On Stationary and Non-stationary Porous Flow in Coarse Granular Materials

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ON STATIONARY AND NON-STATIONARY POROUS FLOW IN COARSE GRANULAR MATERIALS

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by

H.F. Burcharth [1] and Claus Christensen [2]

1. INTRODUCTION AND SUMMARY

Traditionally the hydrodynamic response of rubble mound structures are studied in physical models scaled according to the Froude scaling law which neglects viscous forces. This introduces scale effect because the inherent length scaling of the stone diameters creates laminar flow in regions which in prototypes are dominated by turbulent flow. Numerical modelling of the flow do not have this draw back and is therefore attractive. One important part of a numerical model is the basic description of the non stationary wave generated flow in the porous structure of the breakwater. This flow differs from ground water flow in that fairly large accelerations and velocities are present. The present paper deals with the description of this kind of flow.

A critical discussion based on physical considerations is given of the mathematical description of steady flow. It is argued that the conventional use of the Forchheimer equation is not suitable in case of fully turbulent flow, which is the relevant flow regime for rubble mound breakwaters.

The relationship between the bulk velocity and the pressure gradient in steady and non-steady flow was studied in U-tube permeameter tests for various stone sizes and gradations.

Moreover, a mathematical model of the non-stationary flow in the U-tube permeameter was established in order to provide more insight in the sensitivity of the system to variations in the parameters. In this way also the experimental limitations of the U-tube permeameter technique was explored.

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Results from stationary flow tests confirmed an apparent validity of the Forchheimer equation but demonstrated also the inconsistency in using this equation when extrapolating results from one flow regime to another. A proposal is given for a formula covering the fully turbulent flow range relevant to flows in breakwaters including the effect of the surface characteristics of the stones.

Results from unsteady flow tests were related to a generalized Forchheimer equation which includes an inertia term. However, it was not possible with the technique used to determine the related added mass coefficient with reasonable accuracy within the tested range of accelerations. Thus the general importance of the inertia effect is still a question although the present research indicate the effect to be marginal. It is concluded that answer to this question demands a different experimental technique where the period of large acceleration are increased compared to present tests.

2. STEADY FLOW. THEORETICAL AND EXPERIMENTAL BACKGROUND

The approach used in the research presented in this paper was macroscopic in the sense that a relationship between the bulk (discharge) velocity, V, and the hydraulic pressure gradient, I, was investigated. Related to this approach there are two theoretically sound models which are generally used for the steady flow case.

2.1 The friction factor model

This model, which is one form of the general exponential form $I = a\, v^m$, is based on the general force balance equation

$$I = f \frac{v^2}{2gR} \ , \qquad (1)$$

where v is a characteristic velocity, e.g. $v = V/n$, where n is the porosity. R is a characteristic length (e.g. the hydraulic radius $R = d/4$ in case of pipe flow, here taken as proportional to a stone diameter), and f is a friction factor which is dependent on the geometry of the granular body and on a Reynolds' number, e.g. $Re = VR/\nu$, where ν is the kinematic viscosity.

The problem is to establish an analytical expression for the variation of f. A rather complicated expression is to be expected because it must cover the whole range from laminar flow through the transition to fully turbulent rough flow. Moreover, it must include the relevant characteristics of the granular system. The latter presents a real problem because the hydraulic resistance is sensitive to changes valid for smooth pipe flow, is an example of this type of expression, but is relatively simple due to the fact that the geometry of the system can be given only by two parameters namely the pipe diameter and the pipe surface roughness.

Although the character of porous flow is different from pipe flow there are similarities with respect to the flow regimes commonly recognised in the literature. Consequently, it might be illustrative to show the well explored but rather complicated variation of f with Re in pipe flow, shown in Figure 1.

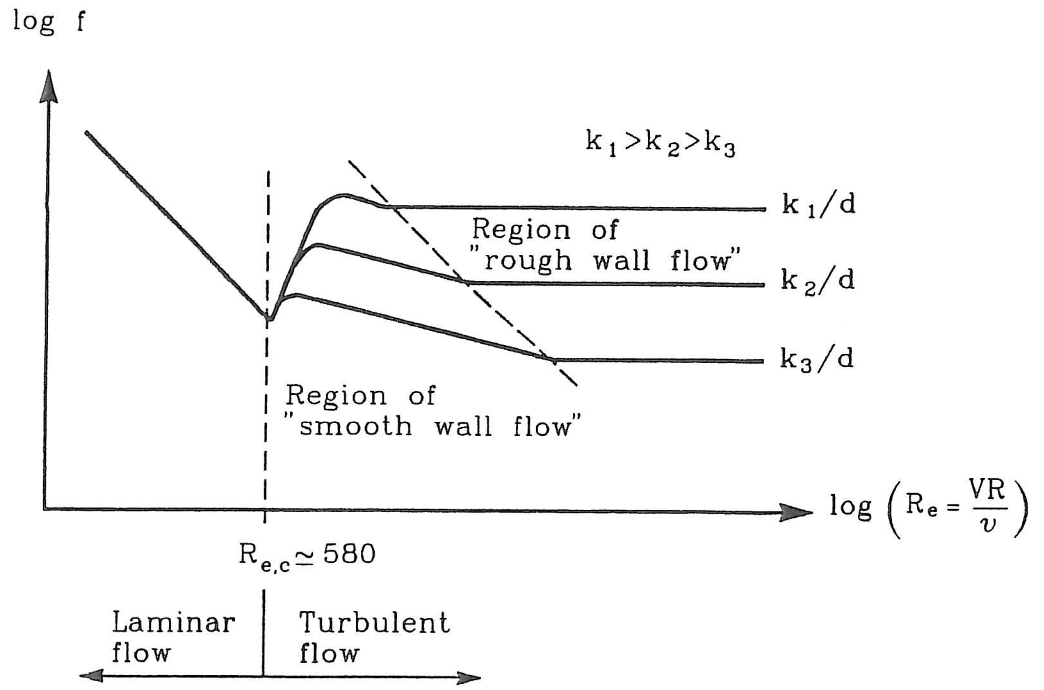

Fig. 1. *Variation of the friction factor f with the Reynolds number in pipe flow.*

