



Aalborg Universitet

AALBORG UNIVERSITY  
DENMARK

## Solution Methods for Structures with Random Properties Subject to Random Excitation

Köylüoğlu, H. U.; Nielsen, Søren R. K.; Cakmak, A. S.

*Publication date:*  
1994

*Document Version*  
Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*  
Köylüoğlu, H. U., Nielsen, S. R. K., & Cakmak, A. S. (1994). *Solution Methods for Structures with Random Properties Subject to Random Excitation*. Dept. of Building Technology and Structural Engineering. Structural Reliability Theory Vol. R9444 No. 136

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- ? Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- ? You may not further distribute the material or use it for any profit-making activity or commercial gain
- ? You may freely distribute the URL identifying the publication in the public portal ?

### Take down policy

If you believe that this document breaches copyright please contact us at [vbn@aub.aau.dk](mailto:vbn@aub.aau.dk) providing details, and we will remove access to the work immediately and investigate your claim.



---

**INSTITUTTET FOR BYGNINGSTEKNIK**  
DEPT. OF BUILDING TECHNOLOGY AND STRUCTURAL ENGINEERING  
AALBORG UNIVERSITET • AUC • AALBORG • DANMARK

---

**STRUCTURAL RELIABILITY THEORY**  
**PAPER NO. 136**

To be presented at the ASCE Specialty Conference, Colorado, June 1995

---

**H. U. KÖYLÜOĞLU, S. R. K. NIELSEN, A. Ş. ÇAKMAK**  
**SOLUTION METHODS FOR STRUCTURES WITH RANDOM PROPERTIES SUBJECT TO RANDOM EXCITATION**  
DECEMBER 1994

---

ISSN 0902-7513 R9444



The STRUCTURAL RELIABILITY THEORY papers are issued for early dissemination of research results from the Structural Reliability Group at the Department of Building Technology and Structural Engineering, University of Aalborg. These papers are generally submitted to scientific meetings, conferences or journals and should therefore not be widely distributed. Whenever possible reference should be given to the final publications (proceedings, journals, etc.) and not to the Structural Reliability Theory papers.



---

**INSTITUTTET FOR BYGNINGSTEKNIK**  
DEPT. OF BUILDING TECHNOLOGY AND STRUCTURAL ENGINEERING  
AALBORG UNIVERSITET • AUC • AALBORG • DANMARK

---

**STRUCTURAL RELIABILITY THEORY**  
**PAPER NO. 136**

To be presented at the ASCE Specialty Conference, Colorado, June 1995

---

**H. U. KÖYLÜOĞLU, S. R. K. NIELSEN, A. Ş. ÇAKMAK**  
**SOLUTION METHODS FOR STRUCTURES WITH RANDOM PROPERTIES SUBJECT TO RANDOM EXCITATION**  
**DECEMBER 1994**

---

ISSN 0902-7513 R9444

---



# Solution Methods for Structures with Random Properties Subject to Random Excitation

**H.U. Köylüoğlu**

*Department of Civil Engineering and Operations Research,  
Princeton University, Princeton, NJ 08544, USA*

**S.R.K. Nielsen**

*Department of Building Technology and Structural Engineering,  
Aalborg University, Sohngaardsholmsvej 57, DK-9000 Aalborg, Denmark*

**A.Ş. Çakmak**

*Department of Civil Engineering and Operations Research,  
Princeton University, Princeton, NJ 08544, USA*

## ABSTRACT

This paper deals with the lower order statistical moments of the response of structures with random stiffness and random damping properties subject to random excitation. The arising stochastic differential equations (SDE) with random coefficients are solved by two methods, a second order perturbation approach and a Markovian method. The second order perturbation approach is grounded on the total probability theorem and can be compactly written. Moreover, the problem to be solved is independent of the dimension of the random variables involved. The Markovian approach suggests transforming the SDE with random coefficients with deterministic initial conditions to an equivalent nonlinear SDE with deterministic coefficients and random initial conditions. In both methods, the statistical moment equations are used. Hierarchy of statistical moments in the Markovian approach is closed by the cumulant neglect closure method applied at the fourth order level.

