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Sørensen, Troels Bundgaard

# Slow Fading Cross-Correlation versus Azimuth Separation of Base Stations

T.B. Sørensen

Abstract: A model is proposed to describe the cross-correlation of slow fading signals received from non co-located base stations. The model summarises the results obtained from a bootstrap analysis of 900MHz urban area propagation measurements.

Introduction: System level simulations of mobile cellular systems have shown that the network performance is affected by the cross-correlation between the random component of the local mean signal power (slow fading) of the serving base station and that of the handover base station candidates. An example study was given in **[1]**, where the outage probability, subject to handover algorithm performance, was analysed for a synthetic cross-correlation model.
Previous experimental results for the cross-correlation reported by Graziano [2] and Mawira [3] seem to indicate that the correlation drops fast for small AAD (Angle of Arrival Difference) in urban and suburban areas. For larger AAD Mawira has observed a linear decrease, whereas Graziano's results were more irregular in shape. In this communication, the previous findings of cross-correlation versus AAD will be supported by providing new urban area measurement results with confidence intervals for the correlation coefficient estimates. The results are summarised in a simple model.

Measurements: The measurements used for the analysis were conducted in Aarhus (Denmark). A measurement van with a roof mounted quarter-wavelength omnidirectional antenna provided simultaneous measurements of the received signal power from three operational base stations located in the downtown area. The relative position of the base stations and the measurement area is depitched in Fig. 1. The measurement area is characterised by almost uniform building height of about 4-5 storeys, and a gently rolling terrain. A total of 35 measurement routes, with an average length of approx. 400 m, were defined by driving around a single or a group of apartment blocks. For each route, samples were taken at an equidistant interval of one third of a wavelength (0.1m). For nearly all of the measurement routes the receiver van was in NLOS (Non Line-Of-Sight) with respect to the base stations.
The signals received from the base stations were separated by means of different transmit frequencies: 951.4, 951.8 and 955.8 MHz. Each of the base stations used a directional antenna pointing south, as indicated in Fig. 1.
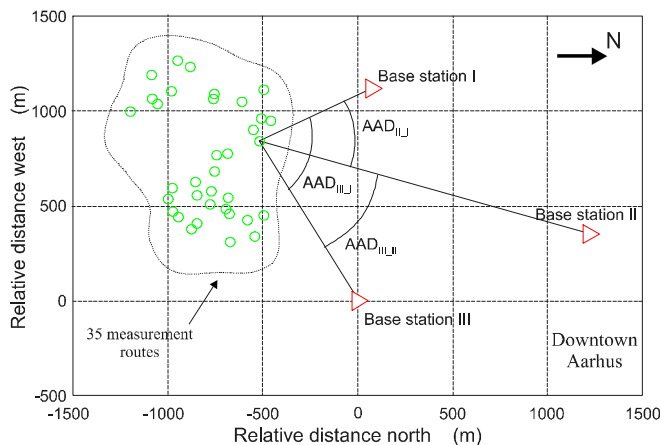
Fig. 1 *Relative position of base stations and measurement area (encircled area).*

Correlation analysis: For each measurement route shown in Fig. 1 three AAD angles may be defined corresponding to any two of three base stations. This gives 3×35 pairs of data records which provide the statistical data for analysing cross correlation as a function of AAD. An AAD bin size of 5° was considered to be the smallest possible for data clustering.
For each data record (a route) the median value for a 12.7 m section was computed as an estimate of the local mean. From previous analysis the standard deviation of the slow fading within the measurement area was found to be approx. 5dB, and the decorrelation distance was estimated to be 5.5 m **[4]**. For any pair of computed local mean values a subset was selected in which each sample pair was separated from adjacent pairs by 25.6 m. This step was taken to avoid any specific assumptions on the process autocorrelation in calculating confidence intervals for the correlation coefficient. With this separation, sample pairs can be assumed to be nearly uncorrelated with autocorrelation below 0.2, according to the empirically derived model in **[4]**.

The above procedure leaves only a small number of samples in each pair for estimation of the cross-correlation coefficient, and for such a small data set we cannot expect asymptotic results to apply. For this reason, the 'independent non-parametric data bootstrap' procedure has been applied in order to estimate a confidence interval for the correlation coefficient r **[5]**. Specifically, Fisher's 'z-transform' was used to stabilise the variance of the parameter estimate $r^{\sim}$ and the 'transformed percentile-t method' **[6]** was used to calculate a *(1-a)×100%* confidence interval for the correlation coefficient.
The correlation coefficient is defined in equation (1) and based on logarithmic (dB) values ('cov' is covariance). An estimate of the coefficient is calculated from the subset of sample pairs P_i, P_j representing the local mean power received from base station i and j, respectively.

$$r^{\sim}_{ij} = \frac{\mathrm{cov}(P_i, P_j)}{\sqrt{\mathrm{cov}(P_i, P_i) \cdot \mathrm{cov}(P_j, P_j)}} \qquad (1)$$

Within a particular AAD bin of 5° a number of different data sets measured at different ranges can be present. To combine the correlation coefficient estimates of the different data sets, and to allow some statistical averaging, the weighted averaging process proposed by Graziano **[2]** is applied before the data bootstrap.
Due to the distances used for the median calculation and the subset selection it is possible to select two distinct subsets of sample pairs based on different samples within the data records - each with its own estimate of correlation coefficient and confidence interval. If the subsets are correlated, as implied in the previous paragraph, one is to expect coincident estimates. On the other hand, if they are nearly uncorrelated, coincident estimates give proof that the estimate is good. For the purpose of this discussion consistency in the analysis is of prime concern and the estimates must agree whichever is the case.

