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Chaos Control in the Nonlinear Bloch Equations using Recursive Active Control

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abstract

The problem of chaos control in the nonlinear Bloch equations is considered based on a modified active control technique. In the proposed control scheme a recursive approach and active control mechanism are combined to design control functions that drive the nonliner Bloch equations to a steady state as well as track a desired trajectory in a systematic way. The effeciency of the proposed Recursive Active Control (RAC) is demonstrated with numerical simulations.

Keywords: Chaos; Bloch equations; recursive backstepping; recursive active control
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I. INTRODUCTION

Many physical systems can exhibit chaotic dynamics under certain conditions. Chaotic behaviours could be beneficial feature in some cases, but can be undesirable in some engineering and other physical applications; and therefore it is often desired that chaos should be controlled, so as to improve the system performance. Thus, it is of considerable interest and potential utility, to devise control techniques capable of forcing a system to maintain a desired dynamical behaviour even when intrisically chaotic. The control of chaos and bifurcation is concerned with using some designed control input(s) to modify the characteristics of a parameterized nonlinear system. The control can be static or dynamic feedback control, or open-loop control. In most cases, the goal could be the stabilization and reduction of the amplitude of bifurcation orbital solutions, optimization of a performance index near bifurcation, reshapening of the bifurcation diagram or a combination of these[1−3].

For almost two decades, there has been intense research activities devoted to the design of effective chaos control techniques. A large number of the proposed methods are based on the Ott, Grebogi and Yorke (OGY) closed-loop feedback method[4] and the Pyragas time-delayed auto-synchronization (TDAS) method[5]. In the recent times, numerous linear and nonlinear control methods have emerged. In particular, recursive backstepping nonlinear control scheme has been employed recently for controlling, tracking and synchronizing chaotic systems[6−12]. Recursive backstepping which is a systematic design approach that consists of a recursive procedure that skillfully interlaces the choice of a Lyapunov function with the control; was proposed by Harb and Harb[6] for the third-order phase-locked-loops.

In another development, Bai and Lonngren proposed an active control method for chaos synchronization[13]. The active control scheme has in the last one decade received considerable attention due to its simplicity and has been widely accepted as an effecient technique for synchronization of identical and non-identical chaotic systems (See for example Refs.[14–23] and refs. therein). Very recently, we reported the control of directed transports arising from co-existing attractors in ratchet motion using the active control mechanism[23]. Zhang et al[10] introduced an active-backstepping mechanism of controlling and tracking chaotic hyperchaotic systems by combining the active control and backstepping control methods.

It is known that chaos synchronization is closely related to observer problem in control theory[24]. Previous applications of the active control techniques tackles the observer problem from synchronization point of view; where a drive-response system configuration is employed[10–23]. Here, we tackle the problem based on suppression and tracking approach that does not require a response system as employed in[10]. To achieve this, a recursive active control (RAC) for controlling chaos is proposed for the nonlinear Bloch equations (NBE). The method combines recursive approach with active control technique to design control functions that can suppress chaos as well track any desired trajectory in the NBE.

II. THE NONLINEAR BLOCH EQUATIONS

In view of the need to interpret various anomalies that had been observed in nuclear magnetic resonance (NMR) experiments, in terms of chaos theory, Abergel[25], recently examined the linear set of equations originally proposed by Bloch to describe the dynamics of an ensemble of spins with minimal coupling. The model incorporates certain nonlinear effects arising from radiation damping based on feedback and consists of the three nonlinear modified Bloch equations (NBE) given in dimensionless units as

$$\begin{aligned}
\dot{x} &= \delta y + \lambda z(x\sin\psi - y\cos\psi) - \frac{x}{\tau_2},\\
\dot{y} &= -\delta x - z + \lambda z(x\cos\psi + y\sin\psi) - \frac{y}{\tau_2},\\
\dot{z} &= y - \lambda\sin\psi(x^2 + y^2) - \frac{z-1}{\tau_1},
\end{aligned} \tag{2.1}$$

