Parameter-Dependent Robust H_∞ Filtering for Uncertain Discrete-Time Systems: A Polynomial Approach

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Abstract—**this paper proposes a method for robust H_∞ filtering for discrete systems with polytopic uncertainties. The parameter uncertainties considered in this paper are assumed to be of polytopic type. A new structured polynomially parameter-dependent method is utilized, which is based on homogeneous polynomially parameter-dependent matrices of arbitrary degree. The proposed method includes results in the quadratic framework and the linearly parameter-dependent framework as special cases for zeroth degree and first degree, respectively. The filter parameters can be obtained from the solution of convex optimization problems in terms of linear matrix inequalities. A numerical example illustrates the feasibility and advantage of the proposed filter design methods.**

Keywords-Discrete systems, uncertain systems, H_∞ filtering, linear matrix inequality (LMI), homogeneous polynomially parameter-dependent (HPPD).

I. INTRODUCTION

The H_∞ filtering issue is introduced by the authors in [1], and its main aim is to design an estimator to minimize the H_∞ norm of the filtering error system so as to ensure that the L_2-induced gain from the noise signals to the estimation error will be less than a prescribed level. In contrast with the traditional Kalman filtering, the H_∞ filtering approach does not require the exact knowledge noise signals, which renders this approach very appropriate in many applications in biology, economics, aerospace, population analysis etc. However, when a priori information on the external noise is not available, the Kalman filtering scheme is no longer applicable. In such cases, H_∞ filter was introduced in [2], which provides both a guaranteed noise attenuation level and robustness against unmodelled dynamics. A great number of results and the H_∞ filtering have been proposed in the literature in both the deterministic end stochastic contexts; see e.g. [3]-[10], and the references therein. When parameter uncertainties appear in a system model, the robust H_∞ filtering has been investigated, and some results on this topic have been presented; see, e.g. [11]-[18], and the references therein. Note that all these mentioned H_∞ filtering results are obtained in the context of one-dimensional (1D) system.

Thus, based on the structured polynomially parameter dependent method, in this paper, we will complete the robust H_∞ filtering problem for uncertain discrete-time systems. This is carried out by applying a parameter-dependent polynomial filter approach to the robust H_∞ filtering. The idea proposed here exploits the positivity of the uncertain parameters belonging to the unit simplex, constructed in such a way that additional free variables are generated, as the degree g of the polynomial filter matrices increases, defining a sequence of sufficient LMI conditions of increasing precision, which provides a sequence of less conservative evaluations. Moreover, it is shown that if the conditions are fulfilled for a certain degree $\hat{g}$, then a feasible solution exists for all degrees $g > \hat{g}$. The condition proposed reduces when g = 0 to the filter design method in the quadratic framework given in [19], and are equivalent to the sufficient LMI tests based on affine parameter-dependent Lyapunov function for g = 1. For any fixed degree g, the family of LMI condition is constructed following simple rules, in terms of the vertices of the polytope.
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that all these mentioned H_∞ filtering results are obtained in the context of one-dimensional (1D) system.

Notation: The notation used throughout the paper is quite standard. $\Re^n$ is the n-dimensional Euclidean space, and $\Re^{n\times m}$ is the set of $n\times m$ real matrices. Π^T stands for the transpose of a matrix Π, and $\Pi > 0 (< 0)$ means that the symmetric matrix P is positive definite (negative definite), I is the identity matrix with appropriate dimension and the symmetric term in a symmetric matrix is denoted by *, e.g., $\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} = \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}$. Matrices, if not explicitly stated, are assumed to have compatible dimension

