Pedagogical approach of grade 7 teachers in teaching the learning competency of integers

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Abstract. This research addresses the intricate challenges of teaching integers in middle school mathematics. Focusing on grade 7 teachers, the study explores strategies to enhance integers' teaching-learning competency, revealing nuanced approaches employed by three teachers. Through qualitative methods, specifically in-depth interviews, the research uses an exploratory approach to unravel the underlying strategies shaping the teaching of integers. Key findings underscore integers as a formidable learning competency for grade 7 students, particularly in operations like addition and subtraction. The most effective pedagogical approach, encapsulated in the 3Rs – Reaching, Representing, Recommending – involves drill exercises for foundational concepts, integrating manipulatives like algebra tiles and coloured tiles, and advocating for continuous manipulative use with persistent drill exercises and integrating technology. These identified themes emphasize the need for a uniform understanding among teachers during discussions to ensure a cohesive and effective teaching strategy. Recognizing these strategies is crucial for addressing the existing learning gap and advancing grade 7 students' competency in dealing with integers. The research contributes valuable insights to the academic discourse, offering effective pedagogical practices and fostering a more robust foundation for mathematical understanding at the middle school level.

Keywords: integers, teaching strategies, learning competency, manipulatives

1. Introduction

Mathematics has a crucial role in advancing human thinking and serves as the foundation for the development of modern technology at present time [3]. Understanding mathematical concepts is the ability to understand mathematical concepts necessary for learning mathematics concepts [18]. In the dynamic landscape of middle school education, the comprehension of integers stands as a cornerstone for students transitioning into more advanced mathematical concepts. This research explores the pedagogical approaches employed by grade 7 teachers in facilitating the learning competency of integers. By exploring the complexities of teaching this fundamental mathematical concept, the study sheds light on educators' pivotal role in shaping students' mathematical understanding.

In the Philippines, the mathematics curriculum, in both secondary education, aims to instil practical concepts necessary for a comprehensive education. The study of integers in grade 7 lays the groundwork for subsequent mathematical concepts, making it imperative for teachers to employ effective teaching strategies that resonate with the diverse learning styles of their students. Integers, encompassing negative, positive, and zero, are paramount in mathematics.

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Beyond their theoretical significance, integers are a gateway to nuanced problem-solving skills and real-world applications. It is also the basis of learning algebra and is considered a vital pre-condition that only three Mathematics teachers were handling. The need for a more comprehensive understanding of effective teaching strategies becomes evident in this observation. While students may have been introduced to integer concepts in grade 7, the application and mastery of these concepts do not necessarily endure as they move into higher grades. This poses a potential obstacle to their overall mathematical development, as integers are foundational to various advanced mathematical topics.

Grade 7 students commonly grapple with conceptual challenges when introduced to integers. Badarudin and Khalid [4] suggested that teachers use models to teach integers. Negative numbers, absolute values, and zero can hinder their mathematical journey. Teachers stand at the forefront of addressing these challenges, requiring a nuanced understanding of their student's needs and the deployment of tailored pedagogical methods. One way to address this challenge is by using Digital media, which has become integral to children's daily lives across all age groups. Numerous researchers have suggested using smart mobile devices for educational purposes [13, 14]. However, Sarama and Clements [37] found out that virtual manipulation, which is facilitated by computer software, is like physical manipulation, and therefore, using ICT and manipulatives could be equally effective in supporting the learning process, especially in mathematics.

Mathematics is an ever-present subject that significantly influences our abilities and problemsolving skills at different stages of life. Its importance goes beyond the confines of the classroom, impacting our experiences and decision-making processes [34]. Effective teaching methods are the linchpin of successful mathematical education. A solid foundation in integers fosters mathematical proficiency and nurtures students' confidence in tackling more complex mathematical concepts. Therefore, exploring and understanding the various teaching methods becomes essential in optimizing the learning experience for grade 7 students. Practical pedagogical approaches are crucial in ensuring students grasp integers' intricacies. The 3Rs – Reaching, Representing, Recommending – emerge as a practical framework, emphasizing the need for educators to reach students through drill exercises, represent integers using manipulatives, and recommend continuous usage of manipulatives with persistent drill exercises [38]. This approach aligns with the evolving philosophy of mathematics education, emphasizing a deeper understanding of mathematical concepts over rote memorization and procedural knowledge.

