DOI:
10.1109/TFUZZ.2024.3364245

Citation for published version (APA):
Liang, H., Wang, W., Pan, Y., Lam, H-K., & Sun, J. (2024). Synchronous MDADT-Based Fuzzy Adaptive Tracking Control for Switched Multiagent Systems via Modified Self-Triggered Mechanism. *IEEE Transactions on Fuzzy Systems*. https://doi.org/10.1109/TFUZZ.2024.3364245

Synchronous MDADT-Based Fuzzy Adaptive Tracking Control for Switched Multiagent Systems via Modified Self-Triggered Mechanism

Hongjing Liang, *Member, IEEE*, Wenzhe Wang, Yingnan Pan, *Member, IEEE*, Hak-Keung Lam, *Fellow, IEEE*, and Jiayue Sun, *Member, IEEE*

***Abstract*—In this paper, a self-triggered fuzzy adaptive switched control strategy is proposed to address the synchronous tracking issue in switched stochastic multiagent systems (MASs) based on mode-dependent average dwell-time (MDADT) method. Firstly, a synchronous slow switching mechanism is considered in switched stochastic MASs and realized through a class of designed switching signals under MDADT property. By utilizing the information of both specific agents under switching dynamics and observers with switching features, the synchronous switching signals are designed, which reduces the design complexity. Then, a switched state observer via a switching-related output mask is proposed. The information of agents and their preserved neighbors is utilized to construct the observer and the observation performance of states is improved. Moreover, a modified self-triggered mechanism is designed to improve control performance via proposing auxiliary function. Finally, by analysing the relationship between the synchronous switching problem and the different switching features of the followers, the synchronous slow switching mechanism based on MDADT is obtained. Meanwhile, the designed self-triggered controller can guarantee that all signals of the closed-loop system are ultimately bounded under the switching signals. The effectiveness of the designed control method can be verified by some simulation results.**

***Index Terms*—Adaptive control, fuzzy logic systems, self-triggered mechanism, switched stochastic multiagent systems, synchronous mode-dependent average dwell-time method**

I. INTRODUCTION

THE great progress of cooperative control for multiagent systems (MASs), made during the past decades, has been applied in broad fields [1]–[8]. Many meaningful methods are considered to improve the control performance of the system [9]–[21]. Because the switching phenomenon widely exists in practice, there are plenty of results which focus on the MASs with switching characteristics. Most of them concern the co-operative control for MASs under switching topologies [22]–[24]. A few researches about switching stochastic dynamics have been reported. Zou *et al.* [25] investigated the sampled-data consensus problem for heterogeneous MASs in switching dynamics. In [26], authors proposed an adaptive prescribed-time consensus protocol for the stochastic nonstrict-feedback MASs with switched nonlinearities. Actually, the dynamics of agents are switched at times inevitably, such as, the multiple handling robots, the multiple amphibious robots, and so on. Hence, it has the practical significance to investigate the cooperative control problem for switched stochastic MASs. What is noteworthy is that the states of the system are difficult to measure in practice, which brings severe challenges to design controller.

For estimating unmeasured states, plenty of useful methods have been designed to address the problem, such as state observers [27]–[29], and so forth [30]. Considering the synchronization error in tracking control of MASs, many scholars have utilized this information to improve the observation performance. There are some non-switched state observers proposed in nonlinear MASs. For example, the distributed fuzzy state observer was investigated in [29], which included the information of other agents' outputs. In [31], the neighborhood state observer was investigated based on the neighbors' information. Due to the information is transmitted by network, the above designed observers do not consider the privacy of information. When the initial states of agents are disclosed between information exchanges, the preservation performance is not guaranteed [32]. Besides, most switched observers in switched stochastic MASs only use the output information about respective agents, which are not efficient. In order to address these issues, more preserved information should be utilized to improve observation performance.

At present, there are some slow switching strategies used for analyzing the stability of switched systems under the controlled switching signals, such as the common Lyapunov function, average dwell time, and mode-dependent average dwell time (MDADT). The notion of MDADT was first proposed in [33] and utilized in the following works [34], [35]. It is worth emphasizing that the MDADT method only applies in the situation that Lyapunov function V satisfies $\dot{V} \leq -cV$. But there is the constant item in the derivative of the Lyapunov

This work was partially supported by the National Natural Science Foundation of China (62322307), and Sichuan Science and Technology Program (2023NSFSC1968). *(Corresponding author: Hongjing Liang.)*

Hongjing Liang is with the School of Automation Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan, 611731, P. R. China, and also with the Laboratory of Electromagnetic Space Cognition and Intelligent Control, Beijing, 100089, P. R. China (e-mail: lianghongjing@uestc.edu.cn).

Wenzhe Wang is with the College of Control Science and Engineering, Bohai University, Jinzhou, Liaoning, 121013, P. R. China (e-mail: wwz991215@163.com).

Yingnan Pan is with the College of Control Science and Engineering, Bohai University, Jinzhou, Liaoning, 121013, P. R. China (e-mail: panyingnan0803@gmail.com).

Hak-Keung Lam is with the Department of Engineering, King's College London, Strand, London, WC2R 2LS, United Kingdom (e-mail: hak-keung.lam@kcl.ac.uk).

Jiayue Sun is with State Key Laboratory of Synthetical Automation for Process Industries (Northeastern University), Shenyang, Liaoning, 110004, China (e-mail: jyuesun@163.com).

function under the adaptive control scheme. Niu *et al.* [36] proposed the adaptive output-feedback control strategy for switched stochastic nonlinear system based on the modified MDADT method. The MDADT method is achieved in the adaptive control rule. In the tracking control of stochastic MASs with switching dynamics, the different performances are obtained according to whether followers are directly linked to the leader. When switching dynamics are intense, the stabilities of these followers are difficult to guarantee. It is a thorny problem which needs to be addressed. Furthermore, ET mechanisms and self-triggered mechanisms are frequently used in the analysis of switched system, which correspond to save communication resources [37]–[46]. The self-triggered mechanism in [47] may be invalid when the change rate of control signal interval has the issue, especially when the derivative of input signal does not exist. How to achieve the synchronous slow switching in switched stochastic MASs via a modified self-triggered mechanism is worthy considering.

Motivated by the above discussion, a modified self-triggered adaptive tracking control scheme is put forward for the switched stochastic MASs based on the MDADT method. Meanwhile, a class of distributed state observers is designed to address the output-feedback control problem. The privacy preservation technology is used to avoid disclosing the initial states of followers. The main contributions of this paper are shown as below.

1) The synchronous slow switching strategy is firstly designed for adaptive control scheme in switched stochastic MASs. By utilizing the information of specific agents which directly link to the leader, the complexity of the design process is reduced and the synchronous effectiveness of controlled switching signals is guaranteed as well.

2) A switched state observer is designed, which uses the information about followers and their preserved neighbors with switching-related output preservation mechanism. Compared with the observer in [29], it improves the observation performance of states and avoids the disclosure of output signal.

3) The modified self-triggered mechanism is proposed to solve the singularity issue caused by the change rates of control signal interval. The auxiliary function and approximate function are designed to maintain the effectiveness of mechanism. By using these functions, it not only reduces the trigger number but also improves the tracking performance.

The rest of this paper is arranged as below. A few preliminaries are given in Section II. Section III introduces the design processes of self-triggered adaptive controller and MDADT-based stability analysis. Some simulation results are outlined in Section IV. Finally, the conclusions are given in Section V.

II. Preliminaries

A. Graph Theory

The exchanged information between agents is represented by a directed graph. It is defined as $\mathcal{G} = (\mathcal{Y}, \Gamma)$ that $\mathcal{Y} = (\mathcal{Y}_1, ..., \mathcal{Y}_N)$ denotes a nonempty set of agents. The form $(\mathcal{Y}_i, \mathcal{Y}_j) \in \Gamma$ denotes the edge of node i to node j. The neighbors of agent i are expressed as $\mathcal{N}_i = \{\mathcal{Y}_j | (\mathcal{Y}_j, \mathcal{Y}_i) \in \Gamma\}$. Simultaneously, the topology for the weighted graph is represented by the adjacency matrix $\mathcal{A} = [a_{i,j}] \in R^{N\times N}$. Then, if the information of node i can be received by node j, $a_{i,j} > 0$, and otherwise $a_{i,j} = 0$. The absolute in-degree matrix is $\mathcal{D} = \mathrm{diag}\{d_1, ..., d_N\}$ with $d_i = \sum_{j=1}^{N} a_{i,j}$. The Laplacian matrix is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$.

In this paper, it has one leader labeled **0** and a set of N followers denoted as 1 to N. Let $\mathcal{B} = \mathrm{diag}\{b_1, ..., b_N\} \in R^{N\times N}$ be the adjacency matrix of leader, and b_i is the weight of the edge from $\mathcal{Y}_0$ to $\mathcal{Y}_i$.

Assumption 1: [2] If there exists at least one node having a directed path to all other nodes, the directed graph $\mathcal{G}$ is said to have a spanning tree with a leader serving as the root.

B. Problem Formulation

Consider a class of switched stochastic MASs. The dynamic of the ith $(i = 1, \ldots, N)$ agent is defined as follows:

$$\begin{cases} dx_{i,p} = (x_{i,p+1} + \check{f}_{i,p}(\bar{x}_{i,p}))dt + \check{q}_{i,p}(\bar{x}_{i,p})dw \\ dx_{i,n} = (u_i + \check{f}_{i,n}(\bar{x}_{i,n}))dt + \check{q}_{i,n}(\bar{x}_{i,n})dw \\ \quad y_i = x_{i,1} \end{cases} \tag{1}$$

where $\bar{x}_{i,p} = [x_{i,1}, ..., x_{i,p}]^T$ represents the state vector with $p = 1, 2, ..., n-1$. Further, $x_{i,2}, ..., x_{i,n}$ are unmeasurable. u_i is the control input signal, and the output signal of the ith agent is represented as y_i. y_0 is the output signal of the leader. $\check{f}_{i,p}(.) = f_{i,p}^{\sigma(t)}(.)$ and $\check{q}_{i,n}(.) = q_{i,n}^{\sigma(t)}(.)$ are unknown smooth nonlinear functions and satisfy local Lipschitz condition. $\sigma(t) : [0, +\infty) \to \Gamma \overset{def}{=} \{1, ..., M\}$ is a switching signal which is identical for each agents. w denotes an r-dimension standard Brownian motion defined on the complete probability space.

Assumption 2: [48] The desired output y_0 and its first-order derivative are known and bounded.

Assumption 3: [49] Considering the functions $\check{f}_{i,m}(.)$ and $\check{g}_{i,m}(.)$, there exist positive constants $g_{i,m}$ and $h_{i,m}$, such that

$$\begin{aligned} \left|\check{f}_{i,m}(x) - \check{f}_{i,m}(\hat{x})\right| \leq g_{i,m}||x - \hat{x}|| \\ \left|\check{g}_{i,m}(x) - \check{g}_{i,m}(\hat{x})\right| \leq h_{i,m}||x - \hat{x}|| \end{aligned} \tag{2}$$

where $i = 1, 2, ..., n$.