## INTRODUCTION

The response of the structure will be stochastic in the cases uncertainties in initial conditions or external loads or parameters of the constitutive relations are modelled using random variables or random fields which are functions of time or space. In 1980's, the analysis of the stochastic response of stochastic structural systems received a lot of attention, consequently a new field, "Stochastic Finite Element Method (SFEM)" was coined to stochastic mechanics to analyze MDOF large scaled structural systems using discrete approximations. The developments in this field are reviewed by Vanmarcke et al. (1986), Benaroya and Rehak (1988), Ghanem and Spanos (1991), Brenner (1991), Der Kiureghian et al. (1991), Kleiber and Hien (1992).

For the full quantification of uncertainty using random variables and random fields, joint probability density function (pdf) should be assigned. Estimations of the joint pdf for the random models are based on experimental tests, observations and engineering judgement. It is very difficult, usually impossible, to quantify the uncertainty in terms of joint pdf. In practice, only the lower order statistical moments of the joint pdf can be estimated accurately. This study considers that the uncertainty can be quantified only for the first and second order statistics. It, then, calculates the first and second order statistical moments of the stochastic response. Due to page limitations, only linear multi-degree-of-freedom (MDOF) structural systems with random stiffness and damping properties subject to white noise excitation is studied. For extensions to nonlinear systems, non-white noise excitations and other derivations, see Köylüoğlu, Nielsen and Çakmak (1994), Köylüoğlu (1995).

The equations of motion of linear MDOF systems with random parameters subject to white noise excitation multiplied with intensity matrix  $\mathbf{q}$  of dimension  $p \times r$  are

$$\mathbf{m}\ddot{\mathbf{V}}(\mathbf{X}, t) + \mathbf{C}(\mathbf{X})\dot{\mathbf{V}}(\mathbf{X}, t) + \mathbf{K}(\mathbf{X})\mathbf{V}(\mathbf{X}, t) = \mathbf{q}\mathbf{R}(t) \quad \mathbf{V}(\mathbf{X}, 0) = \dot{\mathbf{V}}(\mathbf{X}, 0) = \mathbf{0} \quad (1)$$

where  $\mathbf{m}$ ,  $\mathbf{C}(\mathbf{X})$  and  $\mathbf{K}(\mathbf{X})$  are mass, random damping and random stiffness matrices of dimension  $p \times p$ .  $\{\mathbf{R}(t), t \in [-\infty, \infty]\}$  is a vector of dimension  $r \times 1$  denoting zero-mean stationary white noise processes,  $E[\mathbf{R}(t)] = \mathbf{0}$ , with auto-covariance function  $E[R_\alpha(t_1)R_\beta(t_2)] = \delta(t_2 - t_1)$ ,  $\alpha, \beta = 1, \dots, r$ .  $\mathbf{X}^T = [X_1, \dots, X_d]$  are zero-mean random variables,  $E[\mathbf{X}] = \mathbf{0}$ , with specified covariances  $E[X_i X_j] = \kappa_{X_i X_j}$ .  $X_1, \dots, X_d$  are all assumed to be stochastically independent of  $\mathbf{R}(t)$ . The displacement  $\mathbf{V}(\mathbf{X}, t)$  and velocity  $\dot{\mathbf{V}}(\mathbf{X}, t)$  response processes of the nodal points are random partly because of the functional dependency of the external random excitation process and partly due to the random random variables  $\mathbf{X}$ .

## PERTURBATION METHOD

Consider the Taylor expansion of the random matrices and response quantities with respect to the random variables  $X_1, \dots, X_d$  from the mean value system, e.g.

