On lower existence bounds for the asymptotic parameters of $\mathbb{Z}_{2^l}-$linear codes

In [1], [2] and [3] concatenated codes with fixed inner code and the outer code randomly selected is analyzed. For these codes a lower existence bound on the asymptotic parameters is derived and this bound is compared to the Gilbert-Varshamov bound. Furthermore an upper bound on the weight distribution of these codes is determined and this bound is then used to examine the error performance of the codes when used on the $q-$ary symmetric channel with symbol to symbol error probability $p/(q-1)$ and decoded by the maximum likelihood rule.

In this paper we perform a similar analysis of the so called $\mathbb{Z}_{2^l}-$linear codes (defined in [4]) which are a generalization of $\mathbb{Z}_4-$linear codes. To perform this analysis we consider $\mathbb{Z}_{2^l}-$linear codes as concatenated codes (described in Section 1.C). To carry out this analysis we need to define some basic concepts and show a few preliminary results and this is the contents of the rest of the introduction. First, in Section 1.A, we state some results from [2] which are important to our analysis. Then, in Section 1.B, we discuss Galois rings ($\mathbb{Z}_{2^l}$ is a Galois ring) and linear codes over Galois rings. Finally we introduce $\mathbb{Z}_{2^l}-$linear codes.