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Optimal Reliability-Based Planning of Experiments for POD Curves

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Abstract

Optimal planning of crack detection tests is considered. The tests are used to update the information on the reliability of inspection techniques modelled by probability of detection (P.O.D.) curves. It is shown how cost-optimal and reliability-based test plans can be obtained using First Order Reliability Methods in combination with life-cycle cost-optimal inspection and maintenance planning. The methodology is based on preposterior analyses from Bayesian decision theory. An illustrative example is shown.

1. Introduction

Modern reliability methods, see Madsen et al. [1], have shown large applicability as decision support tools in engineering problems. Examples are optimal structural design, inspection and maintenance planning, requalification and experiment planning. One of the main advantages of reliability-based decision tools is that the result of the analysis can reflect the actual knowledge. Thereby the uncertainty (or certainty) associated with a specific problem, such as parameter values, models for the structural behaviour etc. can directly influence the decision. Furthermore, given that a probabilistic model for the uncertainties has been defined, reliability based decision tools make it possible, in a consistent way, to incorporate any additional information about the uncertainties in the problem. Modern methods for inspection and maintenance planning for offshore structures are to some extent based on estimated expected life cycle costs and/or the structural reliability. It is well known to the operators of offshore structures that one of the most uncertain, and at the same time, also one of the most influential parameters in connection with inspection and maintenance planning is the reliability of the sub-sea inspections. This is in particular the case with the uncertainty associated with crack detections. Therefore it is of utmost importance to include this uncertainty in the inspection and maintenance planning analysis.

In recent studies, see e.g. Madsen [2], the uncertainty in crack detection, i.e. the reliability of the inspection methods, has been included using the so-called probability of detection curves (P.O.D. curves). These P.O.D. curves are estimated from experimental inspection trials under controlled conditions. The experiments are conducted under quasi realistic sub-sea conditions by sending divers down in water filled tanks. The divers then inspect submerged structural details with defects of known size. By counting the number of detections and no-detections for given defect sizes a P.O.D. curve is determined see e.g. Thomson & Chimenti [3].

The quality of an inspection method is usually characterized by the crack length which corresponds to a probability of detection of 90 % where the probability of detection curve used is the lower 95 % confidence limit obtained on the basis of the experiments. Therefore the experiments are planned such that this point can be determined as cheap and accurate as possible. This implies that the quality of the P.O.D. curve is highest around this 90/95 % point. However, when the P.O.D. curve is used in a reliability analysis it is important that the whole P.O.D. curve is accurate, i.e. the experimental points have to cover the whole crack length range. The experiments are rather expensive and it thus becomes an important issue to plan the experiments carefully. Such an experiment plan cannot be performed without taking into account the effect of the P.O.D. curve on the structural life cycle costs. The present paper addresses the problem of optimal planning of experimental inspection trials for a given inspection technique taking into account the structural life cycle costs. This is done by combining recently developed desision theoretical based methods for inspection and maintenance planning and experiment planning, see Kroon [4] and Sørensen et al. [5]. An illustrative example of crack detection experiment planning is presented.

2. Optimal Experiment and Inspection Planning

The basis for determination of the cost optimal experiment and inspection strategy is preposterior analyses from classical decision theory, see e.g. Raiffa & Schlaifer [6] and Benjamin & Cornell [7].

The decision variables defining the crack detection experiment can be the number of crack detection experiments N_{sample} and the actual crack lengths of the specimens to be inspected in the experiment $\mathbf{a}^e = (a_1^e, ..., a_{N_{sample}}^e)$, conveniently collected in the experiment vector $\mathbf{e} = (N_{sample}, \mathbf{a}^e)$. The outcome of the experiments in terms of rates of success (detection / no detection) for given specimen crack lengths are modelled by binominally distributed random variables $\mathbf{Z}$. In the most general case the parameters defining the inspection plan are the number of inspections N_i, the time intervals between inspections $\mathbf{\Delta t} = (\Delta t_1, \cdots, \Delta t_{N_i})$ and the inspection qualities $\mathbf{q} = (q_1, \cdots, q_{N_i})$. These parameters are conveniently collected in the inspection vector $\mathbf{i} = (N_i, \mathbf{\Delta t}, \mathbf{q})$. The outcome of an inspection is assumed to be modelled by a random variable S. S typically models a measured crack length or depth. A decision rule d must then be applied to the outcome of the inspection to decide whether or not a repair should be performed. The different uncertain parameters (stochastic variables) modelling the state of nature are collected in a vector $\mathbf{X}$.

A rational experiment and inspection plan can then be obtained using Bayesian decision analysis, see Raiffa & Schlaifer [6]. The decision tree is shown in figure 1. It is assumed that the decision maker chooses the strategy $\mathbf{e}, \mathbf{i}, d$ which maximizes the expected utility

$$u^* = \max_{\mathbf{e}} E_{\mathbf{Z}|\mathbf{e}}[\max_{\mathbf{i}} \max_{d} \quad E''_{S|\mathbf{i},\mathbf{Z}}[E''_{\mathbf{X}|S,\mathbf{Z}}[u(\mathbf{e}, \mathbf{Z}, \mathbf{i}, S, d(S), \mathbf{X})]]] \qquad (1)$$

where $E_{\mathbf{Z}|\mathbf{e}}$ is the prior expectation based on the prior statistical model for $\mathbf{Z}$.

$E''_{S|\mathbf{i},\mathbf{z}}$ is the posterior expectation with respect to S based on the outcome $\mathbf{z}$ of $\mathbf{Z}$, i.e. S is updated on behalf on the outcome of the crack detection experiment $\mathbf{z}$ and $E''_{\mathbf{X}|S,\mathbf{z}}$ is the posterior expectation of the state of nature $\mathbf{X}$ given the outcome s of the inspection which again is updated on the basis of the outcome $\mathbf{z}$ of the crack detection experiment.

