The Fermi Golden Rule at thresholds

Let H be a Schrödinger operator on a Hilbert space $\mathcal{H}$, such that zero is a nondegenerate threshold eigenvalue of H with eighenfunction Ψ_0. Let W be a bounded selfadjoint operator satisfying $(\Psi_0, W\Psi_0) > 0$. Assume that the resolvent $(H-z)^{-1}$ has an asymptotic expansion around $z=0$ of the form typical for Schrödinger operators on odd-dimensional spaces. Let $H(\varepsilon) = H + \varepsilon W$ for $\varepsilon > 0$ and small. We show under some additional assumptions that the eigenvalue at zero becomes a resonance for $H(\varepsilon)$, in the time-dependent sense introduced by A. Orth. No analytic continuation is needed. We show that the imaginary part of the resonance has a dependence on ε of the form $\varepsilon^{2+(\nu/2)}$ with the integer $\nu \geq -1$ and odd. This shows how the Fermi Golden Rule has to be modified in the case of perturbation of a threshold eighenvalue. We give a number of explicit examples, where we compute the location of the resonance to leading order in ε.