Integrated Fault-Tolerant Control Design with Sampled-Output Measurements for Interval Type-2 Takagi-Sugeno Fuzzy Systems

Hongying Zhou, Hak-Keung Lam, *Fellow, IEEE,* Bo Xiao, *Member, IEEE,* and Chengbin Xuan

***Abstract*—This paper is concerned with the integrated design of fault estimation (FE) and fault-tolerant control (FTC) for uncertain nonlinear systems suffering from actuator faults and external disturbance. The uncertain nonlinear systems are characterized as the interval type-2 (IT2) Takagi-Sugeno (T-S) fuzzy model, and IT2 membership functions are employed to effectively handle uncertainties. A fuzzy observer, utilizing only sampled-output measurements, is applied to simultaneously estimate actuator faults and system states. Based on the estimation, the fault-tolerant controller is designed to ensure the system stability under a predefined H_∞ performance. The sampling behavior complicates the system dynamics and makes the integrated FTC design more challenging. To confront this issue, the discontinuous Lyapunov functional technique is exploited to enhance stability results by considering the sampling characteristic, upon which FE and FTC units are co-designed in the linear matrix inequality (LMI) framework. To further relax stability criteria, the analysis process incorporates the bound information of membership functions through the membership-function-dependent (MFD) method. Additionally, the relationship of mismatched premise variables resulting from the sampling scheme is also taken into account. Moreover, considering the imperfect premise matching (IPM) framework, the proposed fault-tolerant controller provides greater flexibility in selecting the shapes of membership functions and number of fuzzy rules that can vary from the counterpart of the fuzzy system. Finally, the efficacy of the proposed FTC technique is validated through a detailed numerical example.**

***Index Terms*—Interval type-2 (IT2) fuzzy model, fault estimation (FE), fault-tolerant control (FTC), membership-function-dependent (MFD) method, sampled-output measurements.**

I. Introduction

COMPLEX nonlinear systems, posing significant challenges to system analysis and synthesis, are prevalent in real-world applications. The Takagi-Sugeno (T-S) fuzzy model, which consists of a bunch of local linear subsystems blended via membership functions, is shown to be an efficient approach to describe nonlinear plants [?], [?]. The T-S fuzzy model, characterized by a systematic model structure and advantageous properties, enables the analysis and control synthesis of nonlinear plants. This is achieved by integrating fruitful linear system theories alongside fuzzy logic theories [?], [?]. Apart from nonlinearity, uncertainties quite often appear in many cases, such as parameter variation and measurement inaccuracy, which could deteriorate the system control performance if not fully considered in the analysis. While the traditional type-1 fuzzy set possesses prominent capabilities to deal with nonlinearity, it is less capable of handling uncertainties with its definite membership functions directly. Therefore, the subsequent interval type-2 (IT2) fuzzy set [?] expands abilities in handling nonlinearity within uncertain environments, where uncertainties can be directly captured and represented by IT2 membership functions [?]. Benefiting from this favorable property, a large quantity of research results on the IT2 fuzzy framework have been published, such as stability analysis [?], [?], tracking control [?], model reduction [?], [?], and filtering [?], [?]. It is noteworthy that [?] intorduced an effective constructing technique for the IT2 fuzzy model, which was followed by an innovative membership-function-dependent (MFD) method reported in [?] under the imperfect premise matching (IPM) mechanism in which fuzzy rule number and shapes of membership functions of the fuzzy controller and fuzzy model allow to be diverse. In [?], [?], [?], control applications of IT2 fuzzy sets such as video streaming, mobile robots, and power systems can be found.

What should be noted is that the design of fault-tolerant control (FTC) systems [?] is of considerable significance in nonlinear control communities, which maintains the system functionality with an admissible performance in the event of failures, especially when more rigorous demands on system safety and reliability are required in engineering fields, for example, nuclear power plants, underwater vehicles, aircraft, and chemical processes. Existent FTC methods can be generally categorized into two types [?], [?]: passive and active. For both normal and faulty scenarios, the passive FTC methods employ the unaltered controller throughout the whole control process, capable of accommodating a collection of presumed faults. In contrast, the active FTC methods proactively adjust controllers in response to the impact of faults imposed on the plant, and compensate for fault effects by utilizing the online fault estimation (FE), which reveals superior fault tolerance capabilities. Recalling the merits of fuzzy model in tackling nonlinear systems, the fuzzy FTC techniques have attracted great attention and undergone extensive investigation, see, e.g., [?], [?], [?], [?], [?], [?], [?], [?], [?], [?], [?]. However, the reported FTC strategies regarding IT2 fuzzy systems are relatively fewer in comparison to the type-1 counterpart. The works in [?], [?], [?], [?], [?] investigate active FTC issues

This work was supported in part by King's College London, and in part by China Scholarship Council. (*Corresponding author: Hak-Keung Lam.)*

Hongying Zhou, Hak-Keung Lam and Chengbin Xuan are with the Department of Engineering, King's College London, Strand, London, WC2R 2LS, U.K. (e-mail: {hongying.zhou, hak-keung.lam, chengbin.xuan}@kcl.ac.uk).

Bo Xiao is with the Hamlyn Centre for Robotic Surgery, Imperial College London, London SW7 2AZ, U.K. (e-mail: b.xiao@imperial.ac.uk).

for IT2 fuzzy systems, among which only [?] considers the design of the FE and FTC modules simultaneously, and [?], [?], [?], [?] are realized by the two-step design method.

On the other hand, sampled-data control mechanisms play a significant role in theoretical and practical research fields due to widely used digital technologies. Within sampled-data control systems, the control signals hold constants over the sampling interval and update only at the sampling instants [?]. This behavior perplexes the systems dynamics and brings about great challenges in the stability analysis with the discontinuity included. To perform stability analysis for sampled-data control systems, several methods have been presented such as the impulsive model approach [?], the discrete-time approach [?], and the input delay approach [?], [?]. In the last strategy, discrete-time signals are converted to the time-delay forms, enabling continuous-time stability analysis methods available to sampled-data control systems. By virtue of the input delay method, a large amount of research [?], [?], [?], [?], [?] concerning the fuzzy sample-data control strategies has been published. Nevertheless, to the best of the authors' knowledge, no results have been published on the observer-based active FTC issue for IT2 fuzzy systems under the sampled-data mechanism. For the sampled observer-based fuzzy FTC system, there exist bidirectional robustness interactions [?] between the fault-tolerant controller and observer resulting from mismatched membership functions and estimation errors. If the bidirectional robustness interactions are ignored in the analysis, only a suboptimal solution is expected. Hence, the integrated design approach is required where the fault-tolerant controller and observer are designed together. It is beneficial to obtain an optimal solution with the desired fault-tolerant performance, but it makes the sampled-data based integrated FTC problem even more complex.

Inspired by the aforesaid discussions, in this work, we propose a sampled-data based integrated design of FE and FTC for nonlinear plants subject to uncertainties, actuator faults, and external disturbances, depicted by the IT2 T-S fuzzy model. Resorting to the Lyapunov stability theory, the MFD results are developed to ensure the asymptotical stability of the augmented system under the prespecified H_∞ performance. Then, the technique on co-designing the FE and FTC is provided via the one-step linear matrix inequality (LMI) expression. At last, a simulation example is proposed to illuminate the efficacy of the established FTC technique. The key contributions of the paper are shown as below:

1) In this work, to facilitate the digital implementation, an integrated fuzzy FTC design technique is provided for nonlinear systems with uncertainties, actuator faults, and external disturbance via co-designing FE and FTC units based on sampled-output measurements. The proposed FTC strategy is designed to obtain an optimal solution by considering the bidirectional robustness interaction between the observer and controller, different from most existing IT2 fuzzy FTC approaches that ignore such interactions.
2) The discontinuous Lyapunov functional technique, which considers the sampling characteristic, is utilized along with the MFD method to relax stability criteria. Moreover, the mismatched problem of premise variables stemming from sampling behaviors is considered as well to earn tighter membership function bounds, which facilitates the stability analysis.
3) The developed IT2 fuzzy observer estimates both system states and actuator faults using only sampled measurements. In addition, the fault-tolerant controller enjoys more design flexibility with the aid of IPM scheme where the fuzzy rule number and membership functions of the fuzzy controller and the fuzzy system are allowed to be different.