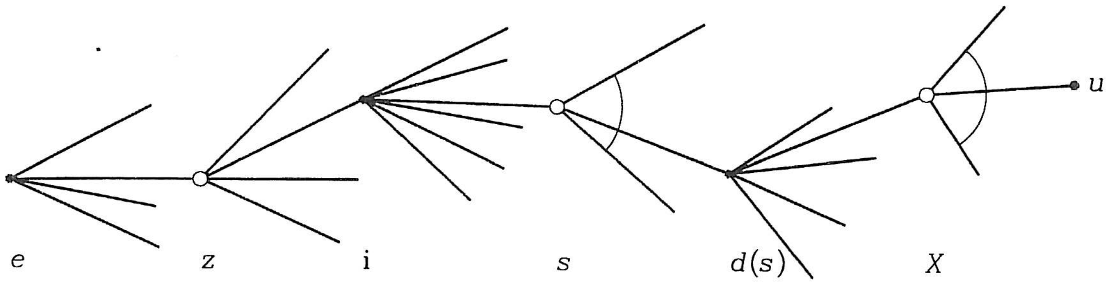

Figure 1. Decision tree.

2.1 Cost optimal inspection planning

If the utility function is related to the total costs and the benefits are neglected (1) can be written

$$C^* = \min_{\mathbf{e}} E_{\mathbf{Z}|\mathbf{e}}[C_{EXP}(\mathbf{e}) + C^*(\mathbf{e}, \mathbf{Z})] \tag{2}$$

where $C_{EXP}(\mathbf{e})$ is the cost related to the crack detection experiment and $C^*(\mathbf{e}, \mathbf{z})$ is the minimum expected inspection, maintenance and failure costs given an experiment plan $\mathbf{e}$ and test result $\mathbf{z}$. If the costs $C(\mathbf{e}, \mathbf{z})$ are divided into inspection, repair and failure costs and a constraint related to a minimum level of reliability is added then $C^*(\mathbf{e}, \mathbf{z})$ can be determined from the optimization problem

$$C^*(\mathbf{e}, \mathbf{z}) = \min_{\mathbf{i},d} \quad C^*(\mathbf{e}, \mathbf{z}, \mathbf{i}, d) = C_{IN}(\mathbf{e}, \mathbf{z}, \mathbf{i}, d) + C_R(\mathbf{e}, \mathbf{z}, \mathbf{i}, d) + C_F(\mathbf{e}, \mathbf{z}, \mathbf{i}, d) \tag{3}$$

$$s.t. \quad \beta(T_L, \mathbf{e}, \mathbf{z}, \mathbf{i}, d) \geq \beta_{\min} \tag{4}$$

$C^*(\mathbf{e}, \mathbf{z}, \mathbf{i}, d)$ is the total expected inspection and maintenance cost in the design lifetime ($T_{N+1} = T_L$). C_{IN} is the expected inspection cost, C_R is the expected cost of repair and C_F is the expected failure cost.

$\beta(T, \mathbf{e}, \mathbf{z}, \mathbf{i}, d)$ is the generalized reliability index defined by

$$\beta(T, \mathbf{e}, \mathbf{z}, \mathbf{i}, d) = -\Phi^{-1}(P''_F(T, \mathbf{e}, \mathbf{z}, \mathbf{i}, d)) \tag{5}$$

where Φ is the standardized normal distribution function and $P''_F(T, \mathbf{e}, \mathbf{z}, \mathbf{i}, d)$ is the probability of failure in the time interval $[0, T]$ given experiment plan $\mathbf{e}$, result $\mathbf{z}$, inspection strategi $\mathbf{i}$ and repair decision d.

The constraint on the minimum reliability (4) is somewhat unnecessary since the reliability is already incorporated in the objective function through the expected cost of the failure term but it is included to take account of future prefixed code demands set up by the authorities.

Other constraints, e.g. on the maximum of the individual costs or direct bounds on the optimization variables can be included in the problem, if necessary.

In (3) the expected inspection, repair and failure costs must be modelled as functions of the decision variables.

The total capitalized expected inspection costs are modelled by

$$C_{IN}(\mathbf{e},\mathbf{z},\mathbf{i},d) = \sum_{i=1}^{N_i} C_{IN_i}(\mathbf{q})(1 - P_F''(T_i,\mathbf{e},\mathbf{z},\mathbf{i},d))\frac{1}{(1+r)^{T_i}} \tag{6}$$

The ith term represents the capitalized inspection costs at the ith inspection when failure has not occurred earlier. Here it is assumed that if failure occurs then the component cannot be repaired. $C_{IN_i}(\mathbf{q})$ is the inspection cost of the ith inspection, $P_F''(T_i,\mathbf{e},\mathbf{z},\mathbf{i},d)$ is the probability of failure in the time interval $[0,T_i]$ given experiment plan $\mathbf{e}$ and result $\mathbf{z}$. r is the real rate of interest.

$$C_R(\mathbf{e},\mathbf{z},\mathbf{i},d) = \sum_{i=1}^{N_i} C_{R_i} P_{R_i}(\mathbf{e},\mathbf{z},\mathbf{i},d)\frac{1}{(1+r)^{T_i}} \tag{7}$$

is the total capitalized expected repair costs. The ith term represents the capitalized expected repair costs at the ith inspection. C_{R_i} is the cost of a repair at the ith inspection and $P_{R_i}(\mathbf{e},\mathbf{z},\mathbf{i},d)$ is the probability of performing a repair after the ith inspection given experiment plan $\mathbf{e}$ and result $\mathbf{z}$ when failure has not occurred earlier.

