On equality in Berge's classical bound for the domination number

Let $\gamma(G)$ denote the cardinality of a minimum dominating set of a graph G. A well-known upper bound for $\gamma(G)$, due to Berge (1962), states that for any graph G of order n and maximum degree $\Delta, \gamma(G) \leq n - \Delta$. Similarly, Hedetniemi and Laskar (1984) proved $\gamma_c(G) \leq n - \Delta$, where $\gamma_c(G)$ denotes the cardinality of a minimum connected dominating set of G. In this paper, we characterize the regular graphs with $\gamma(G) = n - \Delta$, the regular graphs with $\gamma_c(G) = n - \Delta$ and the triangle-free graphs with $\gamma_c(G) = n - \Delta$. Moreover, we prove that both the problem of deciding whether $\gamma(G) = n - \Delta$ and the problem of deciding whether $\gamma_c(G) = n - \Delta$ are $co - NP$-complete.