



AALBORG UNIVERSITY
DENMARK

Aalborg Universitet

Mutual Information Profile of a BISMC with Applications

Huber, Johannes; Hehn, Thorsten; Land, Ingmar Rüdiger; Hoeher, Peter Adam

Published in:

Proceedings Conference in Information Sciences and Systems (CISS)

Publication date:

2005

Document Version

Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

Citation for published version (APA):

Huber, J., Hehn, T., Land, I., & Hoeher, P. A. (2005). Mutual Information Profile of a BISMC with Applications. In Proceedings Conference in Information Sciences and Systems (CISS) The Johns Hopkins University, Baltimore, MD.

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- ? Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- ? You may not further distribute the material or use it for any profit-making activity or commercial gain
- ? You may freely distribute the URL identifying the publication in the public portal ?

Take down policy

If you believe that this document breaches copyright please contact us at vbn@aub.aau.dk providing details, and we will remove access to the work immediately and investigate your claim.

Mutual Information Profile of a BISMIC with Applications

Johannes Huber, Thorsten Hehn Ingmar Land, Peter A. Hoeher
 Inst. f. Information Transmission Information & Coding Theory Lab
 Univ. of Erlangen, Germany Univ. of Kiel, Germany
 e-mail: huber,hehn@LNT.de e-mail: ph,il@tf.uni-kiel.de

Abstract — It is shown how to decompose any Binary Input Symmetric Memoryless Channel (BISMIC) into BSCs. We explain why BEC and BSC are the two extreme cases of any BISMIC with respect to the Mutual Information Profile (MIP). Finally, the paper points out useful applications for MIP.

I. INTRODUCTION

By introducing the Mutual Information Profile (MIP) there is a new, alternative way of completely characterizing a BISMIC which is equivalent to the traditional one given by the channels transition probabilities but describes the behaviour of a channel in a more evident way and thus makes some theorems easier to be proved. Thinking in mutual information profiles is especially interesting when talking about information combining.

II. DECOMPOSING BISMICs

It is shown that any Binary Input Symmetric Memoryless Channel (BISMIC) can be decomposed into BSCs (see also [1]). A subchannel of a BISMIC which is not further decomposable has an output alphabet with one or two output symbols. This will be proved by contradiction where we assume that the subchannel a has more than two different output symbols. As the subchannel is strongly symmetric, the transition matrix has only two probabilities p_1, p_2 and all of its columns are permutations of each other (note that there are only two different permutations). If $p_1 = p_2$, the transition matrix of the considered subchannel can be further decomposed into single columns, each of them corresponding to a BSC with error probability $1/2$. If $p_1 \neq p_2$ we group the two different columns to submatrices and decompose the BISMIC further into channels corresponding to these matrices (decomposition into BSCs). Decomposition of subchannel a is a contradiction to our assumption (q. e. d.).

Along with the decomposition into BSCs we introduce the MIP $w_I(i)$, which is the probability density function of mutual information. Note that the average mutual information over the BISMIC I is given by the first moment of the MIP.

An AWGN channel with binary input ± 1 is a BISMIC and can be decomposed using $a = |y|$ as a subchannel-indicator (y is the continuous output signal). The pdf of a is

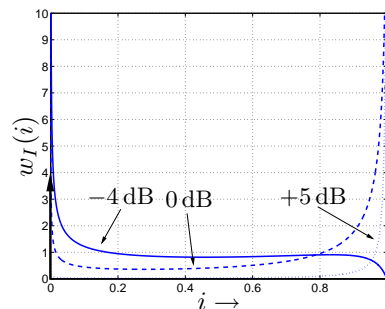
$$f_A(a) = \frac{1}{\sqrt{2\pi}\sigma_n} \left(e^{-\frac{(a+1)^2}{2\sigma_n^2}} + e^{-\frac{(a-1)^2}{2\sigma_n^2}} \right), \quad (1)$$

where $\sigma_n^2 = \frac{1}{2} \cdot \frac{N_0}{E_s}$ is the noise power. The mutual information $I(a)$ of a subchannel and the MIP $w_I(i)$ are given by

$$I(a) = 1 - h \left(\frac{1}{1 + e^{2a/\sigma_n^2}} \right) \quad (2)$$

$$w_I(i) = \frac{f_A(a)}{dI(a)/da} \Big|_{a=I^{-1}(i)}, \quad \frac{dI(a)}{da} = \frac{2 \cdot e^{2a} \cdot \log_2(e^{2a})}{(1 + e^{2a})^2} \quad (3)$$

The figure shows MIPs for an AWGN channel at different values $10 \log_{10}(E_s/N_0)$. For fixed capacity BEC and BSC



are the two extreme cases w. r. t. the MIP, having maximum or minimum (zero) second moment, respectively. Thus, BEC and BSC are extreme cases for all bounds derived by means of Jensen's inequalities, e. g. information combining [2, 3]. Using these extreme cases, a lot of bounds like Fano's and Hellman-Raviv-bound can be derived in a very illustrative way. For high SNR, the AWGN channel can be approximated very accurately by a BEC with equal capacity (see also the plot for $10 \log_{10}(E_s/N_0) = 5$ dB, well approximated by two Dirac δ -functions for $i = 0$ and $i = 1$).

III. APPLICATIONS

High-rate codes are used in high signal to noise ratio scenarios. As for high SNR an AWGN channel can be well approximated by a BEC, high-rate codes can be designed for BECs, e. g. by very efficient density evolution and used for AWGN channels, like shown in the following table:

Rate	0.2	0.5	0.99
Rel. AWGN Cap. Gap	0.00582	0.00357	0.00078

In [4] another case has been studied: Nonoptimized codes perform equally well for AWGN and BEC at any SNR.

REFERENCES

- [1] I. Land, "Reliability Information in Channel Decoding - Practical Aspects and Information Theoretic Bounds", *Ph.D. Thesis*, University of Kiel, Kiel, August 2004.
- [2] I. Land, S. Huettinger, P. A. Hoeher, and J. Huber, "Bounds on Information Combining", *accepted for IEEE Transactions on Information Theory*.
- [3] I. Land, S. Huettinger, P. A. Hoeher, and J. Huber, "Bounds on mutual Information for simple Codes using information combining", *Annales des télécommunications*, vol. 60, no. 1-2, pp. 191-221, 2005 (to appear)
- [4] M. Franceschini, G. Ferrari, and R. Raheli, "Does the Performance of LDPC Codes depend on the Channel?", *Proceedings on ISITA 2004*, pp. 55-59, Parma, Italy, 2004.