\begin{aligned}\mathbb{V}[T_i|\mathcal{H}_1] = \sum_{m=1}^{M-i} \Bigg[& \frac{\tilde{\sigma}_m^2 \tilde{\sigma}_{m+i}^2}{K} + |\rho_i|^2 \sigma_{sh}^4 \\ & - \frac{\pi}{4K}\tilde{\sigma}_m^2 \tilde{\sigma}_{m+i}^2 \Psi^2\left(-\frac{K\sigma_{sh}^4 |\rho_i|^2}{\tilde{\sigma}_m^2 \tilde{\sigma}_{m+i}^2}\right)\Bigg]. \quad (23)\end{aligned}$$

Combining (18), (19), (22) and (23) yields the mean (denoted by μ_1) and variance (denoted by σ_1^2) of T_{IWCD} under $\mathcal{H}_1$, respectively, as

$$\begin{aligned}\mu_1 = & \sum_{i=0}^{M-1} \mathbb{E}[\omega_i T_i|\mathcal{H}_1] = \omega_0 \sum_{m=1}^{M} \tilde{\sigma}_m^2 \\ & + \sum_{i=1}^{M-1} \omega_i \sum_{m=1}^{M-i} \sqrt{\frac{\pi}{4K}}\tilde{\sigma}_m \tilde{\sigma}_{m+i} \Psi\left(-\frac{K\sigma_{sh}^4 |\rho_i|^2}{\tilde{\sigma}_m^2 \tilde{\sigma}_{m+i}^2}\right), \quad (24)\end{aligned}$$

$$\begin{aligned}\sigma_1^2 = & \sum_{i=0}^{M-1} \mathbb{V}[\omega_i T_i|\mathcal{H}_1] = \frac{\omega_0^2}{K} \sum_{m=1}^{M} \tilde{\sigma}_m^4 + \sum_{i=1}^{M-1} \omega_i^2 \sum_{m=1}^{M-i} \Bigg[\frac{\tilde{\sigma}_m^2 \tilde{\sigma}_{m+i}^2}{K} \\ & + |\rho_i|^2 \sigma_{sh}^4 - \frac{\pi}{4K}\tilde{\sigma}_m^2 \tilde{\sigma}_{m+i}^2 \Psi^2\left(-\frac{K\sigma_{sh}^4 |\rho_i|^2}{\tilde{\sigma}_m^2 \tilde{\sigma}_{m+i}^2}\right)\Bigg]. \quad (25)\end{aligned}$$

In view of the CLT, the distribution of IWCD can then be approximated as

$$T_{\text{IWCD}}|\mathcal{H}_1 \sim \mathcal{N}\left(\mu_1, \sigma_1^2\right). \quad (26)$$

The detection probability can thus be obtained with the given threshold λ, as

$$P_d = \mathrm{Q}\left(\frac{\lambda - \mu_1}{\sigma_1}\right). \quad (27)$$

D. Optimal Weights

Several possible performance indices, such as detection probability, receiver operating characteristic (ROC) curve, asymptotic relative efficiency and deflection coefficient (DC) [20], are available for performance optimization of a detector, among which, the DC appeals interesting due to its easy calculation and near-optimal manner. However, it has been pointed out in [20] that the DC might not be a good indicator of performance when the sample size is very low. To circumvent this drawback, a heuristic but efficient approach namely modified deflection coefficient is proposed in [17], which measures the variance-normalized distance between the centers of two PDFs under hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$. The optimal weight vector is able to be found with low computational complexity by optimizing the MDC. Let $\boldsymbol{\omega} = [\omega_0, \omega_1, \ldots, \omega_{M-1}]^T$, $\boldsymbol{\mu_i} = [\mathbb{E}(T_0), \mathbb{E}(T_1), \ldots, \mathbb{E}(T_{M-1})]^T|\mathcal{H}_i$, $i = 0, 1$, $\boldsymbol{f} = \boldsymbol{\mu_1} - \boldsymbol{\mu_0}$, and $\mathbf{\Lambda} = \mathrm{diag}\{\mathbb{V}(T_0|\mathcal{H}_1), \mathbb{V}(T_1|\mathcal{H}_1), \ldots, \mathbb{V}(T_{M-1}|\mathcal{H}_1)\}$. The optimization problem with respect to the maximization of MDC can be stated as [17]

$$\max_{\boldsymbol{\omega}} d_m^2(\boldsymbol{\omega}) = \frac{(\mu_1 - \mu_0)^2}{\boldsymbol{\omega}^T \mathbf{\Lambda} \boldsymbol{\omega}} = \frac{\left(\boldsymbol{f}^T \boldsymbol{\omega}\right)^2}{\boldsymbol{\omega}^T \mathbf{\Lambda} \boldsymbol{\omega}}, \text{ s. t. } |\boldsymbol{\omega}| = 1. \tag{28}$$

