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Abstract: A structural system identification experiment design method is formulated in the light of decision theory, structural reliability theory and optimization theory. The experiment design method is based on a preposterior analysis, well-known from the classical decision theory. I.e. the decisions concerning the experiment design are not based on obtained experimental data. Instead the decisions are based on the expected experimental data assumed to be obtained from the measurements, estimated based on prior information and engineering judgement. The design method provides a system identification experiment design reflecting the costs of the experiment and the value of obtained additional information. An example concerning design of an experiment for parametric identification of a single degree of freedom structural system shows the applicability of the experiment design method.

1. Introduction

When performing system identification experiments one faces the problem of choosing the experimental conditions (test signals, sampling strategy, location of sensors etc.) so that the information provided by the experiment is maximized. The problem of experiment design has been given much attention in the literature. The theory of design of static experiments originated in the early thirties, see e.g. Fisher [1], and has been considerably developed in the statistical literature after the second world war. Fedorov [2] and Silvey [3] can be mentioned as basic references. However, the models considered in the statistical literature are generally static and their applicability to dynamic models has become clear only recently. Design of experiments for parametric identification of dynamic systems has been a subject of research during the last decades mainly developed in relation to identification of electrical systems. The problem of experiment design can be regarded as a generalisation of the problem of optimal input signal design that has been comprehensively treated in the literature, see e.g. Mehra [4]. Representative surveys of the problem of experiment design for dynamic system identification are given in the system identification textbooks Goodwin et al. [5], Kalaba et al. [6], Ljung [7], Norton [8] and Söderström [9]. Beyond these textbooks many research papers exist, mainly on the problem of optimal input design for system identification. The paper Goodwin [10] may be noticed as a contribution to the literature concerned with experiment design for dynamic system identification. Design of system identification experiments in relation to structural problems seems to be a subject which only has

received little attention during the last decade and will be a subject of research in the future, see e.g. Kirkegaard [11]. System identification experiment design for parametric identification of structural systems is the problem of determining the experimental conditions implying that optimal information of the parameter vector $\overline{\theta}$ can be obtained from the measured output of the sensors included in the vector $\overline{y}^m(t)$ given by the measuring equation

$$\overline{y}^m(t) = \overline{y}(t|\overline{\theta}) + \overline{e}(t) \tag{1}$$

$\overline{y}(t|\overline{\theta})$ denotes a prediction of $\overline{y}^m(t)$ based on a model and the parameter vector $\overline{\theta}$. The additive noise $\overline{e}(t)$ is normally assumed to be Gaussian stationary white noise both in space and time parameters. Since the estimate $\hat{\overline{\theta}}_N$ of the parameter vector $\overline{\theta}$ is dependent on random processes the accuracy of $\hat{\overline{\theta}}_N$ must be considered in a statistical sense. For experiment design purposes, it is normally assumed that the accuracy of the parameter estimate is most conveniently expressed in terms of the parameter covariance matrix $\overline{\overline{C}}_{\hat{\overline{\theta}}_N}$. Many authors postulate the existence of an asymptotically efficient unbiased estimator as a basis for the experiment design. This implies that there is a lower bound, the Cramer-Rao lower bound, on the achievable covariance of the estimate $\hat{\overline{\theta}}_N$ irrespective of the estimator algorithm used. This leads to a great simplification, since the minimum variance given by the Cramer-Rao lower bound, equal the inverse of the Fisher information matrix, can be easily computed in several estimation problems. The Cramer-Rao lower bound is given by

$$\overline{\overline{C}}_{\hat{\overline{\theta}}_N} \geq \overline{\overline{J}}^{-1} \tag{2}$$

$$\overline{\overline{J}} = E_{\overline{Y}^m|\overline{\theta}}\left[\left(\frac{\partial \log f_{\overline{Y}^m}(\overline{y}^m|\overline{\theta})}{\partial \overline{\theta}}\right)\left(\frac{\partial \log f_{\overline{Y}^m}(\overline{y}^m|\overline{\theta})}{\partial \overline{\theta}}\right)^T\right] \tag{3}$$

where $\overline{\overline{J}}$ is the Fisher information matrix, see e.g. Goodwin et al. [5]. $f_{\overline{Y}^m}(\overline{y}^m|\overline{\theta})$ is the conditional joint probability density function. $E_{\overline{Y}^m|\overline{\theta}}[\cdot]$ is an expectation operator.

For comparing different experiments it is necessary to have a measure of the applicability of the experiment. A logical approach is to choose a measure related to the expected accuracy of the parameter estimate. Clearly, the parameter accuracy depends on the experimental conditions $\mathcal{H}$. Formally, the problem of optimal parametric identification experiment design could be stated as

$$\min_{\mathcal{H}} \mathcal{A}\left(\overline{\overline{C}}_{\hat{\overline{\theta}}_N}(\mathcal{H})\right) = \min_{\mathcal{H}} \mathcal{A}\left(\overline{\overline{J}}^{-1}(\mathcal{H})\right) \tag{4}$$

where $\mathcal{A}(\cdot)$ is a scalar function of the covariance matrix. Typically, such scalar functions are e.g. the determinant (D-OPTIMUM), the trace (A-OPTIMUM) or the maximal eigenvalue of the covariance matrix (E-OPTIMUM), see e.g. Goodwin [10], Mehra [4], Ljung [7] and Zarrop [12]. In Pazman [13] a detailed discussion of design criteria related to experiment design is given.

In the literature, see e.g. Kirkegaard [11], Udwadia [14], Bayard et al. [15] and Cottin [16] various designs of experiment for parametric identification of structural systems, based on a scalar measure of the inverse of the Fisher information matrix, have been given. By using this approach it is possible to design experiments where the quality of the design is expressed by a scalar measure of an expected estimated parameter covariance matrix. Such a quality measure can be used to compare different designs, but it cannot tell, how much one design is better than another design. This means that e.g. design of an experiment concerning optimal location of sensors is possible, but it is not possible to estimate the optimal number of sensors. More general, it can be said that the traditional design method discussed above does not make it possible to investigate the increase in the value of the information expected to be obtained by changing an experiment design in direction of a more optimal design. This implies that the acquisition of additional information by performing a full-scale measuring of a structure can result in unnecessary use of resources. This is because time, energy and financial resources are not reflected in the experiment design. However, if these quantities should be reflected in the design, the expected utility of performing the experiment should be known before the experiment is performed. If this utility is expressed in a monetary value it will be possible to make a trade-off study between the increased monetary value and the costs of performing the experiment.