where the dots denotes time derivatives, δ, λ, and ψ are the system parameters; and τ_1 and τ_2 are longitudinal time and transverse relaxation time respectively. The dynamics of system (2.1) has been extensively studied in Ref.[17,25] for a fixed subset of the system parameters $(\delta, \lambda, \tau_1, \tau_2)$ and for a space area range of the radiation damping feedback ψ. The regions of ψ that would admit chaotic solutions were obtained. For example, the NBE exhibits chaotic behaviour for $\delta = -0.4\pi$, $\lambda = 30, \psi = 0.173, \tau_1 = 5$ and $\tau_2 = 2.5$ as shown in Fig. 1 and Fig. 2.

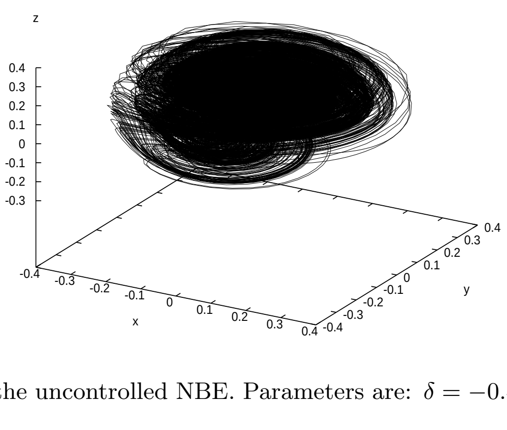

FIG. 1. Chaotic dynamics of the uncontrolled NBE. Parameters are: $\delta = -0.4\pi$, $\lambda = 30, \psi = 0.173, \tau_1 = 5$ and $\tau_2 = 2.5$

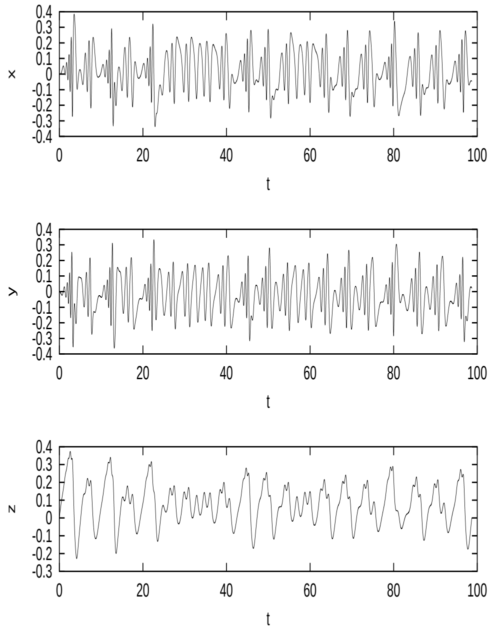

FIG. 2. Time history of the NBE system for the same parameter set as in Fig. 1

In recent study, Moukam Kakmeni, Nguenang and Kofane[27], examined the dynamics of a variant NBE, extended to account for both the bi-axial property of the magnets to which the set of spins belongs and the presence of a back action from the probe. In[17,26,27], the synchronization behaviours of the NBE were also reported. For instance, Ucar et al.[17] studied the synchronization of drive-reponse system of the NBE with non-identical parameters using active control while in Ref.[26], Park studied the synchronization of the NBE with uncertain parameters. Moukam Kakmeni et. al.[27], considered the synchronization problem based on adaptive approach, using both linear and nonlinear feedback couplings. In all these reports, the control of the NBE chaotic behaviour to regular dynamics has not been addressed. In this paper, we set up a modified active control for the NBE chaotic system.

