NICHE HYPERGRAPHS

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Abstract

If $D = (V, A)$ is a digraph, its *niche hypergraph* $N\mathcal{H}(D) = (V, \mathcal{E})$ has the edge set $\mathcal{E} = \{e \subseteq V \mid |e| \geq 2 \wedge \exists v \in V : e = N_D^-(v) \vee e = N_D^+(v)\}$. Niche hypergraphs generalize the well-known niche graphs (see [11]) and are closely related to competition hypergraphs (see [40]) as well as double competition hypergraphs (see [33]). We present several properties of niche hypergraphs of acyclic digraphs.

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