Using the hydraulic radius $R = \frac{d}{4}$ as characteristic length the transition Reynolds number Re_c between the laminar and the smooth wall turbulent region is app. 580. The smooth flow occurs when the wall roughness elements are covered by a laminar boundary layer, but the main flow is turbulent. In this range f is weakly dependent on Re or viscosity. The rough flow corresponds to turbulent boundary layer, i.e. the laminar boundary layer thickness is reduced to a thin film following the surface of the wall roughness elements which then protrude into the turbulent flow. In this range the friction factor f will be influenced only by the relative roughness k/d and not by the viscosity or Re. The Reynolds number range for the smooth wall turbulent flow depends on the relative roughness of the pipe, i.e. the smooth wall turbulent flow depends on the relative roughness of the pipe, i.e. the ratio between the surface roughness, k and the pipe diameter. The larger the value of k/d, the smaller the Reynolds number range for smooth flow. For very rough pipes this range is almost insignificant and consequently the transition Reynolds number between smooth and rough flow is close to 580.

Several researchers have tried the friction factor model approach for porous flow but generally without much success (Bakhmeteff 1937, De Lara 1955, Rumer 1966, Barends 1978).

2.2 The Forchheimer model

The model is given by a two term series expression

$$I = aV + bV^2 \tag{2}$$

where V is the bulk velocity and a and b are supposed to be constants for a given fluid viscosity and granular body geometry. Eq. (2) is often denoted the Forchheimer equation because Forchheimer (1901) was probably the first to suggest this type of equation.

The general interpretation of the equation is that the linear term constitutes the contribution from the laminar flow, for which reason the factor a depends on the viscosity. The nonlinear term represents the fully turbulent flow contribution, i.e. the factor b is independent on the viscosity. It is an open question to which degree eq. (2) can describe the transition between laminar and turbulent flow, because the variation of I in the transition is most probably dependent on the surface charactersistic of the grain matrix, cf. the well known variation of f (in eq. (1)) in pipe flow. The nonlinear second term in eq. (2) signifying the turbulent flow contribution has the same character as eq. (1), but is different in that b depends only on the granular matrix geometry while f is dependent also on a Reynolds' number, i.e. the viscosity, because eq. (1) covers also the laminar and the transition regions.

If the nonlinear term is neglected we obtain the Darcy equation for laminar flow.

2.3 Flow regimes and related Reynolds number ranges

A conventional way of relating eq. (2) to the various flow regimes is depicted in Fig. 2 for one specific porous matrix and viscosity.