The total capitalized expected costs due to failure are estimated from

$$C_F(\mathbf{e},\mathbf{z},\mathbf{i},d) = \sum_{i=1}^{N_i+1} C_F(T_i)(P_F''(T_i,\mathbf{e},\mathbf{z},\mathbf{i},d) - P_F''(T_{i-1},\mathbf{e},\mathbf{z},\mathbf{i},d))\frac{1}{(1+r)^{T_i}} \tag{8}$$

$C_F(T)$ is the cost of failure at the time T.

2.2 Simplified optimal inspection planning

Sometimes it can be difficult to obtain the data to model the costs needed in a cost optimal inspection planning as described above. Instead a simplified inspection planning can be performed using a minimum reliability level (eventually code specified) and estimates of the reliability as function of time taking inspections into account, see e.g. [5] and [8]. In figure 2 the generalised reliability index $\beta = -\Phi^{-1}(P_F(T))$ is shown as a function of time. When $\beta(t)$ decreases to the minimum reliability level $\beta_{\min}$ an inspection has to be performed. The next inspection time is determined assuming that no defects are found by the inspection. Based on this assumption and taking into account the inspection uncertainty modelled by A_d the next inspection time is found as the time T_2 where the updated reliability index decreases to $\beta_{\min}$. In this way an inspection plan can be determined.

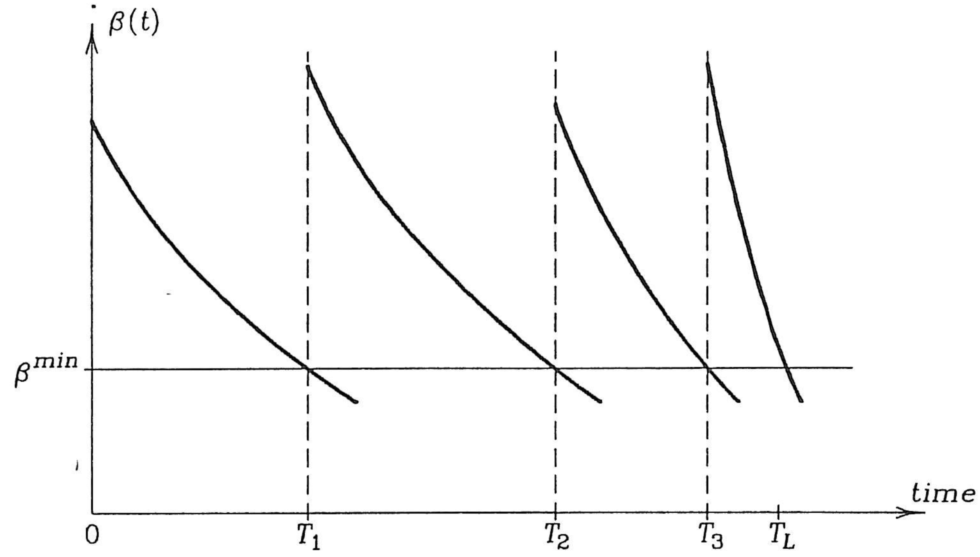

Figure 2. Simplified inspection planning.

In order to decide if experiments should be performed to update the p.o.d. curve the costs of the experiment could be compared with potential savings in inspection costs for the platforms operated by the experiment planner and inspected with the actual inspection method considered in the experiment. If experiments are performed then the generalised reliability index can be updated, see figure 3, where $\beta'' = -\Phi^{-1}(P''_F(T))$, see (5), indicates the reliability index updated on the basis of the results of the experiments and on no-find of defects at the inspection times. Based on the updated reliability index the inspection plan can be updated and the potential savings can be compared with the experiment costs. In figure 3 two inspections has to be performed at times T'_1 and T'_2 if experiments are performed and three inspections has to be performed at T_1, T_2 and T_3 if no experiments are performed. However, it should be noted that although it can be expected that in general the number of inspections is descreased if experiments are performed, it can also happen that more inspections have to be performed if the crack detection experiments show that the p.o.d. is more uncertain than expected a priori.

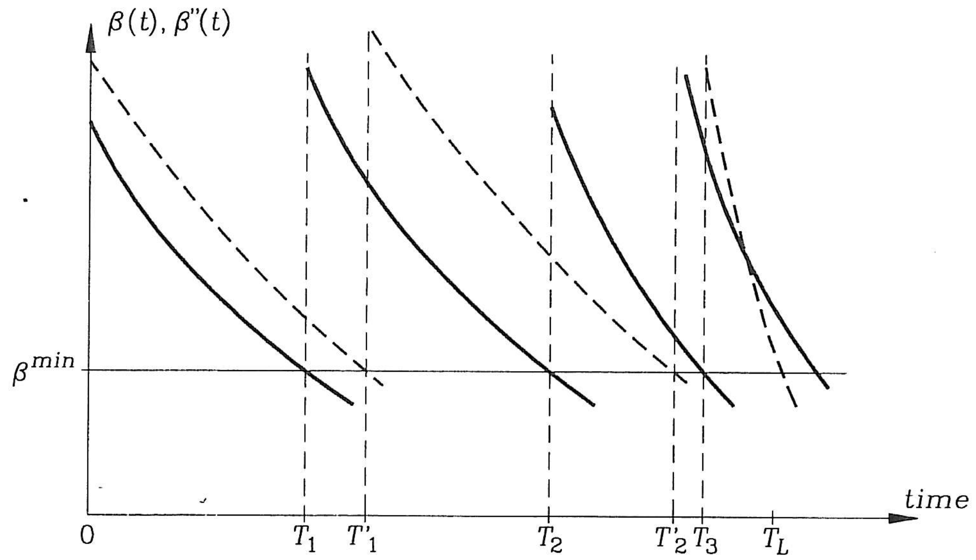

Figure 3. Updated simplified inspection planning. —— : $\beta(t)$ and - - - : $\beta''(t)$.