Lavidas, Apostolou and Papadakis [25] also seemed to prefer mathematical activities based on Connections and Representation processes in their digital classrooms. Moreover, the role of manipulatives in the learning process cannot be overstated. These tangible tools provide students with a bridge between abstract mathematical concepts and real-world applications, fostering a more meaningful understanding of integers. While manipulatives have proven effective in enhancing comprehension, challenges persist in linking this concrete knowledge to mathematical symbolization and abstraction [21].

In the last two decades, research has increasingly underscored the significance of grounding students' mathematical work in real-world contexts to aid the process of abstraction [39]. Various models, including the number line, balloons and weights, and two-colour tiles, have been devised to assist students in understanding integers. However, the effectiveness of these models remains a subject of ongoing exploration and refinement. The findings support that computers, especially

tablets, when combined with the use of developmentally appropriate software in the children's daily routines, may substantially contribute to early childhood students' comprehension of numbers [33].

Innovative approaches, such as Miller's [29] incorporation of the net worth concept, exemplify ongoing efforts to communicate operational integer concepts meaningfully. By introducing the idea of assets and debts to convey net worth, Miller's approach offers a fresh perspective on integer comprehension.

The National Council of Teachers of Mathematics [32] advocates for incorporating realworld contexts, tangible aids, and collaborative working practices in teaching. This prompts the question of how these varied approaches can be effectively implemented in the school classroom to foster meaningful learning. Therefore, teachers wield considerable influence in shaping students' mathematical understanding. Their instructional methods, classroom strategies, and pedagogical choices contribute significantly to students' ability to comprehend and apply integer concepts. Recognizing the impactful role of teachers, this study aims to unravel the diverse approaches employed by grade 7 educators. Examples encompass the integration of physical activities [40], using multiple representations in mathematics instruction [16], employing problem-solving strategies [24], utilizing computer-aided teaching methods [44], incorporating video and computer games [9], adopting collaborative learning approaches [10], fostering mathematical discussion and interaction [35], and addressing specific contexts such as learning disabilities [5].

The existing literature highlights the importance of integers in mathematics education but leaves room for deeper exploration of the pedagogical approaches tailored explicitly to grade 7. This study addresses the gaps in the current understanding, providing a nuanced perspective on teaching strategies and their impact on student learning outcomes. This underscores the significance of the teacher's proficiency in mathematics and pedagogy, essential prerequisites for effectively guiding students through the learning process [30].

The identified research gap highlights a persistent challenge in mathematics education as students progress to higher grades, specifically in grades 8 to 10. Teachers in these grades have observed ongoing confusion among students when faced with integer operations problems. This observation points to a critical gap in understanding and applying integer concepts, indicating that the foundational comprehension established in earlier grades may need to be more robust to support students as they advance in their mathematical education. This is supported by the claim of Keşan, Yılmaz and Altınok [22], Kinach [23], Vlassis and Demonty [43] that the concept of negative numbers and operations with integers involved conceptual challenges.

The research gap emphasizes the importance of addressing the continuity and progression of integer comprehension across grade levels. The traditional methods or approaches used in teaching integers in grade 7 may not seamlessly transition to subsequent grades, leading to persistent confusion among students. This gap calls for a closer examination of the teaching strategies employed in grade 7 and exploring how these strategies may be adapted or enhanced to ensure a more sustained and effective learning trajectory. Akyüz, Stephan and Dixon [1] determined that the students could deal with integer problems when they related them to real life in the context of the profit-loss model and the vertical number line.

In response to this research gap, the proposed study aims to delve into the specific pedagogical approaches used by grade 7 teachers in teaching the learning competency of integers. By

focusing on grade 7, which serves as a foundational year for integer comprehension, the research seeks to understand how teachers in this crucial grade approach the teaching of integers. The emphasis on exploring educators' perspectives is crucial, as it allows for an in-depth understanding of the rationale behind specific strategies and the perceived impacts on student learning.