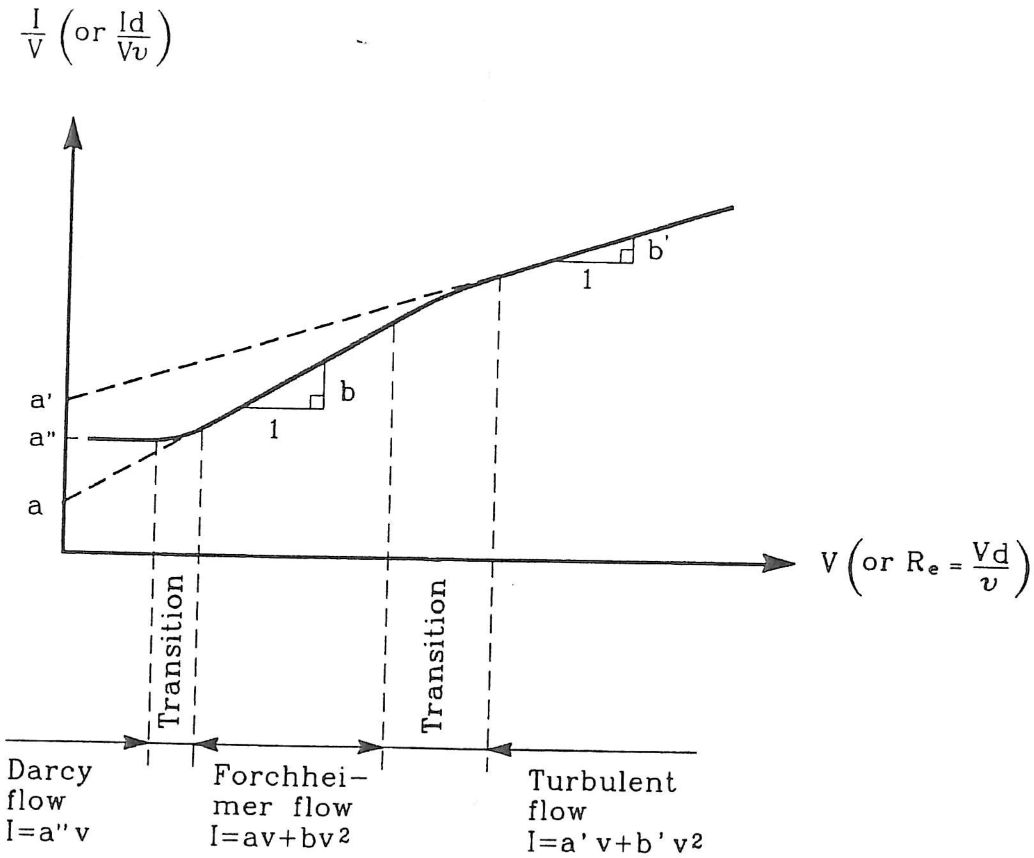

Fig. 2. Conventional representation of flow regimes for porous flow based on a Forchheimer equation analysis. Values of a , a' and a" depend on the granular matrix and the viscosity. Values of b and b' depend on the granular matrix.

The Reynolds numbers corresponding to the transitions between the various regimes were studied by Fand et al., 1987. Based on experiments with uni-size and mixed-size glass spheres they found the ranges of the diameter - Reynolds number given in Table 1. The applied definition of the Reynolds number is

$$Re = \frac{Vd}{\nu} \tag{3}$$

where d is the sphere diameter in case of uni-size spheres. In case of mixed-size spheres d is a weighted "mean" diameter defined as

$$d = \frac{1}{\sum_{i=1}^{n} \frac{F_i}{d_i}} \tag{4}$$

where F_i is the mass fraction for particles having the diameter d_i and n is the total number of fractions applied to the sample. Eq. (4) assumes that all particles have the same density.

Table 1. Diameter - Reynolds number ranges for various flow regimes. Results of experiments by Fand et al. (1986) with 2, 3, and 4 mm glass spheres.

	uni-size					multi-size				
Darcy flow			Re	$\leq$	2.3			Re	$\leq$	1.6 – 2.1
Forchheimer flow	5	$\leq$	Re	$\leq$	80	2.7 – 3.7	$\leq$	Re	$\leq$	55 – 74
Turbulent flow			Re	$\geq$	120			Re	$\geq$	120

It should be noticed that the Re-range for turbulent flow was found identical for uni-size and multi-size media when the definitions eqs. (3) and (4) are used.

Dybbs et al. (1975) found on the basis of tests with plexiglass spheres in a hexagonal packing the Reynolds number regimes shown in Table 2. They use the "pore-size" Reynolds number based on average pore size and average pore velocity as characteristic length and velocity, respectively.

Table 2. Pore-size-Reynolds number ranges for various flow regimes. Results of experiments by Dybbs et al. (1975) with plexiglass spheres in hexagonal packing.

Darcy flow			Re	$<$	1 – 10
Forchheimer flow	1 – 10	$<$	Re	$\leq$	200
Turbulent flow			Re	$>$	350

When considering the ratio of characteristic grain diameter to characteristic pore diameters to be in the order of 2 - 3 and the ratio between bulk velocity and average pore velocity to be in the order of 4 it is seen that the results given in Tables 1 and 2 are rather consistent, the values in Table 1 being somewhat on the low side. Test results for stone samples indicate higher values than those given for spheres.

The relationship between the diameter Reynolds number and the pore-size-Reynolds number might be explored by considering an analogy between the flow in a pipe with large roughness, cf. Fig. 1., and the porous flow.

For the porous flow, like in pipe flow, a relevant characteristic length in the Reynolds number might be a hydraulic radius, R, which for pipe flow is $\frac{D}{4}$, where D is the pipe diameter. For porous flow R is often defined as the ratio of the pore volume to the total surface area of the grains within a unit volume. If we consider a sample of uni-size spheres we get

$$R = \frac{\text{pore volume}}{\text{number of spheres} \times \text{area of a sphere}} = \frac{n}{\frac{1-n}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} \cdot 4\pi\left(\frac{d}{2}\right)^2} = \frac{dn}{6(1-n)} \qquad (5)$$

where n is the porosity and d the sphere diameter.

For samples of uni-size spheres having $n \simeq 0.36$ we obtain $R \simeq 0.1d$. By comparing this to $R = \frac{D}{4}$ used for pipe flow it is seen that on "equivalent pipe diameter" for porous flow would be $0.4d$, since $R = \frac{0.4d}{4} = 0.1d$. This might support the use of an average "pore" size of $0.4d$ as characteristic length in the Reynolds number, cf. the discussion given above of the results by Dybbs et al. It might then be concluded that the diameter Reynolds number range for the transition between Forchheimer flow (smooth wall flow) and the turbulent flow (rough wall flow) in the case of spheres is in the range $80 < Re < 120$. A wider range is to be expected for real stone samples. Moreover, it is to be expected that the shape and the relative surface roughness influences the various transition values of the Reynolds number in the non-Darcy flow ranges.

Engelund (1953) argues that a conventional Reynolds number like the one defined by eq. (3) cannot adequately describe the ratio between the inertial and the viscous forces because it is not dependent on the porosity and the shape of the grains. As a logic solution to this problem Engelund uses a Reynolds number defined as

$$\xi = \frac{bV}{a} \quad , \qquad (6)$$

i.e. the ratio between the turbulent and the laminar terms in eq. (2). As an example Engelund found $\xi \simeq 0.07$ for the transition between Darcy and Forchheimer flow.

