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RECENT DEVELOPMENTS OF THE "DISPLACEMENT METHOD"

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ABSTRACT: The recent development of the "displacement method" (Lemonnier, 1995), for the design of geosynthetically reinforced slopes, is based on the application of the variational approach. Concerning the external stability, the variational limit equilibrium method has been used in the case of the reinforced slopes. It has been shown that the critical shape of the failure surface is a log-spiral. Considering the same basic assumptions as the ones of the original "displacement method" (Gourc et al, 1986) for the internal stability, the variational calculus has been applied to the equilibrium of the membrane zone. The originality of this approach is to allow the determination of the optimum position of the layer (log spiral shape for the membrane zone), in order to obtain an extremal tension in the reinforcement at the failure surface. A comparative case study with two previous models of the "Displacement method" is presented and discussed.

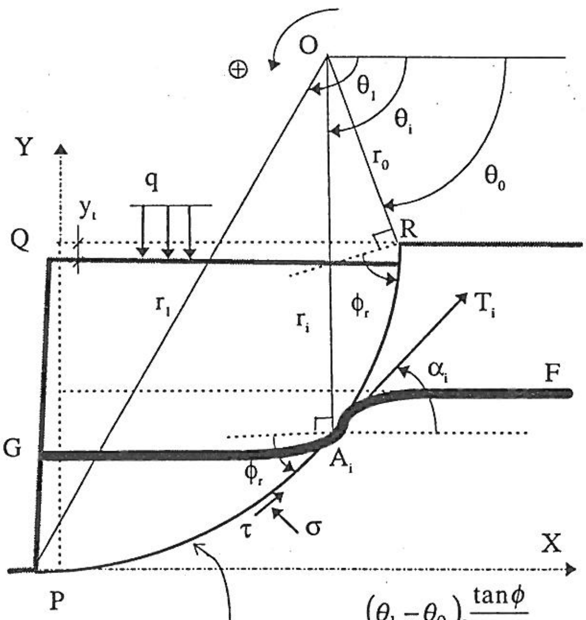
KEYWORDS: Design, Limit equilibrium methods, Geosynthetic reinforcement, Slope stability analysis.

1 INTRODUCTION

The "displacement method", initially developed in France, is now being widely used for more than ten years in many countries to design geosynthetically reinforced structures (Figure 1). The geotechnical engineers have built up this reliance from several experimental investigations and theoretical validations all over the world (Gourc et al, 1988; Yoshioka et al, 1990; Fidler et al, 1994). The advantage of this method is its ability to take into account the extensibility of the reinforcements in the design, which is a major characteristic of the geosynthetics. It is based on the principle of soil limit equilibrium, associated with the geosynthetic behavior as an anchored membrane (Figure 2). The original method considers a simplified mechanism for this membrane zone. Later on, a more rigorous mechanism has been proposed.

A recent development of the "displacement method", which is in the process of being developed in Denmark by the first author (Lemonnier et al, 1997), is based on the application of the variational calculus (therefore called "variational displacement method" in the following).

Firstly, the original "displacement method" together with its two different developments are briefly presented. Secondly, a comparative study on a 6m high geosynthetically reinforced wall, is presented and discussed.



Log Spiral: $r_i = r_0 . e^{(\theta_1 - \theta_0).\frac{\tan\phi}{FS}}$

Figure 1. Global stability of the reinforced slope. Extremal failure surface.

2 THE "DISPLACEMENT METHOD"

2.1 Standard "Displacement Method"

The original "displacement method" (Gourc et al, 1986, "Cartage" software) is based on the principle of soil limit equilibrium, and on the consideration of (i) a

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displacement field along the failure surface defined by a vertical top displacement y_t (see Figure 1), (ii) the anchored membrane concept (see Figure 2), and (iii) the geosynthetic extensibility. The local equilibrium of each sheet is considered, coupled with the tension constitutive model of the reinforcement and of that in friction of the soil-geosynthetic interface (elasto-perfectly plastic behavior). Faced with the problem of the determination of the soil stiffness modulus, a simplified mechanism of the membrane behavior has been considered by the authors: (a) The reinforcement tension is constant all along the membrane zone B_1B_2 (see Figure 2) and is equal to the ones at B_1 and B_2; (b) This tension is either assumed to act in the horizontal direction or in the direction of the tangent to the failure surface (maximum inclination); And (c) the modeling of the geosynthetic behavior is considered either with a small or large displacements assumption.
Note that the corresponding software "Cartage" is widely used in France for practical design work.

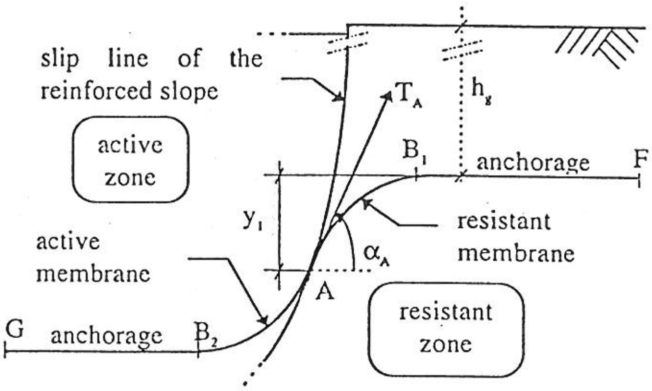

Figure 2: Anchored membrane concept.