Grade 7 serves as a critical juncture in students' mathematical progression. The complexities of integers lay the foundation for subsequent years, making this grade pivotal in shaping students' mathematical journeys. Understanding the pedagogical approaches employed at this stage holds the potential to create a ripple effect on students' mathematical proficiency in the years to come. For several reasons, the study's focus on identifying the most effective strategies used by grade 7 teachers is essential. [31] teachers' pedagogical knowledge of teaching integers is also an excellent factor for learners to understand the concept of integers.

The findings of this research are anticipated to contribute significantly to enhancing student learning outcomes. Insights into effective integer teaching methods can improve student performance, foster greater engagement, and cultivate a lasting interest in mathematics. Furthermore, the research offers recommendations for refining and enhancing these approaches. This forward-looking approach is crucial for adapting teaching strategies to the evolving needs of students as they progress through their mathematical education. By bridging the gap between abstract and concrete understanding, the study seeks to provide practical insights that can inform and elevate mathematics education not only in grade 7 but also in subsequent grades.

Likewise, the identified research gap underscores the need for a more comprehensive understanding of effective teaching strategies to address persistent confusion among students as they progress to higher grades. By focusing on grade 7 teachers, the proposed research aims to uncover successful pedagogical approaches, assess their impacts, and provide recommendations for refining and enhancing these strategies. Ultimately, the overarching goal is to contribute valuable insights that inform and elevate mathematics education, ensuring a more seamless and effective transition in comprehending and applying integer concepts across grade levels.

The following questions will guide the research:

- 1. How do participants view integers as a learning competency?
- 2. What is the best pedagogical approach teachers use to teach these challenging critical areas in learning the competency of integers?
- 3. What are the perceived impacts of this pedagogical approach on the teachers in attaining the learning competency of integers for grade 7 students?

2. Methodology

This research adopts a qualitative phenomenological design to enhance the quality of mathematics education, particularly in teaching the learning competency of integers to grade 7 students. As students' mathematical journey progresses, challenges often persist, as observed by teachers in grades 8 to 10, highlighting a notable gap in the sustained comprehension of integer concepts. Addressing this gap requires a closer examination of the pedagogical approaches employed by grade 7 teachers, who play a foundational role in shaping students' understanding of integers. Phenomenological research, renowned for its emphasis on understanding human experiences, perceptions, and lived realities, becomes the lens through which we explore the teaching strategies implemented by grade 7 teachers. This research design aligns with the need to delve into the intentional, conscious, and first-person perspectives inherent in the teaching of integers. By employing phenomenology, we aim to unravel the nuanced and subjective dimensions of the teachers' approaches, providing valuable insights into the intricate landscape of teaching and learning integers.

The subsequent sections detail the participants, instruments, data collection procedures, and data analysis, providing a comprehensive roadmap for this qualitative exploration into the pedagogical approaches employed by grade 7 teachers in teaching the learning competency of integers.

2.1. Participants

The participants in this study consisted of three grade 7 teachers from Pines City National High School (hereinafter indicated as T1, T2, and T3). Since only three Mathematics teachers were handling the grade 7 classes, they were all considered in this research. The sample size was determined to be sufficient for the qualitative research design employed in this study, aligning with the recommendations of qualitative research experts such as Guest, Bunce and Johnson [17].

2.2. Instruments

This study's primary data collection instrument was informal interviews with grade 7 teachers. The interview questions were specifically formulated to address the research problem, focusing on exploring the pedagogical approaches employed by the teachers in teaching the learning competency of integers. A set of a priori codes was established, and questions were developed based on these codes. Follow-up questions were also incorporated to seek clarification and gather in-depth responses. The interview process involved note-taking, subsequent literature review for result validation, and direct inquiry by the researcher to the participants for triangulation.

2.3. Data collection procedures

The qualitative research method guided the data collection procedures. Informal interviews were conducted with the grade 7 teachers, with questions designed to elicit insights into their pedagogical approaches. A priori codes formed the basis for interview questions, ensuring alignment with the research objectives. During the interviews, note-taking was employed to capture responses, and follow-up questions were used to enhance clarity and depth. The interview transcripts were then translated from the vernacular language to English. The key participants validated the transcribed data to ensure accuracy.