$$\mathbf{C}(\mathbf{X}) = \mathbf{c}_0 + \mathbf{c}_i X_i + \frac{1}{2} \mathbf{c}_{ij} X_i X_j + \dots \quad (2)$$

$$V_m(\mathbf{X}, t) \simeq V_m(\mathbf{0}, t) + V_{m, x_i}(\mathbf{0}, t) X_i + \frac{1}{2} V_{m, x_i x_j}(\mathbf{0}, t) X_i X_j + \dots \quad m = 1, 2, \dots, p \quad (3)$$

where  $\mathbf{c}_0 = \mathbf{C}(\mathbf{0})$ ,  $\mathbf{c}_i = \frac{\partial}{\partial x_i} \mathbf{C}(\mathbf{0})$ , etc.  $\mathbf{V}(\mathbf{0}, t)$  indicate the response processes on condition of  $\mathbf{X} = \mathbf{0}$ .  $V_{m, x_i}(\mathbf{0}, t) = \frac{\partial}{\partial x_i} V_m(\mathbf{0}, t)$ , etc. Further, summation convention has been applied for the dummy indices  $i, j = 1, \dots, d$ . Use of (2) and (3) and retaining terms up to second order in the random variables provides the following approximation for the unconditional covariance  $\kappa_{V_m V_n}(t)$  of the response processes,

$$\begin{aligned}
\kappa_{V_m V_n}(t) &= E[V_m(\mathbf{X}, t)V_n(\mathbf{X}, t)] \simeq E[V_m(\mathbf{0}, t)V_n(\mathbf{0}, t)] + \\
&\left\{ E[V_{m,x_i}(\mathbf{0}, t)V_{n,x_j}(\mathbf{0}, t)] + \frac{1}{2}E[V_m(\mathbf{0}, t)V_{n,x_i x_j}(\mathbf{0}, t)] + \right. \\
&\left. \frac{1}{2}E[V_n(\mathbf{0}, t)V_{m,x_i x_j}(\mathbf{0}, t)] \right\} \kappa_{X_i X_j}
\end{aligned} \tag{4}$$

In order to evaluate the expectations on the right sides, SDE must be formulated specifying the development of  $\mathbf{V}(\mathbf{0}, t)$ ,  $\dot{\mathbf{V}}(\mathbf{0}, t)$  and of the partial derivatives  $\mathbf{V}_{,x_i}(\mathbf{0}, t)$ ,  $\dot{\mathbf{V}}_{,x_j}(\mathbf{0}, t)$ ,  $\mathbf{V}_{,x_i x_j}(\mathbf{0}, t)$ ,  $\dot{\mathbf{V}}_{,x_i x_j}(\mathbf{0}, t)$ . These are obtained from partial differentiation of (1) with respect the random variables, evaluated at the mean structure  $\mathbf{X} = \mathbf{0}$ . All of these equations can next be cast into the following closed system of first order SDE with state vector  $\mathbf{Z}(t)$ . Note that  $\mathbf{Z}(0) = \mathbf{0}$ .

$$\dot{\mathbf{Z}}(t) = \mathbf{a}(\mathbf{Z}, t) + \mathbf{b}\mathbf{R}(t), \quad \mathbf{Z}(t) = \begin{bmatrix} \mathbf{V}(\mathbf{0}, t) \\ \dot{\mathbf{V}}(\mathbf{0}, t) \\ \mathbf{V}_{,x_i}(\mathbf{0}, t) \\ \dot{\mathbf{V}}_{,x_i}(\mathbf{0}, t) \text{ fill} \\ \mathbf{V}_{,x_j}(\mathbf{0}, t) \\ \dot{\mathbf{V}}_{,x_j}(\mathbf{0}, t) \\ \mathbf{V}_{,x_i x_j}(\mathbf{0}, t) \\ \dot{\mathbf{V}}_{,x_i x_j}(\mathbf{0}, t) \end{bmatrix}, \quad \mathbf{a}(\mathbf{Z}, t) = \mathbf{A} \begin{bmatrix} \mathbf{V}(\mathbf{0}, t) \\ \dot{\mathbf{V}}(\mathbf{0}, t) \\ \mathbf{V}_{,x_i}(\mathbf{0}, t) \\ \dot{\mathbf{V}}_{,x_i}(\mathbf{0}, t) \\ \mathbf{V}_{,x_j}(\mathbf{0}, t) \\ \dot{\mathbf{V}}_{,x_j}(\mathbf{0}, t) \\ \mathbf{V}_{,x_i x_j}(\mathbf{0}, t) \\ \dot{\mathbf{V}}_{,x_i x_j}(\mathbf{0}, t) \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} \mathbf{0} \\ \mathbf{m}_0^{-1} \mathbf{q}_0 \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \tag{5}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{m}_0^{-1} \mathbf{k}_0 & -\mathbf{m}_0^{-1} \mathbf{c}_0 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{m}_0^{-1} \mathbf{k}_i & -\mathbf{m}_0^{-1} \mathbf{c}_i & -\mathbf{m}_0^{-1} \mathbf{k}_0 & -\mathbf{m}_0^{-1} \mathbf{c}_0 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ -\mathbf{m}_0^{-1} \mathbf{k}_j & -\mathbf{m}_0^{-1} \mathbf{c}_j & \mathbf{0} & \mathbf{0} & -\mathbf{m}_0^{-1} \mathbf{k}_0 & -\mathbf{m}_0^{-1} \mathbf{c}_0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \\ -\mathbf{m}_0^{-1} \mathbf{k}_{ij} & -\mathbf{m}_0^{-1} \mathbf{c}_{ij} & -\mathbf{m}_0^{-1} \mathbf{k}_j & -\mathbf{m}_0^{-1} \mathbf{c}_j & -\mathbf{m}_0^{-1} \mathbf{k}_i & -\mathbf{m}_0^{-1} \mathbf{c}_i & -\mathbf{m}_0^{-1} \mathbf{k}_0 & -\mathbf{m}_0^{-1} \mathbf{c}_0 \end{bmatrix} \tag{6}$$