Definition 1: [33] Consider $\sigma(t)$ as a switching signal in this paper. For any $T \geq t \geq 0$, the switching numbers of the pth activated subsystem during the interval $[t, T]$ is denoted as $N_{\sigma p}(T, t)$. The whole running time of the pth subsystem during the interval $[t, T]$ are represented as $T_p(T, t)$, $p \in \Gamma$. If there are positive numbers N_{0p} which can be called as the mode-dependent chatter bounds and τ_{ap} such that

$$N_{\sigma p}(T, t) \leq N_{0p} + \frac{T_p(T, t)}{\tau_{ap}}, \quad \forall T \geq t \geq 0 \tag{3}$$

The conclusion that $\sigma(t)$ has an MDADT τ_{ap} can be drawn.

C. Privacy Preservation

Due to the distributed cooperative control for MASs, privacy preservation is utilized to mask the initial states of the agents. Since the output y_i is a little available information for designing controller in this paper, the output mask is taken into account.

Consider the following masked system

$$\begin{aligned} \dot{x} &= h(y) \\ y &= \bar{\varphi}(t, x, \bar{\Lambda}) \end{aligned} \tag{4}$$

where $y = [y_1, ..., y_n]^T$ denotes the output vector which has the same dimension as x. $\bar{\varphi}$ is a continuously time-varying function, and $\bar{\Lambda} \in \mathbb{R}$ is a vector of parameters.

A switching-related additive mask is designed as

$$\bar{\varphi}_i(t, x_i, \bar{\Lambda}_i) = x_i + \varphi_i \tag{5}$$

where

$$\varphi_i = \begin{cases} \gamma_i(\cos\dfrac{\pi(t+t_0)}{2t_0} + 1), & t \le t_0 \\ 0, & t > t_0 \end{cases} \tag{6}$$

where $\bar{\Lambda}_i = \{\gamma_i, t_0\}$. t_0 is the time instant of first switch, and $\gamma_i > 0$ is the constant which can be designed by users.

Different from the result in [32], the function $\bar{\varphi}(.)$ is a piecewise function, and original states are gotten when $t \to t_0$ rather than $t \to \infty$. After achieving the preservation task, the negative influence is avoided as well. It is worth noting that the output masks are arbitrarily decided by each agent i. Further, the levels of privacy are arbitrarily set by choosing different parameters Λ_i for agents.

The synchronization error is given as

$$s_{i,1} = \sum_{j=1}^{N} a_{i,j}(y_i - y_j) + b_i(y_i - y_0) \tag{7}$$

After applying the privacy preservation technology to the outputs of followers, the masked synchronization error is defined as

$$\underline{s}_{i,1} = \sum_{j=1}^{N} a_{i,j}(\underline{y}_i - \underline{y}_j) + b_i(\underline{y}_i - y_0) \tag{8}$$

where $\underline{y}_i = \bar{\varphi}_i$ and $\underline{y}_j = \bar{\varphi}_j$.

Assumption 4: [32] The system (4) is such that $\{\mathcal{N}_i \cup \{i\}\} \nsubseteq \{\mathcal{N}_j \cup \{j\}\}$ for $\forall i, j = 1, ..., n, i \neq j$.

Remark 1: The topology of the directed graph has the limitation that all nodes cannot get the complete information about what is going on at the other nodes. When Assumption 4 is satisfied, the masked system (4) is considered as a dynamically private version of the original system, and its level of privacy can be guaranteed arbitrarily.

D. Modified Self-Triggered Mechanism

For saving communication resources and improving the tracking performance, the modified self-triggered mechanism is designed as

$$u_i(t) = \omega_i(t_k), \forall x \in [t_{i,k}, t_{i,k+1}) \tag{9}$$

$$t_{i,k+1} = t_{i,k} + \frac{\eta_i |u_i(t_k)| + m_i}{\max\{\hbar_i(t), |\zeta_i(t)|\}} \tag{10}$$

where $t_{i,k}, t_{i,k+1} \in N^+$ and $0 < \eta_i < 1$. m_i is a positive constant. $\omega_i(t)$ and $\hbar_i(t)$ are designed continuous functions. $u_i(t)$ is the self-triggered control signal, and $\eta_i|u_i(t)| + m_i$ represents the control signal interval between two adjacent triggered instants. $\max\{\hbar_i(t), |\zeta_i(t)|\}$ shows the change rates of input signal interval. Moreover, the designed functions $\zeta_i(t)$ and $\hbar_i(t)$ are given as

$$\begin{aligned} \zeta_i(t) &= \frac{u_i(t_{i,k}) - \omega_i(t)}{t_{i,k} - t} \\ \hbar_i(t) &= r_i e^{\frac{s_{i,1}^2}{2\iota_i}} \end{aligned} \tag{11}$$

where r_i and ι_i are positive constants. In the process of self-triggered controller design, $\omega_i(t)$ is the actual continuous control signal which needs users to construct. $u_i(t_k)$ is assigned as $\omega_i(t_k)$ when the trigger instant $t_{i,k}$ is obtained, and the value remains constant during the period of time $t \in [t_{i,k}, t_{i,k+1})$.

Obviously, the function $u_i(t)$ is continuous, but may not be derivable. It is improper to directly use $\dot{u}_i(t)$ to construct the change rate of signal interval. Considering the characters of self-triggered mechanism, the new function $\zeta_i(t)$ and $\hbar_i(t)$ are selected to address the problem. $\zeta_i(t)$ is designed to describe the change degree of control interval. $\hbar_i(t)$ is designed to improve the control performance. According to its expression, it has the positive correlation with the synchronization error, which can effectively adjust the trigger interval.

Algorithm 1 Modified self-triggered algorithm for $t_{i,k} \to t_{i,k+1}$ of agent i

```
Initialization t_i > 0, u_i(t), ω_i(t); predefined parameters η_i, m_i, r_i, ι_i;
§_i = 0; §_i is used to represent whether the trigger instant has arrived;
while δ_i > 0 do
    ζ_i(t) ← (ω_i(t) − u_i(t_k)) / (t − t_{i,k}); ħ_i(t) ← r_i e^(s_{i,1}^2 / (2ι_i));
    if §_i = 1 then
        u_i(t) ← ω_i(t_k); ∇_i ← η_i|u_i(t_k)| + m_i;
        get ζ_i(t_k) and ω̇_i(t_k);
        if ζ_i(t) > ħ_i(t) then
            compute δ_i ← ∇_i / ζ_i;
        else
            compute δ_i ← ∇_i / ħ_i;
        end if
        t_{i,k+1} ← t_{i,k} + δ_i;
    else
        u_i(t) and ∇_i remain as previous constants;
    end if
end while
Return t_{i,k+1}
```

Remark 2: Different from the existing self-triggered mechanisms in [47] and [50], the new function about change rate of control signal interval is constructed. By using correlation between $\omega_i(t)$ and $u_i(t)$, $\zeta_i(t)$ is designed to describe the change degree of control interval. The function $\hbar_i(t)$ establishes the contact with the synchronization error By adding the functions $\hbar_i(t)$ and $\zeta_i(t)$, the mechanism is modified.

E. Switched State Observer

Considering that the system states $x_{i,2}, ..., x_{i,n}$ are unmeasurable, a state observer is designed to address this problem.

To get more precise observation performance, the output information about neighbors is used in the process of constructing observer which is different from the previous result in [36]. The form of switched observer is given as

$$\begin{aligned} \dot{\hat{x}}_{i,1} &= \hat{x}_{i,2} - \check{l}_{i,1}\tilde{e}_i \\ \dot{\hat{x}}_{i,2} &= \hat{x}_{i,3} - \check{l}_{i,2}\tilde{y}_i \\ &\vdots \\ \dot{\hat{x}}_{i,n} &= u_i - \check{l}_{i,n}\tilde{y}_i \end{aligned} \tag{12}$$

where $\hat{x}_{i,k}$ represents the estimation of $x_{i,k}$, $2 \le k \le n$. $\check{l}_{i,k}$ is the switched gain. $\tilde{y}_i$ and $\tilde{e}_i = \sum_{j=1}^{N} a_{i,j}(\underline{y}_i - \underline{y}_j) + b_i\underline{y}_i - (\sum_{j=1}^{N} a_{i,j}(\hat{y}_i - \underline{y}_j) + b_i\hat{y}_i)$ are the estimation errors of y_i and e_i, respectively. Let $\tilde{x}_{i,k} = x_{i,k} - \hat{x}_{i,k}$, and one has

$$d\tilde{x}_i = (\Xi_i\tilde{x}_i + F_i)dt + Q_i dw \tag{13}$$

where $F_i = [\check{f}_{i,1}(\bar{x}_{i,1}) - \hat{f}_{i,1}(\hat{x}_{i,1}), ..., \check{f}_{i,n}(\bar{x}_{i,n}) - \hat{f}_{i,n}(\hat{x}_{i,n})]^T$, $Q_i = [\check{q}_{i,1}(\bar{x}_{i,1}) - \hat{q}_{i,1}(\hat{x}_{i,1}), ..., \check{q}_{i,n}(\bar{x}_{i,n}) - \hat{q}_{i,n}(\hat{x}_{i,n})]$ and

$$\Xi_i = \begin{bmatrix} -(b_i+d_i)\check{l}_{i,1} & & & \\ -\check{l}_{i,2} & & & \\ \vdots & & I_{n-1} & \\ -\check{l}_{i,n} & 0 & \cdots & 0 \end{bmatrix} \tag{14}$$

The matrix Ξ_i should be strictly Hurwitz by choosing rational constants $\check{l}_{i,1}, ..., \check{l}_{i,n}$. In view of any definite symmetric matrices Γ_i, there are some positive-definite matrices Ξ_i to satisfy the following equation.

$$\Xi_i^T P_i + P_i\Xi_i = -\Gamma_i \tag{15}$$

Finally, the following equation can be obtained.

$$\begin{cases} dy_i = (\hat{x}_{i,1} + \tilde{x}_{i,1} + \check{f}_{i,1})dt + \check{q}_{i,1}dw \\ d\hat{x}_{i,1} = (\hat{x}_{i,2} - \check{l}_{i,1}\tilde{e}_i)dt \\ d\hat{x}_{i,k} = (\hat{x}_{i,k+1} - \check{l}_{i,k}\tilde{y}_i)dt \\ d\hat{x}_{i,n} = (u_i - \check{l}_{i,n}\tilde{y}_i)dt \end{cases} \tag{16}$$

It is noted that the distributed state observers were investigated in [29] which are constructed by the output information of each followers. Since more relative information is utilized to design observers, the construction process will be tedious and complex. Compared with the existing observers, the output information of the agents and their neighbors is only used in estimating $\hat{x}_{i,1}$. Not only this process can be simplified, but the observation performance is guaranteed.

Remark 3: When the system is considered as the switched system, the desired observation performance can not be guaranteed by a handful of information. For addressing the problem, the extra output information about preserved neighbors is utilized at the same time, which is different from the result in [36] and [29].

Lemma 1: [2] Considering $\mathcal{B} = \text{diag}\{b_i\} \in R^{N\times N}$, $\mathcal{L}+\mathcal{B}$ is nonsingular if exists at least $b_i > 0$.

$\Im^i$ represents the set of all functions with continuous ith partial derivatives. $\Im^{2,1}$ denotes the family of all nonnegative function $V(x,t)$ which are $\Im^2$ in x and $\Im^1$ in t.