2.4. Data analysis

The analysis of the collected data involved a multi-step process. Firstly, a cool analysis was conducted to identify significant answers provided by the key participants. Subsequently, a

warm analysis was performed to identify commonalities in the responses of the key informants. The collected answers were then grouped based on emerging themes, and a thematic approach was employed for an organization. Thematic analysis, as described by Boyatzis [8], involves encoding qualitative information and developing codes, words, or phrases as labels for data sections. The flexibility and communicative advantages of thematic analysis made it the most appropriate mode for this study, allowing for a comprehensive understanding of the phenomenon under investigation.

2.5. Triangulation

Triangulation was employed in this research to enhance the validity of the findings. According to Creswell and Miller [12], triangulation involves seeking convergence among multiple and diverse sources of information. In this study, triangulation occurred through three primary sources: the interviews with the three teachers, a literature review to validate results, and direct observations made by the researcher during the lesson proper. This comprehensive approach aimed to strengthen the credibility and reliability of the research findings.

2.6. Ethical considerations

Ethical considerations were paramount throughout the research process. The participants were provided with an oral request and a formal letter explaining the nature and objectives of the study. They were informed that their participation was voluntary and that the data collected would be used for research purposes aimed at improving education. Confidentiality was assured from the outset, and participants were encouraged to ask questions and raise concerns. The participants were also given the option to withdraw from the research at any point if they felt uncomfortable. Any threats to their rights and welfare were promptly addressed, fostering an ethical and respectful research environment.

3. Results and discussion

The journey through the exploration of teaching integers to grade 7 students has unearthed a myriad of challenges, strategies, and impactful insights. As we navigate the complex landscape of mathematical education, this discussion encapsulates the essence of our findings, shedding light on how educators grapple with the abstract concept of integers and employ pedagogical approaches to bridge the gap between theory and comprehension. Through the lens of developmental theories, teaching strategies, and the perceived impacts on educators and learners, the following discourse aims to unravel the intricacies of teaching integers and contribute to the broader conversation on effective mathematics education. The subtopics delve into the heart of these research findings, where theory and practice converge to shape the future of mathematical pedagogy.

3.1. Understanding integers as a learning competency

The research underscored the intricate nature of teaching integers, especially in grade 7 students. Participants recognized integers as mathematical entities and as crucial skills with real-life applications. The participants' observations echoed developmental theories, such as Piaget's, emphasizing the transition from concrete to abstract thinking at 11 [26]. The struggles encountered, particularly in operations involving dissimilar signs, shed light on teachers' challenges in guiding students through this abstract realm.

The importance of integers extends beyond theoretical learning, finding practical applications in various real-life situations. Acknowledging the cognitive development of 11-year-olds, Hill [20] notes their transition from concrete to abstract thinking, a stage marked by symbolic manipulation and the potential for formal reasoning, as elucidated by Piaget. However, introducing integers as an abstract concept in grade 7 poses a substantial challenge. Participants in this study underscored the complexities surrounding operations on integers and fractions, particularly in the context of addition and subtraction.

Almeida and Bruno [2] further supports these challenges, asserting that students struggle to conceptualize negative numbers with different signs from positive ones, creating a barrier to understanding real-world applications. The participants' collective experience in teaching grade 7 reveals a notable hurdle, with nearly 60-70% of students encountering difficulties in mastering the learning competency associated with integers. This translates to 36-42 learners in a class grappling with the intricacies of the concept, especially in the context of addition and subtraction of dissimilar signs, as emphasized by Teacher 3 (T3).

Recognizing the foundational importance of understanding integers, particularly as the first encounter with abstract numbers, participants advocate for visual representation as a critical pedagogical strategy. Addressing the challenge of comprehending addition and subtraction operations, teachers employed a summary of pedagogical approaches to provide clarity amidst the conceptual complexity.

3.2. Pedagogical approach: the 3Rs framework

Adopting the 3Rs pedagogical approach revealed a thoughtful strategy for addressing the complexities of teaching integers. Researchers Gallardo [15], and [43], who highlighted the challenges of understanding numbers less than zero, find resonance in the 3Rs framework. The hierarchical steps, represented by the triangular symbol, align with Vygotsky's Zone of Proximal Development, emphasizing the role of a knowledgeable guide in navigating challenging concepts.