The state vector  $\mathbf{Z}(t)$  would have a dimension  $N = 2p + 4(pd) + 2(pd^2)$  if it is constructed using all the random variables of the system. Then, the method would, certainly, suffer from the high dimension  $N$  and becomes not practicable, if  $p$  and, especially,  $d$  are large. Indeed, only state vectors of smaller dimension  $N = 8p$  are needed to be considered where the state vector for certain  $i$  and  $j$  is used to calculate all the corresponding coefficients to  $\kappa_{X_i X_j}$  in equation 4, one by one. There will be  $\frac{d(d+1)}{2}$  many smaller systems to consider. (5) is a linear SDE with deterministic coefficients.  $E[\mathbf{Z}(t)] = \mathbf{0}$  and covariances are obtained from moment equations.



## MARKOVIAN METHOD

Since the random variables of the structural system has been assumed to be time-invariant, the following SDE represent the random variables with probability 1.

$$\dot{\mathbf{S}}(t) = \mathbf{0} \quad , \quad \mathbf{S}(0) = \mathbf{X} \quad (7)$$

where  $\mathbf{S}(t)$  is a time-invariant dummy random process and the initial conditions  $\mathbf{S}(0)$  are random with the same probabilistic structure as the random variables  $\mathbf{X}$ . (1) and (7) can next be combined into a closed system of equivalent 1st order SDE as in (5), but, the state vector  $\mathbf{Z}(t)$  is now made up of the displacement and velocity vector and the random variables of the structural system.

$$\mathbf{Z}(t) = \begin{bmatrix} \mathbf{V}(t) \\ \dot{\mathbf{V}}(t) \\ \mathbf{S}(t) \end{bmatrix} \quad , \quad \mathbf{Z}_0 = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{X} \end{bmatrix} \quad , \quad \mathbf{a}(\mathbf{Z}, t) = \begin{bmatrix} \dot{\mathbf{V}}(t) \\ -\mathbf{m}^{-1}\mathbf{C}(\mathbf{S})\dot{\mathbf{V}}(t) - \mathbf{m}^{-1}\mathbf{K}(\mathbf{S})\mathbf{V}(t) \\ \mathbf{0} \end{bmatrix} \quad ,$$

$$\mathbf{b} = \begin{bmatrix} \mathbf{0} \\ \mathbf{m}^{-1}\mathbf{q} \\ \mathbf{0} \end{bmatrix} \quad (8)$$

Due to  $\mathbf{R}(t)$  and as  $\mathbf{Z}_0$  is independent of  $\mathbf{R}(t)$ , the state vector  $\mathbf{Z}(t)$  is Markovian. The new state vector  $\mathbf{Z}(t)$  will have a dimension  $N = 2p + d$ . Hence, the dimension of the state vector is proportional to the number of random variables. For very large  $d$ , this could be a drawback compared to the perturbation method.