II. PROBLEM FORMULATION

Considering the following stable discrete-time linear system:

$$\begin{cases} x(k+1) = A(\alpha)x(k) + B(\alpha)w(k) \\ y(k) = C(\alpha)x(k) + D(\alpha)w(k) \\ z(k) = L(\alpha)x(k) \end{cases} \tag{1}$$

where $x(0)=0$, $x(k)\in\Re^n$ is the state variable, $y(k)\in\Re^m$ is the measurement output, $z(k)\in\Re^p$ is the signal to be estimated, $w(k)\in\Re^l$ is the disturbance input belongs to $l_2[0,\infty)$. $A(\alpha)$, $B(\alpha)$, $C(\alpha)$, $D(\alpha)$ and $L(\alpha)$ are uncertain matrices with appropriate dimensions. It is assumed that (α) is time-invariant and

$$\mathcal{P} \triangleq \left\{ \begin{bmatrix} A(\alpha) & B(\alpha) \\ C(\alpha) & D(\alpha) \\ L(\alpha) & 0 \end{bmatrix} = \sum_{i=1}^{N} \alpha_i \begin{pmatrix} A_i & B_i \\ C_i & D_i \\ L_i & 0 \end{pmatrix}, \quad \alpha \in \Omega \right\}..$$

Where Ω is the unit simplex

$$\Omega = \left\{ (\alpha_1, \alpha_2, ..., \alpha_N) : \sum_{i=1}^{N} \alpha_i = 1, \alpha_i \geq 0 \right\}$$

the robust H_∞ filtering problem considered in this paper is to estimate the signal $z(k)$ by using a parameter-dependent filter described by

$$\begin{cases} x_f(k+1) = A_f(\alpha)x_f(k) + B_f(\alpha)y(k) \\ z_f(k) = C_f(\alpha)x_f(k) + D_f y(k) \end{cases} \tag{2}$$

where $x_f(0)=0$, $x_f(k)\in\Re^{n_f}$, and $z_f(k)\in\Re^{p_f}$, are the state and the output of the filter respectively. The matrices $A_f(\alpha)$, $B_f(\alpha)$, $C_f(\alpha)$, and $D_f(\alpha)$ are parameter-dependent filter matrices with appropriate dimensions to be determined.

Remark 1: When $n_f = n$ the filter model (2) is a full order, and when $1 \leq n_f < n$, , that is a reduced-order filter, but in this paper we will study only the full-order i.e. $n_f = n$.

Defining the augmented state vector $\xi(0)=0$, $\xi(k) = \begin{bmatrix} x^T(k) & x_f^T(k) \end{bmatrix}^T$ and $e(k) = z(k) - z_f(k)$, we can obtain the following filtering error system

$$\begin{cases} \xi(k+1) = \bar{A}(\alpha)\xi(k) + \bar{B}(\alpha)w(k) \\ e(k) = \bar{C}(\alpha)\xi(k) + \bar{D}w(k) \end{cases} \tag{3}$$

Where

$$\bar{A}(\alpha) = \begin{bmatrix} A(\alpha) & 0 \\ B_f(\alpha)C(\alpha) & A_f(\alpha) \end{bmatrix},$$

$$\bar{B}(\alpha) = \begin{bmatrix} B(\alpha) \\ B_f(\alpha)D(\alpha) \end{bmatrix},$$

$$\bar{C}(\alpha) = \begin{bmatrix} L(\alpha) - D_f(\alpha)C(\alpha) & -C_f(\alpha) \end{bmatrix},$$

$$\bar{D}(\alpha) = -D_f(\alpha)D(\alpha)$$

The filtering problems to be dealt with can be stated as follows.