Figure 1 shows the representation of the pedagogical approach that was revealed in their teaching process.

The 3Rs symbol is represented as follows: first R – "reaching students understand the skill", second R – "representing integers for easy understanding", and the last R – "recommended strategies for better understanding". It is in the triangular symbol as it denotes a hierarchical step, and the second R – representation – is the highest in the symbol.

The adoption of the 3Rs pedagogical approach (Reaching, Representing, Recommended Strategies) emerges as a thoughtful strategy for addressing the complexities of teaching integers.



Figure 1: The 3Rs symbol.

This approach aligns with Vygotsky's Zone of Proximal Development, emphasizing the role of a knowledgeable guide in navigating challenging concepts. The 3Rs framework is represented hierarchically, denoting steps that include reaching students to understand the skill, representing integers for easy understanding, and recommending strategies for better understanding.

3.3. "Reaching students understand the skill" - teaching strategies

The varied teaching strategies employed by the teachers reflect a cognizance of diverse learning styles. The emphasis on visual aids, including PowerPoint presentations and manipulatives, aligns with constructivist theories, especially Piaget's emphasis on active learning. Continuous practice through daily drills draws from behaviourist principles, advocating for repetition in skill mastery.

"Reaching" in teaching refers to the myriad strategies educators employ to facilitate students' grasp of the learning competency associated with integers. Teaching strategies encompass the diverse structures, system methods, techniques, procedures, and processes instructors utilize during the instructional process. In the dynamic landscape of education, where traditional teaching styles evolve with differentiated instruction, teachers adapt their approaches to align with their students' varied learning styles and needs. Recognizing the pivotal role teachers play in imparting skills to students, particularly in the context of integers, necessitates the exploration of effective strategies to motivate and engage learners.

The findings of this research underscore the significance of teachers employing diverse strategies in introducing the complex concept of integers. Insights from T1, T2, and T3 reveal a spectrum of approaches:

- T1's approach: Incorporating PowerPoint presentations enriched with illustrations of integers in lesson introductions, followed by drills in worksheets and Bell Works.
- T2's approach: Employing a comprehensive strategy involving drills of varying difficulty levels, real-life problems, and using manipulative tiles such as algebra tiles/counters.
- T3's approach: Advocating the effectiveness of daily drills, emphasizing continuous practice for mastery, and recognizing the somewhat effective use of real-life materials and concepts.

Teachers found that introducing drills with escalating difficulty levels enabled students to practice integer operations. The integration of manipulative tools, such as algebra tiles/counters, money, and colour-coded coins, played a vital role in elucidating the abstract nature of integers. As proposed by Cemen [11], the money model contributed to students' understanding by connecting the concept to real-world scenarios of receiving and spending money.

Moreover, the research aligns with existing claims that active engagement in the learning process enhances student understanding, as asserted by Meeks et al. [28]. The endorsement of manipulatives, especially in the introductory phase, aligns with the assertion by Kamina and Iyer [21] that these tools are instrumental in fostering meaningful thinking and reasoning, particularly given the abstract nature of integers.

However, caution is advised in employing time-pressure tactics, such as using a bell to signal the start and end of drills, especially for students with difficulty handling time constraints. In conclusion, the general perspective derived from this exploration underscores the efficacy of employing manipulative tools, particularly in the introduction of integers, as supported by T2's experiences. This study further suggests that teachers leverage manipulatives to visually represent the intricate concepts of positive and negative numbers, facilitating a more profound understanding among students.

3.4. "Representing integers for easy understanding" - visual representations

The challenge of representing integers for easy comprehension resonates across the educational landscape. Teachers' challenges in initially representing integers highlight the importance of finding tangible, relatable models. Utilizing flip coins, money, and group activities aligns with Bruner's constructivist theory [45], emphasizing the significance of various representations in aiding comprehension.

At the onset of this research, educators faced formidable challenges in discerning features of representations that aligned with the evolving meaning of integers. The intricacies of representing integers became apparent early in the discussion, necessitating a nuanced exploration of this critical aspect of mathematical pedagogy.