In this paper a method for design of optimal experiments for parametric identification of structural dynamic systems is proposed. The experiment design method is based on a preposterior analysis. A preposterior analysis can be used when the additional costs by performing an experiment have to be reflected in the design. The method takes uncertainties in the experiment design problem into account in a consistent manner implying that it is possible to make decisions concerning experiment design in a rational way. This means that the experiment design is based on a probabilistic analysis instead of a traditional deterministic analysis. The experiment design problem is generally based on uncertain parameters, lack of information, predictions and information containing uncertainty. Therefore, an experiment design method based on a probabilistic analysis is interesting. Through probabilistic modelling and analysis, uncertainties may be modelled and assessed properly, and their effects on a given decision concerning the experiment design can be handled systematically. The traditional experiment design method, mentioned above, does not make it possible to take information containing uncertainties into account. This implies that the experiment will be designed based on incomplete information. By using the proposed method it is possible to consider the following problem: Design of an optimal experiment including optimal use of time, energy and financial costs. Further, it is also possible to consider the question: Should additional information be obtained ? Additional information should be obtained if the additional costs for the new information are justified by an elimination of a significant part of uncertainty.

In section 2 the experiment design method is formulated in the light of decision theory, structural reliability theory and optimization theory. The utility in monetary values expected to be obtained by performing an experiment will be expressed by the expected updated structural reliability. When experiments are performed additional information about the uncertain parameters is obtained. This implies that the updated structural

reliability is changed, caused of a reduction of the uncertainty. The updated structural reliability is estimated as a function of the reduction of uncertainty in the problem. The reduction of uncertainty, expected to be obtained if the experiment is performed, is expressed as a function of the experiment design variables. Section 3 deals with calculation procedures. Section 4 is concerned with a simple example showing how the method can be used for design of system identification experiments. At last in sections 5 and 6, respectively, conclusions and references are given.

2. The System Identification Experiment Design Method

In the following the system identification experiment design problem is formulated as a decision problem in the light of decision theory thoroughly presented in e.g. Ang et al. [17] and Raiffa et al. [18]. Decision analysis is the framework which can be used when decisions have to be based on uncertain information.

The various components of a decision problem may be integrated into a formal layout as a decision tree, consisting of a sequence of decisions. In other words, the decision tree integrates the relevant components of the decision analysis in a systematic manner. The decision tree model is introduced to identify the necessary components of a decision problem consisting of:

- Feasible alternatives, including the acquisition of additional information, if appropriate.
- The possible outcomes associated with each alternative.
- The corresponding probability assignments to the outcomes.
- The consequences, measured by its utility value, associated with each combination of alternative and outcome.

In brief, the decision tree provides an organized outline of all the information used for a systematic decision analysis.

For a decision problem where an expected utility value is associated with each combination of alternative and outcome it can be shown, see Von Neumann et al. [19], that the alternative, among the feasible, to be selected is the alternative giving maximum of expected utility.

A decision analysis based entirely on existing information is called a prior analysis. If such an analysis is updated subsequently with additional new information, the latter is called a terminal analysis. A decision analysis with additional information is similar to the prior analysis, except that the updated probabilities, probabilities conditional on the experimental outcomes, are used in the computations. In a terminal analysis, the analysis assumes that the information is available. However, additional information, obtained by performing an experiment, involves the additional time, energy and financial resources which have to be reflected in the design of the experiment. Such a

decision problem calls for a so-called preposterior analysis. Design of a system identification experiment can be considered as a decision problem involving whether and how additional information should be obtained and may then be solved by a preposterior analysis. It may be noticed that in a preposterior analysis the experimental data are not available. Instead, decisions are made based on the experimental data assumed to be obtained from the measurements. These experimental data are estimated based on prior information and engineering judgement.

2.1 The Experiment Design Optimization Problem

Figure 1 shows a decision tree used in this paper to formulate a method for optimal design of experiments for parametric identification of civil engineering structures.

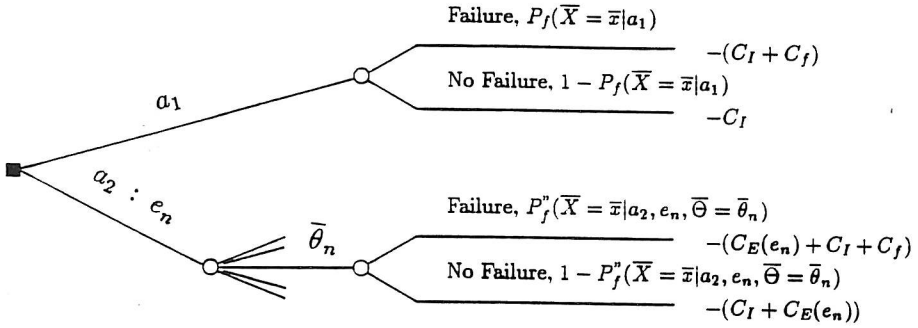

Figure: 1 A decision tree.

The decision tree shows that at the decision node, the square node, a decision whether or not to proceed with an experiment is required, i.e. a choice between an experiment $a_2 : e_n$ and no experiment a_1. $a_2 : e_n$ indicates alternative a_2 with experiment e_n to get additional information. If an experiment is chosen the experimental outcomes of the experiment $\overline{\theta}_n$ can be obtained. The experimental outcome $\overline{\theta}_n$ of the random vector $\overline{\Theta}$ is a vector including the parameters to be determined by the experiment number n. The realization of the random vector is shown by a circular node called a chance node. The decision tree shows that two outcomes follow the experimental outcomes, viz., structural failure and no structural failure. These alternatives are results of the outcomes of the stochastic variables included in the random vector $\overline{X}$. $P_f''(\overline{X} = \overline{x}|a_2, e_n, \overline{\Theta} = \overline{\theta}_n)$ is the posterior probability of structural failure. It is also seen from the decision tree that the alternative a_1 is also followed by the two possible outcomes structural failure and no structural failure. $P_f(\overline{X} = \overline{x}|a_1)$ is the prior probability of structural failure is.