Define $\boldsymbol{\omega}' \triangleq \mathbf{\Lambda}^{-\frac{T}{2}} \boldsymbol{f}$. Then, the optimal wights $\boldsymbol{\omega}^o$ follows from [17], as

$$\boldsymbol{\omega}^o = \frac{\mathbf{\Lambda}^{-\frac{1}{2}} \boldsymbol{\omega}'}{|\mathbf{\Lambda}^{-\frac{1}{2}} \boldsymbol{\omega}'|}. \tag{29}$$

Noticing the diagonal stricture of $\mathbf{\Lambda}$ gives

$$\omega_{i-1}^o = \frac{f_i}{\Lambda_{ii}} \left[\sum_{m=1}^{M} \frac{f_m^2}{\Lambda_{mm}^2} \right]^{-1/2}, \quad i = 1, 2, \ldots, M. \tag{30}$$

Remark 1: For the sake of illustration, we define $\varsigma_i \triangleq \rho_i \sigma_{sh}^2$ for $i = 1, \ldots, M-1$. It is obvious from (30) that the acquisition of optimal weights involve the prior knowledge $(\sigma_m, \tilde{\sigma}_m, \varsigma_i, m = 1, 2, \ldots, M, \ i = 1, \ldots, M-1)$ of the observed data under both hypotheses, which is difficult to obtain in practice. Assume that there are $M \times K$ noise-only sample $[\mathbf{x}^{(0)}(1), \ldots, \mathbf{x}^{(0)}(K)]$ (the noise-only sample are available in possible [21]) and noise-bearing sample $[\mathbf{x}^{(1)}(1), \ldots, \mathbf{x}^{(1)}(K)]$, the relevant unknown parameters can thus be estimated, i.e.,

$$\hat{\sigma}_m^2 = \frac{1}{K} \sum_{k=1}^{K} \mathrm{x}_m^{(0)}(k) \left(\mathrm{x}_m^{(0)}(k)\right)^*, \tag{31}$$

$$\hat{\tilde{\sigma}}_m^2 = \frac{1}{K} \sum_{k=1}^{K} \mathrm{x}_m^{(1)}(k) \left(\mathrm{x}_m^{(1)}(k)\right)^*, \tag{32}$$

$$\hat{\varsigma}_i = \frac{1}{K(M-i)} \sum_{k=1}^{K} \sum_{m=1}^{M-i} \mathrm{x}_m^{(1)}(k) \left(\mathrm{x}_{m+i}^{(1)}(k)\right)^*. \tag{33}$$

In such case, the estimated optimal weights are computed as

$$\hat{\omega}_{i-1}^o = \frac{\hat{f}_i}{\hat{\Lambda}_{ii}} \left[\sum_{m=1}^{M} \frac{\hat{f}_m^2}{\hat{\Lambda}_{mm}^2} \right]^{-1/2}, \quad i = 1, 2, \ldots, M. \tag{34}$$

where

$$\hat{f}_1 = \sum_{m=1}^{M} \left(\hat{\tilde{\sigma}}_m^2 - \hat{\sigma}_m^2\right), \quad \hat{\Lambda}_{11} = \frac{1}{K} \sum_{m=1}^{M} \hat{\tilde{\sigma}}_m^4, \tag{35}$$

$$\hat{f}_i = \sqrt{\frac{\pi}{4K}} \sum_{m=1}^{M-i+1} \left[\hat{\tilde{\sigma}}_m \hat{\tilde{\sigma}}_{m+i-1} \Psi \left(- \frac{K|\hat{\varsigma}_{i-1}|^2}{\hat{\tilde{\sigma}}_m^2 \hat{\tilde{\sigma}}_{m+i-1}^2} \right) \right.$$
$$\left. - \hat{\sigma}_m \hat{\sigma}_{m+i-1} \right], \ i = 2, 3, \ldots, M, \tag{36}$$

$$\hat{\Lambda}_{ii} = \sum_{m=1}^{M-i+1} \left[\frac{\hat{\tilde{\sigma}}_m^2 \hat{\tilde{\sigma}}_{m+i-1}^2}{4K} \left(4 - \pi \Psi^2 \left(\frac{-K|\hat{\varsigma}_{i-1}|^2}{\hat{\tilde{\sigma}}_m^2 \hat{\tilde{\sigma}}_{m+i-1}^2} \right) \right) \right.$$
$$\left. + |\hat{\varsigma}_{i-1}|^2 \right], \ i = 2, 3, \ldots, M. \tag{37}$$