Miller's work in 2013 substantiates this concern, revealing that her curriculum provided a lucid understanding of the abstract concept of subtracting a negative value, leading to a more positive answer. Notably, this clarity stands in contrast to other integer pedagogies that often engender confusion rather than elucidation among students, as expounded by Miller [29].

Bernardo [6] underscores the fundamental nature of integers as the first numbers encountered by students who demand to reason with numbers beyond physical modelling. This assertion aligns with the teachers' accounts during interviews, emphasizing the need for student engagement in carefully planned learning activities to facilitate meaningful comprehension.

Insights from T2 and T3 shed light on practical strategies employed by teachers to overcome these challenges:

- **Bell Works**: Integrated as a task before or during lessons, Bell Works serves as a preparatory exercise to introduce students to the forthcoming content.
- **Manipulation**: Employed during lesson introductions, manipulation techniques are implemented through group work and games, fostering a hands-on approach to learning.

• **Real-life word problems**: Immediate integration of integers into real-life applications serves as both motivation and a means of presenting the lesson or concept, offering students tangible contexts for understanding.

T2 and T3's use of manipulatives, such as money and flip coins, emerges as a practical solution to the representation challenge. T3's utilization of flip coins, with the blue side denoting positive integers and the red side representing negatives, offers a tangible and visually distinct representation, aiding student comprehension.

Research indicates many materials available for representing integers, including balloons, money, number lines, weights, and two-colour tiles, as highlighted by Cemen [11]. However, T2's acknowledgement of the time and effort involved in preparing these manipulatives unveils practical constraints educators face, particularly in larger classes exceeding 60 students.

This exploration underscores the multifaceted nature of representing integers and the diverse strategies educators employ to surmount associated challenges. As we delve into this nuanced realm, it becomes evident that the choice of representation significantly influences students' understanding, and teachers must navigate practical constraints to ensure effective pedagogical delivery.

Teachers' challenges in initially representing integers highlight the importance of finding tangible, relatable models. Utilizing flip coins, money, and group activities aligns with Bruner's constructivist theory, emphasizing the significance of various representations in aiding comprehension.

3.5. "Recommended strategies for better understanding" - continuous engagement

The teachers' recommendations for continuous engagement through manipulatives, games, and drills echo empirical insights and align with established psychological principles. Ebbinghaus's forgetting curve supports the need for continuous engagement to reinforce learning, preventing the rapid decline of acquired knowledge.

In consonance with the insights of Bernardo [6], Hartshorn and Boren [19], Ross and Kurtz [36], the efficacy of students' engagement in lessons and hands-on activities emerges as a focal point in promoting abstract concept acquisition. Given the inherently abstract nature of integers, using manipulative tools is advocated as a valuable strategy to enhance visualization and understanding.

In the aftermath of insightful interviews, educators unequivocally endorse the incorporation of manipulatives in the pedagogical approach to teaching integers. T1, T2, and T3 offer nuanced perspectives on the benefits and practical considerations associated with this recommendation:

- T1 underscores the potential of manipulation, particularly with colour-coded materials representing positive and negative integers. Despite practical constraints in the current academic year, the commitment to exploring manipulative tools in subsequent teaching endeavours is evident.
- T2 affirms the efficacy of using manipulatives during the introduction of integers, emphasizing their role in facilitating comprehension of more abstract concepts.

• T3 advocates for a comprehensive approach that extends beyond mere introduction, urging educators to ensure the retention of integer concepts. The integration of drills and games, exemplified by the playful application of darts to reinforce integer operations, stands out as a pedagogical strategy.

T3's additional recommendation emphasizes the need for teachers to introduce lessons and master the skills themselves, fostering a more profound understanding that can be imparted to students. This strategic approach aligns with the broader goal of ensuring uniformity in comprehension among learners.

The perceived impacts of this pedagogical approach on teachers in attaining the learning competency of grade 7 students regarding integers are multifaceted. Educators acknowledge the inherent challenges posed by the abstract nature of integers, recognizing the pivotal role of foundational understanding for students progressing to higher mathematical concepts. The recommendations put forth by teachers reflect a collective acknowledgement of the complexity inherent in teaching integers and a commitment to adopting diverse strategies, such as manipulatives, drills, and games, to enhance comprehension and retention.