The remainder of the paper is organized as below. In Section II, some preliminaries are presented. Section III introduces the main results. Section IV gives a simulation example to demonstrate the validity of the designed FTC technique. In Section V, the conclusions are summarized.

Notations: I and 0 indicate the identity matrix and zero matrix with compatible dimensions, respectively; $P > 0 (\geq 0)$ represents that P is a real symmetric and positive definite (semidefinite) matrix; the superscript "T" represents matrix transposition; $\star$ denotes a term produced by symmetry; $diag\{\dots\}$ is used to describe a block-diagonal matrix.

II. Preliminaries

A. IT2 T-S Fuzzy Model

Consider a family of nonlinear systems affected by additive actuator faults and external disturbance along with uncertainties, which are depicted as the IT2 T-S fuzzy model as below:

Rule i: IF $\psi_1(\varpi(t))$ is $\tilde{\mho}_1^i$ and $\psi_2(\varpi(t))$ is $\tilde{\mho}_2^i$ and $\cdots$ and $\psi_\Psi(\varpi(t))$ is $\tilde{\mho}_\Psi^i$, THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i\big(u(t) + f(t)\big) + D_i w(t), \\ y(t) = Cx(t), \end{cases}$$

in which $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^p$ stand for the system state, control input, and measurement output, respectively. $w(t) \in \mathbb{R}^d$ indicates the external disturbance that belong to $\mathcal{L}_2[0,\infty)$. $f(t) \in \mathbb{R}^m$ signifies actuator faults with an assumption that $\dot{f}(t)$ belongs to $\mathcal{L}_2[0,\infty)$. $\psi_\alpha(\varpi(t))$ represents the premise variables and $\tilde{\mho}_\alpha^i$ stands for IT2 fuzzy sets where $\alpha = 1, 2, \dots, \Psi$ and $i = 1, 2, \dots, r$. $\varpi(t)$ denotes the measurable system states. A_i, B_i, D_i, and C are known constant matrices. The subsequent interval sets depict the firing strength of rule i

$$\mathscr{W}_i(\varpi(t)) = [\underline{w}_i(\varpi(t)),\ \bar{w}_i(\varpi(t))], \quad i = 1, 2, \dots, r$$

in which $\underline{w}_i(\varpi(t)) = \prod_{\alpha=1}^{\Psi} \underline{\mu}_{\tilde{\mho}_\alpha^i}(\psi_\alpha(\varpi(t)))$ and $\bar{w}_i(\varpi(t)) = \prod_{\alpha=1}^{\Psi} \bar{\mu}_{\tilde{\mho}_\alpha^i}(\psi_\alpha(\varpi(t)))$ denote the lower and upper grades of membership, respectively. $\underline{\mu}_{\tilde{\mho}_\alpha^i}(\psi_\alpha(\varpi(t)))$ and $\bar{\mu}_{\tilde{\mho}_\alpha^i}(\psi_\alpha(\varpi(t)))$ signify the lower and upper membership functions, respectively. Upon definitions of IT2 membership functions, $\bar{\mu}_{\tilde{\mho}_\alpha^i}(\psi_\alpha(\varpi(t))) \geq \underline{\mu}_{\tilde{\mho}_\alpha^i}(\psi_\alpha(\varpi(t))) \geq 0$, which results in the holding of $\bar{w}_i(\varpi(t)) \geq \underline{w}_i(\varpi(t)) \geq 0$ for all i. The inferred IT2 fuzzy model is shown as

$$\begin{cases} \dot{x}(t) = \displaystyle\sum_{i=1}^{r} w_i(\varpi(t))\big[A_i x(t) + B_i\big(u(t) + f(t)\big) + D_i w(t)\big], \\ y(t) = Cx(t), \end{cases} \tag{1}$$

in which $w_i(\varpi(t)) = \underline{\epsilon}_i(\varpi(t))\underline{w}_i(\varpi(t)) + \bar{\epsilon}_i(\varpi(t))\bar{w}_i(\varpi(t)) \geq 0$, furnished with the performance that $\sum_{i=1}^{r} w_i(\varpi(t)) = 1$. $0 \leq \underline{\epsilon}_i(\varpi(t)) \leq 1$ and $0 \leq \bar{\epsilon}_i(\varpi(t)) \leq 1$ indicate nonlinear weighting functions with $\underline{\epsilon}_i(\varpi(t)) + \bar{\epsilon}_i(\varpi(t)) = 1$ for all i. In this work, the fuzzy system (1) is assumed to be controllable and observable, which allows existence of controllers and observers to attain the expected FTC performance.

Remark 1. *In this work, the actuator is only supposed to suffer partial damage. Under such a condition, the faults may be compensated by adjusting the actuator action.*

B. IT2 Fuzzy Observer With Sampled-Output Measurements

In this work, the designed observer relies only on the sampled-output measurements to simultaneously estimate system states and actuator faults. In addition, as system membership functions are uncertain, the proposed fuzzy observer only shares the known bound expressions of membership functions with the fuzzy system. Thus, the rule j of the IT2 fuzzy observer is represented by

Rule j: IF $\psi_1(\varpi(t_s))$ is $\tilde{\mho}_1^j$ and $\psi_2(\varpi(t_s))$ is $\tilde{\mho}_2^j$ and $\cdots$ and $\psi_\Psi(\varpi(t_s))$ is $\tilde{\mho}_\Psi^j$, THEN

$$\begin{cases} \dot{\hat{x}}(t) = A_j\hat{x}(t) + B_j\big(u(t) + \hat{f}(t)\big) + L_j\big(y(t_s) - \hat{y}(t_s)\big), \\ \dot{\hat{f}}(t) = F_j\big(y(t_s) - \hat{y}(t_s)\big), \\ \hat{y}(t) = C\hat{x}(t), \end{cases}$$

where $\hat{x}(t) \in \mathbb{R}^n$, $\hat{f}(t) \in \mathbb{R}^m$, $\hat{y}(t) \in \mathbb{R}^p$ are the observer state, fault estimation, and observer output, respectively. t_s denotes the sth sampling time with $t_{s+1} - t_s \leq h$. L_j and F_j denote the fuzzy observer gains that need to be decided with $j = 1, 2, \ldots, r$. The IT2 fuzzy observer is represented as

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{j=1}^{r} \theta_j(\varpi(t_s))\big[A_j\hat{x}(t) + B_j\big(u(t) + \hat{f}(t)\big) \\ \qquad + L_j\big(y(t_s) - \hat{y}(t_s)\big)\big], \\ \dot{\hat{f}}(t) = \sum_{j=1}^{r} \theta_j(\varpi(t_s))\big[F_j\big(y(t_s) - \hat{y}(t_s)\big)\big], \\ \hat{y}(t) = C\hat{x}(t), \end{cases} \tag{2}$$

where $\theta_j(\varpi(t_s)) = \frac{\underline{\alpha}_j(\varpi(t_s))\underline{w}_j(\varpi(t_s)) + \bar{\alpha}_j(\varpi(t_s))\bar{w}_j(\varpi(t_s))}{\sum_{k=1}^{r}\big(\underline{\alpha}_k(\varpi(t_s))\underline{w}_k(\varpi(t_s)) + \bar{\alpha}_k(\varpi(t_s))\bar{w}_k(\varpi(t_s))\big)}$ with $\sum_{j=1}^{r} \theta_j(\varpi(t_s)) = 1$. $0 \leq \underline{\alpha}_j(\varpi(t_s)) \leq 1$ and $0 \leq \bar{\alpha}_j(\varpi(t_s)) \leq 1$ denote nonlinear weighting functions with $\underline{\alpha}_j(\varpi(t_s)) + \bar{\alpha}_j(\varpi(t_s)) = 1$ for all j, which can be determined in light of the practical demand [**?**].