The consequence associated with each combination of alternatives and outcomes is expressed by a utility value in monetary value. The utility corresponding to no structural failure and no experiment performed is the negative value of initial structural cost $-C_I$. If the costs of failure C_f is known then the expected monetary value $C(a_1)$ corresponding to the alternative a_1 is

$$C(a_1) = -C_I - C_f P_f(\overline{X} = \overline{x}|a_1) \tag{5}$$

In the same way the expected monetary value $C(a_2, e_n^\star)$ can be obtained for the alter-

native a_2 if the costs of the experiment are $C_E(e_n)$

$$C(a_2, e_n^\star) = \max_n E_{\overline{\Theta}}[-C_I - C_E(e_n) - C_f P_f^{"}(\overline{X} = \overline{x}|a_2, e_n, \overline{\Theta} = \overline{\theta}_n)] \tag{6}$$

where the expectation $E_{\overline{\Theta}}[\cdot]$ is obtained with respect to prior probability of $\overline{\Theta}$. The optimal experiment $e_n^\star$ is obtained by maximizing the expected utility with regard to the different experiments e_n. Comparing the expected utility of a_1 (5) with that of a_2 (6), the optimal alternative at the decision node can be selected. Thus, the decision between whether an experiment should be performed or not can be made. This decision will be discussed in section 2.2.

Maximum of the expected monetary value $C(a_2, e_n^\star)$ is seen to be a function of the experiment e_n. The experiment is described by the experiment design variables $\overline{Z}$. In the following, the experimental design variables $\overline{Z}$ will be used instead of e_n. This means that the maximizing with respect to the experiment e_n now can be substituted with a maximizing with respect to $\overline{Z}$. This implies that an optimal experiment can be obtained by solving the following optimization problem obtained from (6)

$$\min_{\overline{Z}} \quad C(a_2, \overline{Z}) = E_{\overline{\Theta}}[C_E(\overline{Z}) + C_f P_f^{"}(\overline{X} = \overline{x}|a_2, \overline{Z}, \overline{\Theta} = \overline{\theta}_n)] \tag{7}$$

$$s.t \quad C_E(\overline{Z}) \leq C_{max}$$

$$Z_i^l \leq Z_i \leq Z_i^u \qquad i = 1, 2, ..., N_D \tag{8}$$

N_D is the number of design variables. The expected monetary value $C(a_2, \overline{Z})$ is the objective function. As constraints upper Z_i^u and lower Z_i^l limits on the design variables $\overline{Z}$ and an upper limit C_{max} of the costs of the experiment $C_E(\overline{Z})$ are given.

One of the difficulties with the above optimization problem is the modelling of C_f and $C_E(\overline{Z})$. When a structure fails one is faced with various costs such as repair costs, reconstruction costs, clean-up costs, loss of income, costs due to loss of social prestige and possible deaths. The total costs of failure C_f may range from e.g. 2 to 5 times the initial costs of a structure. The costs of obtaining the new information $C_E(\overline{Z})$ have to cover not only the sample records and instrumentation but also the costs of statistical analysis of the information and an appropriate share of costs of planning. A simple and useful function for the costs of an experiment is e.g. a linear function. Sometimes a complicated cost function can be used, e.g. when a learning effect is introduced in the statistical analysis.

2.2 Value of Information

The solution of the optimization problem in (7)-(8) implies that the maximum expected utility of the alternative a_2 can be obtained. I.e. the optimal experiment design can be provided. The next question is: "Should the experiment be performed" ? This question can be answered by comparing the expected utility of alternatives a_1 and a_2.

An indicator of the expected utility by performing the optimal experiment is the term value of information VI defined as

$$VI = C(a_2, \overline{Z}) - C_E(\overline{Z}) - C(a_1, \overline{X}) \tag{9}$$

The term VI corresponds to the difference between the expected monetary value of alternative a_2, excluding the costs of the measurements, and the expected monetary value of alternative a_1. The value of information tells two things. Firstly, if VI exceeds the costs of the experiment, the experiment should be performed. Secondly, by considering the value of information an indicator of the expected gain to be obtained by performing an experiment is available. It is seen from the definition of the value of information that the costs of an experiment are bounded by a limit referred to as the value of perfect information VPI. Perfect information is obtained by an experiment giving measurements from which the parameter estimates only including an inherent uncertainty can be obtained.

The above introduction of the value of information shows that it is possible to get an indicator of the value of information. Based on this indicator a choice between the two alternatives a_1 and a_2 in the decision tree can be done. Of course, such a decision is conditioned on prior information. This means that the sensitivity of the choice of an optimal alternative at the decision node is of interest. Therefore, a sensitivity analysis has to be performed before a final decision can be made.

2.3 Structural Reliability Theory.

The probabilities of structural failure $P_f(\cdot)$ and $P_f^{"}(\cdot)$ in (5)-(7) are estimated by using modern reliability methods extensively applied in the last decades, where considerable progress has been made in the area of structural reliability theory. Especially, the development of the so-called first-order reliability methods (FORM) and the second-order reliability methods (SORM) have been very important, see e.g. Madsen et al. [20], Thoft-Christensen et al. [21] and Ditlevsen et al. [22]. These methods are especially developed to estimate the reliability of structural elements and systems. Further, the reliability methods are also an excellent tool to determine important sources of uncertainty.

2.3.1 Structural Reliability Analysis Based on Prior Information.

A reliability analysis is based on a reliability model of the structural system. The elements in the reliability model are failure elements, modelling potential failure modes of the structural system, e.g. fatigue failure, yielding failure, buckling failure etc. Each failure element is described by a failure function $g(\overline{x}, \overline{p}) = 0$ in terms of a realization $\overline{x}$ of a random vector $\overline{X} = (X_1, X_2, .., X_{n_X})$, and deterministic parameters $\overline{p}$, i.e. deterministic design parameters and parameters describing the stochastic variables, (expected value and standard deviation). $\overline{X}$ is assumed to contain n_X stochastic variables, e.g. variables describing the loads, strength, geometry, model uncertainty etc. Realizations $\overline{x}$ of $\overline{X}$ where $g(\overline{x}, \overline{p}) \leq 0$ correspond to failure states in the n-dimensional basic variable space, while $g(\overline{x}, \overline{p}) > 0$ correspond to safe states.