Lemma 2: [36] $V(x,t) \in \Im^{2,1}$ is a continuously differentiable function, and define its differential operator L as follows:

$$LV = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}f + \frac{1}{2}\text{Tr}\left\{q^T\frac{\partial^2 V}{\partial x^2}q\right\} \tag{17}$$

If there exist positive constants μ_i, ς_i, and κ_∞-functions β_1, β_2, such that

$$\begin{cases} \beta_1(|x|) \le V(x,t) \le \beta_2(|x|) \\ LV_{i,n} \le -\mu_i V_{i,n} + \varsigma_i \end{cases} \tag{18}$$

where $\forall x \in R^n$ and $\forall t > 0$. Then, a unique solution of system (1) can be got which satisfies

$$E[V(t)] \le e^{-\mu t}V(0) + \frac{\varsigma}{\mu} \tag{19}$$

Lemma 3: [51] The continuous function $g(X)$ is defined on a compact set $\Omega \in R^q$, and it can be estimated by Fuzzy-Logic Systems (FLSs). For any constant $\delta > 0$, the estimation form is given by such FLSs $\varrho^T\phi(X)$

$$\sup_{X\in\Omega} |g(X) - \varrho^T\phi(X)| \le \delta \tag{20}$$

where ${\varrho^*}^T = [\varrho_1, \varrho_2, ..., \varrho_k]$ is the ideal weight vector, and $k > 1$ is the number of fuzzy rules. $\phi(X) = [\phi_1(X), \phi_2(X), ..., \phi_k(X)]^T / \sum_{m=1}^{N}\phi_i(X)$ is fuzzy basic function vector.

Lemma 4: [52] For $k \ge 2$, the second-order sliding mode integral filter is designed as follows:

$$\begin{cases} \dot{H}_{i,k10} = -\dfrac{H_{i,k10} - I(t)}{o_{i,k10}} - \dfrac{v_{i,k10}(H_{i,k10} - I(t))}{||H_{i,k10} - I(t)|| + r_{i,k10}} \\ \dot{H}_{i,k20} = -\dfrac{H_{i,k20} - \dot{H}_{i,k10}}{o_{i,k20}} - \dfrac{v_{i,k20}(H_{i,k20} - \dot{H}_{i,k10})}{||H_{i,k20} - \dot{H}_{i,k10}|| + r_{i,k20}} \end{cases}$$

where $I(t)$ is the input signal of filter. $H_{i,k10}$ and $H_{i,k20}$ are the states of filter. Others parameters like $o_{i,k10}$, $o_{i,k20}$, $v_{i,k10}$, $v_{i,k20}$, $r_{i,k10}$ and $r_{i,k20}$ are designed constants.

III. Main results

A. Adaptive Self-Triggered Controller Design

Based on the backstepping technique and self-triggered mechanism, the adaptive controller will be designed in this section. Concentrating on agent i in MASs, Fig. 1 shows the details of the designed control scheme.

Firstly, define the unknown constant ξ_i as

$$\xi_i = \max\left\{||\varrho_{i,m}||^2\right\}, \quad m = 0, 1, ..., n \tag{21}$$

The estimation of ξ_i is defined as $\hat{\xi}_i$, and there exists the estimation error $\tilde{\xi}_i$ such that $\tilde{\xi}_i = \xi_i - \hat{\xi}_i$.

Define the following transformation

$$s_{i,k} = \hat{x}_{i,k} - \alpha_{i,k-1} \tag{22}$$

where $\hat{x}_{i,k}$ denotes the estimation of $x_{i,k}$, and $\alpha_{i,k-1}$ is the virtual control signal.

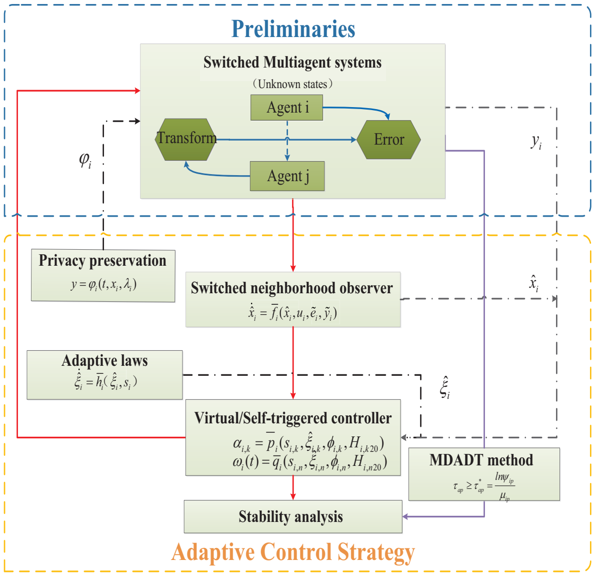

Fig. 1. The block diagram about control scheme.

Step 1: The derivative of the masked synchronization error is given as

$$\begin{aligned} d\underline{s}_{i,1} =& [(b_i+d_i)(\tilde{x}_{i,2}+s_{i,2}+\alpha_{i,1}+\check{f}_{i,1}(x_{i,1})+\dot{\varphi}_i \\ &- b_i\dot{y}_0) - \sum_{j=1}^{N} a_{i,j}(\tilde{x}_{j,2}+\hat{x}_{j,2}+f_{j,1}(x_{j,1})+\dot{\varphi}_j)]dt \\ &+ [(b_i+d_i)\check{q}_{i,1}(x_{i,1}) - \sum_{j=1}^{N} a_{i,j}\check{q}_{j,1}(x_{j,1})]dw \end{aligned} \tag{23}$$

Select the Lyapunov function as

$$V_{i,1p} = \frac{\bar{\imath}}{2}(\tilde{x}_i^T P_{ip}\tilde{x}_i)^2 + \frac{1}{4}\underline{s}_{i,1}^4 + \frac{1}{2}\tilde{\xi}_{i,1}^2 \tag{24}$$

Where $\bar{\imath}$ is a positive constant. According to (17), one gets

$$\begin{aligned} LV_{i,1p} =& -\bar{\imath}\tilde{x}_i^T P_{ip}\tilde{x}_i\tilde{x}_i^T Q_{ip}\tilde{x}_i + 2\bar{\imath}\tilde{x}_i^T P_{ip}\tilde{x}_i\tilde{x}_i^T P_{ip}F_{ip} \\ &+ 2\bar{\imath}\mathrm{Tr}\left\{Q_{ip}^T(2P_{ip}\tilde{x}_i\tilde{x}_i^T P_{ip} + \tilde{x}_i^T P_{ip}\tilde{x}_i P_{ip})Q_{ip}\right\} \\ &+ \underline{s}_{i,1}^3[(b_i+d_i)(\tilde{x}_{i,2}+s_{i,2}+\alpha_{i,1}+\check{f}_{i,1}(x_{i,1}) \\ &+ \dot{\varphi}_i) - \sum_{j=1}^{N} a_{i,j}(\tilde{x}_{j,2}+\hat{x}_{j,2}+\check{f}_{j,1}(x_{j,1})+\dot{\varphi}_j) \\ &- b_i\dot{y}_0] + \frac{3}{2}\underline{s}_{i,1}^2\check{\varphi}_{i,1}^T\check{\varphi}_{i,1} - \tilde{\xi}_{i,1}\dot{\hat{\xi}}_{i,1} \end{aligned} \tag{25}$$

where

$$\check{\varphi}_{i,1} = (b_i+d_i)\check{q}_{i,1} - \sum_{j=1}^{N} a_{i,j}\check{q}_{j,1}$$

Considering (13), one has $F_{ip} = \tilde{F}_{ip} + \hat{F}_{ip}$, and δ_{0p} is a positive constant. According to Young's inequality and Assumption 3, one has

$$\begin{aligned} 2\bar{\imath}\tilde{x}_i^T P_{ip}\tilde{x}_i\tilde{x}_i^T P_{ip}\hat{F}_{ip} \le& \frac{3\bar{\imath}}{2}\tau_{i,0}^{\frac{4}{3}}||P_{ip}||^{\frac{8}{3}}||\tilde{x}_i||^4 + \frac{\bar{\imath}}{2\tau_{i,0}^4}||\hat{F}_{ip}||^4 \\ \le& \frac{3\bar{\imath}}{2}\tau_{i,0}^{\frac{4}{3}}||P_{ip}||^{\frac{8}{3}}||\tilde{x}_i||^4 \\ &+ \frac{\bar{\imath}}{2\tau_{i,0}^4}(\xi_{i,0p}^2+\delta_{i,0p}^4) \end{aligned} \tag{26}$$

$$\begin{aligned} 2\bar{\imath}\tilde{x}_i^T P_{ip}\tilde{x}_i\tilde{x}_i^T P_{ip}\tilde{F}_{ip} \le& \frac{3\bar{\imath}}{2}\tau_{i,0}^{\frac{4}{3}}||P_{ip}||^{\frac{8}{3}}||\tilde{x}_i||^4 + \frac{\bar{\imath}}{2\tau_{i,0}^4}||\tilde{F}_{ip}||^4 \\ \le& \frac{3\bar{\imath}}{2}\tau_{i,0}^{\frac{4}{3}}||P_{ip}||^{\frac{8}{3}}||\tilde{x}_i||^4 \\ &+ \frac{\bar{\imath}}{2\tau_{i,0}^4}(\sum_{m=1}^{n} g_{i,m}^2)^2||\tilde{x}_i||^4 \end{aligned} \tag{27}$$

$$\begin{aligned} &2\bar{\imath}\mathrm{Tr}\left\{Q_{ip}^T(2P_{ip}\tilde{x}_i\tilde{x}_i^T P_{ip} + \tilde{x}_i^T P_{ip}\tilde{x}_i P_{ip})Q_{ip}\right\} \\ &\le 6\bar{\imath}n^{\frac{3}{2}}||P_{ip}||^2||\tilde{x}_i||^2||Q_{ip}||^2 \\ &\le 3\bar{\imath}\epsilon_{i,0}n^{\frac{3}{2}}||P_{ip}||^4||\tilde{x}_i||^4 + \frac{3\bar{\imath}}{\epsilon_{i,0}}n^{\frac{3}{2}}||Q_{ip}||^4 \\ &\le 3\bar{\imath}\epsilon_{i,0}n^{\frac{3}{2}}||P_{ip}||^4||\tilde{x}_i||^4 \\ &\quad + \frac{3\bar{\imath}}{\epsilon_{i,0}}n^{\frac{3}{2}}(\sum_{m=1}^{n} h_{i,m}^2)^2||\tilde{x}_i||^4 \end{aligned} \tag{28}$$

where $\tau_{i,0}$ and $\epsilon_{i,0}$ are positive constants. n denotes the dimension of states. According to (6), there is $|\dot{\varphi}_i| \le \vartheta_{i,1}$. $\vartheta_{i,1}$ is a non-negative constant. Then, one has

$$\begin{aligned} \underline{s}_{i,1}^3\tilde{x}_{i,2} \le& \frac{3}{4}\tau_{i,1}\underline{s}_{i,1}^4 + \frac{1}{4\tau_{i,1}^3}||\tilde{x}_i||^4 \\ \frac{3}{2}\underline{s}_{i,1}^2\check{\varphi}_{i,1}^T\check{\varphi}_{i,1} \le& \frac{3}{4}\jmath_{i,1}^2 + \frac{3}{4}\underline{s}_{i,1}^4||\check{\varphi}_{i,1}||^4\jmath_{i,1}^{-2} \\ (b_i+d_i)\underline{s}_{i,1}^3\dot{\varphi}_i \le& \frac{3}{4}(b_i+d_i)\epsilon_{i,1}\underline{s}_{i,1}^4 + \frac{1}{4\epsilon_{i,1}^3}(b_i+d_i)\vartheta_{i,1}^4 \\ (b_i+d_i)\underline{s}_{i,1}^3 s_{i,2} \le& \frac{3}{4}(b_i+d_i)\underline{s}_{i,1}^4 + \frac{1}{4}(b_i+d_i)s_{i,2}^4 \end{aligned} \tag{29}$$