III. RAC FOR THE NONLINEAR BLOCH EQUATION

To control the NBE chaotic attractor, we introduce the control functions $u_i (i = 1, 2, 3)$ as follows

$$\begin{aligned} \dot{x} &= \delta y + \lambda z(x \sin\psi - y\cos\psi) - \frac{x}{\tau_2} + u_1, \\ \dot{y} &= -\delta x - z + \lambda z(x\cos\psi + y\sin\psi) - \frac{y}{\tau_2} + u_2, \\ \dot{z} &= y - \lambda \sin\psi(x^2 + y^2) - \frac{z-1}{\tau_1} + u_3, \end{aligned} \tag{3.1}$$

and define the error dynamics as

$$\begin{aligned} e_x &= x - x_{1d}, \\ e_y &= y - x_{2d}, \\ e_z &= z - x_{3d}. \end{aligned} \tag{3.2}$$

For simplicity let

$$\begin{aligned} x_{1d} &= 0, \\ x_{2d} &= c_1 e_x, \\ x_{3d} &= c_2 e_x + c_3 e_y, \end{aligned} \tag{3.3}$$

where the $c_i's(i = 1, 2, 3)$ are arbitrary control gains; x_{1d} is the reference output; $x_{2d},\ x_{3d}$ are recursively introduced control inputs. Now, differentiating eq. (3.2) and (3.3); and substituting eqs. (3.1) into the resulting equations, we obtain the following error dynamics system:

$$\begin{aligned}
\dot{e}_x &= \delta e_y + \lambda e_z(e_x \sin\psi - e_y \cos\psi) - \frac{e_x}{\tau_2} + u_1,\\
\dot{e}_y &= -\delta e_x - e_z + \lambda e_z(e_x \cos\psi + e_y \sin\psi) - \frac{e_y}{\tau_2} + u_2,\\
\dot{e}_z &= e_y - \lambda \sin\psi(e_x^2 + e_y^2) - \frac{e_z - 1}{\tau_1} + u_3.
\end{aligned} \tag{3.4}$$

In (3.4), the $c_i's(i = 1, 2, 3)$ have be chosen so that the $\dot{e}_j(j = x, y, z)$ terms on the RHS varnishes. Since the $c_i's(i = 1, 2, 3)$ are arbitrary control gains, it is convenient, without loss of generality, to set $c_1 = c_2 = c_3 = 0$. In the absence of the control $u_i(i = 1, 2, 3)$, eq. (3.4) would have an equilibrium at $(0, 0, 0)$. If a $u_i(i = 1, 2, 3)$ is chosen such that the equilibrium remains unchanged, then the problem can be transformed to that of realizing asymptotic stabilization of system (3.4). Thus, the goal is to find the controls such that the system (3.4) is stabilized at the origin. Following the original method of active control, we re-define the control functions as follows

$$\begin{aligned}
u_1 &= V_1 - \lambda e_z(e_x \sin\psi - e_y \cos\psi),\\
u_2 &= V_2 - \lambda e_z(e_x \cos\psi + e_y \sin\psi),\\
u_3 &= V_3 + \lambda \sin\psi(e_x^2 + e_y^2) - \frac{1}{\tau_1}.
\end{aligned} \tag{3.5}$$

With (3.5), the error dynamics (3.4) becomes:

$$\begin{aligned}
\dot{e}_x &= \delta e_y - \frac{e_x}{\tau_2} + V_1,\\
\dot{e}_y &= -\delta e_x - e_z - \frac{e_y}{\tau_2} + V_2,\\
\dot{e}_z &= e_y - \frac{e_z}{\tau_1} + V_3.
\end{aligned} \tag{3.6}$$

We choose a feedback matrix **A** which will control the error dynamics (3.6) such that

$$\begin{pmatrix} V_1(t) \\ V_2(t) \\ V_3(t) \end{pmatrix} = \mathbf{A} \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix} \tag{3.7}$$

With

$$\mathbf{A} = \begin{pmatrix} k_1 + \frac{1}{\tau_2} & -\delta & 0 \\ \delta & k_2 + \frac{1}{\tau_2} & 1 \\ 0 & -1 & k_3 + \frac{1}{\tau_1} \end{pmatrix} \tag{3.8}$$