Problem 1: Find matrices $A_f \in \Re^{n_f\times n_f}$, $B_f \in \Re^{n_f\times m}$, $C_f \in \Re^{p_f\times n_f}$ and $D_f \in \Re^{p_f\times m}$ of the filter realization (2), such that the estimation error system (3) is asymptotically stable, and an upper bound γ to the H_∞ estimation error performance is assured, that is,

$$\sup_{\|w\|_2 \neq 0, w(k)\in L[0,\infty)} \frac{\|e(k)\|_2}{\|w(k)\|_2} < \gamma, \forall \alpha \in \Omega. \tag{4}$$

Definition 1: The estimation error dynamics (3) is said to be quadratically stable with an H_∞ guaranteed cost γ if there exist $\gamma > 0$ and matrices (G, P, F) with $P = P^T > 0$ such that

$$\begin{bmatrix} G+G^T+P & * & * & * \\ 0 & I & * & * \\ \bar{A}^T G^T - F & \bar{C}^T & P - F\bar{A} - \bar{A}^T F^T & * \\ \bar{B}^T G^T & \bar{D}^T & -\bar{B}^T F^T & \gamma_\infty^2 I \end{bmatrix} > 0 \qquad (5)$$

Lemma 1: Given a scalar $\gamma > 0$, the discrete-time system is asymptotically stable and satisfies the H_∞ performance $\frac{\|e(k)\|_2}{\|w(k)\|_2} < \gamma$ if there exist matrices $P_{11}(\alpha),\ P_{12}(\alpha),\ P_{22}(\alpha)$, $G_{11},\ G_{21},\ G_2,\ F_{11},\ F_{21},\ S_a(\alpha),\ S_b(\alpha),\ S_c(\alpha)$ and $S_d(\alpha)$ such that the following LMI holds:

$$\begin{bmatrix} \Upsilon(\alpha) & G_2+G_{21}^T-P_{12}(\alpha) & 0 & \Pi(\alpha) & S_a(\alpha)-F_{21}^T & G_{11}B(\alpha)+S_b(\alpha)D(\alpha) \\ * & G_2+G_2^T-P_{22}(\alpha) & 0 & \Phi(\alpha) & S_a(\alpha)-\lambda_2 G_2^T & G_{21}B(\alpha)+S_b(\alpha)D(\alpha) \\ * & * & I & \Xi(\alpha) & -S_c(\alpha) & -S_d(\alpha)D(\alpha) \\ * & * & * & \Psi_{11}(\alpha) & \Psi_{12}(\alpha) & -F_{11}B(\alpha)-\lambda_1 S_b(\alpha)D(\alpha) \\ * & * & * & * & \Psi_{22}(\alpha) & -F_{21}B(\alpha)-\lambda_2 S_b(\alpha)D(\alpha) \\ * & * & * & * & * & \gamma_\infty^2 I \end{bmatrix} > 0 \qquad (6)$$

Where

$$\begin{aligned} \Upsilon(\alpha) &= G_{11}+G_{11}^T-P_{11}(\alpha) \\ \Pi(\alpha) &= G_{11}A(\alpha)+S_b(\alpha)C(\alpha)-F_{11}^T \\ \Psi_{11}(\alpha) &= P_{11}(\alpha)-F_{11}A(\alpha)-A^T(\alpha)F_{11}^T \\ &\quad -\lambda_2(S_b(\alpha)C(\alpha)+C^T(\alpha)S_b^T(\alpha)) \\ \Psi_{12}(\alpha) &= P_{12}(\alpha)-\lambda_1 S_a(\alpha)+A^T(\alpha)F_{21}^T-\lambda_2 C^T(\alpha)S_b^T(\alpha) \\ \Psi_{22}(\alpha) &= P_{22}(\alpha)-\lambda_2(S_a(\alpha)+S_a^T(\alpha)) \\ \Xi(\alpha) &= L(\alpha)-S_d(\alpha)C(\alpha) \\ \Phi(\alpha) &= G_{21}A(\alpha)+S_b(\alpha)C(\alpha)-\lambda_1 G_2^T \end{aligned}$$