where $\epsilon_{i,1}$, $\tau_{i,1}$ and $\jmath_{i,1}$ are positive. Then, one obtains

$$\begin{aligned} LV_{i,1p} \le& -\bar{\imath}||\tilde{x}_i||^4\Bigg(\lambda_{\min}(P_{ip})\lambda_{\min}(Q_{ip}) - 3\tau_{i,0}^{\frac{4}{3}}||P_{ip}||^{\frac{8}{3}} \\ &- \frac{1}{2\tau_{i,0}^4}(\sum_{m=1}^{n} g_{i,m}^2)^2 - \frac{3}{\epsilon_{i,0}}n^{\frac{3}{2}}(\sum_{m=1}^{n} h_{i,m}^2)^2 \\ &- \frac{1}{4\tau_{i,1}^4} - 3\epsilon_{i,0}n^{\frac{3}{2}}||P_{ip}||^4\Bigg) + \frac{\bar{\imath}}{2\tau_{i,0}^4}(\xi_{i,0l}^2+\delta_{i,0l}^4) \\ &+ \underline{s}_{i,1}^3[(b_i+d_i)(\alpha_{i,1}+\frac{3}{4}\underline{s}_{i,1}) + \bar{f}_{i,1} - \sum_{j=1}^{N} a_{i,j}\hat{x}_{j,2} \\ &- b_i\dot{y}_0] + \frac{1}{4}(b_i+d_i)s_{i,2}^4 - \tilde{\xi}_{i,1}\dot{\hat{\xi}}_{i,1} + \frac{3}{4}\jmath_{i,1}^2 \\ &+ \frac{1}{4\epsilon_{i,1}^3}(b_i+d_i)\vartheta_{i,1}^4 \end{aligned} \tag{30}$$

where $\bar{f}_{i,1}(X_{i,1}) = (b_i+d_i)\check{f}_{i,1}(x_{i,1})\frac{3}{4}(b_i+d_i)\epsilon_{i,1}\underline{s}_{i,1}^4 + \frac{3}{4}\tau_{i,1}^{\frac{4}{3}}\underline{s}_{i,1} - \sum_{j=1}^{N} a_{i,j}(\tilde{x}_{j,2}+\check{f}_{j,1}(x_{j,1})+\dot{\varphi}_j) - \frac{3}{4}\underline{s}_{i,1}||\check{\varphi}_{i,1}||^4\jmath_{i,1}^{-2}$. By utilizing FLSs, for $\forall\delta_{i,1} > 0$, one has

$$\bar{f}_{i,1}(X_{i,1}) = \varrho_{i,1}^T\phi_{i,1}(X_{i,1}) + \varepsilon_{i,1}(X_{i,1}), |\varepsilon_{i,1}(X_{i,1})| \le \delta_{i,1} \tag{31}$$

Then, one gets

$$\underline{s}_{i,1}^3\bar{f}_{i,1}(X_{i,1}) = \underline{s}_{i,1}^3(\varrho_{i,1}^T\phi_{i,1}(X_{i,1}) + \varepsilon_{i,1}(X_{i,1}))$$

$$\leq \frac{\xi_{i,1}}{2\upsilon_{i,1}^2}\underline{s}_{i,1}^6\phi_{i,1}^T\phi_{i,1} + \frac{1}{2}\upsilon_{i,1}^2 + \frac{3}{4}\underline{s}_{i,1}^4 + \frac{1}{4}\delta_{i,1}^4 \tag{32}$$

where $\upsilon_{i,1}$ is the designed constant. The virtual controller and the adaptive law are designed as

$$\alpha_{i,1} = \frac{1}{b_i+d_i}[-c_{i,1}\underline{s}_{i,1} - \sum_{j=1}^{N} a_{i,j}\hat{x}_{j,2} + b_i\dot{y}_0 - \frac{\hat{\xi}_{i,1}}{2\upsilon_{i,1}^2}\underline{s}_{i,1}^3\phi_{i,1}^T\phi_{i,1}] - \frac{3}{4}\underline{s}_{i,1} \tag{33}$$

$$\dot{\hat{\xi}}_{i,1} = \frac{1}{2\upsilon_{i,1}^2}\underline{s}_{i,1}^6 - \aleph_{i,1}\hat{\xi}_{i,1} \tag{34}$$

where $c_{i,1}$ and $\aleph_{i,1}$ are positive parameters. Then, (30) can be rewritten as

$$\begin{aligned} LV_{i,1p} \leq & -\bar{\imath}||\tilde{x}_i||^4\Bigg(\lambda_{\min}(P_{ip})\lambda_{\min}(Q_{ip}) - 3\tau_{i,0}^{\frac{4}{3}}||P_{ip}||^{\frac{8}{3}} \\ & - \frac{1}{2\tau_{i,0}^4}(\sum_{m=1}^{n} g_{i,m}^2)^2 - \frac{3}{\epsilon_{i,0}}n^{\frac{3}{2}}(\sum_{m=1}^{n} h_{i,m}^2)^2 \\ & - 3\epsilon_{i,0}n^{\frac{3}{2}}||P_{ip}||^4 - \frac{1}{4\tau_{i,1}^4}\Bigg) - c_{i,1}\underline{s}_{i,1}^4 \\ & + \frac{1}{4}(b_i+d_i)s_{i,2}^4 + \Delta_{i,1} - \frac{\aleph_{i,1}}{2}\tilde{\xi}_{i,1}^2 \end{aligned}$$

where

$$\Delta_{i,1} = \frac{\bar{\imath}}{2\tau_{i,0}^4}(\xi_{i,0p}^2 + \delta_{i,0p}^4) + \frac{3}{4}\jmath_{i,1}^2 + \frac{1}{4\epsilon_{i,1}^3}(b_i+d_i)\vartheta_{i,1}^4 + \frac{1}{2}\upsilon_{i,1}^2 + \frac{1}{4}\delta_{i,1}^4 + \frac{\aleph_{i,1}}{2}\xi_{i,1}^2$$

Step k: By using (22), one has

$$\begin{aligned} ds_{i,k} &= d\hat{x}_{i,k} - d\alpha_{i,k-1} \\ &= (\hat{x}_{i,k+1} - \check{l}_{i,k}y_i - L\alpha_{i,k-1})dt \\ &\quad - (\sum_{m=1}^{k-1}\frac{\partial\alpha_{i,k-1}}{\partial\hat{x}_{i,m}}\check{q}_{i,m})dw \end{aligned} \tag{35}$$

where $L\alpha_{i,k-1}$ has following definition

$$\begin{aligned} L\alpha_{i,k-1} = & \frac{\partial\alpha_{i,k-1}}{\partial y_i}(\hat{x}_{i,1} + \tilde{x}_{i,1} + \check{f}_{i,1}) + \sum_{m=1}^{k-1}\frac{\partial\alpha_{i,k-1}}{\partial\hat{\xi}_{i,m}}\dot{\hat{\xi}}_{i,m} \\ & + \sum_{m=1}^{k-2}\frac{\partial\alpha_{i,k-1}}{\partial H_{i,m20}}\dot{H}_{i,m20} + \sum_{m=2}^{k-1}\frac{\partial\alpha_{i,k-1}}{\partial\hat{x}_{i,m}}\dot{\hat{x}}_{i,m} \\ & + \frac{1}{2}\sum_{p,t=1}^{k-1}\frac{\partial^2\alpha_{i,k-1}}{\partial\hat{x}_{i,p}\partial\hat{x}_{i,t}}\check{q}_{i,p}\check{q}_{i,t}, \quad k = 3, ..., n-1 \end{aligned}$$

Especially, one has

$$\begin{aligned} L\alpha_{i,1} = & \frac{\partial\alpha_{i,1}}{\partial y_i}(\hat{x}_{i,1} + \tilde{x}_{i,1} + \check{f}_{i,1}) + \frac{1}{2}\frac{\partial^2\alpha_{i,1}}{\partial x_{i,1}^2}\check{q}_{i,1}^2 \\ & + \frac{\partial\alpha_{i,1}}{\partial\hat{\xi}_{i,1}}\dot{\hat{\xi}}_{i,1} + \frac{\partial\alpha_{i,1}}{\partial y_0}\dot{y}_0 \end{aligned}$$

From Lemma 4, the derivative of virtual signal can be estimated by the introduced filter

$$H_{i,k20} - L\alpha_{i,k-1} = \tilde{H}_{i,k-1} \tag{36}$$

where $\tilde{H}_{i,k-1}$ represents the error of estimation which satisfies $|\tilde{H}_{i,k-1}| \leq H_{i,km}$ with $H_{i,km} > 0$.

Select the Lyapunov function as

$$V_{i,k} = V_{i,k-1} + \frac{1}{4}s_{i,k}^4 + \frac{1}{2}\tilde{\xi}_{i,k}^2 \tag{37}$$

Then, one gets

$$\begin{aligned} LV_{i,k} = & LV_{i,k-1} + s_{i,k}^3(s_{i,k+1} + \alpha_{i,k} - \check{l}_{i,k}y_i - H_{i,k20} \\ & + H_{i,k-1}) + \frac{3}{2}s_{i,k}^2\check{\varphi}_{i,k}^T\check{\varphi}_{i,k} - \tilde{\xi}_{i,k}\dot{\hat{\xi}}_{i,k} \end{aligned} \tag{38}$$

where $\check{\varphi}_{i,k} = -\sum_{m=1}^{k-1}\frac{\partial\alpha_{i,k-1}}{\partial\hat{x}_{i,m}}\check{q}_{i,m}$. By Young's inequalities, one has

$$\begin{aligned} s_{i,k}^3 s_{i,k+1} &\leq \frac{3}{4}s_{i,k}^4 + \frac{1}{4}s_{i,k+1}^4 \\ s_{i,k}^3\tilde{H}_{i,k-1} &\leq \frac{3}{4}s_{i,k}^4 + \frac{1}{4}H_{i,km}^4 \\ \frac{3}{2}s_{i,k}^2\check{\varphi}_{i,k}^T\check{\varphi}_{i,k} &\leq \frac{3}{4}\jmath_k^2 + \frac{3}{4}s_{i,k}^4||\check{\varphi}_{i,k}||^4\jmath_k^{-2} \end{aligned} \tag{39}$$

The unknown nonlinear function is selected as $\bar{f}_{i,k} = \frac{3}{4}s_{i,k}^4||\check{\varphi}_{i,k}||^4\jmath_k^{-2} - \check{l}_{i,k}y_i$ According to FLSs, one has

$$s_{i,k}^3\bar{f}_{i,k} \leq \frac{\xi_{i,k}}{2\upsilon_{i,k}^2}s_{i,k}^6\phi_{i,k}^T\phi_{i,k} + \frac{1}{2}\upsilon_{i,k}^2 + \frac{3}{4}s_{i,k}^4 + \frac{1}{4}\delta_{i,k}^4 \tag{40}$$