C. IT2 Fuzzy Fault-Tolerant Controller

Upon the estimated information on actuator faults and system states, an IT2 fuzzy fault-tolerant controller will be developed to ensure the stability of the uncertain nonlinear system suffering from actuator faults in the form of IT2 fuzzy model (1). To furnish the fuzzy controller with more design freedom, the constructed controller is capable of having its particular membership functions, potentially differing from those of the concerned fuzzy system. The rule l of the IT2 fuzzy fault-tolerant controller is shown by

Rule l: IF $\varphi_1(\varpi(t_s))$ is $\tilde{\aleph}_1^l$ and $\varphi_2(\varpi(t_s))$ is $\tilde{\aleph}_2^l$ and $\cdots$ and $\varphi_\Omega(\varpi(t_s))$ is $\tilde{\aleph}_\Omega^l$, THEN

$$u(t) = K_l\hat{x}(t) - \hat{f}(t),$$

in which $\varphi_\upsilon(\varpi(t_s))$ and $\tilde{\aleph}_\upsilon^l$, $\upsilon = 1, 2, \ldots, \Omega$, $l = 1, 2, \ldots, c$, denote the premise variables and IT2 fuzzy sets, respectively. K_l is the fuzzy controller gain that needs to be decided. The subsequent interval sets depict the firing strength of rule l

$$\mathscr{M}_l(\varpi(t_s)) = [\underline{m}_l(\varpi(t_s)),\ \bar{m}_l(\varpi(t_s))], \quad l = 1, 2, \ldots, c$$

in which $\underline{m}_l(\varpi(t_s)) = \prod_{\upsilon=1}^{\Omega} \underline{\mu}_{\tilde{\aleph}_\upsilon^l}(\varphi_\upsilon(\varpi(t_s))$ and $\bar{m}_l(\varpi(t_s)) = \prod_{\upsilon=1}^{\Omega} \bar{\mu}_{\tilde{\aleph}_\upsilon^l}(\varphi_\upsilon(\varpi(t_s))$ indicate the lower and upper grades of membership, respectively. $\underline{\mu}_{\tilde{\aleph}_\upsilon^l}(\varphi_\upsilon(\varpi(t_s)))$ and $\bar{\mu}_{\tilde{\aleph}_\upsilon^l}(\varphi_\upsilon(\varpi(t_s)))$ represent the lower and upper membership functions, respectively. Upon definitions of IT2 membership functions, $\bar{\mu}_{\tilde{\aleph}_\upsilon^l}(\varphi_\upsilon(\varpi(t_s))) \geq \underline{\mu}_{\tilde{\aleph}_\upsilon^l}(\varphi_\upsilon(\varpi(t_s))) \geq 0$, which results in the holding of $\bar{m}_l(\varpi(t_s)) \geq \underline{m}_l(\varpi(t_s)) \geq 0$ for all l. The inferred IT2 fuzzy fault-tolerant controller is shown by

$$u(t) = \sum_{l=1}^{c} m_l(\varpi(t_s))\big[K_l\hat{x}(t) - \hat{f}(t)\big], \tag{3}$$

where $m_l(\varpi(t_s)) = \frac{\underline{\beta}_l(\varpi(t_s))\underline{m}_l(\varpi(t_s)) + \bar{\beta}_l(\varpi(t_s))\bar{m}_l(\varpi(t_s))}{\sum_{k=1}^{c}\big(\underline{\beta}_k(\varpi(t_s))\underline{m}_k(\varpi(t_s)) + \bar{\beta}_k(\varpi(t_s))\bar{m}_k(\varpi(t_s))\big)}$ and $\sum_{l=1}^{c} m_l(\varpi(t_s)) = 1$. $\underline{\beta}_l(\varpi(t_s)) \in [0,1]$ and $\bar{\beta}_l(\varpi(t_s)) \in [0,1]$ denote nonlinear weighting functions having the property $\underline{\beta}_l(\varpi(t_s)) + \bar{\beta}_l(\varpi(t_s)) = 1$ for all l.

Remark 2. *Note that the proposed fuzzy FTC technique utilizes only sampled-output measurements to stabilize the closed-loop system, which is greatly practical and significant in engineering applications due to widely used digital technologies. The observer-based FTC design with sampled-output measurements is the first time to be considered for uncertain nonlinear systems in the IT2 fuzzy model. Furthermore, the proposed fuzzy fault-tolerant controller enjoys a high degree of freedom in selecting the number of fuzzy rules and membership functions not constrained to the counterpart of the fuzzy system by virtue of the IPM scheme.*

D. Useful Lemmas

Lemma 1. *[?] Let $g(x) \in \Upsilon[\iota_1\ \iota_2)$ with $g(\iota_1) = 0$. Then for any matrix $M > 0$, the inequality below is satisfied:*

$$\int_{\iota_1}^{\iota_2} g^T(x)Mg(x)dx \leq \frac{4(\iota_2 - \iota_1)^2}{\pi^2}\int_{\iota_1}^{\iota_2} \dot{g}^T(x)M\dot{g}(x)dx.$$

Lemma 2. *[?] With symmetric matrix X, positive definite matrix W and scalar ς, the following inequality holds:*

$$-XW^{-1}X \leq \varsigma^2 W - 2\varsigma X.$$

Lemma 3. *[?] For any matrix $\begin{bmatrix} U & V \\ \star & U \end{bmatrix} \geq 0$, scalar function $\vartheta(t) \in [0, h]$, and vector function $\xi(t) : [-h, 0] \longrightarrow \mathbb{R}^{2n+m}$*

so that the integration in the inequality below is well defined, then

$$-h\int_{t-h}^{t}\dot{\xi}^T(\nu)U\dot{\xi}(\nu)d\nu \le \eta^T(t)\triangle\eta(t),$$

in which

$$\triangle = \begin{bmatrix} -U & U-V & V \\ \star & -2U+V+V^T & U-V \\ \star & \star & -U \end{bmatrix},$$

$$\eta(t) = \left[\, \xi^T(t)\ \xi^T(t-\vartheta(t))\ \xi^T(t-h) \,\right]^T.$$

III. Main Results

Upon the preliminaries aforementioned, the stability analysis of the whole closed-loop system in the IT2 fuzzy framework will be conducted with the prescribed H_∞ performance in this section. Then, resorting to the obtained MFD stability criteria, the integrated design strategy of the FE and FTC will be proposed in the form of the LMI.

A. Stability and H_∞ Performance Analysis

Denote $e_x(t) = x(t) - \hat{x}(t)$, $e_f(t) = f(t) - \hat{f}(t)$, $e(t) = [e_x^T(t)\ e_f^T(t)]^T$, and $v(t) = [w^T(t)\ \dot{f}^T(t)]^T$. In this work, the sampling behavior is handled by the input delay approach. Thus, defining $\vartheta(t) = t - t_s$ for $t_s \le t < t_{s+1}$, we can get $0 \le \vartheta(t) < h$. Applying $t_s = t - \vartheta(t)$ and combining (1), (2) and (3), we have

$$\begin{aligned}\dot{x}(t) =& \sum_{i=1}^{r}\sum_{l=1}^{c} w_i(\varpi(t))m_l(\varpi(t_s))\big[(A_i+B_iK_l)\hat{x}(t) \\ &+ A_ie_x(t) + B_ie_f(t) + D_iw(t)\big], \end{aligned} \tag{4}$$

$$\begin{aligned}\dot{\hat{x}}(t) =& \sum_{j=1}^{r}\sum_{l=1}^{c} \theta_j(\varpi(t_s))m_l(\varpi(t_s))\big[(A_j+B_jK_l)\hat{x}(t) \\ &+ L_jCe_x(t-\vartheta(t))\big], \end{aligned} \tag{5}$$

$$\begin{aligned}\dot{e}(t) =& \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{c} w_i(\varpi(t))\theta_j(\varpi(t_s))m_l(\varpi(t_s)\big[\mathscr{A}_{ijl}^{21}\hat{x}(t) \\ &+ \mathscr{A}_i^{22}e(t) + \mathscr{A}_{dj}^{22}e(t-\vartheta(t)) + \mathscr{E}_i^{21}v\big], \end{aligned} \tag{6}$$

where

$$\mathscr{A}_{ijl}^{21} = \begin{bmatrix} A_i - A_j + \big(B_i - B_j\big)K_l \\ 0 \end{bmatrix}, \mathscr{A}_i^{22} = \begin{bmatrix} A_i & B_i \\ 0 & 0 \end{bmatrix},$$

$$\mathscr{A}_{dj}^{22} = \begin{bmatrix} -L_jC & 0 \\ -F_jC & 0 \end{bmatrix}, \mathscr{E}_i^{21} = \begin{bmatrix} D_i & 0 \\ 0 & I \end{bmatrix}.$$