Results and conclusion: The results of the bootstrap analysis is shown in Fig. 2 for AAD ranging from 10 to 85°. The correlation coefficient estimates are marked with a diamond symbol and the related 90% confidence interval with a vertical line through the symbol. A bootstrap resample size of 1000 has been used with a nested variance estimation bootstrap of size 25; see [5] for details. An estimate is shown for each of the two distinct, but possibly correlated, data subsets. Sufficient agreement can be seen to gain confidence in the applicability of the bootstrap procedure for cross correlation estimation. The standard deviation (right scale in Fig. 2) is found to be approximately 0.1 on average, and largely dependent on the number of samples in a bin; this is particularly evident for the first bin which contains the smallest number of samples.
In general, the results confirm the observations in [2,3]. The actual shape of ρ versus AAD, however, is likely to change

for a different urban location. Despite of this, a piecewise continuous model is proposed, which summarises the information contained in Fig. 2:

$$r = \begin{cases} 0.78 - 0.0056 \cdot AAD & , & 0^o \le |AAD| < 15^o \\ 0.48 - 0.0056 \cdot AAD & , & 15^o \le |AAD| < 60^o \\ 0 & , & otherwise \end{cases} \quad (2)$$

This model (thick solid curve in the figure) includes the main characteristics which are likely to be observed in an urban area; certainly, the synthetic models used in [1] do not (see the example in Fig. 2). One possible variation of the model is to change the position of the first discontinuity, say within 10-40°. This is believed to have the most dominant effect on system level simulations.
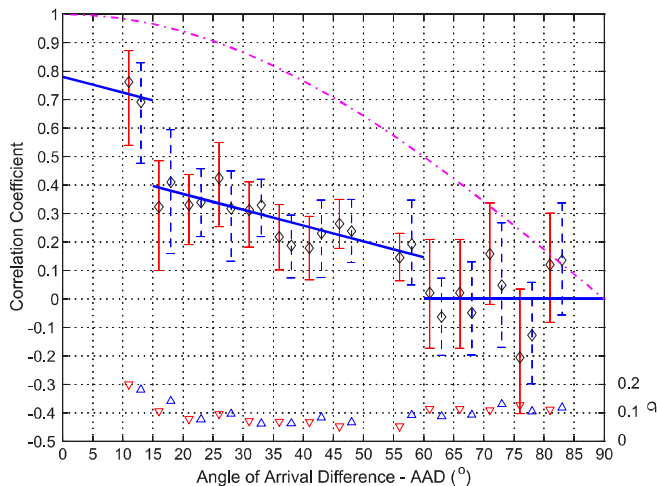
Fig. 2 *Correlation coefficient and 90% confidence interval estimates; points at the bottom are the standard deviation* **s** *of the estimate (right scale).*

¾¾¾ *Subset 1*   - - - - - *Subset 2*
- ×- ×- *Synthetic model* **[1]**   ———— *Proposed model*

References

[1] F. Graziosi, M. Pratesi, M. Ruggieri, F. Santucci, "Joint Characterization of Outage Probability and Handover Performance in Cellular Mobile Networks", IEEE Proc. Vehicular Technology Conference (VTC98), May 1998, pp. 1054-1058

[2] V. Graziano, "Propagation Correlations at 900 MHz", IEEE Trans. on Vehicular Technology, Vol. VT-27, No. 4, November 1978, pp.182-189

[3] A. Mawira, "Models for the spatial correlation functions of the (Log)-Normal component of the variability of VHF/UHF field strength in Urban environment", PIMRC'92, Boston, USA, pp. 1503-1507

[4] T.B. Sørensen, "Correlation Model for Slow Fading in a Small Urban Macrocell", IEEE Proc. Personal, Indoor and Mobile Radio Communications (PIMRC'98), September 1998, pp. 1161-1165

[5] A. M. Zoubir, B. Boashash, "The Bootstrap and its Application in Signal Processing", IEEE Signal Processing Magazine, Vol. 15, No. 1, January 1998, pp. 56-76

[6] P. Hall, M. A. Martin, "Better Nonparametric Bootstrap Confidence Intervals for the Correlation Coefficient", Journal of statistical computation and simulation, Vol. 33, No. 16, 1989, 161-172