Since the drift vector is nonlinear in  $\mathbf{Z}$ , the statistical moment equations for this state vector are ordinary nonlinear differential equations where hierarchy of moments appear. In this study, the hierarchy is closed by the cumulant neglect closure method applied at the fourth order statistical moment level.

## NUMERICAL EXAMPLE

A numerical example is worked out for a SDOF oscillator. The random parameters  $C$  and  $K$  are assumed to be mutually stochastically independent with the following following mean values ( $E[\cdot]$ ) and variational coefficients ( $v[\cdot]$ ).  $m = 1.0$ ,  $E[C] = c_0 = 0.1$ ,  $v_C = 0.3$ ,  $E[K] = k_0 = 1.0$ ,  $v_K = 0.3$ ,  $q = \sqrt{2c_0k_0}$ . The corresponding variances of the displacement and velocity of the mean linear oscillator are both equal to 1.  $\kappa_{VV}(t)$  obtained by the presented methods are compared to the so-called exact ones in Figures 1 and 2. Given  $f_{\mathbf{X}}(\mathbf{x})$ , the exact unconditional nonstationary variances  $\kappa_{VV}(t)$  can be obtained by the application of the total probability theorem on the conditional nonstationary variances  $\kappa_{VV}(\mathbf{X} = \mathbf{x})$ .

$$\kappa_{VV}(t) = \int_{\mathbf{x}} \kappa_{VV}(\mathbf{X} = \mathbf{x}, t) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \quad (9)$$

$f_{\mathbf{X}}(\mathbf{x})$  is assigned as uniform and triangular shaped distributions for  $C$  and  $K$ .

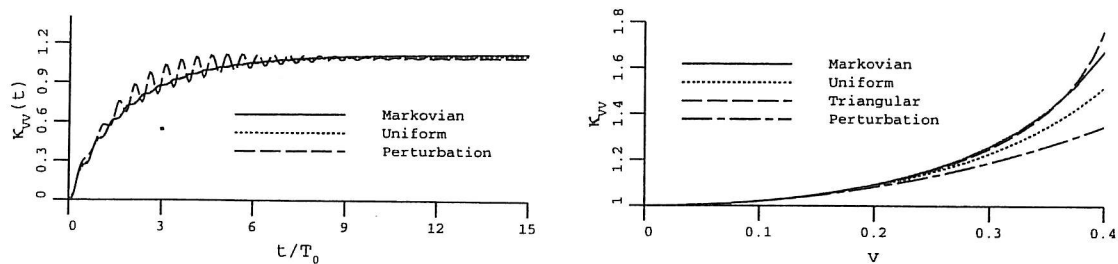


Figure 1)  $\kappa_{VV}(t)$  versus  $\frac{t}{T_0}$ . Only  $K$  is random. Figure 2) Stationary  $\kappa_{VV}$  versus  $\nu = \nu_C = \nu_K$ .

## CONCLUSION

The SDE with random coefficients resulting in dynamic SFEM problems are attacked by two methods, a second order perturbation approach and a Markovian method. Perturbation method is grounded on total probability theorem, yields a formulation independent of the dimension of the random variables involved, possesses divergent secular terms under the governing control of damping in the nonstationary regime and may handle coefficient of variations up to 25-30 percent. The Markovian approach results in hierarchy in the statistical moment equations. Closure by the cumulant neglect closure method applied at the fourth order level seems to be promising to attack problems with larger variability.