Where $\delta_{i,k}$ and $\upsilon_{i,k}$ are positive constants. The virtual control signal and the adaptive law are designed as

$$\begin{aligned} \alpha_{i,k} = & -c_{i,k}s_{i,k} - \frac{9}{4}s_{i,k} - \frac{1}{4}\varpi_{i,k}s_{i,k} + H_{i,k20} \\ & - \frac{\hat{\xi}_{i,k}}{2\upsilon_{i,k}^2}s_{i,k}^3\phi_{i,k}^T\phi_{i,k} \end{aligned} \tag{41}$$

$$\dot{\hat{\xi}}_{i,k} = \frac{1}{2\upsilon_{i,k}^2}s_{i,k}^6 - \aleph_{i,k}\hat{\xi}_{i,k} \tag{42}$$

where $c_{i,k}$, $\aleph_{i,k}$ are positive constants, and define $\varpi_{i,k}$ as

$$\varpi_{i,k} = \begin{cases} b_i + d_i, & k = 2 \\ 1, & otherwise \end{cases}$$

Then, one has

$$\begin{aligned} LV_{i,k} \leq & -\bar{\imath}||\tilde{x}_i||^4\Bigg(\lambda_{\min}(P_{ip})\lambda_{\min}(Q_{ip}) - 3\tau_{i,0}^{\frac{4}{3}}||P_{ip}||^{\frac{8}{3}} \\ & - \frac{1}{2\tau_{i,0}^4}(\sum_{m=1}^{n} g_{i,m}^2)^2 - \frac{3}{\epsilon_{i,0}}n^{\frac{3}{2}}(\sum_{m=1}^{n} h_{i,m}^2)^2 \\ & - 3\epsilon_{i,0}n^{\frac{3}{2}}||P_{ip}||^4 - \frac{1}{4\tau_{i,1}^4}\Bigg) + \Delta_{i,k} \\ & - \sum_{m=1}^{k}(c_{i,m}s_{i,m}^4 + \frac{\aleph_{i,m}}{2}\tilde{\xi}_{i,m}^2) + \frac{1}{4}s_{i,k+1}^4 \end{aligned} \tag{43}$$

where

$$\Delta_{i,k} = \Delta_{i,k-1} + \frac{3}{4}\jmath_{i,k}^2 + \frac{1}{2}\upsilon_{i,k}^2 + \frac{1}{4}\delta_{i,k}^4 + \frac{\aleph_{i,k}}{2}\xi_{i,k}^2 + \frac{1}{4}H_{i,km}^4$$

Step n: The self-triggered adaptive controller is designed in this step. Then, one has

$$\begin{aligned} ds_{i,n} &= d\hat{x}_{i,n} - d\alpha_{i,n-1} \\ &= (u_i - \check{l}_{i,n}\tilde{y}_i - L\alpha_{i,n-1})dt \end{aligned}$$

$$-(\sum_{m=1}^{n-1}\frac{\partial\alpha_{i,n-1}}{\partial\hat{x}_{i,m}}\check{q}_{i,m})dw \tag{44}$$

where

$$L\alpha_{i,n-1}=\frac{\partial\alpha_{i,n-1}}{\partial y_i}(\hat{x}_{i,1}+\tilde{x}_{i,1}+\check{f}_{i,1})+\sum_{m=1}^{n-1}\frac{\partial\alpha_{i,n-1}}{\partial\hat{\xi}_{i,m}}\dot{\hat{\xi}}_{i,m}$$
$$+\sum_{m=1}^{n-2}\frac{\partial\alpha_{i,n-1}}{\partial H_{i,m20}}\dot{H}_{i,m20}+\sum_{m=0}^{n-1}\frac{\partial\alpha_{i,n-1}}{\partial\hat{x}_{i,m}}\dot{\hat{x}}_{i,m}$$
$$+\frac{1}{2}\sum_{p,t=1}^{n-1}\frac{\partial^2\alpha_{i,n-1}}{\partial x_{i,p}\partial x_{i,t}}\check{q}_{i,p}\check{q}_{i,t}$$

Let the introduced filter estimate the derivative of virtual signal, and one has

$$H_{i,n20}-L\alpha_{i,n-1}=\tilde{H}_{i,n-1} \tag{45}$$

where $\tilde{H}_{i,n-1}$ represents the estimate error of the filter which satisfies $|\tilde{H}_{i,n-1}|\leq H_{i,nm}$.

Select the Lyapunov function as

$$V_{i,n}=\frac{1}{4}s_{i,n}^4+\frac{1}{2}\tilde{\xi}_{i,n}^2+V_{i,n-1} \tag{46}$$

Then, one gets

$$LV_{i,n}=s_{i,n}^3(u_i-\check{l}_{i,n}\tilde{y}_i-H_{i,n20}+H_{i,n-1})$$
$$+\frac{3}{2}s_{i,n}^2\check{\varphi}_{i,n}^T\check{\varphi}_{i,n}-\tilde{\xi}_{i,n}\dot{\hat{\xi}}_{i,n}+LV_{i,n_i-1} \tag{47}$$

where $\check{\varphi}_{i,n}=-\sum_{j=1}^{n-1}\frac{\partial\alpha_{i,n-1}}{\partial x_{i,j}}q_{i,j}$. By utilizing Young's inequality, one gets

$$\frac{3}{2}s_{i,n}^2\check{\varphi}_{i,n}^T\check{\varphi}_{i,n}\leq\frac{3}{4}\jmath_n^2+\frac{3}{4}s_{i,n}^4||\check{\varphi}_{i,n}||^4\jmath_n^{-2} \tag{48}$$

The unknown nonlinear function is selected as $\bar{f}_{i,n}=\frac{3}{4}s_{i,n}^4||\check{\varphi}_{i,n}||^4\jmath_n^{-2}-\check{l}_{i,n}y_i$. For $\forall\delta_{i,n},\upsilon_{i,n}>0$, one obtains

$$s_{i,n}^3\tilde{H}_{i,n-1}\leq\frac{3}{4}s_{i,n}^4+\frac{1}{4}H_{i,nm}^4$$
$$s_{i,n}^3\bar{f}_{i,n}\leq\frac{\xi_{i,n}}{2\upsilon_{i,n}^2}s_{i,n}^6\phi_{i,n}^T\phi_{i,n}+\frac{1}{2}\upsilon_{i,n}^2 \tag{49}$$
$$+\frac{3}{4}s_{i,n}^4+\frac{1}{4}\delta_{i,n}^4$$

One has

$$LV_{i,n}\leq s_{i,n}^3(u_i-H_{i,n20}+\frac{\xi_{i,n}}{2\upsilon_{i,n}^2}s_{i,n}^3\phi_{i,n}^T\phi_{i,n})-\tilde{\xi}_{i,n}\dot{\hat{\xi}}_{i,n}$$
$$+\frac{3}{4}\jmath_n^2+\frac{1}{2}\upsilon_{i,n}^2+\frac{1}{4}\delta_{i,n}^4+\frac{1}{4}H_{i,nm}^2+LV_{i,n-1} \tag{50}$$

From (9) and (10), there exists an inequality when $t\in[t_{i,k},t_{i,k+1})$

$$|\omega_i(t)-u_i(t)|\leq\eta_i|u_i(t)|+m_i \tag{51}$$

Then, one has

$$u_i(t)=\frac{\omega_i(t)-\rho_2(t)m_i}{1+\rho_1(t)\eta_i} \tag{52}$$

where $\rho_1(t)$ and $\rho_2(t)$ are time-varying functions which also have some properties: (a) $\rho_1(t_k)=\rho_2(t_k)=0$. (b) $\rho_1(t_{k+1})=\rho_2(t_{k+1})=1$. (c) $|\rho_1(t)|\leq 1$, $|\rho_2(t)|\leq 1$ and $t\in[t_k,t_{k+1})$. Obviously, one has $1+\rho_1\eta\geq 1-\eta$ so that

$$-\frac{\rho_2 m_i}{1+\rho_1\eta_i}\leq|\frac{m_i}{1-\eta_i}|$$

By applying Young's inequalities

$$s_{i,n}^3|\frac{m_i}{1-\eta_i}|\leq\frac{3}{4}s_{i,n}^4+\frac{1}{4}(\frac{m_i}{1-\eta_i})^4 \tag{53}$$

Then, one gets

$$LV_{i,n}\leq s_{i,n}^3(\frac{\xi_{i,n}}{2\upsilon_{i,n}^2}s_{i,n}^3\phi_{i,n}^T\phi_{i,n}-H_{i,n20}+\frac{1}{(1+J_i)}\omega_i(t))$$
$$-\tilde{\xi}_{i,n}\dot{\hat{\xi}}_{i,n}+LV_{i,n-1}+\frac{1}{4}(\frac{m_i}{1-\eta_i})^4$$
$$+\frac{3}{4}\jmath_n^2+\frac{1}{2}\upsilon_{i,n}^2+\frac{1}{4}\delta_{i,n}^4+\frac{1}{4}H_{i,nm}^2 \tag{54}$$

where $J_i=\rho_1\eta_i$. The continuous function and the adaptive law are selected as follows:

$$\omega_i(t)=-(1+J_i)(c_{i,n}s_{i,n}+\frac{\hat{\xi}_{i,n}}{2\upsilon_{i,n}^2}s_{i,n}^3\phi_{i,n}^T\phi_{i,n}-H_{i,n20}) \tag{55}$$

$$\dot{\hat{\xi}}_{i,n}=\frac{1}{2\upsilon_{i,n}^2}s_{i,n}^6-\aleph_{i,n}\hat{\xi}_{i,n} \tag{56}$$

where $c_{i,n}$ and $\aleph_{i,n}$ are positive constants. Eventually, one obtains

$$LV_{i,n}\leq-\bar{\imath}||\tilde{x}_i||^4\Bigg(\lambda_{\min}(P_{ip})\lambda_{\min}(Q_{ip})-3\tau_{i,0}^{\frac{4}{3}}||P_{ip}||^{\frac{8}{3}}$$
$$-\frac{1}{2\tau_{i,0}^4}(\sum_{m=1}^{n}g_{i,m}^2)^2-\frac{3}{\epsilon_{i,0}}n^{\frac{3}{2}}(\sum_{m=1}^{n}h_{i,m}^2)^2$$
$$-3\epsilon_{i,0}n^{\frac{3}{2}}||P_{ip}||^4-\frac{1}{4\tau_{i,1}^4}\Bigg)+\frac{1}{4}(\frac{m_i}{1-\eta_i})^4$$
$$-\sum_{m=1}^{n}(c_{i,m}s_{i,m}^4+\frac{\aleph_{i,m}}{2}\tilde{\xi}_{i,m}^2)+\frac{\aleph_{i,n}}{2}\xi_{i,n}^2$$
$$+\frac{3}{4}\jmath_n^2+\frac{1}{2}\upsilon_{i,n}^2+\frac{1}{4}\delta_{i,n}^4+\frac{1}{4}H_{i,nm}^2+\Delta_{i,n-1} \tag{57}$$

B. MDADT Method and Stability Analysis

Define an unknown positive constant vector $\xi_i=M^*(\xi_{i,1},\xi_{i,2},...,\xi_{i,M})\in R^{n\xi_i}$, and $M^*(\cdot)=(M_1^*(\cdot),M_2^*(\cdot),...,M_n^*(\cdot))^T$ is positive-definite function. Let $\tilde{\xi}_i=\xi_i-\hat{\xi}_i$ denote the estimation error. Then, the following theorem can be got.