Denote $\mathscr{U}(t) = [\hat{x}^T(t)\ e^T(t)]^T$. Combining (5) and (6), an augmented system is depicted as following:

$$\begin{cases} \dot{\mathscr{U}}(t) = \displaystyle\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{c} w_i(\varpi(t))\theta_j(\varpi(t_s))m_l(\varpi(t_s))\big[\mathscr{A}_{ijl}\mathscr{U}(t) \\ \qquad + \mathscr{A}_{dj}\mathscr{U}(t-\vartheta(t)) + \mathscr{E}_iv(t)\big], \\ z_p(t) = C_x\hat{x}(t) + C_ee(t), \end{cases} \tag{7}$$

where

$$\mathscr{A}_{ijl} = \begin{bmatrix} \mathscr{A}_{jl}^{11} & 0 \\ \mathscr{A}_{ijl}^{21} & \mathscr{A}_i^{22} \end{bmatrix},\quad \mathscr{A}_{dj} = \begin{bmatrix} 0 & \mathscr{A}_{dj}^{12} \\ 0 & \mathscr{A}_{dj}^{22} \end{bmatrix},\quad \mathscr{E}_i = \begin{bmatrix} 0 \\ \mathscr{E}_i^{21} \end{bmatrix},$$

$$\mathscr{A}_{jl}^{11} = A_j + B_jK_l,\quad \mathscr{A}_{dj}^{12} = \left[\, L_jC\ \ 0 \,\right],$$

$\mathscr{A}_{ijl}^{21}$, $\mathscr{A}_i^{22}$, $\mathscr{A}_{dj}^{22}$ and $\mathscr{E}_i^{21}$ have been defined under (6). $z_p(t) \in \mathbb{R}^{2n+m}$ represents the performance output where weighting matrices C_x and C_e are given by the user.

In the subsequent analysis, for notational convenience, $w_i(\varpi(t))$, $\theta_j(\varpi(t_s))$ and $m_l(\varpi(t_s))$ are represented by w_i, θ_j^s and m_l^s, respectively. The augmented system (7) is rewritten in a concise form as

$$\begin{cases} \dot{\mathscr{U}}(t) = \displaystyle\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{c} w_i\theta_j^s m_l^s\Gamma_{ijl}\zeta(t), \\ z_p(t) = \bar{C}\mathscr{U}(t), \end{cases} \tag{8}$$

where $\Gamma_{ijl} = [\mathscr{A}_{ijl}\ \mathscr{A}_{dj}\ 0\ \mathscr{E}_i]$, $\zeta(t) = [\mathscr{U}^T(t)\ \mathscr{U}^T(t-\vartheta(t))\ \mathscr{U}^T(t-h)\ v^T(t)]^T$, and $\bar{C} = [C_x\ C_e]$.

Remark 3. *According to (4) and (6), we can find that the bidirectional robustness interaction exists between the control system and the observer. Consequently, it is infeasible to conduct the controller design independent of the observer design. To get rid of this problem, we integrate together the design of fault-tolerant controller and observer, which is conducive to attaining an optimal solution with the desired fault-tolerant performance.*

The target of the article is to develop the fault-tolerant controller based on FE using sampled-output measurements so that augmented system (7) is asymptotically stable despite the occurrence of actuator faults and satisfies the prescribed H_∞ performance $\int_0^\infty z_p^T(t)z_p(t)dt \le \gamma^2\int_0^\infty v^T(t)v(t)dt$ under the zero initial condition.

Theorem 1. *With given scalars $\gamma > 0$, $\underline{h}_{ijlu}$, δ_{ijlu}, fuzzy observer gain matrices $L_j \in \mathbb{R}^{n\times p}$, $F_j \in \mathbb{R}^{m\times p}$, and fuzzy controller gain matrix $K_l \in \mathbb{R}^{m\times n}$, the augmented system (7) is guaranteed to be asymptotically stable under the prespecified H_∞ performance index γ, if there exist positive definite matrices $P \in \mathbb{R}^{(2n+m)\times(2n+m)}$, $Q \in \mathbb{R}^{(2n+m)\times(2n+m)}$, $R \in \mathbb{R}^{(2n+m)\times(2n+m)}$, $Y \in \mathbb{R}^{(2n+m)\times(2n+m)}$, non-negative definite matrix $\Pi_{ijlu} \in \mathbb{R}^{(12n+7m+d)\times(12n+7m+d)}$, and matrix $S \in \mathbb{R}^{(2n+m)\times(2n+m)}$ such that the following inequalities are satisfied for $i, j = 1, 2, \ldots, r$, $l = 1, 2, \ldots, c$, $u = 1, 2, \ldots, \mathcal{U}$:*

$$\begin{bmatrix} R & S \\ \star & R \end{bmatrix} \ge 0, \tag{9}$$

$$\Xi_{ijl} - \Pi_{ijlu} \le 0,\quad \forall i, j, l, u \tag{10}$$

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{c}\big(\underline{h}_{ijlu}\Xi_{ijl} + \delta_{ijlu}\Pi_{ijlu}\big) < 0,\quad \forall u \tag{11}$$

where

$$\Xi_{ijl} = \begin{bmatrix} \bar{\Phi}_{ijl} & h\Gamma_{ijl}^TR & h\Gamma_{ijl}^TY & \bar{C}^T \\ \star & -R & 0 & 0 \\ \star & \star & -Y & 0 \\ \star & \star & \star & -I \end{bmatrix},$$

$$\bar{\Phi}_{ijl} = \begin{bmatrix} \bar{\Phi}_{ijl}^{11} & \Phi_j^{12} & S & P\mathscr{E}_i \\ \star & \Phi^{22} & R-S & 0 \\ \star & \star & -Q-R & 0 \\ \star & \star & \star & -\gamma^2I \end{bmatrix},$$

$$\bar{\Phi}_{ijl}^{11} = P\mathscr{A}_{ijl} + \mathscr{A}_{ijl}^TP + Q - R - \frac{\pi^2}{4}Y,$$

$$\Phi_j^{12} = P\mathscr{A}_{dj} + R - S + \frac{\pi^2}{4}Y,$$
$$\Phi^{22} = -2R + S + S^T - \frac{\pi^2}{4}Y.$$

Proof. Consider the discontinuous Lyapunov-Krasovskii functional candidate as below

$$V(t) = \sum_{k=1}^{4} V_k(t), \tag{12}$$

in which

$$\begin{aligned}
V_1(t) =& \mathscr{U}^T(t)P\mathscr{U}(t),\\
V_2(t) =& \int_{t-h}^{t} \mathscr{U}^T(\nu)Q\mathscr{U}(\nu)d\nu,\\
V_3(t) =& h\int_{-h}^{0}\int_{t+w}^{t} \dot{\mathscr{U}}^T(\nu)R\dot{\mathscr{U}}(\nu)d\nu dw,\\
V_4(t) =& h^2\int_{t_s}^{t} \dot{\mathscr{U}}^T(\nu)Y\dot{\mathscr{U}}(\nu)d\nu\\
& - \frac{\pi^2}{4}\int_{t_s}^{t} \big(\mathscr{U}(\nu) - \mathscr{U}(t_s)\big)^T Y\big(\mathscr{U}(\nu) - \mathscr{U}(t_s)\big)d\nu,
\end{aligned}$$

with positive matrices P, Q, R, and Y. By virtue of Lemma 1, it is obvious that $V_4(t) \geq 0$. In addition, $V_4(t)$ reduces to zero at $t = t_s$ so that we have $\lim_{t\to t_s^-} V(t) \geq V(t_s)$.