2.2 Modified "Displacement Method"

This method considers a more rigorous mechanism (circular shape) of the membrane behavior. The determination of the reinforcement tension T_A (see Figure 2), which is also assumed to be constant all along the circular membrane zone, is based on the local equilibrium of this zone. Thus, the reinforcement tension and its inclination α_A depend on the vertical top displacement y_t and the soil stiffness modulus. Note that the corresponding software "Membrane" is used essentially for research (Gotteland, 1991).

2.3 "Variational Displacement Method"

The problem has been split into two different parts:

- Global equilibrium of the sliding mass

Applying the variational limit equilibrium method, elaborated by Baker and Garber (1977) in the case of unreinforced slopes, to the one of reinforced slopes, it has been shown that (i) the trace of the critical failure surface is a log spiral (see Figure 1), and (ii) the only moment equilibrium equation of all forces acting on the sliding mass is enough for the determination of the safety factor *FS*, defined with respect to the shear strength parameters of the soil.

- Local equilibrium of the reinforcement sheets

In order to assess the reinforcement contribution to the stability, the variational calculus has been applied to the equilibrium of the membrane zone in the neighborhood of the failure surface, considering the principle of the "displacement method" (i.e.: the anchored membrane concept, see Figure 2). The variational approach allows the determination of the optimal shape (log spiral) as well as the optimal position of the sheet for which one obtains the extremal tension at the intersection with the failure surface. Thus, the tension is no longer assumed to be constant along the membrane zone. Furthermore, this method provides the tension distribution and the relative soil-geosynthetic displacements all along the sheets.

2.4 Comparison Of The Different Models

Table 1 shows the comparisons of some principles of the three above mentioned models, regarding (a) the global equilibrium of the sliding mass (columns 1 to 3) and (b) the local equilibrium of the reinforcements (columns 4 to 6). Note that the listed principles are those which differ from each other. It appears that the "Displacement Method" has gained in rigour in its two latter developments (i.e. less a priori assumptions). The improvement provided by the "Modified" model consists of a more rigorous determination of the reinforcement tensions. With the present model, it provides an improvement on the determination of both the safety factor (no a priori assumptions concerning the failure surface shape, nor the normal stress distribution along this surface), and the reinforcement tension (no a priori assumptions concerning the shape of the membrane zone, nor its critical position).

3 COMPARATIVE CASE STUDY

In order to show the developments of the "displacement method", the three above mentioned models have been applied to a 6m high wall reinforced with 11 geosynthetic layers. All other data required for the analysis are presented in Figure 3. For all three models, the safety factor *FS* of the soil in shear is set to 1.5. Note that all "Cartage" results correspond to the assumption of large displacements with a horizontal reinforcement tension (see §2.1).

Table 1. Comparison of some principles of the three models

Name	1. Limit equilibrium method	2. Failure surface shape	3. Search for critical failure surface	4. Critical position of the membrane zone	5. Tension in B_1B2 (see Fig.2)	6. Inclination of tensions
"Cartage"	"Perturbation"	any (circular more used)	manual	(as rest)	(constant)	(null) or (maximum)
"Membrane"	"Perturbation" Fellenius Bishop Jambu	(circular) (bilinear) (mixed)	manual	(fixed)	(constant)	(fixed)
Present model	Variational	Log-spiral*	automatic	result of minimisation process*	non constant*	result of minimisation process*

Notes: Terms in parentheses () refer to a priori assumptions, and the one followed by * refers to analytical result.

3.1 Failure Surfaces

Concerning the two first models (i.e. "standard" and "modified"), and for each potential slip line (circular shape) considered, a critical vertical top displacement y_{tc} corresponding to FS=1.5 is determined. The critical slip circle (represented in a dotted line in Figure 3), which is the same in both models, corresponds to the larger y_{tc} value considering six different potential slip circles (Gourc et al, 1989).
For the present model, the slip line shape is a log spiral and the critical position (represented in Figure 3) corresponds to the critical y_t value (called y_{tc}) which provides a safety factor FS=1.5 as its minimum value, considering 860 different log-spirals passing through the toe of the wall.
Note that these two lines are rather close to each other in the lower half part of the wall, then the log-spiral is placed further away from the facing than the circle. Nevertheless, regarding the difference in the determination of the critical position of the failure surface in each model, the scatter between these positions is small.

3.2 Critical Vertical Top Displacements

Concerning the critical vertical top displacements y_{tc}, the standard model gives 26mm, the modified one gives 50mm, and the present one gives 88mm (see Figure 3). Thus, it seems that there is a tendency in the development of the method for an increase of this key parameter.