The reliability R of the failure element can now be written

$$R = 1 - P_f = 1 - \int_{g(\overline{x},\overline{p})\leq=0} f_{\overline{X}}(\overline{x})d\overline{x} \approx 1 - \Phi(-\beta_i) \tag{10}$$

where P_f is the probability of failure. $f_{\overline{X}}(\overline{x})$ is the joint probability density function of $\overline{X}$ and $\Phi(\cdot)$ is the one-dimensional standard normal distribution function. The reliability index β_i is estimated by using first-order reliability methods (FORM) where the approximation in (10) is obtained by using a transformation $\overline{T}$, see e.g. Madsen et al. [20] of the generally correlated and non-normally distributed variables $\overline{X}$ into standardized, normally distributed variables $\overline{U} = (U_1, U_2, .., U_{n_X})$. Let $\overline{U} = \overline{T}^{-1}(\overline{X}, \overline{p})$. In the $\overline{u}$-space the reliability index β_i is defined as

$$\beta_i = \min_{g(\overline{T}(\overline{u}),\overline{p})=0} (\overline{u}^T\overline{u})^{\frac{1}{2}} \tag{11}$$

The solution point $\overline{u}^\star$ of the optimization problem in (11) is the point on the failure surface $g(\overline{x}, \overline{p})$ closest to the origin in the $\overline{u}$-space and is called the design point, or β-point. The reliability index β_i is thus determined by solving an optimization problem with one constraint. The optimization problem is generally non-linear and can in principle be solved using any general non-linear optimization algorithm, but the iteration algorithm developed by Rackwitz and Fiessler, see e.g. Madsen et al. [20], is traditionally used in FORM since it has shown to be fast and effective in FORM analysis. It is seen that the reliability index β_i is introduced as a measure of the reliability which can be estimated based only on second moment information of the uncertainties entering the reliability problem.

It should be noticed that a better reliability estimate can be obtained by an improved approximation of the failure surface. A quadratic approximation of the failure surface at the design point is called a second-order reliability method (SORM). Computation of SORM estimates can be costly when $\overline{X}$ is large and the failure function involves complicated numerical algorithms, e.g. finite element analysis, numerical integration etc. because the second order derivatives at the design point are required in SORM.

If the whole structure is modelled, as a series system or a parallel system by failure elements then the probability of failure can be determined by a so-called generalized systems reliability index of the series or the parallel system, see e.g. Madsen et al. [20]. Since these generalized systems indices require time consuming or more or less impossible numerical calculations a number of different methods has been developed to make approximately calculations such as the Ditlevsen bounds, the simple bounds, average correlation coefficient approximation and the Hohenbichler approximation, see e.g Madsen et al. [20] and Enevoldsen [23]. It is also possible to have a series system of parallel systems see e.g. Thoft-Christensen et al. [21], Enevoldsen [23] and Guenard et al. [24].

It may be noticed that since the experiment design is estimated based on a reliability index a failure function $g(\overline{x}, \overline{p})$ corresponding to the parameters to be estimated from the measurements shall be used. This implies that a choice of a fatigue failure function

is relevant since the parameters normally estimated from full-scale measurements on dynamically sensitive structures are related to the fatigue failure mode. Generally, this failure mode is the most important failure mode for dynamically sensitive structures.

2.3.2 Structural Reliability Analysis Based on Preposterior Information.

In the following it will be shown how the posterior probability of failure $P_f''(\cdot)$ in (7) will be estimated by using the modern reliability methods. These methods provide a rational tool for updating a reliability analysis when additional information becomes available. Updating of a structural reliability analysis means to couple additional information from fabrication and service of a structure to the design information, prior information, in order to obtain a posterior information so that the posterior probability of failure $P_f''(\cdot)$ can be estimated. Service information is achieved from proof loading, inspection, vibration measurements, repair etc.. The updating of the reliability analysis can be used when decisions have to be made concerning e.g. extension of life time, inspection planning etc. Examples of such applications can be found in e.g. Diamantidis [25] where reliability assessment of existing offshore structures is considered and Madsen et al. [26] consider inspection planning. The great interest for developing methods for estimation of the reliability of existing structures is caused by the fact that much of the decision making performed of engineers in evaluating the safety of existing structures is made on the engineer's judgement. The theory of updating of a reliability analysis is given in e.g. Ditlevsen et al. [22].

The framework which can be used when additional information obtained from experiments shall be coupled to prior information is the Bayesian statistical model, see e.g. Ditlevsen [27]. By using the Bayesian statistical model it can be shown by assuming Gaussian parameters and Gaussian vibration measurements, that a Gaussian posterior density function can be given by the updated mean value and the updated covariance matrix $\overline{\overline{C}}^{u}_{\hat{\theta}_N}$ given by, see e.g. Sprandel [28]

$$\overline{\overline{C}}^{u}_{\hat{\theta}_N} = \left(\overline{\overline{C}}^{-1}_{\hat{\theta}_p} + \overline{\overline{C}}^{-1}_{\hat{\theta}_{new}} \right)^{-1} \tag{12}$$

$\overline{\overline{C}}_{\hat{\theta}_p}$ is the prior covariance matrix of the parameter $\overline{\theta}$. $\overline{\overline{C}}_{\hat{\theta}_{new}}$ is the covariance matrix of the parameter $\overline{\theta}$ determined from the vibration measurements. (12) indicates that the inverse of the updated covariance matrix can be obtained by adding the inverse of the covariance matrix of the prior information and a term which in certain circumstances corresponds to the Fisher information matrix. This corresponds to what one intuitively would have expected and it also corresponds to a result known from information theory which expresses that information is additive for independent events, see e.g. Reza [29]. This means that the updated Fisher information matrix $\overline{\overline{J}}^{u}$ is given by a sum of the information matrix corresponding to prior information $\overline{\overline{J}}_p$ and an information matrix $\overline{\overline{J}}_{new}$ corresponding to new information from the vibration measurements.