Now before presenting the Lemma 1 in HPPD, some definitions and preliminaries are needed to represent and to handle products and sums of homogeneous polynomials.
First, define the HPPD matrices of arbitrary degree g by

$$S_{a_g}(\alpha) = \sum_{j=1}^{J(g)} \alpha_1^{k_1}\alpha_2^{k_2}\ldots\alpha_N^{k_N} S_{a_{\Re_j(g)}} \qquad (7)$$

$$S_{b_g}(\alpha) = \sum_{j=1}^{J(g)} \alpha_1^{k_1}\alpha_2^{k_2}\ldots\alpha_N^{k_N} S_{b_{\Re_j(g)}} \qquad (8)$$

$$S_{c_g}(\alpha) = \sum_{j=1}^{J(g)} \alpha_1^{k_1}\alpha_2^{k_2}\ldots\alpha_N^{k_N} S_{c_{\Re_j(g)}} \qquad (9)$$

$$S_{d_g}(\alpha) = \sum_{j=1}^{J(g)} \alpha_1^{k_1}\alpha_2^{k_2}\ldots\alpha_N^{k_N} S_{d_{\Re_j(g)}} \qquad (10)$$

with

$$k_1 k_2 \ldots k_N = \Re_j(g)$$

The notations in the above are explained as follows. $\alpha_1^{k_1}\alpha_2^{k_2}\ldots\alpha_N^{k_N}$, $\alpha \in \Omega$, $k_i \in \mathbb{N}$, $i=1,\ldots,N$ are the monomials, $S_{a_{\Re_j(g)}}$, $S_{b_{\Re_j(g)}}$, $S_{c_{\Re_j(g)}}$, and $S_{d_{\Re_j(g)}}$, are matrices valued coefficients. Here, by definition, $\Re_j(g)$ is the jth N-tuples of $\Re(g)$ which is lexically ordered, $j=1,\ldots,\Im(g)$ and $\Re(g)$ is the set of N-tuples obtained as all possible combinations of $k_1 k_2 \ldots k_N$, $k_i \in \mathbb{N}$, $i=1,\ldots,N$ such that $k_1+k_2+\ldots+k_N = g$. Since the number of vertices in the polytope $\mathcal{P}$ is equal to N, the number of elements in $\Re(g)$ is given by $\Im(g) = (N+g-1)!/(g!(N-1)!)$.

For each $i=1,\ldots,N$ define the N-tuples $\Re_j^i(g)$, that are equal to $\Re_j(g)$, but with $k_i > 0$ replaced by $k_i - 1$. Note that the N-tuples $\Re_j^i(g)$ are defined only in the cases where the corresponding k_i is positive. Note also that, when applied to the elements of $\Re(g+1)$, the N-tuples $\Re_j^i(g+1)$ define subscripts $k_1 k_2 \ldots k_N$ of matrices $S_{a_{k_1k_2\ldots k_N}}$, $S_{b_{k_1k_2\ldots k_N}}$, $S_{c_{k_1k_2\ldots k_N}}$, and $S_{d_{k_1k_2\ldots k_N}}$, associated to homogeneous polynomial parameter-dependent matrices of degree g. Finally, define the scalar constant coefficients $\beta_j^i(g+1) = g!/(k_1!k_2!\ldots k_N!)$, with $k_1 k_2 \ldots k_N \in \Re_j^i(g+1)$.

Using this notation we now present the Theorem1.

Theorem 1: Given a scalar $\gamma > 0$, the discrete-time system is asymptotically stable and satisfies the H_∞ performance

$\dfrac{\left\| e(k) \right\|_2}{\left\| w(k) \right\|_2} < \gamma$ if there exist matrices $P_{11_{\Re_j(g)}}$, $P_{12_{\Re_j(g)}}$, $P_{22_{\Re_j(g)}}$, G_{11}, G_{21}, G_2, F_{11}, F_{21}, $S_{a_{\Re_j(g)}}$, $S_{b_{\Re_j(g)}}$, $S_{c_{\Re_j(g)}}$ and $S_{d_{\Re_j(g)}}$, such that $\forall \Re_l(g+l) \in \Re(g+l)$, $l = 1,...,\Im(g+1)$ satisfying the following LMI holds:

$$\sum_{i \in \Re_l(g+1)} \begin{bmatrix} \Upsilon_{\Re_l^i(g+1)} & \Gamma_{1_{\Re_l^i(g+1)}} & 0 & \Pi_{\Re_l^i(g+1)} & \Gamma_{3_{\Re_l^i(g+1)}} & \beta_l^i(g+1)G_{11}B_i + S_{b_{\Re_l^i(g+1)}}D_i \\ * & \Gamma_{2_{\Re_l^i(g+1)}} & 0 & \Phi_{\Re_l^i(g+1)} & \Gamma_{4_{\Re_l^i(g+1)}} & \beta_l^i(g+1)G_{21}B_i + S_{b_{\Re_l^i(g+1)}}D_i \\ * & * & \beta_l^i(g+1)I & \Xi_{\Re_l^i(g+1)} & -S_{c_{\Re_l^i(g+1)}} & -S_{d_{\Re_l^i(g+1)}}D_i \\ * & * & * & \Psi_{11_{\Re_l^i(g+1)}} & \Psi_{12_{\Re_l^i(g+1)}} & -\beta_l^i(g+1)F_{11}B_i - \lambda_1 S_{b_{\Re_l^i(g+1)}}D_i \\ * & * & * & * & \Psi_{22_{\Re_l^i(g+1)}} & -\beta_l^i(g+1)F_{21}B_i - \lambda_2 S_{b_{\Re_l^i(g+1)}}D_i \\ * & * & * & * & * & \beta_l^i(g+1)\gamma_\infty^2 I \end{bmatrix} > 0 \quad (11)$$

where

$$\Gamma_{1_{\Re_l^i(g+1)}} = \beta_l^i(g+1)(G_2 + G_{21}^T) - P_{12_{\Re_l^i(g+1)}}$$

$$\Gamma_{2_{\Re_l^i(g+1)}} = \beta_l^i(g+1)(G_2 + G_2^T) - P_{22_{\Re_l^i(g+1)}}$$

$$\Gamma_{3_{\Re_l^i(g+1)}} = S_{a_{\Re_l^i(g+1)}} - \beta_l^i(g+1)F_{21}^T$$

$$\Gamma_{4_{\Re_l^i(g+1)}} = S_{a_{\Re_l^i(g+1)}} - \lambda_2 \beta_l^i(g+1)G_2^T$$

$$\Upsilon_{\Re_l^i(g+1)} = \beta_l^i(g+1)(G_{11} + G_{11}^T) - P_{11_{\Re_l^i(g+1)}}$$

$$\Pi_{\Re_l^i(g+1)} = \beta_l^i(g+1)(G_{11}A_i - F_{11}^T) - S_{b_{\Re_l^i(g+1)}}C_i$$

$$\Psi_{11_{\Re_l^i(g+1)}} = P_{11_{\Re_l^i(g+1)}} - \beta_l^i(g+1)(F_{11}A_i + A_i^T F_{11}^T) - \lambda_2(S_{b_{\Re_l^i(g+1)}}C_i + C_i^T S_{b_{\Re_l^i(g+1)}}^T)$$

$$\Psi_{12_{\Re_l^i(g+1)}} = P_{12_{\Re_l^i(g+1)}} - \lambda_1 S_{a_{b_{\Re_l^i(g+1)}}} + A_i^T F_{21}^T - \lambda_2 C_i^T S_{b_{\Re_l^i(g+1)}}^T$$

$$\Psi_{22_{\Re_l^i(g+1)}} = P_{22_{\Re_l^i(g+1)}} - \lambda_2(S_{a_{b_{\Re_l^i(g+1)}}} + S_{a_{\Re_l^i(g+1)}}^T)$$

$$\Xi_{\Re_l^i(g+1)} = \beta_l^i(g+1)L_i - S_{d_{\Re_l^i(g+1)}}C_i$$

$$\Phi_{\Re_l^i(g+1)} = \beta_l^i(g+1)(G_{21}A_i - \lambda_1 G_2^T) + S_{b_{\Re_l^i(g+1)}}C_i$$