3.3 Critical Tension Distributions

Figure 4 shows the distribution of the critical tensions and of their inclinations along the failure surfaces. It is interesting to note that the tension distribution of the present model is close to the one of "Cartage" in the upper half part of the wall, and close to the one of "Membrane" in the lower half part. Plus, the present model gives the lowest maximal tension T_{max} (presented in Figure 4), which is reached in the lowest sheet in each model. Nevertheless, the scatter between these tension distributions is rather small.

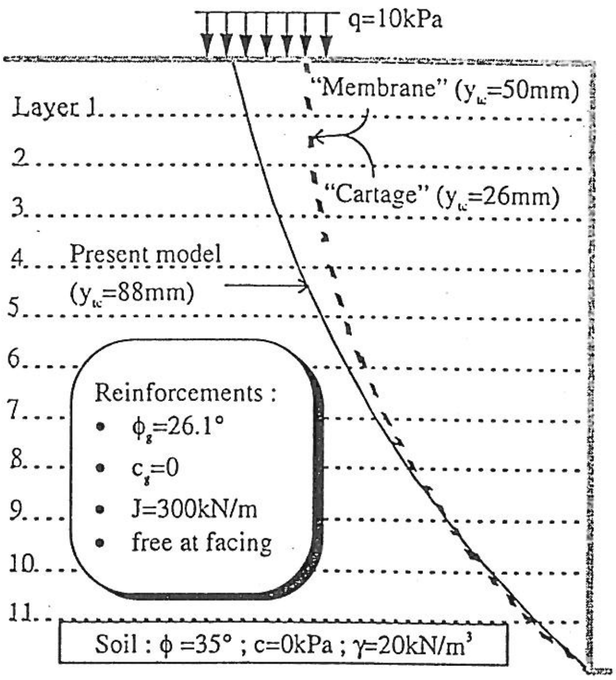

Figure 3. Case study - Critical positions of the theoretical failure surfaces

3.4 Critical Tension Inclination Distributions

Concerning the tension inclinations, the distributions as given by "Membrane" and the present model (null for "Cartage") are very different from each other. "Membrane" distribution decreases continuously with the depth, while the one of the present model passes through a minimum value at the second layer (see

Figure 4), then from the 8th layer and deeper it reaches the maximal inclination distribution, which corresponds to tensions tangent to the failure surface (the log-spiral of Figure 3).

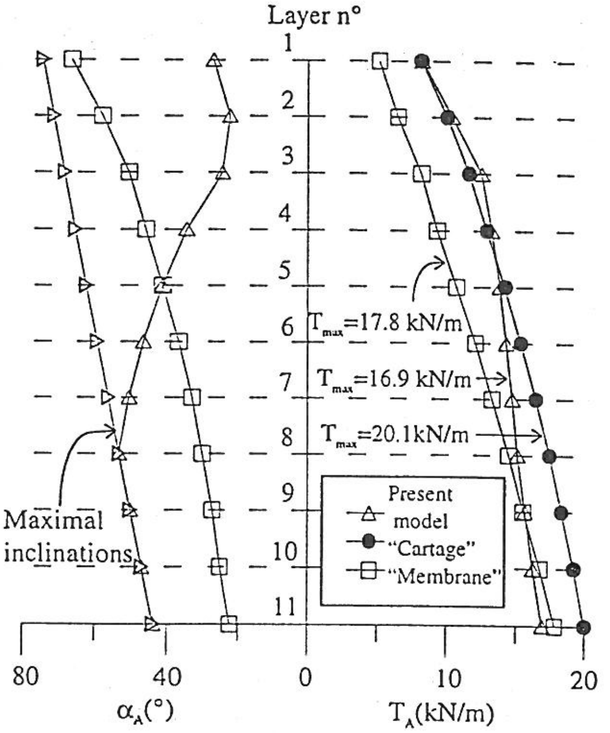

Figure 4. Comparative study (case study of Figure 3)

3.5 Discussion

The present comparison study shows that (i) the critical failure surfaces obtained with the three models are close to each other (see Figure 3), (ii) the reinforcement tension distributions along the failure surface are similar (see Figure 4), but (iii) the obtained critical vertical top displacements y_{tc} has been increased significantly from one model to its following development (92% between the two first, and 76% between the two last). However, the latter parameter is difficult to correlate to actual measurements, and further research should be performed on the validation of these models on real structures build and tested to failure.

4 CONCLUSION

This paper presents the recent developments of the "displacement method". This method, which is the standard in France for the design of geosynthetically reinforced earth structures, has since been improved at least twice. The first improvement concerns the determination of the reinforcement tensions, the second one also concerns the determination of the safety factor of the structure. Indeed, the originality of the latter is the application of the variational calculus on both the equilibrium of the sliding mass and the one of the membrane zone in the vicinity of the failure surface. This analysis allows a significant decrease in the number of a priori assumptions considered in the previous models. A comparative study on a 6m high wall has shown that the rigour tends to increase the obtained critical top displacement.

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