$$\overline{\overline{J}}^{u} = \overline{\overline{J}}_p + \overline{\overline{J}}_{new} \tag{13}$$

2.3.3 Sensitivity Analysis

Prior to the experiment a sensitivity analysis of the element or systems reliability indices has to be performed in order to point out parameters valuable to obtain more information about. The derivative of the reliability index shall also be determined if the optimization problem in (7)-(8) shall be solved. The derivative of an element reliability index to variations of parameters $\overline{p}$ where $\overline{p}$ is a parameter vector including statistical parameters (mean value and standard deviation) describing the random variables in $\overline{X}$ follows from, see e.g. Madsen et al. [20]

$$\frac{\partial \beta_i}{\partial p_j} = \frac{1}{\beta_i} \sum_{l=1}^{n_X} u_l^\star \frac{\partial \{T_l^{-1}(\overline{x}^\star, \overline{p})\}}{\partial p_j} \tag{14}$$

It is seen that the gradients of the reliability index which are generally time-consuming to estimate numerically can be determined semi-analytically.

Estimation of the derivatives of systems reliability indices is also possible, see e.g. Sørensen [30]. However, it may be noticed that reliability based optimization with derivatives of systems reliability indices can be expected to be rather costly. Further systems reliability indices are generally estimated by approximations. This implies that the estimates of derivatives of systems reliability indices based on these approximations do not have accuracy necessary for optimization. Therefore numerical derivatives may be used implying that the computation times become unacceptably large. Because of that alternative optimization procedures have been proposed where the time-consuming approximation of the systems reliability index can be avoided. Such procedures are derived in e.g. Sørensen [30] and Enevoldsen [23].

3. Calculation Procedures

The experiment design method is in section 2.1 formulated as an optimization problem. It may be realized that in certain circumstances a full optimization is unrealistic. It is due to the fact that the optimization problem in (7)-(8) can imply expensive calculations (long calculation time) and that accurate estimates of the gradients of the objective function are generally required to achieve convergence of a mathematical optimization algorithm.

In the following it will be explained how design of an experiment can be done by a sequential procedure. This can be used if a full optimization is impossible. Further, the sequential design procedure is presented to outline the steps in the experiment design method proposed in this paper. Design by a sequential procedure means that different experiment designs are chosen and then the value of information is estimated for each of these. Based on these estimates of value of information the most optimal design among them can be chosen.

3.1 Sequential Design

The experiment design procedure can be divided into the following steps:

1) For the structure under consideration a structural model, prior statistical characteristics of the parameters in the model and excitation are specified based on prior information and engineering judgement.
2) The parameters $\overline{\theta}$ to be estimated from the vibration measurements can now be determined by sensitivity analysis.
3) Estimate the costs of structural failure.
4) Design an experiment if the sensitivity analysis shows that it can be valuable to obtain new information. A chosen design implies that the number of sensors, the location of sensors, the excitation signal etc. are known. This means that the whole identification problem is completely specified.
5) Calculate the updated covariance matrix $\overline{\overline{C}}^{u}_{\hat{\theta}_N}$ based on the updated information matrix $\overline{\overline{J}}^{u}$ (13). The updated covariance matrix is estimated based on the chosen model, prior information and the proposed experiment design.
6) Estimate the updated probability of failure $P_f^{"}(\cdot)$ based on the updated covariance matrix. It may be noticed that it is assumed that the updated mean value corresponds to the mean value corresponding to prior information.
7) Calculate the expected monetary value.
8) Repeat 5)-7) with different proposed experiment designs.
9) Calculate and compare the value of information for the proposed experiment designs and choose the most optimal. Make a sensitivity analysis of the decision with respect to the prior information.
10) Make a final decision based on the sensitivity analysis for the value of information. The final decision gives an answer to the question: "Should the most optimal experiment design among the proposed designs be performed" ?

The steps 1)-10) outline the experiment design method proposed in this chapter. The procedure outlined can be used to choose between different proposed designs. However an optimal experiment design is obtained by solving the optimization problem in (7)-(8). This means that the steps 4)-7) shall be repeated until convergence is achieved.

4. EXAMPLE: Optimal Choice of Sampling Interval for Identifying a SDOF System

The aim of the example is to show how the proposed system identification experiment design method can be used. In the example a single-degree-of-freedom (SDOF) system assumed to model a cantilever steel construction, see figure 2, is considered. This simple example is chosen since it gives a good description of the principle of the system identification design method.

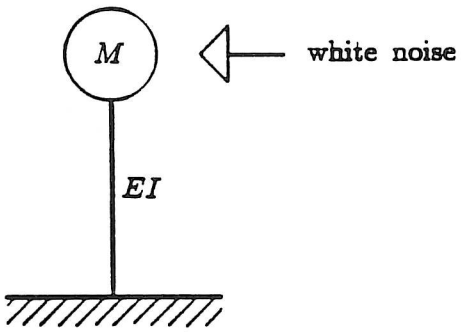

Figure 2: Model of a single-degree-of-freedom system.

It is assumed that the system can be modelled as a SDOF subjected to Gaussian white noise. The length, mass and stiffness are modelled such the undamped angular eigenfrequency $\omega_0 = 2\pi$ rad/sec and the damping ratio $\zeta = 0.04$.

4.1 Reliability Analysis Based on Prior Information

A fatigue reliability analysis is performed to determine the parameters $\overline{\theta}$ to be estimated from the vibration measurements. It is assumed that the governing failure mode is the fatigue failure mode. This implies that the structure is modelled by one fatigue failure element at the clamped end.

A fatigue failure function can be established by different damage accumulation models. Here, the model is based on Miner's rule combined with the so-called S-N approach. Among other models, a crack growth model based on fracture mechanics can be mentioned where the most used law is the well-known Paris Law. Assuming a narrow-banded stress process the fatigue failure function based on Miner's rule and S-N curves is written, see e.g. Wirsching [31]

$$g(\overline{x},\overline{p}) = \ln(D_{Fail}) + \ln(K) - \ln(T_L) - k\ln(2\sqrt{2}) - \ln(\Gamma(1+\frac{k}{2})) \\ - \ln\frac{\sigma_s^k}{T_0} - \frac{k}{4}\ln(\frac{t}{22}) \tag{15}$$

Here, the expected lifetime T_L is 25 years and k is modelled as a constant, $k = 3$, and K is modelled as a random variable. Stress concentration is neglected. The standard

deviation of the stress process σ_s and the zero-upcrossing period T_0 are estimated by spectral moments, see e.g. Thomsen [32]. It may be noticed that (15) is developed under a narrow-banded assumption which can be adjusted due to the wide-banded stress process, see e.g Wirsching [31]. For simplicity this is not done here.