The definition (6) works very well in the Forchheimer flow regime but is very difficult to use for the turbulent regime because a is ill-determined as a very small quantity compared to bV.

Consequently, it is recommended to use the Reynolds number definition given by eq. (3) for turbulent flow in breakwaters. The argument by Engelund is not so important in this case because the influence of the normal variations in the porosity and the shape of the grains will not cause significant changes in Re. Alternatively a Reynolds number defined by $Re = \frac{VR}{\nu}$, where R is given by eq. (5) could be used.

In the following sections Re means the diameter-Reynolds number given by eq. (3).

2.4 Evaluation of the factors in the Forchheimer equation

Many researchers have tried to develop generally applicable empirical or semi-empirical expressions for a and b (Lindquist 1933, Carman 1937, Muskat 1946, Ergun 1952, Engelund 1953, Irmay 1958, Scheidegger 1960, Ward 1964, Ahmed et al., 1969, Hanoura et al., 1978).

2.4.1 Dimensional analysis

A rational approach is to use dimensional analysis.
From eq. (1) it is seen that

$$I = I\left(\frac{V}{n}\ ,\ \nu\ ,\ g\ ,\ R\ ,\ \text{geometry}\right) \tag{7}$$

The geometry might be expressed partly by a surface roughness element height, k characterizing the surface characteristics of the grains and partly by a gradation parameter, G. In case of laminar flow the surface roughness element height k can be neglected and for the turbulent flow the viscosity ν can be neglected. Thus by dimensional analysis

$$I_{lam} = I_{lam}\left(\frac{\nu}{gR^2}\frac{v}{n}\ ,\ G\right) \tag{8}$$

$$I_{turb} = I_{turb}\left(\frac{\left(\frac{k}{R}\right)^N}{gR}\left(\frac{V}{n}\right)^2\ ,\ G\right) \qquad \text{for all } N \tag{9}$$

If for R eq. (5) is applied we obtain

$$I_{lam} = I_{lam}\left(\frac{(1-n)^2\nu}{n^3gd^2}V\ ,\ G\right) \tag{10}$$

$$I_{turb} = I_{turb}\left(\frac{1-n}{n^3gd}V^2\left(\frac{1-n}{n}\cdot\frac{k}{d}\right)^N\ ,\ G\right) \tag{11}$$

The most simple interpretation of (10) and (11) would be

$$I_{lam} = \alpha\,\frac{(1-n)^2}{n^3}\,\frac{\nu}{gd^2}\,V \tag{12}$$

$$I_{turb} = \beta\,\frac{1-n}{n^3}\,\frac{1}{gd}\,V^2 \tag{13}$$

and consequently the Forchheimer equation (2) becomes

$$I = \alpha \frac{(1-n)^2}{n^3} \frac{\nu}{gd^2} V + \beta \frac{1-n}{n^3} \frac{1}{gd} V^2 \tag{14}$$

in which case α depends only on the gradation, G and β depends both on the relative surface roughness, $\frac{k}{d}$ and the gradation, G.

Another way of deriving eq. (13) is by considering the total drag force on the grains within a sample of volume $dy\,dx\,dz$. The force on each grain is $\approx \frac{1}{2}\rho\left(\frac{v}{n}\right)^2 d^2$ where $\approx$ means proportional to. The number of grains within the volume is $\approx \frac{1-n}{d^3}\,dx\,dy\,dz$. The total force of grains is then $\approx \frac{1-n}{d}\,dx\,dy\,dz\,\rho\,\frac{v^2}{n^2}$. This force is balanced by (equal to) the differential pore water pressure force in two consequtive sections spaced dx in the bulk velocity direction, i.e. $\approx \Delta p\,n\,dy\,dz$ where the pore water area of a unit area plane section as of fairly close assumption is taken as n. Using $I = \frac{\Delta p}{\rho\,g\,dx}$ we arrive at, $I \approx \frac{1-n}{n^3 g\,d}\,v^2$.

One or both of the simple expressions given by eqs. (12) and (13) have been used by several researchers, e.g. Ergun (1952), Lindquist (1933), Fand et al. (1987), Dudgeon (1966).

Table 3 lists examples of experimental values of α and β related mainly to the Forchheimer flow Reynolds number range. α and β are often denoted shape factors, because researchers generally explain α and β as dependent on the geometry of the granular medium. However, no distinction is made between the α- and the β-dependency as explained by the above given dimensional analysis.

Table 3. Examples of α and β values (defined by eq. (14)) covering mainly the Forchheimer flow range.

Researcher	Particles	Porosity	Diameter range or d_{50} mm	Re	α	β
Fand et al. 1987	uniform glass spheres	0.360	2-4	5-80	~ 182	~ 1.92
Lindquist *) 1933	shot	0.383	1-5	4 - 263	184	1.82
Dudgeon **) 1966	uniform glass spheres	0.415	16	< 400	164	1.7
	— " —	0.385	29	< 180	193	2.4
	river gravel	0.367	16	< 85	329	4.7
	— " —	0.406	110	< 7000	922	2.0
	angular rock	0.455	16	< 400	622	5.4
	— " —	0.515	14	< 200	479	4.0
	— " —	0.438	25	< 400	425	5.3
	— " —	0.483	37	< 500	92	10.8
Engelund 1953	flinty, calcareous sand and of uniform size	0.395	1.4-2.6	25-150	335	3.57

*) Data taken from Ahmed et al., 1969.