Theorem 1: Based on the switched stochastic nonlinear MASs (1), consider μ_{ip}, ς_i, and $\psi_{ip}\geq 1$ as positive bounded constants where p satisfies $p\in\Gamma$. Assume that there exist positive definite, radially unbounded function $V_{ip}(x_i,\hat{\xi}_i)\in C^1$, smooth functions $\Phi_i(x_i,\hat{\xi}_i)$, adaptive laws and controllers

$$\dot{\hat{\xi}}_i=\Phi_i(x_i,\hat{\xi}_i) \tag{58}$$
$$u_{ip}=u_{ip}(x_i,\hat{\xi}_i) \tag{59}$$

so that $\forall p\in\Gamma$

$$LV_{ip}(x_i,\hat{\xi}_i)\leq-\mu_pV_{ip}(x_i,\hat{\xi}_i)+\varsigma_i \tag{60}$$

where $\mu_p = \min\{\mu_{ip}\}$. For $\forall(\sigma(t_i) = p, \sigma(t_i^-) = l) \in \Gamma \times \Gamma, p \neq l$

$$V_{ip}(x_i, \hat{\xi}_i) \leq \psi_{ip} V_{il}(x_i, \hat{\xi}_i) \leq \psi_p V_{il}(x_i, \hat{\xi}_i) \tag{61}$$

where $\psi_p = \max\{\psi_{ip}\}$. For any switching signals based synchronous MDADT, all signals in the closed-loop system (1) are bounded

$$\tau_{ap} \geq \tau_{ap}^* = \frac{\ln\psi_p}{\mu_p} \tag{62}$$

Proof: Considering $T > 0$ and $t_0 = 0$, let $t_1, t_2, ...t_i, t_{i+1}, ...t_{N_\sigma(T,0)}$ denote the switching times during the period $[0, T]$ where it has $N_\sigma(T, 0) = \sum_{p=1}^{M} N_{\sigma p}(T, 0)$. Construct $\Theta_i(t) = e^{a_{\sigma(t)}t} V_{i\sigma(t)}(X(t))$ which is piecewise differentiable along the solution of the system. When $t \in [t_j, t_{j+1})$, one obtains

$$\begin{aligned}\dot{\Theta}_i(t) &= a_{\sigma(t)} e^{a_{\sigma(t)}t} V_{i\sigma(t)}(X(t)) + e^{a_{\sigma(t)}t} \dot{V}_{i\sigma(t)}(X(t)) \\ &\leq \varsigma_i e^{a_{\sigma(t)}t}\end{aligned} \tag{63}$$

By utilizing $E\{dw(t)\} = 0$, one has

$$\begin{aligned}E\left\{\frac{d\Theta_i(t)}{dt}\right\} &= E\left\{a_{\sigma(t)} e^{a_{\sigma(t)}t} V_{i\sigma(t)}(X(t))\right\} \\ &\quad + E\left\{e^{a_{\sigma(t)}t} \dot{V}_{i\sigma(t)}(X(t))\right\} \\ &\leq E\left\{\varsigma_i e^{a_{\sigma(t)}t}\right\}\end{aligned} \tag{64}$$

Then, one obtains

$$\begin{aligned}E\left\{\int_{t_j}^{t_{j+1}} \dot{\Theta}_i(t) dt\right\} &= E\left\{\Theta_i(t_{j+1}^-)\right\} - E\left\{\Theta_i(t_j)\right\} \\ &\leq E\left\{\int_{t_j}^{t_{j+1}} \varsigma_i e^{a_{\sigma(t)}t} dt\right\}\end{aligned} \tag{65}$$

Considering $V_{ip}(t) \leq \psi_p V_{il}(t)$, it naturally holds that

$$\begin{aligned}E\left\{\Theta_i(t_{j+1})\right\} \leq E&\left\{\prod_{l=0}^{j} \psi_{\sigma(t_{l+1})} \Theta_i(t_0)\right. \\ &\left.\times e^{\sum_{l=0}^{j}[a_{\sigma(t_{l+1})} - a_{\sigma(t_l)}]t_{l+1}}\right\} \\ &+ E\left\{\sum_{s=0}^{j}\left\{\prod_{l=s}^{j} \int_{t_s}^{t_{s+1}} \varsigma_i e^{a_{\sigma(t_s)}t} dt\right\}\right. \\ &\left.\times e^{\sum_{l=0}^{j}[a_{\sigma(t_{l+1})} - a_{\sigma(t_l)}]t_{l+1}}\right\}\end{aligned} \tag{66}$$

Then, one has

$$\begin{aligned}E\left\{\Theta_i(T^-)\right\} \leq E&\left\{\prod_{l=0}^{N_\sigma(T,0)-1} \psi_{\sigma(t_{l+1})} \Theta_i(t_0)\right. \\ &\left.\times e^{\sum_{l=0}^{N_\sigma(T,0)-1}[a_{\sigma(t_{l+1})} - a_{\sigma(t_l)}]t_{l+1}}\right\} \\ &+ E\left\{\sum_{s=0}^{N_\sigma(T,0)-1}\left\{\prod_{l=s}^{N_\sigma(T,0)-1} \int_{t_s}^{t_{s+1}} \varsigma_i e^{a_{\sigma(t_s)}t} dt\right\}\right. \\ &\left.\times e^{\sum_{l=0}^{N_\sigma(T,0)-1}[a_{\sigma(t_{l+1})} - a_{\sigma(t_l)}]t_{l+1}}\right\} \\ &+ E\left\{\int_{t_{N_\sigma(T,0)}}^{T} \varsigma_i e^{a_{\sigma(N_\sigma(T,0))}t} dt\right\}\end{aligned} \tag{67}$$

The following inequality can be obtained.

$$\begin{aligned}&E\left\{V_{i\sigma(T^-)}(X(T))\right\} \\ &\leq E\left\{e^{\sum_{p=1}^{M} N_{0p} ln\psi_p} e^{\sum_{p=1}^{M} N_{0p}(\frac{T_p}{\tau_{ap}}\ln\psi_p - \mu_p T_p)}\right. \\ &\quad\left.\times V_{\sigma(0)}(X(0))\right\} \\ &\quad + E\left\{\sum_{s=0}^{N_{\sigma(T,0)}-1}\left\{\prod_{p=l}^{M} \psi_p^{N_{\sigma p}(T, t_{s+1})} e^{-\sum_{p=l}^{M} \mu_{ip} T_p(T, t_{s+1})}\right\}\right. \\ &\quad\left.\times e^{-\nu_{\min} t_{s+1}} \int_{t_s}^{t_{s+1}} \varsigma_i e^{\nu_{\min} t} dt\right\} \\ &\quad + E\left\{e^{-\nu_{\min} T} \int_{t_{N_\sigma(T,0)}}^{T} \varsigma_i e^{-\nu_{\min} t} dt\right\}\end{aligned} \tag{68}$$

Where $\nu_{\min} = \min\{\nu_p, p \in \Gamma\}$ and $\nu_p \in (0, (\mu_p - \ln\psi_p/\tau_{ap}))$. For $\forall \nu_p$, one has $\tau_{ap} \geq \ln\psi_p/(\mu_p - \nu_p)$. According to Definition 1, one gets

$$N_{\sigma p}(T, t) \leq N_{0p} + \frac{(\mu_p - \nu_p) T_p(T, t)}{\ln\psi_p} \tag{69}$$

Further, the following inequality holds

$$\psi_p^{N_{\sigma p}(T, t_{s+1})} \leq \psi_p^{N_{0p}} e^{(\mu_p - \nu_p) T_P(T, t_{s+1})} \tag{70}$$

Substituting (70) into (68) yields

$$\begin{aligned}E\left\{V_{i\sigma(T^-)}(X(T))\right\} &\leq e^{\sum_{p=1}^{M} N_{0p}\ln\psi_p} e^{\max\limits_{p\in\Omega}(\frac{\ln\psi_p}{\tau_{ap}} - \mu_p)T} \\ &\quad \times E\left\{V_{i\sigma(0)}(X(0))\right\} \\ &\quad + \prod_{p=1}^{M} \psi_p^{N_{0p}} \frac{\varsigma_i}{\nu_{min}}\end{aligned} \tag{71}$$

Based on the above analysis, if there exists the MDADT which satisfies $\tau_{ap} \geq \frac{\ln\psi_p}{\mu_p}$, $V_{i\sigma(T^-)}(X(T))$ can converge to the small neighborhood close to zero when $T \to \infty$. Further, the signals in the system (1) can be bounded.

Theorem 2: Consider the switched stochastic MASs (1) based on the privacy preservation. If the Assumptions 1-3 hold and the MDADT satisfies the condition $\tau_{ap} \geq \frac{\ln\psi_p}{\mu_p}$, all the signals are bounded by utilizing the switched neighborhood observer (12), adaptive laws (34), (42), (56), and the self-triggered controller (55).

Proof: Construct the Lyapunov functions as follows:

$$V_{ip} = \frac{\bar{\iota}}{2}(\tilde{x}_i^T P_p \tilde{x}_i)^2 + \frac{1}{4}\sum_{m=1}^{n} s_{i,m}^4 + \frac{1}{2}\sum_{m=1}^{n} \tilde{\xi}_{i,m}^2 \tag{72}$$

In view of $\lambda_{\min}(P_{ip})\lambda_{\min}(Q_{ip}) - 3\tau_{i,0}^{\frac{4}{3}}||P_{ip}||^{\frac{8}{3}} - \frac{1}{4\tau_{i,1}^4} - \frac{1}{2\tau_{i,0}^4}(\sum_{m=1}^{n} g_{i,m}^2)^2 - \frac{3}{\epsilon_{i,0}} n^{\frac{3}{2}}(\sum_{m=1}^{n} h_{i,m}^2)^2 - 3\epsilon_{i,0} n^{\frac{3}{2}}||P_{ip}||^4 \geq c_{i,0} > 0$, one has

$$LV_{ip} \leq -c_{i,0}||\tilde{x}_i||^4 - \sum_{m=1}^{n}(c_{i,m}s_{i,m}^4 + \frac{\aleph_{i,m}}{2}\tilde{\xi}_{i,m}^2)$$

$$+\Delta_{i,n-1}+\frac{1}{4}(\frac{\eta_i}{1-\eta_i})^4+\frac{1}{4}(\frac{m_i}{1-\eta_i})^4+\frac{\aleph_{i,n}}{2}\xi_{i,n}^2$$
$$+\frac{3}{4}\jmath_n^2+\frac{1}{2}\upsilon_{i,n}^2+\frac{1}{4}\delta_{i,n}^4+\frac{1}{4}H_{i,nm}^2$$
$$\leq -\mu_{ip}V_{ip}+\varsigma_i \tag{73}$$

where

$$\mu_{ip}=\min\left\{(2c_{i,0}/[\bar{\imath}\lambda_{\max}^2(P_{ip})]),4c_{i,m},\aleph_{i,m}\right\}$$
$$\varsigma_i=\frac{\bar{\imath}}{2\tau_{i,0}^4}(\xi_{i,0p}^2+\delta_{i,0p}^4)+\sum_{m=1}^{n}\frac{\aleph_{i,n}}{2}\xi_{i,m}^2+\frac{3}{4}\sum_{m=1}^{n}\jmath_{i,m}^2$$
$$+\frac{1}{4\epsilon_{i,1}^3}(b_i+d_i)\vartheta_{i,1}^4+\frac{1}{2}\sum_{m=1}^{n}\upsilon_{i,m}^2+\frac{1}{4}\sum_{m=1}^{n}\delta_{i,m}^4$$
$$+\frac{1}{4}(\frac{\eta_i}{1-\eta_i})^4+\frac{1}{4}(\frac{m_i}{1-\eta_i})^4+\frac{1}{4}\sum_{m=2}^{n}H_{i,mm}^2$$

Then, let $\psi_{ip}=e^{\max\left\{[\lambda_{\max}^{\flat}(P_{ip})]/[\lambda_{\min}^{\flat}(P_{il})]\right\}}$ where $\forall p,l\in\Gamma$ and $\flat$ is a positive constant. Obviously there is $\psi_p>1$. Then, one obtains

$$V_{ip}(t)\leq\psi_p V_{il}(t) \tag{74}$$

According to Theorem 1, since the MDADT $\tau_{ap}\geq\frac{\ln\psi_p}{\mu_p}$ and (74) are satisfied respectively, the signals in the system (1) are bounded.