The random variables taken into account are

Variable	Designation	Distrib.	Exp. value	Var. Coeff
M	Mass	N	1.0	0.1
EI	Stiffness	N	4.0	0.1
ζ	Damping ratio	N	0.04	0.5
K	Constant in SN-curve	LN	6400	1024

Table 1: Statistical characteristics (N : Normal, LN : Lognormal)

The random variables are assumed to be mutually independent. For simplicity D_{fail} is deterministically modelled and model uncertainty is neglected.

The above modelling of the structure implies that the reliability of the structure becomes $\beta_i = 1.09$.

The reliability calculations in this paper are performed with the computer program PRADSS (Program for Reliability Analysis and Design of Structural Systems), see Sørensen [33].

A sensitivity analysis gives the sensitivities shown in table 2

Variable	$\frac{\partial \beta_i}{\partial \mu_j} \frac{\mu_j}{100}$	$\frac{\partial \beta_i}{\partial \sigma_j} \frac{\sigma_j}{100}$
M	-0.0136	-0.00068
EI	0.0140	-0.00022
ζ	0.0182	-0.00904
K	0.0092	-0.00307

Table 2: Sensitivity of the element reliability index to variations of the mean values μ_j and standard deviations σ_j for the basic variables.

Table 2 shows that the largest contributions to the overall uncertainty are due to the damping ratio ζ and the parameter K describing the fatigue strength.

4.2 Reliability Analysis Based on Preposterior Information

In the following an experiment will be designed in order to determine the parameter $\theta = \zeta$ from vibration measurements. It is assumed that the structure can be identified by an ARMA(2,1) model.

In Kirkegaard [11] an analytical solution for the Fisher information matrix is given as function of the sampling time Δt and the number of data N when it is assumed that an ARMA(2,1) model is used. Using this information matrix and the expression (13)

for the updated information matrix, the updated variance of the damping ratio can be estimated as function of the sampling interval and number of data. The updated variance can now be used in the reliability calculations instead of the prior variance of the damping ratio.

In figure 3a the variation of the expected updated reliability is shown as a function of the sampling time Δt for $N = 5000$ and in figure 3b as a function the number of data points N for $\Delta t = 0.499$. It is assumed that the updated mean value of the damping ratio corresponds to the prior mean value of the damping ratio. This implies that the sensitivity of the results with regard to the mean value has to be investigated. This is done in section 4.2.1.

a) b)

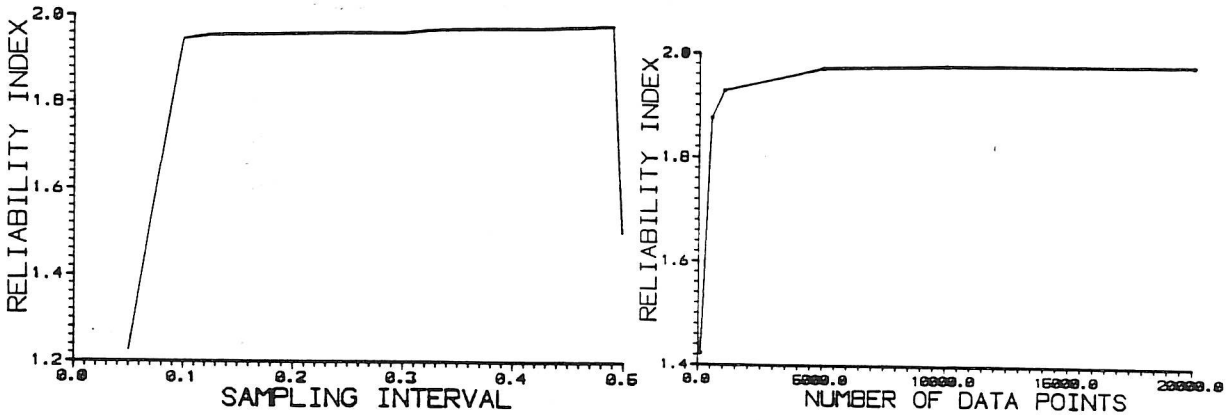

Figure 3: a)The updated reliability as function of the sampling time Δt for $N = 5000$. b) The updated reliability as function of number of data points N for $\Delta t = 0.499$.

It is seen from figure 3a that the curve of the updated reliability has a maximum. This maximum corresponds to a minimum of the updated variance of the damping ratio. This minimum is obtained for $\Delta t = 0.499$. Figure 3b shows that only a limited improvement can be obtained when the number of data points have reached a given magnitude. It may be noticed from this example that the updated reliability can be used as a design criterion instead of e.g. the determinant criterion. It is seen that the updated reliability index is a function of the experiment design variables Δt (sampling interval) and N (number of data points). In the following it will be shown how an optimal number of N can be determined.

According to (7)-(8) the optimization problem becomes

$$\min_{\Delta t, N} \quad C(\Delta t, N) = E_{\overline{\Theta}}[C_E(N, N_s) + C_f P_f''(\Delta t, N, N_s)] \tag{16}$$

$$s.t \quad \Delta t^l \leq \Delta t \leq \Delta^u$$
$$N^l \leq N \leq N^u \tag{17}$$

N_s is the number of sensors. Here, one sensor is assumed to be used.

From above it is known that the optimal sampling time is $\Delta t = 0.499$. Therefore, the only optimization variable will be N.

The costs of the experiment are modelled by

$$C_E(N, N_s) = C_0 + C_1 N + C_2 N_s \tag{18}$$

C_0 is the costs of instrumentation and planning. C_1 is the cost of an additional sample point and C_2 is the cost of an additional sensor. It may be noticed that the costs are deterministically modelled.