Computing the derivative of the $V_k(t)$ $(k = 1, 2, 3, 4)$ regarding time t and considering $t_s = t - \vartheta(t)$, we can have

$$\dot{V}_1(t) = \dot{\mathscr{U}}^T(t)P\mathscr{U}(t) + \mathscr{U}^T(t)P\dot{\mathscr{U}}(t), \tag{13}$$
$$\dot{V}_2(t) = \mathscr{U}^T(t)Q\mathscr{U}(t) - \mathscr{U}^T(t-h)Q\mathscr{U}(t-h), \tag{14}$$
$$\dot{V}_3(t) = h^2\dot{\mathscr{U}}^T(t)R\dot{\mathscr{U}}(t) - h\int_{t-h}^{t} \dot{\mathscr{U}}^T(\nu)R\dot{\mathscr{U}}(\nu)d\nu, \tag{15}$$
$$\begin{aligned}\dot{V}_4(t) =& h^2\dot{\mathscr{U}}^T(t)Y\dot{\mathscr{U}}(t) - \frac{\pi^2}{4}\big(\mathscr{U}(t) - \mathscr{U}(t-\vartheta(t))\big)^T\\
&\times Y\big(\mathscr{U}(t) - \mathscr{U}(t-\vartheta(t))\big).\end{aligned} \tag{16}$$

Resorting to Lemma 3 to address the integral term inside $\dot{V}_3(t)$ in (15), it yields that

$$\dot{V}_3(t) \leq h^2\dot{\mathscr{U}}^T(t)R\dot{\mathscr{U}}(t) - \zeta^T(t)\Lambda\zeta(t), \tag{17}$$

where

$$\Lambda = \begin{bmatrix} -R & R-S & S & 0\\ \star & -2R+S+S^T & R-S & 0\\ \star & \star & -R & 0\\ \star & \star & \star & 0 \end{bmatrix}.$$

To study the H_∞ performance of augmented system (7), the following index function is introduced

$$J(t) = \int_0^\infty \big(z_p^T(t)z_p(t) - \gamma^2 v^T(t)v(t)\big)dt. \tag{18}$$

It can be seen that the H_∞ performance is guaranteed if the condition $J(t) \leq 0$ holds. Under zero initial conditions, namely $V(t)|_{t=0} = 0$, we can find that

$$\begin{aligned}
J(t) =& \int_0^\infty\big(\dot{V}(t) + z_p^T(t)z_p(t) - \gamma^2 v^T(t)v(t)\big)dt - \int_0^\infty \dot{V}(t)dt\\
=& \int_0^\infty\big(\dot{V}(t) + z_p^T(t)z_p(t) - \gamma^2 v^T(t)v(t)\big)dt - V(\infty) + V(0)\\
\leq& \int_0^\infty\big(\dot{V}(t) + z_p^T(t)z_p(t) - \gamma^2 v^T(t)v(t)\big)dt.
\end{aligned} \tag{19}$$

Hence, considering (13)-(19) along with (8), we can conclude that $J(t) \leq 0$ once the condition below holds

$$\begin{aligned}
&\dot{V}(t) + z_p^T(t)z_p(t) - \gamma^2 v^T(t)v(t)\\
\leq& \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{c} w_i\theta_j^s m_l^s\big[\zeta^T(t)\Gamma_{ijl}^T P\mathscr{U}(t) + \mathscr{U}^T(t)P\Gamma_{ijl}\zeta(t)\big]\\
&+ \mathscr{U}^T(t)Q\mathscr{U}(t) - \mathscr{U}^T(t-h)Q\mathscr{U}(t-h) - \zeta^T(t)\Lambda\zeta(t)\\
&- \frac{\pi^2}{4}\big(\mathscr{U}(t) - \mathscr{U}(t-\vartheta(t))\big)^T Y\big(\mathscr{U}(t) - \mathscr{U}(t-\vartheta(t))\big)\\
&+ \mathscr{U}^T(t)\bar{C}^T\bar{C}\mathscr{U}(t) - \gamma^2 v^T(t)v(t)\\
&+ h^2\zeta^T(t)\big(\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{c} w_i\theta_j^s m_l^s\Gamma_{ijl}^T\big)(R+Y)\\
&\times\big(\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{c} w_i\theta_j^s m_l^s\Gamma_{ijl}\big)\zeta(t)\\
=& \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{c} w_i\theta_j^s m_l^s\zeta^T(t)\Phi_{ijl}\zeta(t)\\
&+ h^2\zeta^T(t)\big(\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{c} w_i\theta_j^s m_l^s\Gamma_{ijl}^T R\big)R^{-1}\\
&\times\big(\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{c} w_i\theta_j^s m_l^s R\Gamma_{ijl}\big)\zeta(t)\\
&+ h^2\zeta^T(t)\big(\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{c} w_i\theta_j^s m_l^s\Gamma_{ijl}^T Y\big)Y^{-1}\\
&\times\big(\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{c} w_i\theta_j^s m_l^s Y\Gamma_{ijl}\big)\zeta(t)\\
\leq& 0,
\end{aligned} \tag{20}$$

in which

$$\Phi_{ijl} = \begin{bmatrix} \Phi_{ijl}^{11} & \Phi_j^{12} & S & P\mathscr{E}_i\\ \star & \Phi^{22} & R-S & 0\\ \star & \star & -Q-R & 0\\ \star & \star & \star & -\gamma^2 I \end{bmatrix},$$
$$\Phi_{ijl}^{11} = P\mathscr{A}_{ijl} + \mathscr{A}_{ijl}^T P + Q - R - \frac{\pi^2}{4}Y + \bar{C}^T\bar{C},$$
$$\Phi_j^{12} = P\mathscr{A}_{dj} + R - S + \frac{\pi^2}{4}Y,$$
$$\Phi^{22} = -2R + S + S^T - \frac{\pi^2}{4}Y.$$

Through utilizing Schur complement, the holding of the inequality (20) is implied by the following inequality

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{c} w_i\theta_j^s m_l^s\Xi_{ijl} < 0, \tag{21}$$

in which Ξ_{ijl} is defined in Theorem 1.

For obtaining less conservative results, the MFD approach is employed by introducing the membership function information into the stability criteria. The state space of interest Θ is partitioned into $\mathcal{U}$ connected substate spaces Θ_u, i.e., $\Theta = \cup_{u=1}^{\mathcal{U}}\Theta_u$, so that more local membership function information can be considered. Let us define $h_{ijl}(\varpi(t),\varpi(t_s)) = w_i\theta_j^s m_l^s$ to lighten notations. In each subspace Θ_u, the constant maximum and minimum to enclose the local membership function $h_{ijlu}(\varpi(t),\varpi(t_s))$ are denoted as $\bar{h}_{ijlu}$ and $\underline{h}_{ijlu}$, respectively, which are included into stability conditions with the aid of slack matrices. According to the definition of $\bar{h}_{ijlu}$ and $\underline{h}_{ijlu}$, it is obvious that $0 \le \underline{h}_{ijlu} \le h_{ijlu}(\varpi(t),\varpi(t_s)) \le \bar{h}_{ijlu} \le 1$. Here, we introduce the slack matrices Π_{ijlu}, $i,j = 1,2,\ldots,r$, $l = 1,2,\ldots,c$, $u = 1,2,\ldots,\mathcal{U}$, with the constraint that $\Pi_{ijlu} \ge 0$ and $\Pi_{ijlu} \ge \Xi_{ijl}$. Thereby, within the subspace Θ_u, from (21) we can deduce

$$
\begin{aligned}
&\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{c} h_{ijlu}(\varpi(t),\varpi(t_s))\Xi_{ijl}\\
=&\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{c}\big(h_{ijlu}(\varpi(t),\varpi(t_s)) + \underline{h}_{ijlu} - \underline{h}_{ijlu}\big)\Xi_{ijl}\\
\le&\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{c}\underline{h}_{ijlu}\Xi_{ijl}\\
&+\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{c}\big(h_{ijlu}(\varpi(t),\varpi(t_s)) - \underline{h}_{ijlu}\big)\Xi_{ijl}\\
\le&\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{c}\big(\underline{h}_{ijlu}\Xi_{ijl} + \delta_{ijlu}\Pi_{ijlu}\big),
\end{aligned}
\tag{22}
$$

in which $\delta_{ijlu} = \bar{h}_{ijlu} - \underline{h}_{ijlu}$ are constants relative to the bounds of IT2 membership functions.