With matrix **A** as above, we obtain the following control functions:

$$\begin{aligned}
u_1 &= (k_1 + \frac{1}{\tau_2})e_x - \delta e_y - \lambda e_z(e_x \sin\psi - e_y \cos\psi)\\
u_2 &= \delta e_x + (k_2 + \frac{1}{\tau_2})e_y + e_z - \lambda e_z(e_x \cos\psi + e_y \sin\psi)\\
u_3 &= -e_y + (k_3 + \frac{1}{\tau_1})e_z + \lambda \sin\psi(e_x^2 + e_y^2) - \frac{1}{\tau_1}
\end{aligned} \tag{3.9}$$

The three eigenvalues k_1, k_2 and k_3 in eq. (3.8) and (3.9) play significant role in ensuring stable controlled state. If k_1, k_2 and k_3 are negative definite, a global stability of the controlled state is achieved. In fig. 3, we present the numerical simulation when when $k_1 = k_2 = k_3 = -1$. Here, the controls are simultneously switched on at $t \geq 100s$. It is clear from the numerical simulation shown in

Fig. 3 that the chaotic behaviour has been controlled as soon as the control is activated. We found also that when one of the $k_i (i = 1, 2, 3)$ is set at zero while the others are made negative definite, control is also achieved. However, the dynamics of the state variable associated with the zero eigenvalue in this case is not confined to the zero oscillation; anf if k_i are all zero, no control is achieved.

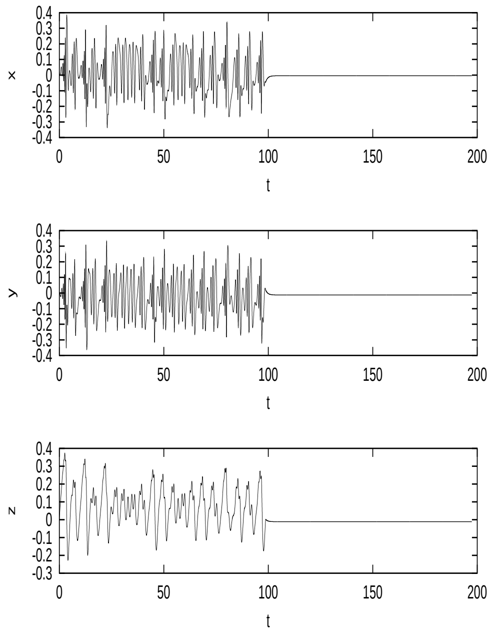

FIG. 3. Time history of the NBE system when control has been activated at $t = 100$ for the same parameter set as in Fig. 1

IV. DESIGN OF RAC FOR TRACKING TRAJECTORIES

Suppose a trajectory, $A \sin \omega t$ is desired, then we can employ the above technique to track the time evolution of its orbit. To show this, we define the error between the state variables and the desired trajectory as follows:

$$\begin{aligned} e_x &= x - x_{1t}, \\ e_y &= y - x_{2t}, \\ e_z &= z - x_{3t}. \end{aligned} \tag{4.1}$$

where

$$\begin{aligned} x_{1t} &= A \sin \omega t, \\ x_{2t} &= c_1 e_x, \\ x_{3t} &= c_2 e_x + c_3 e_y. \end{aligned} \tag{4.2}$$

Proceeding as before, we obtain the following error dynamics

$$\begin{aligned}
\dot{e}_x &= \delta e_y + \lambda e_z[(e_x + A\sin\omega t)\sin\psi - e_y\cos\psi] \\
&\quad -\frac{e_x}{\tau_2} - \frac{A\sin\omega t}{\tau_2} + u_x \\
\dot{e}_y &= -\delta e_x + \lambda e_z[(e_x + A\sin\omega t)\cos\psi + e_y\sin\psi] \\
&\quad -\delta A\sin\omega t - e_z - \frac{e_y}{\tau_2} + u_y \\
\dot{e}_z &= e_y - \lambda\sin\psi[(e_x + A\sin\omega t)^2 + e_y^2] \\
&\quad -\frac{e_z - 1}{\tau_1} + u_z
\end{aligned} \tag{4.3}$$