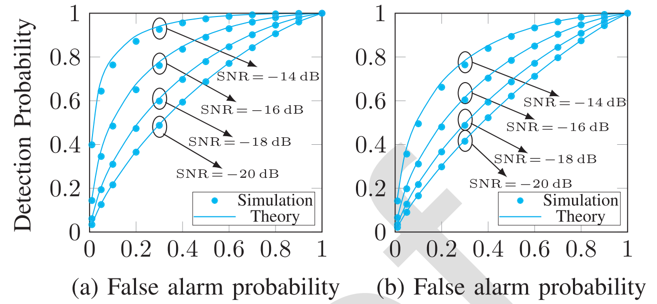

Fig. 1. Verification of theoretical results. (a) uniform noise with $K = 400$, $M = 6$ and $\rho = 0.7 + 0.1\iota$; (b) $K = 300$, $M = 4$ and $\rho = 0.6 + 0.2\iota$ with non-uniform noise variance $[-1, 0, 1.5, -0.5]$ dB.

AQ4

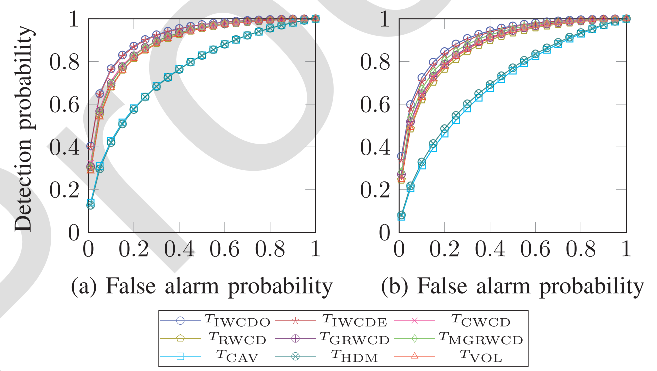

Fig. 2. Comparison of ROC curve with $K = 500$, $M = 4$ and SNR $= -14$ dB. (a) uniform noise with $\rho = 0.8 + 0.4\iota$; (b) non-uniform noise variance $[0.3, -0.4, -0.7, 0.8]$ dB with $\rho = 0.7 + 0.3\iota$.

IV. Numerical Results

This section provides numerical examples to validate the theoretical analyses and compare the performance of the proposed IWCDs obtained by the optimal weights (30) (IWCDO) and estimated weights (34) (IWCDE), to the four WCDS, namely CWCD [11], RWCD [12], GRWCD ($p = \frac{1}{4}$) [13], MGRWCD ($p = \frac{1}{4}$) [13], as well as three popular competitors, namely CAV [6], VOL [7], HDM [8]. In general, the noise power is assumed to be one for uniform noise and the average noise power is set to be one for non-uniform noise.

Fig. 1 validates the asymptotic expressions of P_f (14) and P_d (27) obtained via the optimal weights (30), by comparing the theoretical and simulated ROC curves. The parameter setup is $\rho = 0.7 + 0.1\iota$, $K = 400$ and $M = 6$ for uniform noise and $\rho = 0.6 + 0.2\iota$, $K = 300$ and $M = 4$ for non-uniform noise with variance $[-1, 0, 1.5, -0.5]$ dB, both with respect to four values of SNR $\in \{-14, -16, -18, -20\}$ dB. As expected, the theoretical values agree well with the simulation counterparts, thus verifying the correctness of our derived results.

Fig. 2 depicts the ROC curve of our proposed IWCD methods in comparison with other seven detectors for $M = 4$, $K = 500$ and SNR $= -14$ dB. Two values of high antenna correlation, $\rho = 0.8 + 0.4\iota$ with uniform noise variance and $\rho = 0.7 + 0.3\iota$ with non-uniform noise variance $[0.3, -0.4, -0.7, 0.8]$ dB, are considered in Fig. 2 (a) and Fig. 2 (b), respectively. It is clear that IWCDE of estimated weights performs comparably with IWCDO of optimal weights, both of which perform better than that of comparison approaches.