Accordingly, the holding of criteria (10) and (11) can make sure that $\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{c} h_{ijlu}(\varpi(t),\varpi(t_s))\Xi_{ijl} < 0$, which guarantees the H_∞ performance of the augmented system (7). Furthermore, when $v(t) = 0$, considering (20)-(22), it is obvious that $\dot{V}(t) < 0$ excluding $\mathscr{U}(t) = 0$, which indicates that augmented system (7) is asymptotically stable. This completes the proof. □

Remark 4. *Because of the sampling mechanism adopted, the values of $h_{ijlu}(\varpi(t),\varpi(t_s))$ are dependent on both $\varpi(t)$ and $\varpi(t_s)$. As we deem $\varpi(t_s)$ totally irrelevant to $\varpi(t)$, the injected information of membership functions cannot be extracted well via $\underline{h}_{ijlu}$ and δ_{ijlu}. The potential relation between $\varpi(t)$ and $\varpi(t_s)$ should be explored to calculate lower and upper bounds of $h_{ijlu}(\varpi(t),\varpi(t_s))$ to narrow down the bounds of IT2 membership functions, which helps further relax the MFD stability criteria. Recalling that $t_s = t - \vartheta(t)$ and $0 \le \vartheta(t) < h$ over a sampling period, $\varpi_i(t_s)$ can be estimated via $\varpi_i(t)$ by: $\varpi_i(t_s) \in [\varpi_i(t) - h\dot{\varpi}_{imax} \ \ \varpi_i(t) + h\dot{\varpi}_{imax}]$, where $\varpi_i(t_s)$ and $\varpi_i(t)$ represent the i-th element of vectors $\varpi(t_s)$ and $\varpi(t)$, respectively. $\dot{\varpi}_{imax}$ denotes the maximum value of $|\dot{\varpi}_i(t)|$ amid the dynamic process. Hence, based on the aforementioned estimation, $h_{ijl}(\varpi(t),\varpi(t_s))$ can be estimated by $h_{ijl}(\varpi(t))$ which only depends on the variable $\varpi(t)$.*

B. Integrated Design of IT2 Fuzzy Observer and Fault-Tolerant Controller

Note that the observer and controller gain matrices are coupled with unknown matrix variables in Theorem 1, so that the gain matrices cannot be immediately solved through Theorem 1 by utilizing the LMI toolbox. Hence, according to the results developed in Theorem 1, the procedures on the co-design of the fuzzy fault-tolerant controller and fuzzy observer are provided in what follows.

Theorem 2. *With given scalars $\gamma > 0$, $\underline{h}_{ijlu}$, δ_{ijlu}, ς_1, ς_2, and constant matrix $\mathscr{J} \in \mathbb{R}^{n\times m}$, the augmented system (7) is guaranteed to be asymptotically stable under the prespecified H_∞ performance index γ, if there exist positive definite matrices $W_1 \in \mathbb{R}^{n\times n}$, $W_2 \in \mathbb{R}^{n\times n}$, $W_3 \in \mathbb{R}^{m\times m}$, $\bar{Q} \in \mathbb{R}^{(2n+m)\times(2n+m)}$, $\bar{R} \in \mathbb{R}^{(2n+m)\times(2n+m)}$, $\bar{Y} \in \mathbb{R}^{(2n+m)\times(2n+m)}$, non-negative definite matrix $\bar{\Pi}_{ijlu} \in \mathbb{R}^{(12n+7m+d)\times(12n+7m+d)}$, and matrices $\bar{S} \in \mathbb{R}^{(2n+m)\times(2n+m)}$, $N \in \mathbb{R}^{p\times p}$, $T_j \in \mathbb{R}^{n\times p}$, $G_j \in \mathbb{R}^{m\times p}$, $M_l \in \mathbb{R}^{m\times n}$ for $i,j = 1,2,\ldots,r$, $l = 1,2,\ldots,c$, $u = 1,2,\ldots,\mathcal{U}$, such that*

$$
CW_2 = NC \tag{23}
$$

$$
\begin{bmatrix} \bar{R} & \bar{S} \\ \star & \bar{R} \end{bmatrix} \ge 0, \tag{24}
$$

$$
\breve{\Xi}_{ijl} - \bar{\Pi}_{ijlu} \le 0, \quad \forall i,j,l,u \tag{25}
$$

$$
\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{c}\big(\underline{h}_{ijlu}\breve{\Xi}_{ijl} + \delta_{ijlu}\bar{\Pi}_{ijlu}\big) < 0, \quad \forall u \tag{26}
$$

where

$$
\breve{\Xi}_{ijl} = \begin{bmatrix}
\breve{\Xi}_{ijl}^{11} & \breve{\Xi}_{j}^{12} & \bar{S} & \bar{\mathscr{E}}_i & \breve{\Xi}_{ijl}^{15} & \breve{\Xi}_{ijl}^{16} & W\bar{C}^T \\
\star & \bar{\Xi}^{22} & \bar{\Xi}^{23} & 0 & \breve{\Xi}_{j}^{25} & \breve{\Xi}_{j}^{26} & 0 \\
\star & \star & \bar{\Xi}^{33} & 0 & 0 & 0 & 0 \\
\star & \star & \star & -\gamma^2 I & h\bar{\mathscr{E}}_i^T & h\bar{\mathscr{E}}_i^T & 0 \\
\star & \star & \star & \star & \bar{\Xi}^{55} & 0 & 0 \\
\star & \star & \star & \star & \star & \bar{\Xi}^{66} & 0 \\
\star & \star & \star & \star & \star & \star & -I
\end{bmatrix},
$$

$$
\breve{\Xi}_{ijl}^{11} = \begin{bmatrix} \Sigma_{jl}^{11} & \Sigma_{ijl}^{12} & 0 \\ \star & \Sigma_i^{22} & \Sigma_i^{23} \\ \star & \star & 0 \end{bmatrix} + \bar{Q} - \bar{R} - \frac{\pi^2}{4}\bar{Y},
$$

$$
\breve{\Xi}_{j}^{12} = \begin{bmatrix} 0 & T_jC & T_jC\mathscr{J} \\ 0 & -T_jC & -T_jC\mathscr{J} \\ 0 & -G_jC & -G_jC\mathscr{J} \end{bmatrix} + \bar{R} - \bar{S} + \frac{\pi^2}{4}\bar{Y},
$$

$$
\breve{\Xi}_{ijl}^{15} = \breve{\Xi}_{ijl}^{16} = \begin{bmatrix} \Upsilon_{jl}^{11} & \Upsilon_{ijl}^{12} & 0 \\ 0 & \Upsilon_i^{22} & 0 \\ 0 & \Upsilon_i^{32} & 0 \end{bmatrix}, \ W = \begin{bmatrix} W_1 & 0 & 0 \\ \star & W_2 & W_2\mathscr{J} \\ \star & \star & W_3 \end{bmatrix},
$$

$$
\bar{\Xi}^{22} = -2\bar{R} + \bar{S} + \bar{S}^T - \frac{\pi^2}{4}\bar{Y}, \ \bar{\Xi}^{23} = \bar{R} - \bar{S},
$$

$$
\breve{\Xi}_{j}^{25} = \breve{\Xi}_{j}^{26} = \begin{bmatrix} 0 & 0 & 0 \\ hC^TT_j^T & -hC^TT_j^T & -hC^TG_j^T \\ h\mathscr{J}^TC^TT_j^T & -h\mathscr{J}^TC^TT_j^T & -h\mathscr{J}^TC^TG_j^T \end{bmatrix},
$$

$$
\begin{aligned}
\bar{\Xi}^{33} =& -\bar{Q} - \bar{R}, \quad \bar{\Xi}^{55} = \varsigma_1^2\bar{R} - 2\varsigma_1 W, \quad \bar{\Xi}^{66} = \varsigma_2^2\bar{Y} - 2\varsigma_2 W,\\
\Sigma_{jl}^{11} =& A_jW_1 + B_jM_l + W_1A_j^T + M_l^TB_j^T,\\
\Sigma_{ijl}^{12} =& W_1(A_i - A_j)^T + M_l^T(B_i - B_j)^T,\\
\Sigma_i^{22} =& A_iW_2 + B_i\mathscr{J}^TW_2 + W_2A_i^T + W_2\mathscr{J}B_i^T,\\
\Sigma_i^{23} =& A_iW_2\mathscr{J} + B_iW_3,\\
\Upsilon_{jl}^{11} =& h\big(W_1A_j^T + M_l^TB_j^T\big),\\
\Upsilon_{ijl}^{12} =& h\big(W_1(A_i - A_j)^T + M_l^T(B_i - B_j)^T\big),\\
\Upsilon_i^{22} =& h\big(W_2A_i^T + W_2\mathscr{J}B_i^T\big),
\end{aligned}
$$

$$\Upsilon_i^{32} = h\big(\mathscr{J}^T W_2 A_i^T + W_3 B_i^T\big).$$

Then the observer gains are obtained as $L_j = T_j N^{-1}$, $F_j = G_j N^{-1}$, *and the fault-tolerant controller gain is obtained as* $K_l = M_l W_1^{-1}$.