3. Reliability Modelling

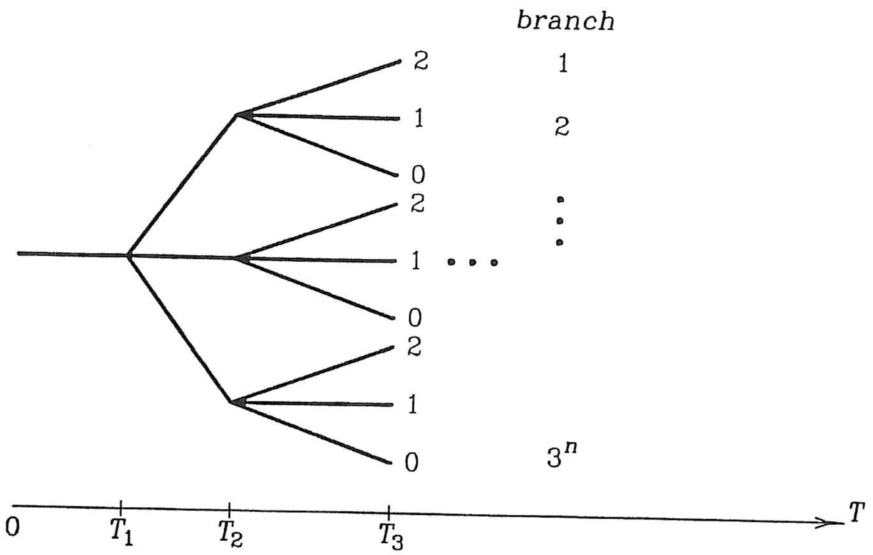

Figure 4. Repair realizations.

The failure and repair probabilities necessary for calculation of the expected costs can be expressed in terms of intersections of the events of inspection, repair and failure. All branches of the repair event tree at the N_i inspection times according to the chosen maintenance strategy must be taken into account when calculating the probabilities.

If repair is assumed to be performed when a defect is detected and has a measured size a larger than a critical level a_r then the total number of repair realizations (branches) is 3^{N_i}, see figure 4, where 0, 1 and 2 signify that no defect has been detected (no repair), defect has been detected, but is too small to be repaired and repair has been performed, respectively.
The probability of failure in the time interval $[0, T_i]$ is for $0 \leq T \leq T_1$:

$$P_F(T) = P(M_F(0) > 0 \cap M_F(T) \leq 0) \tag{9}$$

where $M_F(T)$ is the safety margin modelling failure at the time T.
For $T_1 < T \leq T_2$:

$$\begin{aligned} P_F(T) = P_F(T_1) + P(M_F(0) > 0 \cap B^0 \cap M_F^0(T) \leq 0) \\ + P(M_F(0) > 0 \cap B^1 \cap M_F^0(T) \leq 0) + P(M_F(0) > 0 \cap B^2 \cap M_F^2(T) \leq 0) \end{aligned} \tag{10}$$

where B^0, B^1 and B^2 are the events corresponding to no detection, detection and no repair and detection and repair at the first inspection, respectively:

$$\begin{aligned} B^0 &= \{M_F(T_1) > 0 \cap M_D(T_1) > 0\} \\ B^1 &= \{M_F(T_1) > 0 \cap M_D(T_1) \leq 0 \cap M_R(T_1) > 0\} \\ B^2 &= \{M_F(T_1) > 0 \cap M_D(T_1) \leq 0 \cap M_R(T_1) \leq 0\} \end{aligned} \tag{11}$$

$M_D(T_1)$ is the safety margin modelling detection of a defect and $M_R(T_1)$ is the safety margin modelling repair. $M_F^0(T)$ and $M_F^2(T)$ are safety margins with respect to failure at the time $T > T_1$ corresponding to no repair and repair at the first inspection. Similar expressions are obtained for $T > T_2$.
The probability of repair at the time T_i is determined in a similar manner.
The safety margin $M_F(T)$ modelling failure at time T can typically be written

$$M_F(T) = a_c - a(\mathbf{X}, T) \tag{12}$$

where a_c is the critical crack length and a is the crack length at time T. a is a function of the stochastic variables $\mathbf{X}$. The failure criteria can alternatively be related to the crack depth, see the example in section 5.
The safety margin $M_D(T)$ modelling detection of a crack at time T can be written

$$M_D(T) = A_d - a(\mathbf{X}, T) \tag{13}$$

where A_d is the smallest detectable crack size. A_d is modelled as a stochastic variable with a distribution function $F_{A_d}(a_d)$ equal to the P.O.D. (probability of detection) curve. The probability of failures in (9)-(10) are estimated using updated distribution functions $F''_{A_d}(a_d|\mathbf{e}, \mathbf{z})$ of A_d. The updatings are based on the experiments $\mathbf{e}$ and results $\mathbf{z}$. In section 4 it is shown how F''_{A_d} can be obtained.
The safety margin $M_R(T)$ modelling possible repair at time T can typically be written

$$M_R(T) = a_r - a(\mathbf{X}, T) - \epsilon \tag{14}$$

where a_r is a repair crack length and ϵ is a zero mean stochastic variable modelling the crack measurement error.