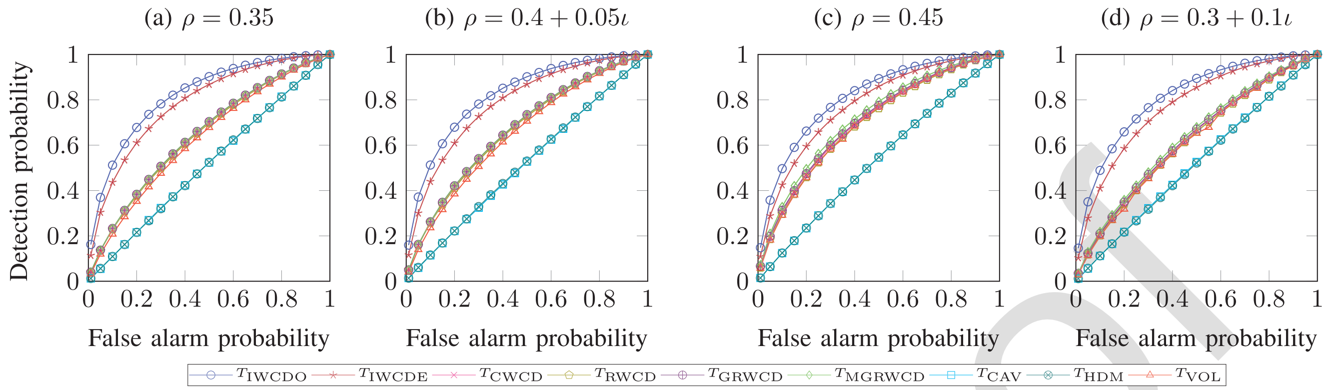

Fig. 3. Comparison of ROC curve for all considered detectors with $K = 300$, $M = 6$ and SNR $= -15$ dB. (a) uniform noise; (b) uniform noise; (c) non-uniform noise with variance [1, −0.5, 0.1, −1.2, 0.6,0] dB. (d) non-uniform noise with variance [1, −0.5, 0.1, −1.2, 0.6,0] dB.

Fig. 3 draws the detection performance with respect to ROC curve, of all considered detectors in the case where the correlation across the receiver antennas is low. Simulation parameters are set as $K = 300$, $M = 6$ and SNR $= -15$ dB. Four correlation coefficients in the forms of real value and complex value are considered for both the uniform and non-uniform background noise. Specifically, for the case where noise variance is identical, Fig. 3 (a) and Fig. 3 (b) shows the results corresponding to $\rho = 0.35$ and $\rho = 0.4 + 0.05\iota$, respectively; whereas the results for $\rho = 0.45$ and $\rho = 0.3 + 0.1\iota$ in the scenarios of non-uniform noise are plotted in Fig. 3 (c) and Fig. 3 (d), respectively. We can deduce from Fig. 3 that our proposed IWCD detectors are superior to other considered methods due to its highest detection probability under a specific false alarm probability. In addition, by comparing Fig. 2 and Fig. 3, the superiority of our proposed detector over other considered detectors can be more evidently observed in the low correlation regime. Compared with optimal weight-aided detector, the estimated weight-aided detector suffers from evident performance loss in the case of low correlation.

V. Conclusion

This letter developed an IWCD detector for cognitive radios with correlated multiple antennas. The proposed method depending on the arbitrary volatile weights, possesses more freedom than the traditional WCDs. The analytic forms with respect to the probabilities of false alarm and detection were derived, facilitating us to determine the optimal weights by maximizing the MDC. Besides, a proper estimator for the optimal weights was devised with the available samples. The superiority of the proposed detector over other state-of-the-art methods was shown via extensive numerical examples.

References

[1] W. Zhao, S. S. Ali, M. Jin, G. Cui, N. Zhao, and S.-J. Yoo, "Extreme eigenvalues-based detectors for spectrum sensing in cognitive radio networks," *IEEE Trans. Commun.*, vol. 70, no. 1, pp. 538–551, Jan. 2022.

[2] M. Ali, M. N. Yasir, D. M. S. Bhatti, and H. Nam, "Optimization of spectrum utilization efficiency in cognitive radio networks," *IEEE Wireless Commun. Lett.*, vol. 12, no. 3, pp. 426–430, Mar. 2023.