Proof. Aiming to attain the convex design criteria that can be handled by convex programming strategies, the congruence transformation is employed to (10)-(11) by pre- and post-multiplication of $\mathcal{X} = diag\{W, W, W, I, R^{-1}, Y^{-1}, I\}$ with the definition $W = P^{-1}$. After performing the congruence transformation, apply Lemma 2 to the terms $-R^{-1} = -W\bar{R}^{-1}W$ and $-Y^{-1} = -W\bar{Y}^{-1}W$ respectively to circumvent the existence of R^{-1} and Y^{-1} in the design conditions. Then, it yields

$$\bar{\Xi}_{ijl} - \bar{\Pi}_{ijlu} \leq 0, \ \forall i, j, l, u \tag{27}$$

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{c}\big(\underline{h}_{ijlu}\bar{\Xi}_{ijlu} + \delta_{ijlu}\bar{\Pi}_{ijl}\big) < 0, \ \forall u \tag{28}$$

where

$$\bar{\Xi}_{ijl} = \begin{bmatrix} \bar{\Xi}_{ijl}^{11} & \bar{\Xi}_j^{12} & \bar{S} & \bar{\mathscr{E}}_i & \bar{\Xi}_{ijl}^{15} & \bar{\Xi}_{ijl}^{16} & W\bar{C}^T \\ \star & \bar{\Xi}^{22} & \bar{\Xi}^{23} & 0 & \bar{\Xi}_j^{25} & \bar{\Xi}_j^{26} & 0 \\ \star & \star & \bar{\Xi}^{33} & 0 & 0 & 0 & 0 \\ \star & \star & \star & -\gamma^2 I & h\bar{\mathscr{E}}_i^T & h\bar{\mathscr{E}}_i^T & 0 \\ \star & \star & \star & \star & \bar{\Xi}^{55} & 0 & 0 \\ \star & \star & \star & \star & \star & \bar{\Xi}^{66} & 0 \\ \star & \star & \star & \star & \star & \star & -I \end{bmatrix},$$

$$\begin{aligned}
\bar{R} =& WRW, \ \bar{S} = WSW, \bar{Q} = WQW, \ \bar{Y} = WYW, \\
\bar{\Xi}_{ijl}^{11} =& \mathscr{A}_{ijl}W + W\mathscr{A}_{ijl}^T + \bar{Q} - \bar{R} - \frac{\pi^2}{4}\bar{Y}, \\
\bar{\Xi}_j^{12} =& \mathscr{A}_{dj}W + \bar{R} - \bar{S} + \frac{\pi^2}{4}\bar{Y}, \ \bar{\Xi}_{ijl}^{15} = \bar{\Xi}_{ijl}^{16} = hW\mathscr{A}_{ijl}^T, \\
\bar{\Xi}_j^{25} =& \bar{\Xi}_j^{26} = hW\mathscr{A}_{dj}^T, \ \bar{\Pi}_{ijlu} = \mathcal{X}\Pi_{ijlu}\mathcal{X}.
\end{aligned}$$

$\bar{\Xi}^{22}$, $\bar{\Xi}^{23}$, $\bar{\Xi}^{33}$, $\bar{\Xi}^{55}$, and $\bar{\Xi}^{66}$ are the same as defined in Theorem 2. Similarly, after pre- and post-multiplication of $diag\{W, W\}$ to (9), we can get the condition (24).

In order to facilitate design synthesis, the positive definite matrix W is specified as

$$W = \begin{bmatrix} W_1 & 0 & 0 \\ \star & W_2 & W_2\mathscr{J} \\ \star & \star & W_3 \end{bmatrix}, \tag{29}$$

where matrix variables $W_1 \in \mathbb{R}^{n\times n}$, $W_2 \in \mathbb{R}^{n\times n}$, and $W_3 \in \mathbb{R}^{m\times m}$ are unknown, while $\mathscr{J} \in \mathbb{R}^{n\times m}$ is the constant matrix chosen by the user. Meanwhile, assume that there exists a nonsingular matrix N to make the equality constraint $CW_2 = NC$ valid. Then, we have

$$\begin{aligned}
L_jCW_2 =& L_jNC \\
:=& T_jC,
\end{aligned} \tag{30}$$

$$\begin{aligned}
F_jCW_2 =& F_jNC \\
:=& G_jC.
\end{aligned} \tag{31}$$

Substituting the matrix variable W constructed in (29) into (27) and (28) and considering the expressions stated in (30) and (31) with $M_l = K_lW_1$, the inequalities (25) and (26) can be derived. This completes the proof. □

Remark 5. *Theorem 2 proposes sufficient conditions for asymptotical stability of the augmented system (7). However, there is an equality constraint (23) in Theorem 2, which is hard to be solved through the LMI toolbox. To obviate this problem, we introduce the following condition:*

$$(CW_2 - NC)^T(CW_2 - NC) < \varrho I, \tag{32}$$

where ϱ *is a positive scalar. By Schur complement, we can see that (32) can be rewritten as*

$$\begin{bmatrix} -\varrho I & (CW_2 - NC)^T \\ \star & -I \end{bmatrix} < 0. \tag{33}$$

In consequence, the issue of solving the conditions (23)-(26) in Theorem 2 is transformed into the optimization problem below

$$\min \varrho \tag{34}$$

subject to (24)-(26) and (33).

IV. Simulation Example

To validate the efficacy of the designed FTC strategy under the sampled-data scheme, a numerical example is presented.

Consider a nonlinear system with the parameter uncertainty, actuator fault and external disturbance, depicted by an IT2 T-S fuzzy model with three fuzzy rules in the form of (1):

$$\begin{aligned}
A_1 =& \begin{bmatrix} 0.59 & -7.29 \\ 0.01 & -2.85 \end{bmatrix}, A_2 = \begin{bmatrix} 0.02 & -4.64 \\ 0.35 & -8.56 \end{bmatrix}, \\
A_3 =& \begin{bmatrix} 0.73 & 8.45 \\ 0.26 & -15.43 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0.3 \end{bmatrix}, \\
B_2 =& \begin{bmatrix} 8 & 2 \end{bmatrix}^T, \ B_3 = \begin{bmatrix} 4 & 0.8 \end{bmatrix}^T, \\
D_1 =& \begin{bmatrix} 0.5 & 0.1 \end{bmatrix}^T, \ D_2 = \begin{bmatrix} 1 & 0.1 \end{bmatrix}^T, \\
D_3 =& \begin{bmatrix} 0.2 & 0.01 \end{bmatrix}^T, \ C = \begin{bmatrix} 1 & 0 \end{bmatrix},
\end{aligned}$$

in which the lower and upper membership functions of the IT2 fuzzy model are depicted as: $\underline{w}_1(x_1(t)) = 1 - 1/(1 + e^{(-x_1(t)-3.5)})$, $\underline{w}_3(x_1(t)) = 1/(1 + e^{(-x_1(t)+3.5)})$, $\bar{w}_2(x_1(t)) = 1 - \underline{w}_1(x_1(t)) - \underline{w}_3(x_1(t))$, $\bar{w}_1(x_1(t)) = 1 - 1/(1 + e^{(-x_1(t)-2.5)})$, $\bar{w}_3(x_1(t)) = 1/(1 + e^{(-x_1(t)+2.5)})$, $\underline{w}_2(x_1(t)) = 1 - \bar{w}_1(x_1(t)) - \bar{w}_3(x_1(t))$. To certify design flexibility of the established FTC technique by using the IPM scheme, a two-rule IT2 fuzzy fault-tolerant controller is provided to stabilize the nonlinear plant subject to the actuator fault with the lower and upper membership functions defined as: $\underline{m}_1(x_1(t_s)) = \{1$, when $x_1(t_s) < -2.2$; $(-x_1(t_s)+1.8)/4$, when $-2.2 \leq x_1(t_s) \leq 1.8$; 0, when $x_1(t_s) > 1.8\}$, $\bar{m}_1(x_1(t_s)) = \{1$, when $x_1(t_s) < -1.8$; $(-x_1(t_s) + 2.2)/4$, when $-1.8 \leq x_1(t_s) \leq 2.2$; 0, when $x_1(t_s) > 2.2\}$, $\bar{m}_2(x_1(t_s)) = 1 - \underline{m}_1(x_1(t_s))$, $\underline{m}_2(x_1(t_s)) = 1 - \bar{m}_1(x_1(t_s))$.