4. Probabilistic Modeling of the Detectable Crack Size

As seen from section 3 the detectable crack size plays an important role in expected costs and reliability based inspection and maintenance planning. Therefore the probabilistic modeling of the size of detectable cracks must be performed carefully using all available information (in the form of available experimental inspection results and subjective information from similar conditions) and formulated such that future additional inspection trials can be used to update the probabilistic models. The detectable crack size A_d is assumed to be modelled by a distribution function $F_{A_d}(a_d)$ equal to the P.O.D. curve. The P.O.D. curve must represent the experimental knowledge about A_d as accurately as possible both in terms of the actual observations (detection/no detection) and in terms of the experimental sample size.

In the following it is first shown how a statistical model of the P.O.D. curve can be obtained on the basis of detection/no detection observations from experiments. Next it is discussed how other types of available information on the P.O.D. curve can be modelled statistically. Finally it is shown how the probabilistic model of the P.O.D. curve can be updated on the basis of new experiments.

It is assumed that an appropriate family of distribution functions for the P.O.D. curve has been identified and are denoted as $F_{A_d}(a_d, \mathbf{p}')$ where $\mathbf{p}'$ are statistical parameters to be determined. The problem is to fit the distribution function to the experimental inspection trials with observations of the type : detection / no detection. The maximum likelihood method (see e.g. Lindley [9]) is used since it not only gives the distribution parameters $\mathbf{p}'$ but also the joint distribution $f_{\mathbf{p}'}(\mathbf{p}')$ of $\mathbf{p}'$ modelling the statistical uncertainty. Since the outcomes of the experimental inspection trials are of the type detection / no detection the likelihood function $L(\mathbf{p}')$ has the following form, corresponding to N_{sample} experimental inspection trials performed with crack sizes $a_i^e, i = 1, \cdots, N_{sample}$

$$L(\mathbf{p}') = \prod_{i=1}^{N_{sample}} P_i(\mathbf{p}') \tag{15}$$

where

$$P_i(\mathbf{p}') = \begin{cases} F_{A_d}(a_i^e|\mathbf{p}') = P(A_d \leq a_i^e|\mathbf{p}') & \text{if detection} \\ 1 - F_{A_d}(a_i^e|\mathbf{p}') = P(A_d > a_i^e|\mathbf{p}') & \text{if no detection} \end{cases} \tag{16}$$

The maximum likelihood estimates $\mathbf{p}'^*$ are obtained by solving the optimization problem

$$\min_{\mathbf{p}'} \; -L(\mathbf{p}') \tag{17}$$

For large sample sizes ($N_{sample} > 20$, see e.g. Lindley [9]) the joint distribution of $\mathbf{p}'$ tends to be Normal distributed with expected values $\mu_{\mathbf{p}'} = \mathbf{p}'^*$ and covariance matrix $\mathbf{C}_{\mathbf{p}'\mathbf{p}'}$

$$\mathbf{C}_{\mathbf{p}'\mathbf{p}'} = \mathbf{H}^{-1} \quad \text{where} \quad H_{i,j} = -\frac{\partial^2 \ln L(\mathbf{p}'^*)}{\partial p'_i \partial p'_j} \tag{18}$$

To evaluate the goodness of fit of the selected family of distribution functions the robustness of the estimates $\mathbf{p}'^*$ should be examined. As the maximum likelihood estimates are statistically robust the estimates $\mathbf{p}'^*$ should at least be insensitive to augmenting the experimental inspection trial sample with one additional sample. If this is not the case, the selected distribution function is not suitable. Otherwise the distribution function $F_{A_D}(a_d|\mathbf{p}')$ can now be used for reliability analysis and/or inspection and maintenance planning.

Next the problem to model available information which can not directly be expressed as a number of experiment trials corresponding to the above N_{sample} experiments. This information can e.g. be subjective information about the P.O.D. curve. Often this information can be expressed by 95 % confidence intervals of the P.O.D. curve.

It is assumed that the information can be modelled by the family of distributions $F_{A_d}(a_d, \mathbf{p}')$ mentioned above. A calibration of the available information to this model can be performed as follows:

a. The available information is modelled by N_0 'fictive' experiments with crack sizes $\mathbf{a}_0^e$ and with the results $\mathbf{z}_0$ modelling detection / no detection of the experiments.

b. The parameters $\mathbf{p}'_0$ are estimated by the maximum likelihood method as described above. The joint normal distribution function of the estimate $\mathbf{p}'_0$ is denoted $f_{\mathbf{p}'_0}$.

c. 95 % confidence intervals of the P.O.D. curve corresponding to the model in step a are estimated as $[\mu(a_d) - 2\sigma(a_d); \mu(a_d) + 2\sigma(a_d)]$ where

$$\mu(a_d) = \int \ldots \int F_{A_d}(a_d|\mathbf{p}'_0 = \mathbf{p}) f_{\mathbf{p}'_0}(\mathbf{p}) d\mathbf{p} \tag{19}$$

$$\sigma^2(a_d) = \int \ldots \int (F_{A_d}(a_d|\mathbf{p}'_0 = \mathbf{p}) - \mu(a_d))^2 f_{\mathbf{p}'_0}(\mathbf{p}) d\mathbf{p} \tag{20}$$

are determined numerically.

d. If the 95 % confidence intervals are not in a satisfactory agreement with the initial knowledge then the model in step a is adjusted and step b-c are repeated. If there is a satisfactory agreement then the 'fictive' experiments in step a are used together with the N_{sample} real experiments.

It should be noted that special care is needed if the number of fictive experiments N_0 is smaller than 20. Alternatively the initial subjective information can be modelled using Bayesian statistics.