where $u_i (i = x, y, z)$ are the tracking control functions to be determined. Re-defining the control functions as follows:

$$\begin{aligned}
u_x &= v_x - \lambda e_z[(e_x + A\sin\omega t)\sin\psi - e_y\cos\psi] \\
&\quad -\frac{A\sin\omega t}{\tau_2} - A\omega\sin\omega t, \\
u_y &= v_y - \lambda e_z[(e_x + A\sin\omega t)\cos\psi + e_y\sin\psi], \\
u_z &= v_z + \lambda\sin\psi[(e_x + A\sin\omega t)^2 + e_y^2] - \frac{1}{\tau_1},
\end{aligned} \tag{4.4}$$

we obtain the following error dynamics system:

$$\begin{aligned}
\dot{e}_x &= \delta e_y - \frac{e_x}{\tau_2} + v_x, \\
\dot{e}_y &= -\delta e_x - e_z - \frac{e_y}{\tau_2} + v_y, \\
\dot{e}_z &= e_y - \frac{e_z}{\tau_1} + v_z.
\end{aligned} \tag{4.5}$$

From eq. (3.7) and (3.8), we propose the following control functions

$$\begin{aligned}
u_x &= (k_1 + \frac{1}{\tau_2})e_x - \delta e_y \\
&\quad -\lambda e_z[(e_x + A\sin\omega t)\sin\psi - e_y\cos\psi] \\
u_y &= \delta e_x + (k_2 + \frac{1}{\tau_2})e_y + e_z \\
&\quad -\lambda e_z[(e_x + A\sin\omega t)\cos\psi + e_y\sin\psi] \\
u_z &= -e_y + (k_3 + \frac{1}{\tau_1})e_z \\
&\quad +\lambda\sin\psi[(e_x + A\sin\omega t)^2 + e_y^2] - \frac{1}{\tau_1}
\end{aligned} \tag{4.6}$$

We perform numerical simulations of the NBE with the controllers (4.6) activated at $t \geq 100s$ for $A = 0.2$ and $\omega = 0.67$; all other parameters are fixed as before. The results are displayed in Fig. 4. The vibration of the z variable appears to exhibit zero oscillation in the presence of the controllers. However, a zoom of the control region dipicts an oscillation with amplitude of 0.2. Thus, comfirming that global tracking of the orbit is achieved.

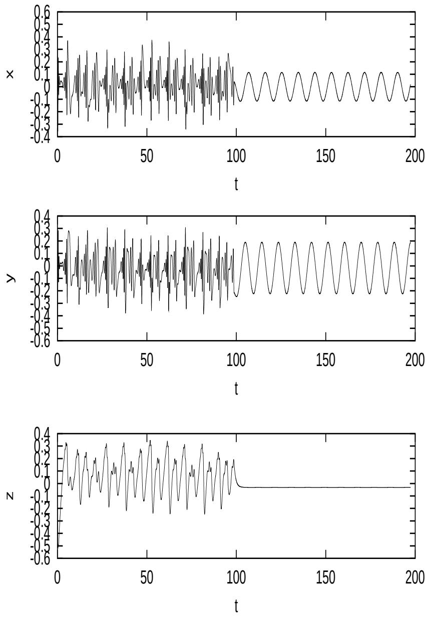

FIG. 4. Time history of the NBE system when tracking trajectory $x_{1,t} = 0.2 \sin 0.67t$ at $t \geq 100$ for the same parameter set as in Fig. 1

V. CONCLUSION

In this paper, a recursive approach combined with the active control technique has been used to formulate a technique for suppressing the chaotic behaviour in nonlinear Bloch equations. The proposed Recursive Active Control (RAC) has also been found effective for tracking a desired trajectory. The proposed method guarantee global stability, excellent transience performance and is simple to implement. Numerical simulations have been employed to confirm our results.

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