Then the homogeneous polynomially parameter-dependent matrices given by (7)-(10) assure (6) for all $\alpha \in \Omega$. Moreover, if the LMI of (11) is fulfilled for a given degree g, then the LMI corresponding to any degree $g > \hat{g}$ are also satisfied. In this case, the matrices of the 2D discrete-time HPPD filter are given by

$$A_{fg}(\alpha) = \sum_{j=1}^{\Im(g)} \alpha_1^{k_1}\alpha_2^{k_2}...\alpha_N^{k_N} A_{f\Re_j(g)} \quad (12)$$

$$B_{fg}(\alpha) = \sum_{j=1}^{\Im(g)} \alpha_1^{k_1}\alpha_2^{k_2}...\alpha_N^{k_N} B_{f\Re_j(g)} \quad (13)$$

$$C_{fg}(\alpha) = \sum_{j=1}^{\Im(g)} \alpha_1^{k_1}\alpha_2^{k_2}...\alpha_N^{k_N} C_{f\Re_j(g)} \quad (14)$$

$$D_{fg}(\alpha) = \sum_{j=1}^{\Im(g)} \alpha_1^{k_1}\alpha_2^{k_2}...\alpha_N^{k_N} D_{f\Re_j(g)} \quad (15)$$

With

$$\begin{aligned} A_{f_{\Re_j(g)}} &= G_2^{-1}S_{a_{\Re_j(g)}}, \\ B_{f_{\Re_j(g)}} &= G_2^{-1}S_{b_{\Re_j(g)}}, \\ C_{f_{\Re_j(g)}} &= S_{c_{\Re_j(g)}}, \\ D_{f_{\Re_j(g)}} &= S_{d_{\Re_j(g)}}, \\ k_1k_2...k_N &= \Re_j(g). \end{aligned} \quad (16)$$

Proof: Note that (6) for ($A(\alpha)$, $B(\alpha)$, $C(\alpha)$, $D(\alpha)$, $C_1(\alpha)$, $D_1(\alpha)$) $\in \mathcal{P}$ and $S_a(\alpha)$, $S_b(\alpha)$, $S_c(\alpha)$, $S_d(\alpha)$ given by (7)-(10) are homogeneous polynomial matrices equations of degree $g+1$ that can be written as

$$\sum_{l=1}^{J(g+1)} \alpha_1^{k_1}\alpha_2^{k_2}\alpha_3^{k_3}...\alpha_N^{k_N} \left\{ \sum_{i\in\Re_l(g+1)} \begin{bmatrix} \Upsilon_{\Re_l^i(g+1)} & \Gamma_{1_{\Re_l^i(g+1)}} & 0 & \Pi_{\Re_l^i(g+1)} & \Gamma_{3_{\Re_l^i(g+1)}} & \beta_l^i(g+1)G_{11}B_i + S_{b_{\Re_l^i(g+1)}}D_i \\ * & \Gamma_{2_{\Re_l^i(g+1)}} & 0 & \Phi_{\Re_l^i(g+1)} & \Gamma_{4_{\Re_l^i(g+1)}} & \beta_l^i(g+1)G_{21}B_i + S_{b_{\Re_l^i(g+1)}}D_i \\ * & * & \beta_l^i(g+1)I & \Xi_{\Re_l^i(g+1)} & -S_{c_{\Re_l^i(g+1)}} & -S_{d_{\Re_l^i(g+1)}}D_i \\ * & * & * & \Psi_{11_{\Re_l^i(g+1)}} & \Psi_{12_{\Re_l^i(g+1)}} & -\beta_l^i(g+1)F_{11}B_i - \lambda_1 S_{b_{\Re_l^i(g+1)}}D_i \\ * & * & * & * & \Psi_{22_{\Re_l^i(g+1)}} & -\beta_l^i(g+1)F_{21}B_i - \lambda_2 S_{b_{\Re_l^i(g+1)}}D_i \\ * & * & * & * & * & \beta_l^i(g+1)\gamma_\infty^2 I \end{bmatrix} \right\} > 0$$