**) Data calculated from Dudgeon's graphs, not from the data points.

2.4.2 Engelund coefficient

Engelund (1953) used eq. (13) but proposed an alternative to eq. (12), namely

$$I_{lam} = \alpha_E \frac{(1-n)^3}{n^2} \frac{\nu}{g d_E^2} V \tag{15}$$

where the indices E stands for Engelund. The characteristic diameter d_E is defined as the diameter of a sphere which has a volume equal to the average volume of the grains. Using eq. (15) the Forchheimer equation reads

$$I = \alpha_E \frac{(1-n)^3}{n^2} \frac{\nu}{g d_E^2} V + \beta \frac{1-n}{n^3} \frac{1}{g d_E} V^2 \tag{16}$$

Engelunds argument for recommending this expression was a better fit to the experimental data available to him (data of Franzini (1951) with uniform spheres covering the porosity range $n = 0.27 - 0.48$ and data by Rose (1945)). However, it should be noticed that for what might be regarded the relevant porosity range for rubble mound structures, $n = 0.37 - 0.48$, there is no significant difference between eqs. (12) and (15) as the corresponding ratio between the porosity factors $\frac{(1-n)^3}{n^2} / \frac{(1-n)^2}{n^3} = n(1-n)$ only varies between 0.233 and 0.250. Moreover, because Engelund's semi-empirical expression eq. (15) was based mainly on experiments with small particles it is recommended to prefer the theoretically better founded eq. (12), especially when dealing with breakwaters where extrapolation to much larger particle diameters is needed.

The α and β values recommended by Engelund are given in Table 4.

Table 4. The Engelund coefficients (defined by eqs. (16).

	α_E	β
Uniform, spherical particles	$\sim$ 780	$\sim$ 1.8
Uniform, rounded sand grains	$\sim$ 1000	$\sim$ 2.8
Irregular, angular grains	up to 1500 or more	up to 3.6 or more

Engelund pointed out that the recommended values of β are based only on very few experiments (by Lindquist, 1933, Givan, 1934 and Chardabellas, 1940).

The Engelund α-coefficients can be transformed to the α value defined by eq. (12) by multiplying with $(1-n)n \simeq 0.24$, cf. Table 5.

Table 5. Engelund coefficients transformed to fit eq. (14).

	α	β
Uniform, spherical particles	~ 190	~ 1.8
Uniform, rounded sand grains	~ 240	~ 2.8
Irregular, angular grains	up to 360 or larger	up to 3.6 or larger

2.4.3 Shih coefficients

Very recently and after the completion of tests presented in this paper Shih (1990) proposed the following expressions for α and β, based on regression analyses of steady flow permeameter test results for single size and wide grade samples of chrushed limestone with stone diameters of up to 75 mm, respectively 40 mm. The values are to be applied in eq. (16) with $d = d_{15}$, i.e. the diameter that 85% of the sample exceeds.

Single size sample:
(Tested ranges: $d_{85}/d_{15} \simeq 1.3$, $5mm < d_{15} < 55mm$)

$$\alpha = 1684 + 3.12 \cdot 10^{-3} \left(\frac{g}{\nu^2}\right)^{2/3} d_{15}^2 \tag{17}$$

$$\beta = 1.72 + 1.57 \, exp\left[-5.10 \cdot 10^{-3} \left(\frac{g}{\nu^2}\right)^{1/3} d_{15}\right] \tag{18}$$

Wide grade sample:
(Tested ranges: $2 < d_{85}/d_{15} < 5$, $4mm < d_{15} < 17mm$)

Eqs. (16), (17) and (18) are used with d_{15} replaced by

$$d_* = d_{15} \left(\frac{d_{15}}{d_{50}}\right)^{-1.11} \left(\frac{d_{50}}{d_{85}}\right)^{0.52} \tag{19}$$

The shape of the limestones and the porosities of the samples used by Shih are not specified.

The tested Reynolds range is app. $50 \leq Re \leq$ app. 6,000, i.e. mainly the fully turbulent range and as such relevant to flow in rubble mound breakwaters.

Apart from not being stringent in accordance with the dimensional analysis, cf. eqs. (8) - (11), eqs. (17) and (18) demonstrates an unexpected dependency of α and β on the grain diameter, cf. eqs. (12) - (14). Fig. 3 shows the experimental results by Shih (1990).