## REFERENCES

1. Vanmarcke, E., Shinozuka, M., Nakagiri, S. "Schueller, G. & Grigoriu, M. (1986) Random Fields and Stochastic Finite-Elements", *Structural Safety* **3**, 143-166.
2. Benaroya, H. & Rehak, M. "Finite Element Methods in Probabilistic Structural Analysis : A Selective Review", (1988) *Appl. Mech. Rev.*, **40**, 201-213.
3. Ghanem, R. & Spanos, P.D. (1991) *Stochastic Finite Elements : A Spectral Approach*, Springer-Verlag, New York.
4. Brenner, C. (1991) *Stochastic Finite Elements (Literature Review)* Internal Working Report No. 35-91, Institute of Engrg. Mech., University of Innsbruck, Austria.
5. Der Kiureghian, A., Li, C.C. & Zhang, Y. (1991) "Recent Developments in Stochastic Finite Element", *Lecture Notes in Engineering IFIP 76, Proc. Fourth IFIG WG 7.5 Conference, Germany*, Ed. R. Rackwitz & P. Thoft-Christensen, Springer-Verlag.
6. Kleiber, M. and Hien, T.D. (1992) *The Stochastic Finite Element Method*, Wiley, Chichester.



7. Köylüoğlu, H.U. (1994) *Stochastic response and reliability analyses of structures with random properties subject to random stationary excitation*, Ph.D. thesis, Dept. of Civil Engineering and Operations Research, Princeton University, 1994.
8. Köylüoğlu, H.U., Nielsen, S.R.K. and Çakmak, A.Ş. (1994) *Solution of random structural system subject to nonstationary excitation : Transforming the equation with random coefficients to one with deterministic coefficients and random initial conditions* Accepted for publication in Soil Dynamics and Earthquake Engineering.



## STRUCTURAL RELIABILITY THEORY SERIES

PAPER NO. 107: H. U. Köylüoğlu, S. R. K. Nielsen & R. Iwankiewicz: *Response and Reliability of Poisson Driven Systems using Path Integration*. ISSN 0902-7513 R9323.

PAPER NO. 108: H. U. Köylüoğlu, S. R. K. Nielsen & R. Iwankiewicz: *Reliability of Non-Linear Oscillators Subject to Poisson Driven Impulses*. ISSN 0902-7513 R9235.

PAPER NO. 109: C. Pedersen & P. Thoft-Christensen: *Reliability Analysis of Prestressed Concrete Beams with Corroded Tendons*. ISSN 0902-7513 R9306.

PAPER NO. 110: F. M. Jensen: *Optimization of Large-Scale Structural Systems*. Ph.D.-Thesis. ISSN 0902-7513 R9310.

PAPER NO. 111: H. U. Köylüoğlu & S. R. K. Nielsen: *Reliability of Dynamically Excited Linear Structures with Random Properties*. ISSN 0902-7513 R9311.

PAPER NO. 112: J. D. Sørensen, M. H. Faber & I. B. Kroon: *Reliability-Based Optimal Design of Experiment Plans for Offshore Structures*. ISSN 0902-7513 R9331.

PAPER NO. 113: J. D. Sørensen & I. Enevoldsen: *Sensitivity Weaknesses in Application of some Statistical Distribution in First Order Reliability Methods*. ISSN 0902-7513 R9302.

PAPER NO. 114: H. U. Köylüoğlu & S. R. K. Nielsen: *Stochastic Dynamics of Linear Structures with Random Stiffness Properties and Random Damping subject to Random Loading*. ISSN 0902-7513 R9308.

PAPER NO. 115: H. U. Köylüoğlu & S. R. K. Nielsen: *New Approximations for SORM Integrals*. ISSN 0902-7513 R9303.

PAPER NO. 116: H. U. Köylüoğlu, S. R. K. Nielsen & A. Ş. Çakmak: *Stochastic Dynamics of Geometrically Non-Linear Structures subject to Stationary Random Excitation*. ISSN 0902-7513 R9418.

PAPER NO 117: H. U. Köylüoğlu, S. R. K. Nielsen & A. Ş. Çakmak: *Perturbation Solutions for Random Linear Structural Systems subject to Random Excitation using Stochastic Differential Equations*. ISSN 0902-7513 R9425.

PAPER NO. 118: I. Enevoldsen & J. D. Sørensen: *Reliability-Based Optimization in Structural Engineering*. ISSN 0902-7513 R9332.