In principle the optimization problem (16)-(17) can be solved using any general non-linear optimization algorithm. In this paper, the optimization problem has been solved by the NLPQL algorithm developed by Schittkowski [34]. The NLPQL algorithm is based on the optimization method by Han, Powell and Wilson, see Gill et al. [35]. The algorithm is an effective method where each iteration consists of two steps. The first step is a determination of the search direction. The second step is a line search.

The NLPQL algorithm requires estimates of the gradients of the objective function (16). The derivative of the objective function $C(\Delta t, N, N_s)$ with respect to the design variable N is

$$\frac{\partial C(\Delta t, N, N_s)}{\partial N} = \frac{\partial P_f''(\Delta t, N, N_s)}{\partial N} C_f + \frac{\partial C_E(N, N_s)}{\partial N} \tag{19}$$

The last term in (19) is easy to estimate analytically but the gradients of the updated probability of failure are more difficult to obtain. Since the probability of failure is estimated by FORM the gradient of the objective function becomes

$$\frac{\partial C(\Delta t, N, N_s)}{\partial N} \approx \varphi(-\beta_i) \frac{\partial(-\beta_i)}{\partial N} C_f + C_1 \tag{20}$$

where $\varphi(\cdot)$ is the standard normal density function.

The derivative of the reliability index with respect to the design variable N is

$$\frac{\partial \beta_i}{\partial N} = \sum_{j=1}^{n_{\overline{\Theta}}} \frac{\partial \beta_i}{\partial \sigma_j} \frac{\partial \sigma_j}{\partial N} \tag{21}$$

where σ_j is the standard deviation of the jth parameter in the parameter vector $\overline{\theta}$ containing $n_{\overline{\theta}}$ parameters. Here, $n_{\overline{\theta}} = 1$ since the only parameter to be determined from the experiment is the damping ratio. $\frac{\partial \beta_i}{\partial \sigma_j}$ is determined by (14). The derivative $\frac{\sigma_j}{\partial N}$ can be estimated numerically from the expression for the updated information matrix $\overline{\overline{J}}^u$. The inverse of the updated information matrix includes the standard deviation σ_j. The updated information matrix is determined by the analytical expression from Kirkegaard [11].

4.2.1 Results

The optimal number of data N^{opt} for an experiment with one sensor for identifying the SDOF can now be estimated.

It is assumed that the cost function can be modelled as:

$$C_0 = 10^4 \ DKK., \quad C_1 = 5 \ DKK., \quad C_2 = 10^4 \ DKK. \tag{22}$$

where DKK is Danish Kroner. The optimal solution of N for various values of cost of failure C_f is shown in table 3.

C_f (DKK.)	$P_f''(\cdot)$	$C(\cdot)$ (DKK.)	$C_E(\cdot)$ (DKK.)	N^{opt}
10^5	$3.06\,10^{-2}$	$2.40\,10^4$	$2.09\,10^4$	181
10^6	$2.59\,10^{-2}$	$4.86\,10^4$	$2.27\,10^4$	534
10^7	$2.44\,10^{-2}$	$2.72\,10^5$	$2.80\,10^4$	1600
10^8	$2.38\,10^{-2}$	$2.43\,10^6$	$4.50\,10^4$	5010
10^9	$2.36\,10^{-2}$	$2.38\,10^7$	$1.02\,10^5$	16500

Table 3: The optimal solution of N for various values of cost of failure C_f.

It is seen, as expected, that N^{opt} increases when C_f increases, which means that acquisition of more information is of course more relevant when cost of failure increases.

In table 4 the value of information corresponding to the values in table 3 is shown. This value of information VI represents the maximum cost that may be allowed for acquisition of additional information. If VI exceeds the costs of the experiment C_E, the experiment should be performed.

$C_f\,(DKK.)$	$C_f P_f''(\cdot)\,(DKK.)$	$C_f P_f(\cdot)\,(DKK.)$	$C_E(\cdot)\,(DKK.)$	$VI(DKK.)$
10^5	$3.06\,10^3$	$1.38\,10^4$	$2.09\,10^4$	$1.07\,10^4$
10^6	$2.60\,10^4$	$1.38\,10^5$	$2.27\,10^4$	$1.12\,10^5$
10^7	$2.44\,10^5$	$1.38\,10^6$	$2.80\,10^4$	$1.14\,10^6$
10^8	$2.38\,10^6$	$1.38\,10^7$	$4.50\,10^4$	$1.14\,10^7$
10^9	$2.37\,10^7$	$1.38\,10^8$	$1.02\,10^5$	$1.14\,10^8$

Table 4: Value of information.

It is seen that the experiment should be performed if the costs of failure of the structure are larger than $10^6\ DKK$.

However, before a final decision further investigations must be performed. It may be noticed that the results obtained above are estimated based on prior information and information which is assumed to be obtained by performing the experiment. This means that the expected monetary value $C(\Delta t, N, N_s)$ is an uncertain quantity.

To investigate the sensitivity of $C(\Delta t, N, N_s)$ with respect to variations of the information, which is used above, a sensitivity study is made. Table 5 shows the sensitivities

of the expected monetary value $C(\Delta t, N, N_s)$ to variations of the mean value and standard deviation for the variables which are stochastically modelled. The sensitivity of $C(\Delta t, N, N_s)$ with respect to a mean value μ_j is

$$\frac{\partial C(\Delta t, N, N_s)}{\partial \mu_j} \approx \varphi(-\beta_i)\frac{\partial(-\beta_i)}{\partial \mu_j}C_f + \frac{\partial C_E(N, N_s)}{\partial \mu_j} \tag{23}$$

The derivative with respect to the standard deviation can be obtained in the same manner. The sensitivities of the reliability index are given in table 5 for an optimal value of N. Here, the sensitivities are shown for the situation where it is assumed that costs of failure $C_f = 10^7$ DKK, i.e $N^{opt} = 1600$. It may be noticed that the sensitivities shown in table 5 and the sensitivities shown in table 2 are different. This is caused of that the standard deviations of the damping ratios are different.

Variable	$\frac{\partial \beta}{\partial \mu_j}\frac{\mu_j}{100}$	$\frac{\partial \beta}{\partial \sigma_j}\frac{\sigma_j}{100}$
M	-0.0311	-0.00156
EI	0.0356	-0.00248
ζ	0.0224	-0.00023
K	0.0317	-0.01699

Table 5: The sensitivities of $C(\Delta t, N, N_s)$ to variations of the mean values μ_j and standard deviations σ_j for the basic variables.