IV. Simulation Results

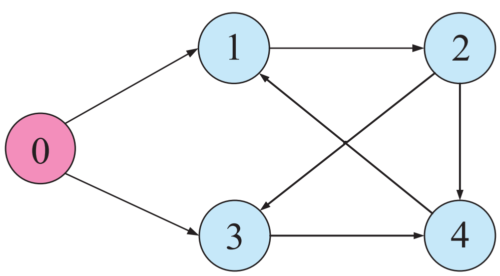

Fig. 2. The topology of the communication.

In this section, a numerical example verifies the effectiveness of the control scheme. We consider a class of stochastic MASs with four followers and one leader. The communication topology of stochastic MASs is shown and illustrated in Fig. 2. The adjacency matrix $\mathcal{A}$ can be written as

$$\mathcal{A}=\begin{bmatrix}0&0&0&1\\1&0&0&0\\0&1&0&0\\0&1&1&0\end{bmatrix} \tag{75}$$

The dynamics of the ith $(i=1,2,3,4)$ agent is defined in the strict-feedback form

$$\begin{cases}dx_{i,1}=(\hat{x}_{i,2}+\check{f}_{i,1}(\bar{x}_{i,1}))dt+\check{q}_{i,1}(\bar{x}_{i,1})dw\\ dx_{i,2}=(u_i+\check{f}_{i,2}(\bar{x}_{i,2}))dt+\check{q}_{i,2}(\bar{x}_{i,2})dw\\ \quad y_i=x_{i,1}\end{cases} \tag{76}$$

where $\hat{x}_{i,2}$ denotes the estimation of $x_{i,2}$. $\check{f}_{i,1},\check{f}_{i,2},\check{q}_{i,1}$ and $\check{q}_{i,2}$ are nonlinear functions for agent i under switching signal. Specifically, one has

$$f_{i,1}^{(1)}(\bar{x}_{i,1})=0.2x_{i,1}^2\text{sin}(x_{i,1}),f_{i,1}^{(2)}(\bar{x}_{i,1})=0.3x_{i,1}\text{cos}(x_{i,1})$$
$$f_{i,2}^{(1)}(\bar{x}_{i,2})=0.1x_{i,1}x_{i,2}\text{sin}(x_{i,2}),f_{i,2}^{(2)}(\bar{x}_{i,2})=0.1x_{i,1}^2\text{cos}(x_{i,2})$$
$$q_{i,1}(\bar{x}_{i,1})=0.5\text{cos}(x_{i,2}),q_{i,2}(\bar{x}_{i,2})=0.5x_{i,1}^2x_{i,2}^2\text{cos}(x_{i,2})$$

The dynamics of the leader node is modeled by as $y_0=\sin(t)$.

Based on the topology in Fig. 2 and the switching signal in Fig. 3, the switched observer parameters are designed as $l_{1,1}^{(1)}=l_{3,1}^{(1)}=l_{4,1}^{(1)}=8$, $l_{2,1}^{(1)}=16$, $l_{1,2}^{(1)}=l_{3,2}^{(1)}=2$ and $l_{2,2}^{(1)}=l_{4,2}^{(1)}=3$. $l_{1,1}^{(2)}=l_{3,1}^{(2)}=l_{4,1}^{(2)}=4$, $l_{2,1}^{(2)}=8$, $l_{1,2}^{(2)}=l_{3,2}^{(2)}=2$ and $l_{2,2}^{(2)}=l_{4,2}^{(2)}=3$. $\Xi_i^{(.)}$ are Hurwitz matrix. When $\digamma_i^{(1)}=10I$ and $\digamma_i^{(2)}=8I$, one has

$$P_1^{(1)}=P_3^{(1)}=\begin{bmatrix}0.94&-5\\-5&40.47\end{bmatrix},P_2^{(1)}=P_4^{(1)}=\begin{bmatrix}1.25&-5\\-5&27.08\end{bmatrix}$$

$$P_1^{(2)}=P_3^{(2)}=\begin{bmatrix}1.5&-4\\-4&16.75\end{bmatrix},P_2^{(2)}=P_4^{(2)}=\begin{bmatrix}2&-4\\-4&11.33\end{bmatrix}$$

Then, one obtains $\lambda_1(P_1^{(1)})=0.3149$, $\lambda_2(P_1^{(1)})=41.0914$, $\lambda_1(P_1^{(2)})=0.5145$, $\lambda_2(P_1^{(2)})=17.7355$, $\lambda_1(P_2^{(1)})=0.3160$, $\lambda_2(P_2^{(1)})=28.0173$, $\lambda_1(P_2^{(2)})=0.5203$ and $\lambda_2(P_2^{(2)})=12.8130$. According to the following designed parameters, one has $\mu_{1p}=\mu_{3p}=0.8$ and $\mu_{2p}=\mu_{4p}=0.8$. Further, MDADT can be obtained as $\tau_{a1}=4.31$ and $\tau_{a2}=3.85$.

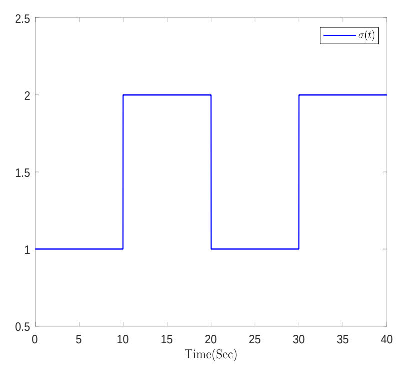

Fig. 3. The switching signal.

The parameters of the controllers are shown as follows. $c_{1,1}=c_{3,1}=13$, $c_{2,1}=c_{4,1}=16$, $c_{1,2}=c_{3,2}=6$, $c_{2,2}=c_{4,2}=8$, $\aleph_{1,1}=\aleph_{2,1}=\aleph_{3,1}=\aleph_{4,1}=0.8$ and $\aleph_{1,2}=\aleph_{2,2}=\aleph_{3,2}=\aleph_{4,2}=0.8$. Simultaneously, for $i=1,2,3,4$, the parameter of filter is $J_i=0.8$, $o_{i,210}=o_{i,220}=2$, $v_{i,210}=v_{i,220}=1$ and $r_{i,210}=r_{i,220}=1$. Further, the initial condition is designed as $x_{i,1}(0)=0.1$, $x_{i,2}(0)=0.1$, $\xi_{i,1}(0)=\xi_{i,2}(0)=0.1$ and $H_{i,210}(0)=H_{i,220}(0)=0.1$.

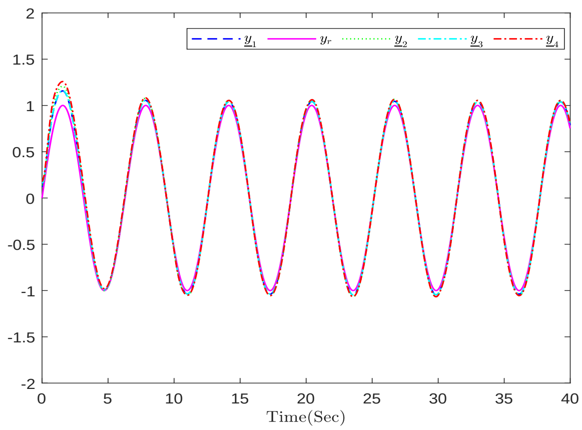

Fig. 4. The tracking performance.

The Fig. 4 and Fig. 5 respectively show the tracking performance and tracking errors of each agent under switching signal during 40 seconds. Both the tracking performance and tracking errors are divided into two different situations, which match up with the designed switching signal in Fig. 3. According to the analysis about MDADT in stochastic MASs under switching dynamics, agent 1 and agent 3 are designed under the bigger gains because they directly link to the leader.

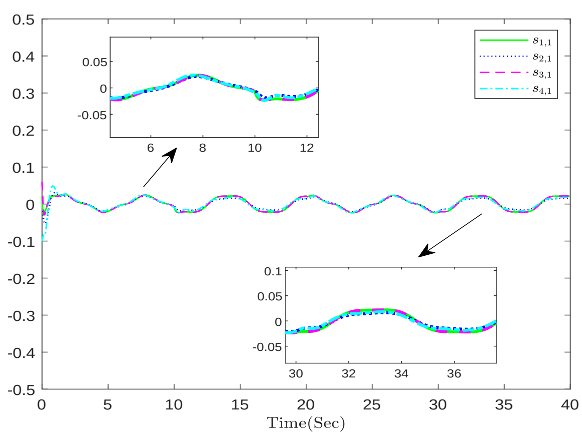

Fig. 5. The tracking errors.

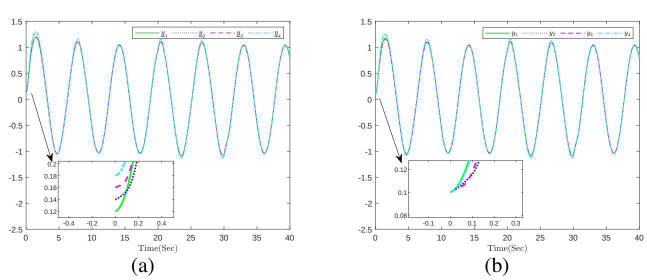

Fig. 6. Privacy preserving results of output. (a) the masked states; (b) the original states.

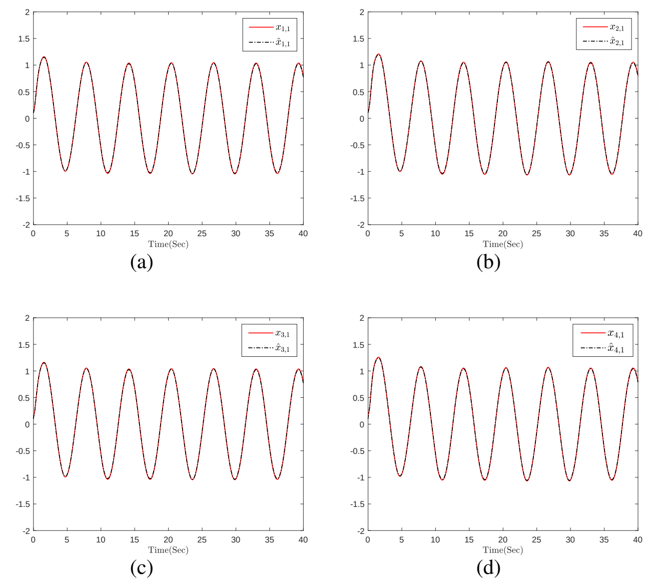

Fig. 7. Curves of states $x_{i,1}$ and $\hat{x}_{i,1}$.