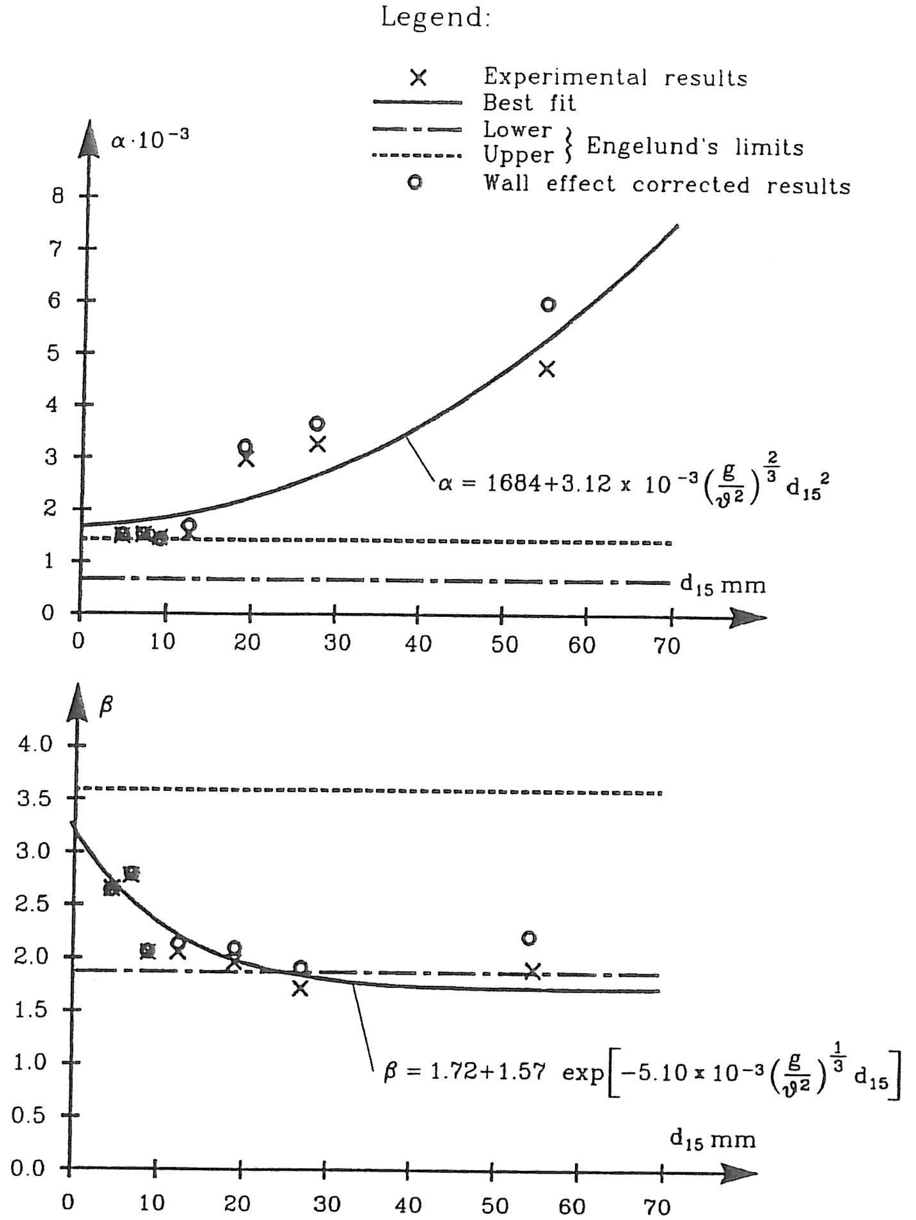

Fig. 3. Experimental results by Shih (1990). Corrections for wall effects according to Fig. 18 also shown.

The surprising but theoretically uncorrect trend of α increasing with the diameter is also observed in the experimental results presented in this paper if a conventional fit to the Forchheimer equation is used, cf. section 9. A similar analysis of the test results by Fand et al. shows the same trend.

The use of the Shih formulae for large size materials, e.g. extrapolation to a real breakwater situation, implies a considerable overestimation of the laminar flow resistance.

A discussion of the appearence of this fictive variation of α and β with the diameter is given in section 2.4.4.

Because of a relative large ratio between the stone diameter and the permeameter diameter in some of the test series by Shih (1990) a correction for wall effects should be made. The circles in Fig. 3 shows the experimental results corrected for wall effects using the correction factors presented in section 8.

2.4.4 Critical review of conventional fit of the Forchheimer equation to experimental data.

It is seen from Fig. 2 that for a given granular matrix the values of the coefficients a and b in the Forchheimer equation generally vary with *Re*. Only within certain ranges (denoted Darcy, Forchheimer and Turbulent) they attain constant (but different) values. Consequently, the asymptotic values of a and b, depends on the *Re*-range covered by the underlaying experimental results.

Taking the α-value as an example it will depend on the lower limit of the *Re*-values applied in the specific test series. Fig. 2 demonstrates clearly that an α-value corresponding to a Forchheimer range *Re*-value is smaller than an α-value corresponding to an upper transition range *Re*-value.

In permeameter tests with samples of different grain diameters the upper and lower *Re* limits usually depend on the diameter. This is due to normal experimental limitations. For example in the case of small diameter samples the max obtainable *Re*-value is smaller than for a large diameter sample and vise versa for the minimum obtainable *Re*-value. Thus the experimental *Re*-range increases with the grain diameter as in the case for almost all published permeameter test results, including the results by Shih. In some cases there might not even be an overlap in the *Re*-ranges for the finest and the coarsest materials applied in one set of published results.

The drastic increase in α with the diameter seen in Fig. 3 can most probably be explained by this effect. This is because the lowest *Re*-values, which are related to the smallest diameter samples, correspond to the upper limit of the Forchheimer flow range where α obtains minimum values, while the larger *Re*-values, which are related to the larger diameter samples, correspond to the fully turbulent flow range characterized by larger α values, cf. Fig. 2.

In this way also the large values of β for small diameter samples seen in Fig. 3 can be explained because the β-values decreases when moving from the Forchheimer flow range to the fully turbulent range, cf. Fig. 2.

It is concluded that it can be very misleading to use Forchheimer equation coefficient values (or α amd β values) outside the Re range corresponding to the underlaying test results.

Correct asymptotic values of the coefficients a'', $a - b$, $a' - b'$ in Fig. 2 can only be found from set of test results completely within each of the related Re-ranges. Simple extrapolation from one Re-range to another is not possible.