[3] H. Ma, X. Yuan, L. Zhou, B. Li, and R. Qin, "Joint block support recovery for sub-Nyquist sampling cooperative spectrum sensing," *IEEE Wireless Commun. Lett.*, vol. 12, no. 1, pp. 85–88, Jan. 2023.

[4] A. Gao, C. Du, S. X. Ng, and W. Liang, "A cooperative spectrum sensing with multi-agent reinforcement learning approach in cognitive radio networks," *IEEE Commun. Lett.*, vol. 25, no. 8, pp. 2604–2608, Aug. 2021.

[5] R. Tandra and A. Sahai, "SNR walls for signal detection," *IEEE Sensors J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 4–17, Feb. 2008.

[6] Y. Zeng and Y.-C. Liang, "Spectrum-sensing algorithms for cognitive radio based on statistical covariances," *IEEE Trans. Veh. Technol.*, vol. 58, no. 4, pp. 1804–1815, May 2009.

[7] L. Huang, C. Qian, Y. Xiao, and Q. Zhang, "Performance analysis of volume-based spectrum sensing for cognitive radio," *IEEE Trans. Wireless Commun.*, vol. 14, no. 1, pp. 317–330, Jan. 2015.

[8] L. Huang, Y. Xiao, H. C. So, and J. Fang, "Accurate performance analysis of hadamard ratio test for robust spectrum sensing," *IEEE Trans. Wireless Commun.*, vol. 14, no. 2, pp. 750–758, Feb. 2015.

[9] J. Xie, J. Fang, C. Liu, and X. Li, "Deep learning-based spectrum sensing in cognitive radio: A CNN-LSTM approach," *IEEE Commun. Lett.*, vol. 24, no. 10, pp. 2196–2200, Oct. 2020.

[10] C. Liu, J. Wang, X. Liu, and Y.-C. Liang, "Deep CM-CNN for spectrum sensing in cognitive radio," *IEEE J. Sel. Areas Commun.*, vol. 37, no. 10, pp. 2306–2321, Oct. 2019.

[11] M. Jin, Q. Guo, J. Xi, Y. Li, Y. Yu, and D. Huang, "Spectrum sensing using weighted covariance matrix in Rayleigh fading channels," *IEEE Trans. Veh. Technol.*, vol. 64, no. 11, pp. 5137–5148, Nov. 2015.

[12] A.-Z. Chen and Z.-P. Shi, "A real-valued weighted covariance-based detection method for cognitive radio networks with correlated multiple antennas," *IEEE Commun. Lett.*, vol. 22, no. 11, pp. 2290–2293, Nov. 2018.

[13] A.-Z. Chen, Z.-P. Shi, and J. Xiong, "Generalized real-valued weighted covariance-based detection methods for cognitive radio networks with correlated multiple antennas," *IEEE Access*, vol. 7, pp. 34373–34382, 2019.

[14] A. Winkelbauer, "Moments and absolute moments of the normal distribution," 2012, *arXiv:1209.4340*.

[15] A. Nafkha and B. Aziz, "Closed-form approximation for the performance of finite sample-based energy detection using correlated receiving antennas," *IEEE Wireless Commun. Lett.*, vol. 3, no. 6, pp. 577–580, Dec. 2014.

[16] A. W. V. D. Vaart, *Asymptotic Statistics* (Cambridge Series in Statistical and Probabilistic Mathematics). Cambridge, U.K.: Cambridge Univ. Press, 1998.

[17] Z. Quan, S. Cui, and A. H. Sayed, "Optimal linear cooperation for spectrum sensing in cognitive radio networks," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 28–40, Feb. 2008.

[18] A.-Z. Chen, Z.-P. Shi, H. Sun, Z.-Q. He, F. Bu, and D. Yang, "A low-complexity spectrum sensing method for noncircular signal in cognitive radio networks with multiple receive antennas," *IEEE Commun. Lett.*, vol. 23, no. 7, pp. 1190–1193, Jul. 2019.

[19] "Rayleigh distribution." [Online]. Available: https://en.jinzhao.wiki/wiki/Rayleigh_distribution AQ5

[20] S. A. Kassam, *Signal Detection in Non-Gaussian Noise*. New York, NY, USA: Springer, 2012.

[21] V. Rakovic, D. Denkovski, V. Atanasovski, P. Mähönen, and L. Gavrilovska, "Capacity-aware cooperative spectrum sensing based on noise power estimation," *IEEE Trans. Commun.*, vol. 63, no. 7, pp. 2428–2441, Jul. 2015.