Finally, the situation is considered where it is investigated if new extra trial experiments should be performed.

If N_{new} new experimental inspection trials are obtained the distribution $F_{A_D}(a_d|\mathbf{p}')$ must be updated. This is done by performing a new maximum likelihood method based fitting with the following likelihood function

$$L(\mathbf{p}'') = \prod_{i=1}^{N_0+N_{sample}+N_{new}} P_i(\mathbf{p}'') \tag{21}$$

where

$$P_i(\mathbf{p}'') = \begin{cases} F_{A_d}(a_i^e|\mathbf{p}'') & \text{if detection} \\ 1 - F_{A_d}(a_i^e|\mathbf{p}'') & \text{if no detection} \end{cases} \tag{22}$$

Again by maximizing the likelihood function the updated joint distribution of distribution parameters for the detectable crack size distribution $f_{\mathbf{p}''}(\mathbf{p}'')$ can be obtained.

As the probabilistic model of the detectable crack size has now been described implementation of the experiment planning for crack detection tests can be considered in relation to the two different schemes of performing inspection and maintenance planning outlined in section 2. Starting with the expected total cost based inspection and maintenance planning the task is to estimate the expected value in equation (2)

$$E_{\mathbf{Z}|\mathbf{e}}[C_{EXP}(\mathbf{e}) + C^*(\mathbf{e}, \mathbf{Z})] \tag{23}$$

where $\mathbf{e}$ describes the planned new crack detection experiments in terms of number of experiments and crack geometries. The experimental outcomes in terms of detection and no detection are modelled by the random variables $\mathbf{Z}$. Hence the probability of detection and no detection at the ith planned experiment can be written as

$$\begin{aligned} P(Z_i = \text{detection}) &= F_{A_D}(a_i^e|\mathbf{p}') \\ P(Z_i = \text{no detection}) &= 1 - F_{A_D}(a_i^e|\mathbf{p}') \end{aligned} \tag{24}$$

where the parameters $\mathbf{p}'$ are random variables with prior distribution function $F_{\mathbf{p}'}(\mathbf{p}')$. For a given experiment outcome $\mathbf{z}_k = (z_{k1}, z_{k2}, .., z_{kN_{new}})^T$ the distribution function $F_{A_D}(a_d|\mathbf{p}')$ can be updated as explained above yielding the distribution function $F_{A_D}(a_d|\mathbf{p}'')$. Using this distribution function the optimal inspection plan $\mathbf{i}_k$ and the corresponding costs $C^*(\mathbf{e}, \mathbf{z}_k)$ can be determined by equation (3) and (4). Since there are $2^{N_{new}}$ possible different outcomes of the crack detection experiment equation (3) - (4) must be solved for all these combinations. The expectation in equation (2) can then be estimated by

$$E_{\mathbf{Z}|\mathbf{e}}[C_{EXP}(\mathbf{e}) + C^*(\mathbf{e}, \mathbf{Z})] = \sum_{k=1}^{2^{N_{new}}} (C_{EXP}(\mathbf{e}) + C^*(\mathbf{e}, \mathbf{Z} = \mathbf{z}_k))P(\mathbf{Z} = \mathbf{z}_k) \tag{25}$$

where

$$P(\mathbf{Z}=\mathbf{z}_k)=\int\ldots\int\prod_{i=1}^{N_{new}}P(Z_i=z_{ki}(\text{detection/no detection})|\mathbf{p}'=\mathbf{p})f_{\mathbf{P}'}(\mathbf{p})d\mathbf{p}$$
$$=\int\ldots\int(\prod_{l=1}^{N_{new,det}}F_{A_D}(a^e_{kl}|\mathbf{p}'=\mathbf{p})\prod_{j=1}^{N_{new,no\ det}}(1-F_{A_D}(a^e_{kj}|\mathbf{p}'=\mathbf{p})))f_{\mathbf{P}'}(\mathbf{p})d\mathbf{p} \quad (26)$$

$N_{new}=N_{new,det}+N_{new,no\ det}$ where $N_{new,det}$ is the number of detections and $N_{new,no\ det}$ is the number of tests leading to no detection in the new experiments. The probabilities in (26) can be estimated by numerical integration if the dimension of $\mathbf{p}'$ is less than 3-4. Alternatively, the probabilities can be evaluated using FORM with the limit state function

$$g=U-\Phi^{-1}P_f(\mathbf{p}) \quad (27)$$

where

$$P_f(\mathbf{p})=\prod_{l=1}^{N_{new,det}}(F_{A_D}(a^e_{kl}|\mathbf{p}'=\mathbf{p})\prod_{j=1}^{N_{new,no\ det}}(1-F_{A_D}(a^e_{kj}|\mathbf{p}'=\mathbf{p}))) \quad (28)$$

and U is an auxilary standard normal variable.

For the reliability index based inspection and maintenance planning scheme the procedure is simpler. As described in section 2.2 the updated reliability index curves are calculated based on assumed detections in the planned experiments. The distribution function for the detectable crack size is estimated using $F_{A_D}(a_d|\mathbf{p}'')$ which is determined using the likelihood function given in equation (21). The updated reliability index $\beta''(t)$ is estimated by

$$\beta''(t)=\sum_{k=1}^{2^{N_{new}}}\beta''_k(t)P(\mathbf{Z}=\mathbf{z}_k) \quad (29)$$

where $\beta''_k(t)$ is the updated reliability index at time t assuming no detection at the inspections and assuming that the outcome of the crack detection experiment is $\mathbf{z}_k$.