As the research unfolds, it becomes evident that teachers' endorsement of hands-on activities and thoughtful engagement strategies aligns with established pedagogical principles. This synergy between theoretical insights and practical recommendations underscores the significance of an adaptive and participatory teaching approach in navigating the intricacies of integer education for grade 7 students.

The chart (figure 2) also shows the process of learning integers in grade 7, along with the resulting framework generated in the outcome of this research.

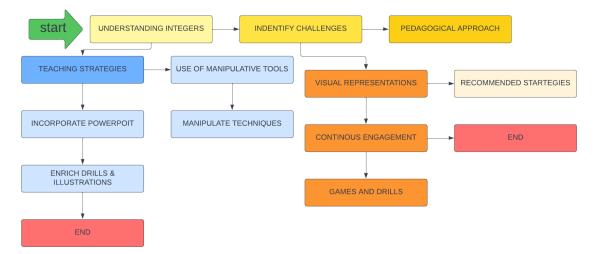


Figure 2: The pedagogical approach diagram.

The research commences by exploring the complexities of teaching integers to grade 7 students, unveiling challenges and insights. A structured pedagogical approach, encapsulated in the 3Rs framework, emerges as a comprehensive strategy. The approach includes specific teaching strategies, such as PowerPoint presentations and enriched drills, visual representations utilizing manipulative tools like algebra tiles, and recommendations for continuous engagement. Teaching strategies are further detailed, emphasizing the use of PowerPoint presentations enriched with illustrations and the importance of enriched drills. Visual representation techniques, including manipulative tools like algebra tiles and flip coins, are identified as crucial components of the pedagogical approach. The flowchart concludes with the implementation of the outlined strategies, providing a comprehensive framework for addressing challenges in teaching integers.

3.6. Perceived impacts on teachers and students

Teachers' recognition of the challenges in teaching integers underscores the vital role of this foundational concept. The perceived impacts align with Maslow's Hierarchy of Needs [27], emphasizing fulfilling the fundamental need for understanding foundational concepts before students can progress to higher-order mathematical thinking.

The journey to attaining learning competency in integers has unveiled a consensus among educators regarding the inherent challenges in teaching this abstract mathematical concept. Teachers, upon reflection, recognize the intricacies involved in effectively imparting the understanding of integers to students. The significance of this competency is underscored by its pivotal role as a foundational knowledge base, essential for students progressing to more advanced mathematical concepts in subsequent academic levels.

Supporting this realization are findings from various studies, including those by Almeida and Bruno [2], Bofferding [7], Vlassis [41, 42], which emphasize the prevalence of challenges associated with negative numbers among high school students. The synthesis of these studies corroborates the teachers' observations, reinforcing the notion that a comprehensive grasp of integers is imperative for students navigating the complexities of mathematical education.

After insightful participant experiences, several pedagogical approaches emerge as recommendations for effectively teaching integers. T1 advocates for the early incorporation of manipulatives to represent integers, particularly emphasizing their utility in elucidating the abstract notions of negative and positive numbers. T2, on the other hand, highlights the effectiveness of Bell Work as a precursor to group games in introducing integers. Additionally, T3 proposes the integration of daily drills post-introduction to fortify the retention of integer concepts among learners.

However, amidst the diversity of teacher approaches, T3 posits a noteworthy recommendation – a call for a uniform teaching approach. This proposition, rooted in recognizing a class's varied needs and challenges, advocates for an organized and standardized instructional strategy. While acknowledging the potential benefits of additional approaches tailored to specific class dynamics, T3 emphasizes the advantages of a cohesive and uniform student understanding. According to T3, this approach offers a measure of guidance and assistance, particularly beneficial for learners who may require additional support.

As the researcher acknowledges the validity of T3's claim, the quest for uniformity in the teaching of integers emerges as a nuanced consideration. While diversity in instructional approaches is acknowledged, an underpinning advantage exists in fostering a common understanding among students. This approach not only promotes consistency but also addresses the varying needs of learners, contributing to a more inclusive and effective learning environment.