$$k_1k_2k_3...k_N = \Re_l(g+1)$$

Condition (11) imposed for all $l = 1,...,\Im(g+1)$ assure condition in (6) for all $\alpha\in\Omega$, and thus the first part is proved.

Suppose that the LMIs of (11) are fulfilled for a certain degree $\hat{g}$, that is, there exist $\Im(\hat{g})$ matrices $S_{a_{\Re_j(\hat{g})}}$, $S_{b_{\Re_j(\hat{g})}}$, $S_{c_{\Re_j(\hat{g})}}$ and $S_{d_{\Re_j(\hat{g})}}$, $j = 1,...,\Im(\hat{g})$ such that $S_{a_{\hat{g}}}(\alpha)$, $S_{b_{\hat{g}}}(\alpha)$, $S_{c_{\Re_j(\hat{g})}}$ and $S_{d_{\hat{g}}}(\alpha)$ are homogeneous polynomially parameter-dependent matrices assuring condition in (6). Then, the terms of the polynomial matrices $S_{a_{\hat{g}+1}}(\alpha) = (\alpha_1+...+\alpha_N)S_{a_{\hat{g}}}(\alpha)$, $S_{b_{\hat{g}+1}}(\alpha) = (\alpha_1+...+\alpha_N)$ $S_{b_{\hat{g}}}(\alpha)$, $S_{d_{\hat{g}+1}}(\alpha) = (\alpha_1+...+\alpha_N)S_{d_{\hat{g}}}(\alpha)$ and $S_{c_{\hat{g}+1}}(\alpha) = (\alpha_1+...+\alpha_N)S_{c_{\hat{g}}}(\alpha)$, satisfy the LMIs of Theorem 1 corresponding to the degree $\hat{g}+1$ which can be obtained in this case by linear combination of the LMIs of Theorem 1 for $\hat{g}$. □

Example 1: Consider the discrete-time system borrowed from [20]:

$$A = \begin{bmatrix} 0 & -0.5 \\ 1 & 1+\delta \end{bmatrix},\ B = \begin{bmatrix} -6 & 0 \\ 1 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} -100 & 10 \end{bmatrix},\quad D = \begin{bmatrix} 0 & 1 \end{bmatrix},\quad L = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

where $\|\delta\| \le 0.45$.

For this example Theorem 1 with ($\lambda_1 = 0.05, \lambda_2 = -0.33$) and $g = 1$, provides a guaranteed H_∞ cost of 1.3711 and with ($\lambda_1 = 0.0465, \lambda_2 =$ -0.0694), $\gamma_{opt} = 1.3527$ while in [6] ($\lambda_1 = -1.43, \lambda_2 = -0.08$) yields 1.6577, [20] ($\lambda_1 = 0.05, \lambda_2 = -0.33$) yields 1.703 and [[4], Corollary 5] provides 1.8600 for $g = 1$ and 1.8208 for $g = 2$. In this case, the H_∞ guaranteed cost obtained by Theorem 1 with $g = 1$ is smaller than the one provided by [4] with polynomial matrices of degree $g = 2$.

III. Conclusion

This paper has studied the robust H_∞ filtering problem for discrete systems, the proposed conditions provide an efficient methodology for the design of parameter-dependent filter design, based on homogeneous polynomially parameter-dependent matrices of arbitrary degree, and are less conservative than the existing ones in the literature. The obtained H_∞ filter design is less conservative, which has been demonstrated by illustrative example.

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