2.5 Proposal for a turbulent flow equation

For prototype rubble mound breakwaters exposed to design waves the flow regime will be turbulent almost without exception.

The discussion in section 2.4.4 shows that exrtapolation of permeameter data from Forchheimer and/or the transitions regions into the turbulent region is not possible. Consequently to arrive at data corresponding to prototype turbulent flow only permeameter data covering the turbulent range can be used.

It follows from the dimensional analysis that for fully turbulent flow it is not correct to use a series expression consisting of both a linear term and a quadratic term, although this is generally the conventional approach, cf. Fig. 2. A more correct approach is depicted in Fig. 4.

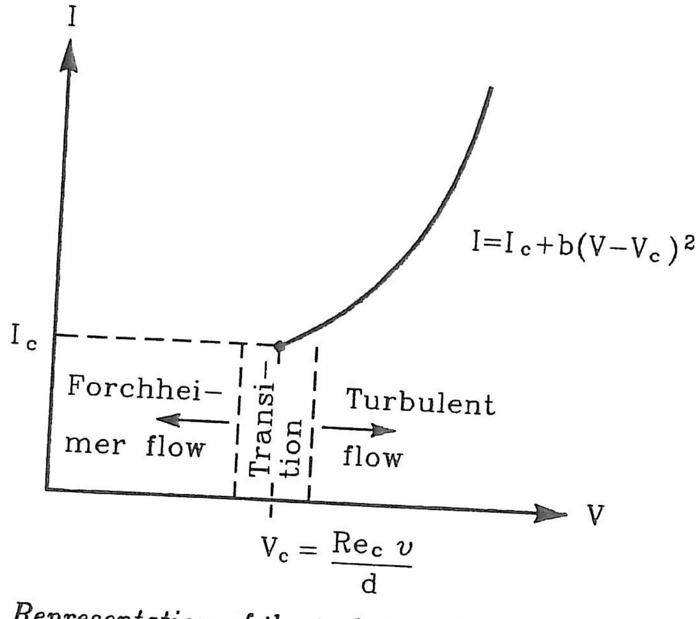

Fig. 4. Representation of the turbulent flow equation.

Re_c is in principle the critical Reynolds number signifying a lower value for the turbulent flow regime and V_c is the corresponding bulk velocity. According to the previous discussion the Reynolds number range for the transition between the Forchheimer flow and the turbulent flow is rather narrow, $80 \leq Re \leq 120$ for spherical particles. For this case it can be assumed as a close approximation that $Re_c = 100$ separates the Forchheimer flow range and the turbulent range. For stone samples the corresponding Reynolds number range are wider and a larger value of Re_c must be chosen, e.g. 300.

The turbulent flow equation is given by

$$I = I_c + b(V - V_c)^2 \tag{20}$$

I_c can be calculated from the Forchheimer flow equation with V equal to

$$V_c = \frac{Re_c\, \nu}{d} \tag{21}$$

Inserting eq. (21) into eq. (14) we obtain

$$I_c = Re_c\, \alpha_F \frac{(1-n)^2}{n^3} \frac{\nu^2}{gd^3} + Re_c^2\, \beta_F \frac{1-n}{n^3} \frac{\nu^2}{gd^3} \tag{22}$$

or

$$I_c = \frac{\nu^2}{gd^3} \frac{1-n}{n^3} \left[\alpha_F (1-n) Re_c + \beta_F Re_c^2\right] \tag{23}$$

where α_F and β_F correspond to the Forchheimer flow range and to the related definition of the characteristic diameter, d.

In order to evaluate eq. (21) Table 6 shows typical values of I_c and the related V_c calculated for various characteristic grain diameters using $\alpha = 500$, $\beta = 5.0$ and $n = 0.45$, which are approximate values for irregular, angular grains, cf. Table 5, $\nu = 1.14 \cdot 10^{-6}\ m^2/s$ and $Re_c = 300$.

Table 6. Typical values of I_c and V_c.

Characteristic diameter, d m	I_c	V_c m/s
0.001	430	0.34
0.01	$43 \cdot 10^{-2}$	0.034
0.03	$1.6 \cdot 10^{-2}$	0.011
0.06	$2.0 \cdot 10^{-3}$	0.006
0.20	$5.3 \cdot 10^{-5}$	0.002

It is seen from Table 6 that for all breakwaters with core material of quarry run ($d > 0.03\ m$) or coarser material I_c will be smaller than app. 10^{-2} and the corresponding critical bulk velocity V_c smaller than app. $10^{-2}\ m/s$. In this case I_c and $V_c \simeq 0$ and eq. (21) reduces to

$$I = \beta \frac{1-n}{n^3} \frac{V^2}{gd} \qquad (24)$$

where β depends on the relative surface roughness of the grains and the grading, cf. eq. (11).

For the quasi-steady flow in breakwater sand cores the viscous effects will be present and consequently the Forchheimer equation (14) with the α and the β values given in Table 5 might be used. The very large I_c-value of 430 given in Table 6 for sand with $d = 0.001\ m$ indicates that fully turbulent flow in sand will never occur in a breakwater situation. Even related to permeameter tests such a large hydraulic gradient is extreme.