Table 5 shows that the dominating contribution to the overall uncertainty is due to the parameter K describing the fatigue strength. Further, if the results in table 5 are compared with the results in table 2, it is seen that the contribution of the damping ratio to the overall uncertainty is reduced while the contributions of the mass and the stiffness are increased. However, K has the largest contribution. This means that K is a variable with a large influence on the optimal design of the experiment and the value of information. Therefore, the influence of the statistical characteristic of K on the experiment design will be investigated closer.

In figure 4a the value of information minus the costs of the experiment $(VI - C_E(\cdot))$ is shown to variations of the mean value μ_K and to variations of the coefficient of variation δ_K of K.

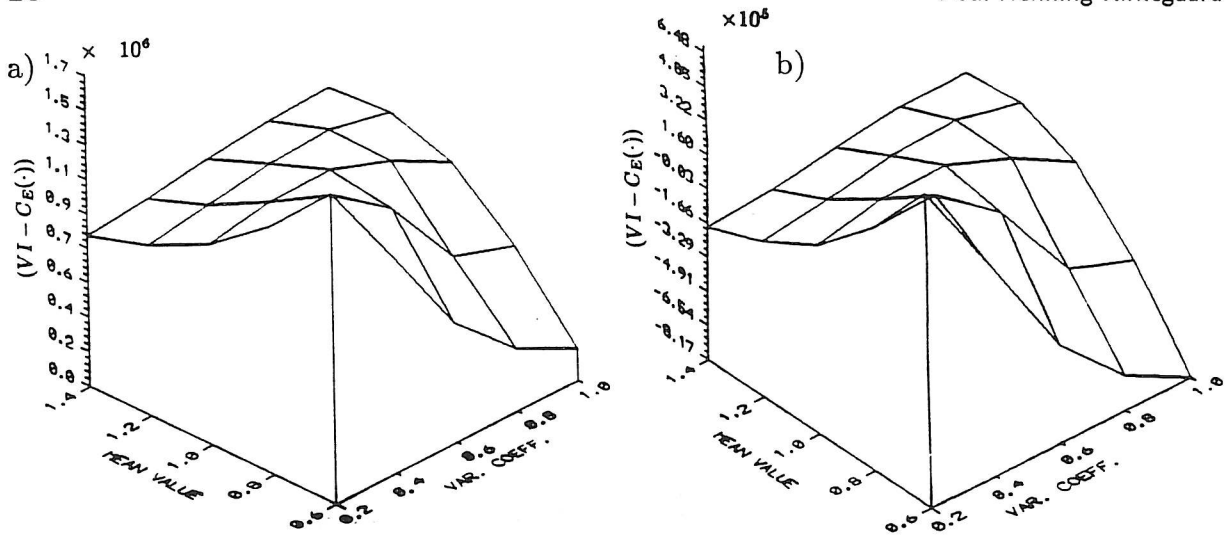

Figure 4: The sensitivity of $(VI - C_E(\cdot))$ to variations of the mean value μ_K and to variations of the coefficient of variation $\delta_K$of K. a), $C_0 = 10^4\ DKK$ b), $C_0 = 10^6\ DKK$.

Figure 4a shows that the value of information minus the costs of the experiment $(VI - C_E(\cdot))$ is positive for all values of the mean value and coefficient of variation. This means that the experiment should be performed. However, it is also seen that $(VI - C_E(\cdot))$ is very sensitive to variations of the mean value and the coefficient of variation. Especially, when $\mu_K < 1.0$ and $\delta_K > 0.4$. This implies that one has to take more care when a decision concerning performing an experiment is based on values in this area. It is seen from figure 4b that the experiment should not be performed, if the mean value μ_K is small and the coefficient of variation δ_K is large.

In figure 5 the sensitivity of the optimal design, N^{opt} is shown to variations of the mean value μ_K and the coefficient of variation δ_K. It is seen that the optimal design is sensitive to the mean value μ_K and the coefficient of variation δ_K. Especially, for a coefficient of variation less than 0.4.

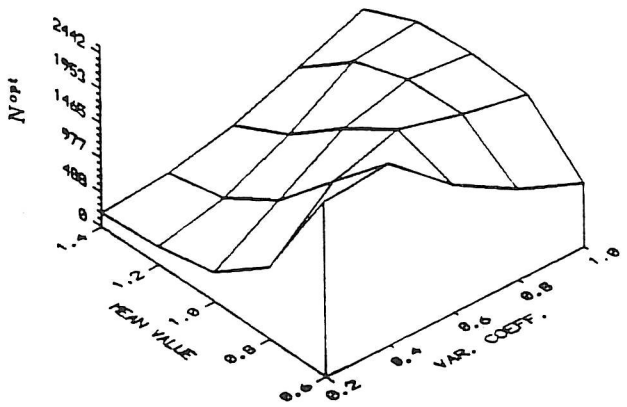

Figure 5: The sensitivity of the optimal design, N^{opt} to variations of the mean value μ_K and to variations of the coefficient of variation δ_K of K..

The conclusion of the example is that the experiment should be performed if the costs of structural failure are larger than $10^6 DKK$. However, if the costs of instrumentation and planning C_0 increases, the mean value and the coefficient of variation should have more attention before a final decision should be made. It may also be noticed that an experiment giving additional information about the random variable K could be performed. It could be interesting to investigate whether additional information should be obtained by performing a system identification experiment, or an experiment, giving additional information about K. Such an investigation could also be formulated as a decision problem.

5. Conclusions

This paper has been devoted to a presentation of a system experiment design method based on a preposterior analysis. The characteristics features of the method are as follows:

- The method is based on a preposterior analysis implying that uncertain quantities can be modelled as random variables.
- The method provides system identification experiment designs reflecting the costs of the experiment and the value of additional information. This implies that cost optimal system identification experiment can be designed. E.g. the system identification experiment design method makes it possible to determine the optimal number of sensors. This is not possible with the well-known system experiment design methods.
- The proposed system identification design method makes it is possible to consider the experiment design problem evolving whether and how additional information should be obtained.

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