4. Conclusion

This research significantly contributes to the theoretical understanding of the challenges associated with the learning competency of integers among grade 7 Mathematics students. The study establishes the 3Rs – Reaching, Representing, and Recommended Strategies – as a valuable conceptual framework, encapsulating teachers' pedagogical approaches. It reveals that the perceived difficulty lies in the addition and subtraction of integers, marking these operations as crucial focal points for educators. Manipulatives emerge as a cornerstone pedagogical tool, unanimously endorsed by teachers, providing a structured and practical approach to teaching abstract mathematical concepts. Integrating manipulatives and drill exercises proves to be a potent combination for enhancing students' comprehension and retention of integer concepts. Despite these valuable contributions, the study acknowledges limitations, primarily focusing on grade 7 Mathematics. It highlights the need for future research to explore the long-term impact of these strategies across different academic levels. The proposed pedagogical framework guides teachers in navigating the complexities of teaching integers and advocates for a uniform teaching approach, emphasizing consistency and inclusivity. The study's significance lies in its practical implications for educators, providing insights into transformative teaching methods that can positively affect students' learning experiences. This research is a foundation for future investigations, encouraging a deeper exploration of effective teaching methodologies and their transferability to diverse mathematical concepts and academic levels. Ultimately, the findings contribute to the ongoing discourse on improving the quality of education by fostering a more robust and engaging learning environment for students grappling with the complexities of mathematical competency in the realm of integers.

5. Limitations and recommendations

In the pursuit of unravelling the complexities associated with the learning competency of integers among grade 7 students, this research has not only provided valuable insights but has also encountered certain limitations that warrant careful consideration. Recognizing and addressing these limitations is essential for refining the study's scope and ensuring the applicability of its findings.

A primary limitation lies in the study's exclusive focus on grade 7 Mathematics teachers. While the insights gained are undoubtedly valuable for this specific academic level, the transferability of the findings to other grades remains uncertain. The dynamics of mathematical comprehension and effective teaching strategies may vary across different grade 7 students in different schools. Therefore, it is imperative for future research to broaden its scope, encompassing different grade 7 learners in different schools, to enrich the understanding of how students across various schools grapple with the intricacies of integer concepts.

Another noteworthy limitation pertains to the predominant emphasis on the 3Rs conceptual framework — Reaching, Representing, and Recommended Strategies. While these components offer a comprehensive understanding of teachers' pedagogical approaches, it is essential to acknowledge that there may be other influential factors and teaching methodologies that impact the learning competency of integers. Future studies should explore additional variables, ensuring

a more nuanced and holistic comprehension of the multifaceted nature of teaching and learning integer concepts.

Furthermore, the research primarily centres on the challenges associated with the addition and subtraction of integers. While these operations undoubtedly form the bedrock of integer comprehension, a comprehensive examination encompassing other mathematical operations involving integers would provide a more holistic perspective on students' difficulties. A more inclusive approach would contribute to a comprehensive understanding of the intricacies associated with various integer operations.

In addition to these considerations, the study has yet to delve deeply into socio-economic and cultural factors that may influence students' learning experiences with integers. Understanding how these external factors intersect with pedagogical approaches could offer valuable insights into tailoring teaching strategies to cater to diverse backgrounds. Future research should consider exploring the intersection of socio-economic and cultural factors with the teaching and learning of integers to foster a more inclusive educational environment.

While this research significantly advances our understanding of teaching integers, it is crucial to view its findings within the context of these acknowledged limitations. These limitations serve as stepping stones for future investigations, guiding researchers toward refining methodologies, expanding the scope, and embracing a more inclusive approach. By addressing these limitations, future research endeavours can contribute to a more robust and nuanced understanding of effective teaching methodologies, ensuring that the benefits of transformative pedagogical approaches extend across diverse mathematical concepts and academic levels.

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A. Guide questions

- 1. What learning competency in the mathematics curriculum do you think is difficult for the learners to understand?
- 2. If we will get the percentage of your students who cannot attain this learning competency, what would it be?
- 3. How do you help these students, then?
- 4. What strategies did you use to teach the concept, ma'am? Do you use manipulatives in teaching integers?
- 5. How do you conduct these strategies?
- 6. What are the results of these strategies for the learners?
- 7. Which strategies will you recommend as the most effective in teaching integers?
- 8. So, what will you recommend in teaching integers for better understanding?