Considering that eq. (22) is expressing the conditions at transition between Forchheimer flow and fully turbulent flow it is surprising that the laminar and the turbulent terms are of almost the same magnitude. It is to be expected that the laminar term should be negligible. The ratio between the two terms are, cf. also the Engelund Reynolds number equation (6),

$$\xi = \frac{\beta_F\, Re_c}{\alpha_F (1-n)} \simeq 5 \qquad \text{for} \quad Re_c = 300 \qquad (25)$$

This indicates that the Reynolds number, $Re^{T,L}$, corresponding to the lower value of the fully turbulent flow regime should be somewhat larger, i.e. $Re_c \simeq 600 - 1000$. However, if this is the case then it can be concluded that the empirically determined α and β values by Engelund and other researchers dealing with sand size grains have not been fitted to results covering the whole regime from Darcy flow to fully turbulent flow, but only the lower Reynolds number range where viscous forces are of importance. Consequently, it is doubtful if the reported small grain β-values by Engelund and others can be taken as the asymptotic values for turbulent flow regime. Instead of this β-values determined from experiments with fully turbulent flow should be used.

As to the α-value = 190 for uniform spherical particles reported by Engelund, cf. Table 5, it represents truly the lower asymptotic value for the Forchheimer flow regime because it is quantitatively identical to the uniform diameter sphere

$$= \rho \left(1 + C_m \frac{(1-n)}{n}\right) \Delta\ell \frac{dV}{dt} \tag{30}$$

The force F_I over a unit area corresponds to a difference in pressure Δp_I over the sample length $\Delta\ell$, i.e.

$$F_I = \Delta p_I = I_I \Delta\ell \rho g \ , \tag{31}$$

where I_I is the gradient corresponding to the inertia forces.

Substituting (31) into (30) we obtain for the last term in (27)

$$I_I = \frac{1 + C_m \left(\frac{1-n}{n}\right)}{g} \frac{dV}{dt} \tag{32}$$

The overall inertia coefficient C_m is expected to be dependent on a Reynolds' number, the shape and the relative surface roughness of the grains, the relative motion of the fluid (usually expressed by a Keulegan-Carpenter number) and the history of the fluid motion.

The complete equation (27) now reads

$$I = aV + bV^2 + \frac{1 + C_m \left(\frac{1-n}{n}\right)}{g} \frac{dV}{dt} \tag{33}$$

The two first terms on the right hand side, which correspond to the drag force, are as an approximation used with values of a and b determined from steady flow tests. Under this assumption it is possible by means of permeameter tests with non-steady flow to determine C_m and thereby explore the significance of the inertia term. Hannoura et al. (1978) used this approach but were not successful in determining consistent values of C_m as they for example found C_m-values in the range $-7 < C_m < 5$ for river gravel of size 1.6 cm. Hannoura et al. used a U-tube permeameter with a 15.2 cm square sample section and a maximum pressure difference corresponding to 150 cm head of water. The fluid velocity (and thus the acceleration) was determined by a TSI laser anemometer in one point a short distance upstream of the sample. The authors explain the reasons for the very large scatter in their results to be partly due to the very short period of "large" acceleration (0.15–0.25 sec) which makes accurate measurements very difficult, and partly due to the use of the steady flow coefficient values of a and b in (23), which might be a bad approximation because the flow regime corresponding to the steady flow Reynolds' number cannot be fully developed during the very short period of fast acceleration starting with the fluid at rest.

coefficient, $36\,\kappa = 36 \cdot 5.34 = 192$, given in the Darcy flow equation

$$I = 36\,\kappa\,\frac{(1-n)^2}{n^3}\,\frac{\nu}{gd^2}\,V \qquad (26)$$

where κ is the Kozeny-Carman constant, see Fand et al. (1990).

3. UNSTEADY FLOW. THEORETICAL AND EXPERIMENTAL BACKGROUND

A model for unsteady flow in porous media was probably first proposed by Polubarinova Kochina (1962) who generalized the Forchheimer equation by adding a time dependent inertia term

$$I = a\,V + b\,V^2 + C\,\frac{dV}{dt} \qquad (27)$$

where a, b and C are taken as constants for a given porous media. However, Polubarinova Kochina did not investigate the inertia term further.

The force F_I corresponding to the inertia term might be formulated as the sum of the contributions from the inertia generated fluid forces on the grains, $F_{I,g}$, and the forces necessary to accelerate the pore water, $F_{I,w}$, i.e. $F_I = F_{I,g} + F_{I,w}$.

$$F_{I,g} = \rho\,v_b\,\frac{dv}{dt} + C_a\,\rho\,v_b\,\frac{dv}{dt} = \rho\,(1 + C_a)\,v_b\,\frac{dv}{dt} = \rho\,C_m\,v_b\,\frac{dv}{dt} \qquad (28)$$

where ρ is the fluid mass density, v_b is the volume of the grains and v is a characteristic fluid velocity. The first term in (28) is the Froude-Krylov force and the second term is the added mass term.

The pore water inertia term might be formulated as

$$F_{I,w} = \rho\,v_w\,\frac{dv}{dt} \qquad (29)$$

where v_w is the pore water volume.

If we consider a sample of length $\Delta\ell$ with a unit area cross section perpendicular to the flow and introduce the volumetric porosity n = void volume/total volume and take the characteristic velocity $v = V/n$, where V is the bulk velocity (Darcy velocity) we obtain

$$F_I \;=\; F_{I,g} + F_{I,w} = \rho\,C_m\left(\frac{1-n}{n}\right)\Delta\ell\,\frac{dV}{dt} + \rho\,\Delta\ell\,\frac{dV}{dt}$$

In the present study the same approach as used by Hannoura et al. (1978) was applied but with a larger cross section U-tube.