5. Example

To illustrate the application of cost optimal experiment planning for P.O.D. curves in combination with optimal inspection and maintenance planning an example from the offshore industry is presented in the following. An offshore structure of the jacket type with tubular structural steel members connected in welded joints is considered. The planning of the P.O.D. tests is performed in relation to a crititical, but representative joint. It is assumed that a spectral stress analysis has been performed and that the results of this analysis are given in terms of a weighted average fatigue stress range ($WASR$), see [10], and a corresponding expected number of fatigue load cycles. In order to describe the events of failure, repair and inspection observations it is necessary to model the crack growth. For this purpose the software module FACTS [10] is used. A tow-dimensional crack growth model is used. If an observed crack depth at an inspection is smaller than 10 % of the chord thickness repair by grinding will be used. Otherwise the crack will be repaired by welding.

The probabilistic model of the variables used in the inspection and maintenance planning is shown in table 1. All dimensions are in [mm] and [N]. ϵ refers to the stochastic variable modelling the measuring uncertainty in (14). All stochastic variables in table 1 are assumed to be independent.

variable	distribution	μ	σ
Initial chord thickness	Weibull	38.9	0.583
Initial crack depth	Weibull	2.0	0.2
Initial crack length	Weibull	8.0	0.02
Chord thickness after grinding	Weibull	35.0	0.583
Crack depth after grinding	Weibull	1.0	0.5
Crack length after grinding	Weibull	4.0	0.1
Chord thickness after welding	Weibull	38.9	0.583
Crack depth after welding	Weibull	2.0	1.0
Crack length after welding	Weibull	8.0	2.0
Initial Paris C	Log-Normal	$5.0 \cdot 10^{-12}$	$5.0 \cdot 10^{-12}$
Initial Paris m	Deterministic	3.1	
Paris C after welding	Log-Normal	$5.0 \cdot 10^{-12}$	$5.0 \cdot 10^{-12}$
Paris m after welding	Deterministic	3.1	
Weigh. Aver. Str. Range ($WASR$)	Normal	15.0	1.0
POD	Weibull	p'_1	p'_2
ϵ	Normal	0.0	0.25
Design lifetime T_L	Deterministic	36 years	
Stress cycles per year	Deterministic	$6 \cdot 10^6$	
Real rate of interest r	Deterministic	0.02	

Table 1. Statistical models for crack growth parameters (all dimensions in m and MPa). μ : expected value and σ : standard deviation.

The inspection method is assumed to correspond to a MPI technique. The prior parameters p'_1 and p'_2 model the size and shape parameters in the Weibull distributed P.O.D. curve. The size parameter is assumed to be normal distributed and the shape parameter to be deterministic. The parameters are estimated as described in section 4. It is assumed that the $N_{sample} = 29$ test results in table 2 are known and that the statistical parameters of p'_1 and p'_2 are estimated from these using (15)-(18).

a^e	$N_{detection}$	$N_{no\ detection}$
0.3	1	5
1.0	2	4
2.0	4	2
5.0	5	0
10.0	6	0

Table 2. Available test results. $N_{detection}$ and $N_{no\ detection}$: number of detections and no detections with this crack size. Crack sizes in [mm].

A simplified approach for inspection and maintenance planning is used where only the next inspection time and repair strategy are optimized. It is assumed that inspections can be performed in one weather window each year. The following discretized inspection times (in years from the inspection planning time) are considered : T_1 = 2, 4, 6, 8,...,32, 34.

The following cost models are used, see (6)-(8):

$$C_{IN}(T_1, \mathbf{e}, \mathbf{z}, d) = C_{IN_0}(1 - P_F(T_1, \mathbf{e}, \mathbf{z}, d))\frac{1}{(1+r)^{T_1}}$$

$$C_F(T_1, \mathbf{e}, \mathbf{z}, d) = C_{F_0}(P_F(T_1, \mathbf{e}, \mathbf{z}, d) - P_F(0, \mathbf{e}, \mathbf{z}, d))\frac{1}{(1+r)^{T_1}} + C_{F_0}(P_F(T_L, \mathbf{e}, \mathbf{z}, d) - P_F(T_1, \mathbf{e}, \mathbf{z}, d))\frac{1}{(1+r)^{T_L}}$$

$$C_R(T_1, \mathbf{e}, \mathbf{z}, d) = C_{R_1}P_{R_1}(T_1)\frac{1}{(1+r)^{T_1}} + C_{R_2}P_{R_2}(T_1)\frac{1}{(1+r)^{T_1}}$$

where P_{R_1} and P_{R_2} are the probabilities that repair is performed by grinding and welding, respectively. Two cost models A and B are used, see cost coefficients in table 3.

		cost, model A	cost, model B
C_{IN_0}	MPI Inspection	$0.1 \cdot 10^6$	$0.1 \cdot 10^6$
C_{R_1}	Grind repair	$0.25 \cdot 10^6$	$0.25 \cdot 10^6$
C_{R_2}	Weld Repair	$5.0 \cdot 10^6$	$5.0 \cdot 10^6$
C_{F_0}	Failure	$100.0 \cdot 10^6$	$500.0 \cdot 10^6$

Table 3. Costs (in ECU) of inspection, repair and failure.

The optimal inspection and maintenance plan for the considered joint is selected as the repair option and the time instant between 2 and 34 years which results in the smallest expected total costs. For given results $\mathbf{z}^k$ of the crack detection tests the statistical parameters p_1' and p_2' in the P.O.D. curve are updated and the updated optimal inspection and maintenance plan are estimated using PREDICT [11].