As is shown in Fig. 6, the influence of privacy preservation is acquired. By choosing parameters arbitrarily, the different masked initial states in Fig. 6 (a) are obtained so that the level of privacy can be guaranteed arbitrarily. Meanwhile, the original initial states are displayed in Fig. 6 (b).

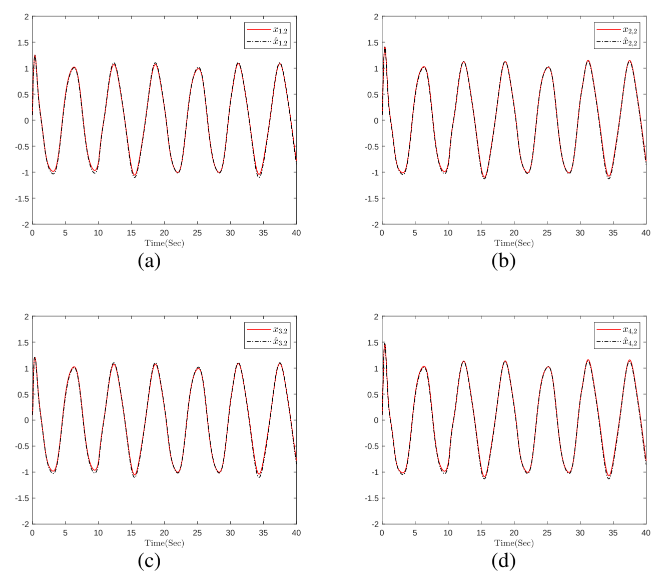

Fig. 8. Curves of states $x_{i,2}$ and $\hat{x}_{i,2}$.

Fig. 7 and Fig. 8 display the curves of switched state observers. The differences between the two observers are revealed in Fig. 9. Since the information of neighborhood is considered to construct the observer, the better observation performance is obtained.

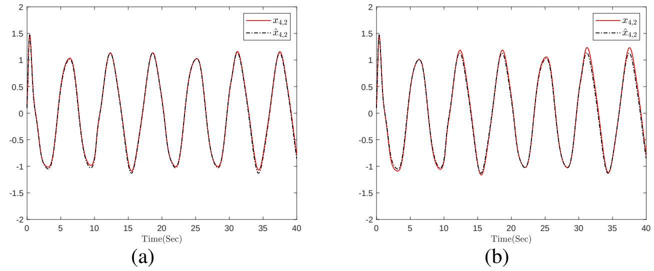

Fig. 9. The comparison of observer curves. (a) switched neighborhood observer; (b) switched observer in [36].

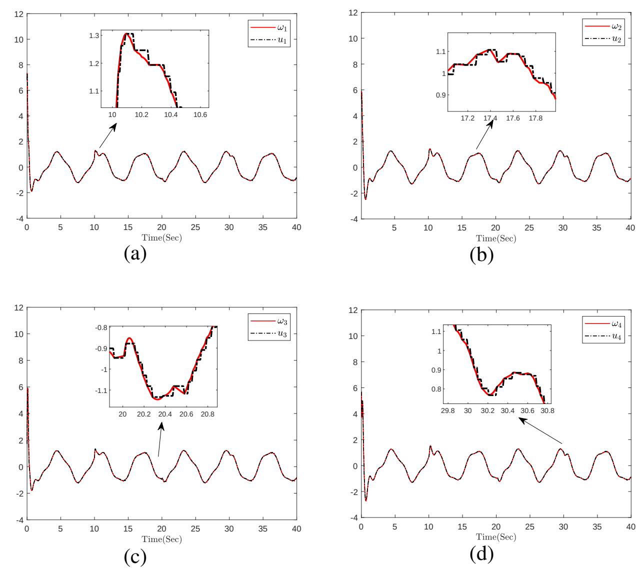

Fig. 10. The self-triggered control signal u_i.

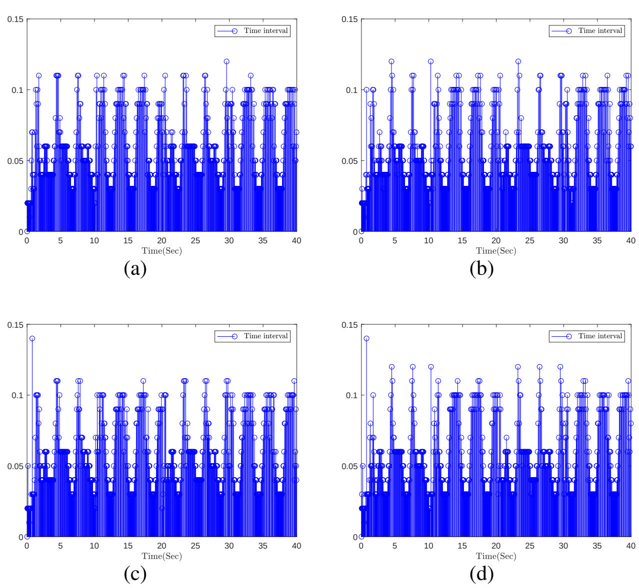

Fig. 11. The triggered times. (a) Agent 1; (b) Agent 2; (c) Agent 3; (d) Agent 4.

Fig. 10 shows the information of self-triggered input signal and the original input signal. Fig. 11 represents the trigger intervals and trigger numbers. Because the switching dynamics are considered in this paper, the control signals and trigger intervals are spontaneously divided into two different situations.

TABLE I
THE TRIGGERED EVENTS OF TWO SELF-TRIGGERED MECHANISMS.

	Agent 1	Agent 2	Agent 3	Agent 4
Modified mechanism	765	788	765	793
Normal mechanism in [47]	859	870	859	872

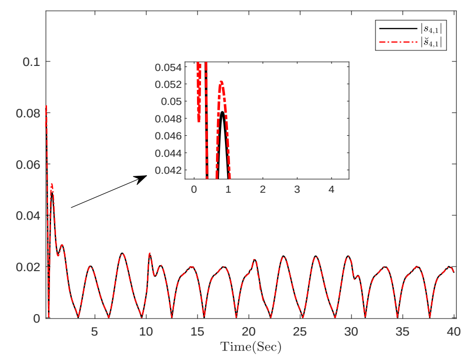

Fig. 12. The tracking errors of agent 4.

Under the modified self-triggered mechanism, the trigger times about each agents are given in Table 1, which are decreased compared with normal self-triggered mechanism in [47]. Further, the tracking errors about agent 4 under these two different self-triggered mechanisms are shown in Fig. 12. Since the auxiliary functions are designed to contact with the synchronization error, the tracking errors well decrease while the trigger times reduce. It can verify the advantages of modified self-triggered mechanism proposed in this paper.

V. CONCLUSIONS

A self-triggered fuzzy adaptive control problem has been considered for switched stochastic MASs under the switching signals with MDADT method. Firstly, synchronous M-DADT method has been considered for stochastic MASs with switched dynamics based on some switching characteristics. Then, the switched state observer has been designed to improve the observed performance of switching states. The initial states of each agent have been masked by using the switching-related additive mask. Next, the singularity problem caused by the derivative of input has been avoided by using the modified self-triggered mechanism. Finally, it has been proven that all signals of the closed-loop system are verified to be ultimately bounded under a class of switching signals with MDADT property by the designed self-triggered control signals. The effectiveness of the designed control method has been verified by some simulation results. In the future, the analysis of stability for stochastic MASs will be further developed.

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Hongjing Liang (Member, IEEE) received the B.S. degree in mathematics from Bohai University, Jinzhou, China, in 2009, and the M.S. degree in fundamental mathematics in 2011, and the Ph.D. degree in control theory and control engineering in 2016, both from Northeastern University, Shenyang, China.

He was a Temporary Research Associate with the Science Program, Texas A&M University, Doha, Qatar. He is currently a Professor with the University of Electronic Science and Technology of China, Chengdu, China, and also with the Laboratory of Electromagnetic Space Cognition and Intelligent Control, Beijing, China. His current research interests include adaptive control, fuzzy control, and cooperative control and their applications.

Dr. Liang was the recipient of the Outstanding Reviewer Award of CAA/Automatica Sinica 2019. He has been in the Editorial Board of *International Journal of Fuzzy Systems*, *Fluctuation and Noise Letters*.

Wenzhe Wang received the B.S. degree in automation and electronic engineering from Qingdao University of Science and Technology, Qingdao, China, in 2021. He is currently pursuing the M.S. degree in control theory and control engineering with Bohai University, Jinzhou, China.

His current research interests include fuzzy control, adaptive control, cooperative control and their applications.

Yingnan Pan (Member, IEEE) received the B.S. degree in mathematics and applied mathematics and the M.S. degree in applied mathematics from Bohai University, Jinzhou, China, in 2012 and 2015, respectively, and the Ph.D. degree in navigation guidance and control from Northeastern University, Shenyang, China, in 2019.

He is currently an associate professor with Bohai University, Jinzhou, China. His current research interests include fuzzy control, adaptive control, event-triggered control and their applications.

Hak-Keung Lam (Fellow, IEEE) received the B.Eng. (Hons.) degree in electronic engineering and Ph.D. degree in electronic and information engineering from the Department of Electronic and Information Engineering, The Hong Kong Polytechnic University, Hong Kong, China, in 1995 and 2000, respectively. During the period of 2000 and 2005, he was a Postdoctoral Fellow and Research Fellow with the Department of Electronic and Information Engineering, The Hong Kong Polytechnic University. He was a Lecturer with King’s College London, London, U.K. in 2005 and is currently a Reader. He has authored or coauthored three monographs: Stability Analysis of Fuzzy-Model-Based Control Systems (Springer, 2011), Polynomial Fuzzy Model Based Control Systems (Springer, 2016), and Analysis and Synthesis for Interval Type-2 Fuzzy-Model-Based Systems (Springer, 2016).

His current research interests include intelligent control (in particular, fuzzy control), computational intelligence, and machine learning (deep learning and reinforcement learning). Dr. Lam has served as a Program Committee Member, International Advisory Board Member, Invited Session Chair, and Publication Chair for various international conferences and a reviewer for various books, international journals, and international conferences. He is an Associate Editor for IEEE Transactions on Fuzzy Systems, IEEE Transactions on Circuits and Systems II: Express Briefs, IET Control Theory and Applications, International Journal of Fuzzy Systems, Neurocomputing, and Nonlinear Dynamics. He also serves as Guest Editor and on the editorial board for a number of international journals. He is a Coeditor of two edited volumes: Control of Chaotic Nonlinear Circuits (World Scientific, 2009) and Computational Intelligence and Its Applications (World Scientific, 2012).

Jiayue Sun (Member, IEEE) received the Ph.D. degree in power electronics and power transmission from Northeastern University in Shenyang, China, under the supervision of IEEE Fellow Huaguang Zhang, in 2021.

She is a postdoctoral fellow under Tianyou Chai (a member of the Chinese Academy of Engineering, IEEE Life Fellow), in the State Key Laboratory of Synthetical Automation for Process Industries, Northeastern University. Now She is a teacher at Northeastern University, a member of the Chinese Association of Automation (CAA) and Chinese Association for Artificial Intelligence(CAAI), and works as the Intelligent Adaptive Learning Committee’s secretary of CAAI. She has authored or co-authored 30 peer-reviewed international journal papers.

Her current research interests include optimization of complex industrial processes, intelligent adaptive learning, and distributed control of multi-agent systems.