PAPER NO. 119: I. Enevoldsen & K. J. Mørk: *Effects of a Vibration Mass Damper in a Wind Turbine Tower*. ISSN 0902-7513 R9334.

PAPER NO. 120: C. Pedersen & P. Thoft-Christensen: *Interactive Quasi-Newton Optimization Algorithms*. ISSN 0902-7513 R9346.

PAPER NO. 121: H. U. Köylüoğlu, S. R. K. Nielsen & A. Ş. Çakmak: *Applications of Interval Mapping for Structural Uncertainties and Pattern Loadings*. ISSN 0902-7513 R9411.



## STRUCTURAL RELIABILITY THEORY SERIES

PAPER NO. 122: H. U. Köylüoğlu, S. R. K. Nielsen & A. Ş. Çakmak: *Fast Cell-to-Cell Mapping (Path Integration) with Probability Tails for the Random Vibration of Nonlinear and Hysteretic Systems*. ISSN 0902-7513 R9410.

PAPER NO. 123: A. Aşkar, H. U. Köylüoğlu, S. R. K. Nielsen & A. Ş. Çakmak: *Faster Simulation Methods for the Nonstationary Random Vibrations of Nonlinear MDOF Systems*. ISSN 0902-7513 R9405.

PAPER NO. 125: H. I. Hansen, P. H. Kirkegaard & S. R. K. Nielsen: *Modeling of Deteriorating RC-Structures under Stochastic Dynamic Loading by Neural Networks*. ISSN 0902-7513 R9409.

PAPER NO. 126: H. U. Köylüoğlu, S. R. K. Nielsen & A. Ş. Çakmak: *Reliability Approximations for MDOF Structures with Random Properties subject to Random Dynamic Excitation in Modal Subspaces*. ISSN 0902-7513 R9440.

PAPER NO. 128: H. U. Köylüoğlu, S. R. K. Nielsen, A. Ş. Çakmak & P. H. Kirkegaard: *Prediction of Global and Localized Damage and Future Reliability for RC Structures subject to Earthquakes*. ISSN 0901-7513 R9426.

PAPER NO. 129: C. Pedersen & P. Thoft-Christensen: *Interactive Structural Optimization with Quasi-Newton Algorithms*. ISSN 0902-7513 R9436.

PAPER NO. 130: I. Enevoldsen & J. D. Sørensen: *Decomposition Techniques and Effective Algorithms in Reliability-Based Optimization*. ISSN 0902-7513 R9412.

PAPER NO. 131: H. U. Köylüoğlu, S. R. K. Nielsen & A. Ş. Çakmak: *Approximate Forward Difference Equations for the Lower Order Non-Stationary Statistics of Geometrically Non-Linear Systems subject to Random Excitation*. ISSN 0902-7513 R9422.

PAPER NO. 132: I. B. Kroon: *Decision Theory applied to Structural Engineering Problems*. Ph.D.-Thesis. ISSN 0902-7513 R9421.

PAPER NO. 134: H. U. Köylüoğlu, S. R. K. Nielsen & A. Ş. Çakmak: *Solution of Random Structural System subject to Non-Stationary Excitation: Transforming the Equation with Random Coefficients to One with Deterministic Coefficients and Random Initial Conditions*. ISSN 0902-7513 R9429.

PAPER NO. 135: S. Englund, J. D. Sørensen & S. Krenk: *Estimation of the Time to Initiation of Corrosion in Existing Uncracked Concrete Structures*. ISSN 0902-7513 R9438.

PAPER NO. 136: H. U. Köylüoğlu, S. R. K. Nielsen & A. Ş. Çakmak: *Solution Methods for Structures with Random Properties subject to Random Excitation*. ISSN 0902-7513 R9444.

PAPER NO. 137: J. D. Sørensen, M. H. Faber & I. B. Kroon: *Optimal Reliability-Based Planning of Experiments for POD Curves*. ISSN 0902-7513 R9455.

Department of Building Technology and Structural Engineering  
Aalborg University, Sohngaardsholmsvej 57, DK 9000 Aalborg  
Telephone: +45 98 15 85 22 Telefax: +45 98 14 82 43