The following three experiment plans for crack detection tests are investigated:
Experiment 1: $\mathbf{e}_1$: $N_{new} = 4$, $\mathbf{a}^e$= (0.3, 0.3, 0.3, 0.3) [mm] ≈ 10 % fractile
Experiment 2: $\mathbf{e}_2$: $N_{new} = 4$, $\mathbf{a}^e$= (1.3, 1.3, 1.3, 1.3) [mm] ≈ 50 % fractile
Experiment 3: $\mathbf{e}_2$: $N_{new} = 4$, $\mathbf{a}^e$= (3.3, 3.3, 3.3, 3.3) [mm] ≈ 90 % fractile
For the three experiment plans, table 4, 5 and 6 show the minimum expected costs of inspection, maintenance and failure both for the different combinations of possible test results and for the total expected value $E_{\mathbf{Z}|\mathbf{e}}[C^*(\mathbf{e},\mathbf{Z})]$. Table 7 shows the minimum expected costs of inspection, maintenance and failure in case of no crack detection tests. The differences in costs between no extra tests and the experiment plans 1, 2 and 3 and are shown in table 8. It is seen that for cost model A it is not economically to perform extra crack detection tests. For model B experiment plan 2 is optimal. For this plan the expected costs of inspection, maintenance and failure are 32 900 ECU smaller if experiments are performed. This number should then be compared with the costs of the experiments.

experiments	$P(\mathbf{Z}=\mathbf{z}^k)$	C_A^*	C_B^*
IIII	$1.33\ 10^{-3}$	$3\ 994\ 10^3$	$11\ 695\ 10^3$
IIIO	$9.48\ 10^{-3}$	$3\ 967\ 10^3$	$11\ 782\ 10^3$
IIOO	$6.84\ 10^{-2}$	$3\ 935\ 10^3$	$11\ 908\ 10^3$
IOOO	$3.05\ 10^{-1}$	$3\ 897\ 10^3$	$12\ 076\ 10^3$
OOOO	$6.16\ 10^{-1}$	$3\ 853\ 10^3$	$12\ 232\ 10^3$
$E_{\mathbf{Z}|\mathbf{e}}[C^*(\mathbf{e},\mathbf{Z})]$		$3\ 872\ 10^3$	$12\ 155\ 10^3$

Table 4. Expected costs of inspection, maintenance and failure for experiment 1. I and O indicate detection and no detections, respectively. C_A^* and C_B^* are the expected costs corresponding to cost model A and B.

experiments	$P(\mathbf{Z}=\mathbf{z}^k)$	C_A^*	C_B^*
IIII	$1.01\ 10^{-1}$	$3\ 963\ 10^3$	$11\ 793\ 10^3$
IIIO	$2.58\ 10^{-1}$	$3\ 921\ 10^3$	$11\ 962\ 10^3$
IIOO	$3.31\ 10^{-1}$	$3\ 874\ 10^3$	$12\ 139\ 10^3$
IOOO	$2.32\ 10^{-1}$	$3\ 822\ 10^3$	$12\ 342\ 10^3$
OOOO	$7.05\ 10^{-2}$	$3\ 767\ 10^3$	$12\ 609\ 10^3$
$E_{\mathbf{Z}|\mathbf{e}}[C^*(\mathbf{e},\mathbf{Z})]$		$3\ 876\ 10^3$	$12\ 134\ 10^3$

Table 5. Expected costs of inspection, maintenance and failure for experiment 2. I and O indicate detection and no detections, respectively. C_A^* and C_B^* are the expected costs corresponding to cost model A and B.

experiments	$P(\mathbf{Z}=\mathbf{z}^k)$	C_A^*	C_B^*
IIII	$6.46\ 10^{-1}$	$3\ 911\ 10^3$	$12\ 006\ 10^3$
IIIO	$2.72\ 10^{-1}$	$3\ 827\ 10^3$	$12\ 310\ 10^3$
IIOO	$7.08\ 10^{-2}$	$3\ 745\ 10^3$	$12\ 658\ 10^3$
IOOO	$1.07\ 10^{-2}$	$3\ 671\ 10^3$	$12\ 959\ 10^3$
OOOO	$7.29\ 10^{-4}$	$3\ 609\ 10^3$	$13\ 211\ 10^3$
$E_{\mathbf{Z}\mid\mathbf{e}}[C^*(\mathbf{e},\mathbf{Z})]$		$3\ 873\ 10^3$	$12\ 144\ 10^3$

Table 6. Expected costs of inspection, maintenance and failure for experiment 3. I and O indicate detection and no detections, respectively. C_A^* and C_B^* are the expected costs corresponding to cost model A and B.

C_A^*	C_B^*
$3\ 871\ 10^3$	$12\ 167\ 10^3$

Table 7. Expected costs of inspection, maintenance and failure for no extra experiments. C_A^* and C_B^* are the expected costs corresponding to cost model A and B.

	Difference - A	Difference - B
Experiment 1	$-1.1\ 10^3$	$11.5\ 10^3$
Experiment 2	$-4.3\ 10^3$	$32.9\ 10^3$
Experiment 3	$-1.7\ 10^3$	$22.5\ 10^3$

Table 8. Expected costs of inspection, maintenance and failure if no tests are performed minus expected costs of inspection, maintenance and failure if crack detection tests are performed.

6. Conclusion

A new methodology for reliability-based optimal planning for crack detection experiments is presented. The theoretical basis and the coupling with reliability-based optimal inspection and maintenance planning are described.

An example is presented illustrating the proposed technique for optimal planning of crack detection experiments applied for a tubular joint in a jacket type offshore structure. In the example different test plans and cost models are considered. It is seen that the cost models as expected have a large influence on the economically optimal decision on performing extra crack detection tests.

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