The operation domain of $x_1(t)$ is assumed within the range of $[-3, 3]$. As discussed before, more abundant information of membership functions will be captured via partitioning the entire operation domain into a series of subdomains. But the more subdomains are, the higher the computation cost will be. Here, the operating domain x_1 is partitioned into 5 uniform subdomains where the suddomain is characterized by

$[-\frac{21}{5} + \frac{6}{5}u, -3 + \frac{6}{5}u]$, $u = 1, 2, \dots, 5$. To apply the FTC approach built in Theorem 2, let us define $\varsigma_1 = 0.1$, $\varsigma_2 = 0.1$, $\mathscr{J} = [1\ \ 1]^T$. Set the sampling interval $h = 0.01s$ and $\dot{x}_{1max} = 10$ which should be verified by simulations. In light of Remark 4, the largest variation of $x_1(t)$ over a sampling period can be calculated by $h\dot{x}_{1max} = 0.1$. It implies that $x_1(t_s)$ locates in the interval of $[x_1(t) - 0.1,\ x_1(t) + 0.1]$. This relationship helps gain more accurate constant parameters $\underline{h}_{ijlu}$ and δ_{ijlu} relevant to bounds of membership functions. By solving the minimization problem (34) under the prespecified H_∞ performance $\gamma = 1.5$, the fuzzy observer and fuzzy fault-tolerant controller gains are computed as: $L_1 = \begin{bmatrix} 7.5160 \\ 2.5580 \end{bmatrix}$, $L_2 = \begin{bmatrix} 32.9597 \\ 5.6985 \end{bmatrix}$, $L_3 = \begin{bmatrix} 14.6504 \\ 5.1413 \end{bmatrix}$, $F_1 = 8.2363$, $F_2 = 15.6380$, $F_3 = 27.3333$, $K_1 = \begin{bmatrix} -2.9992 & 2.5619 \end{bmatrix}$, $K_2 = \begin{bmatrix} -3.1414 & 1.7613 \end{bmatrix}$.

To conduct simulations, we assume that $\underline{\epsilon}_1(x_1(t)) = (\sin(x_1(t)) + 1)/2$, $\bar{\epsilon}_1(x_1(t)) = 1 - \underline{\epsilon}_1(x_1(t))$, $\underline{\epsilon}_3(x_1(t)) = (\cos(x_1(t)) + 1)/2$, $\bar{\epsilon}_3(x_1(t)) = 1 - \underline{\epsilon}_3(x_1(t))$, through which $w_1(x_1(t))$ and $w_3(x_1(t))$ can be obtained. $\underline{\epsilon}_2(x_1(t))$ and $\bar{\epsilon}_2(x_1(t))$ are unnecessary to know as $w_2(x_1(t))$ can be calculated based on the relationship $w_2(x_1(t)) = 1 - w_1(x_1(t)) - w_3(x_1(t))$. Besides, for the observer and fault-tolerant controller, the weighting functions are selected as: $\underline{\alpha}_j(x_1(t_s)) = \bar{\alpha}_j(x_1(t_s)) = 0.5$, $j = 1, 2, 3$, and $\underline{\beta}_l(x_1(t_s)) = \bar{\beta}_l(x_1(t_s)) = 0.5$, $l = 1, 2$, respectively. We first assume the actuator fault $f(t)$ as

$$f(t) = \begin{cases} 0, & t < 5, \\ 2(1 - e^{-0.5(t-5)}), & t \geq 5. \end{cases}$$

The external disturbance is depicted as $w(t) = e^{-0.2t}\sin(0.3t)$. The simulation results are provided by Figs. 1-3 under the initial states $x(0) = [1\ \ -0.8]^T$ and $\hat{x}(0) = [-0.5\ \ 0]^T$. Specifically, Fig. 1 displays the state responses of the open-loop system; Fig. 2 shows the time response of the actuator fault and its estimation; Fig. 3 represents the trajectories of system states and their estimations. From Fig. 1, we can find that the concerned system is unstable without cooperation of the controller. As displayed in Figs. 2 and 3, the obtained sampled-data fuzzy observer can reconstruct the actuator fault and system states with a satisfactory level. And also, it is observed that the obtained fault-tolerant controller can successfully stabilize the closed-loop system with system states asymptotically approaching zero despite the appearance of actuator faults. It demonstrates the efficacy of the provided FE-based FTC approach with sampled-output measurements.

To further demonstrate the robust FTC performance of the presented strategy, we assume the actuator fault $f(t)$ as follows:

$$f(t) = \begin{cases} 1, & 10 < t \leq 20, \\ 1.5 - 0.5e^{(-t+20)}, & 20 < t \leq 40, \\ 0.5 + \cos(0.1t - 4), & 40 < t \leq 123, \\ 0, & otherwise. \end{cases}$$

The simulation results are represented in Figs. 4 and 5. Fig. 4 displays the time response of the actual fault and its estima-

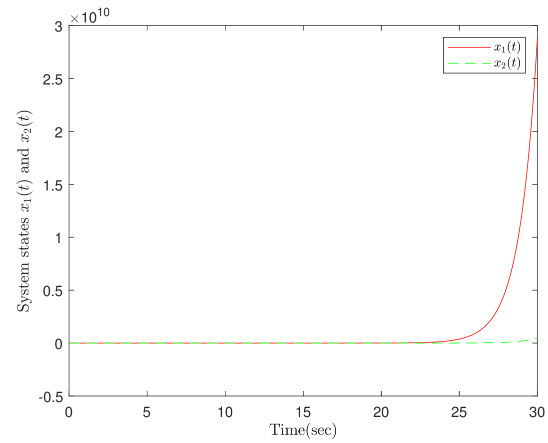

Fig. 1. Time response of system state $x(t)$ of the open-loop system

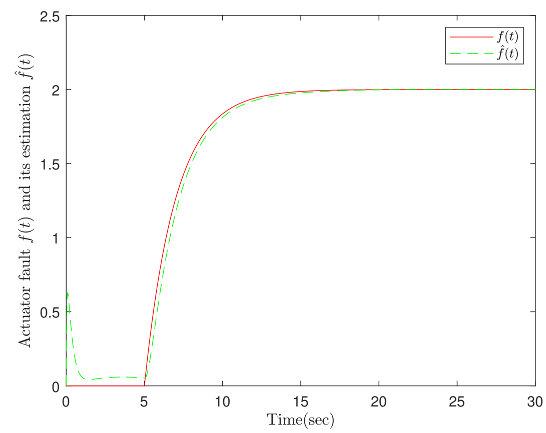

Fig. 2. Time response of actuator fault $f(t)$ and its estimation $\hat{f}(t)$

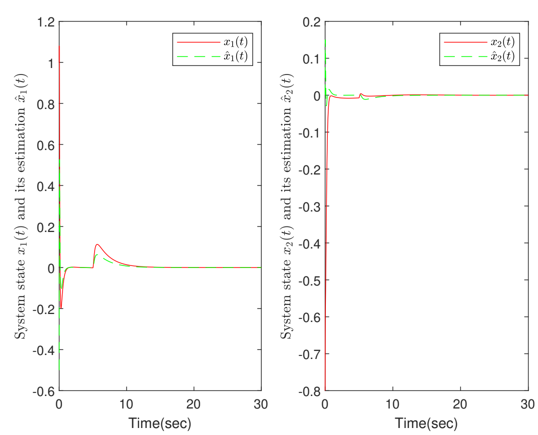

Fig. 3. Time response of system state $x(t)$ and its estimation $\hat{x}(t)$

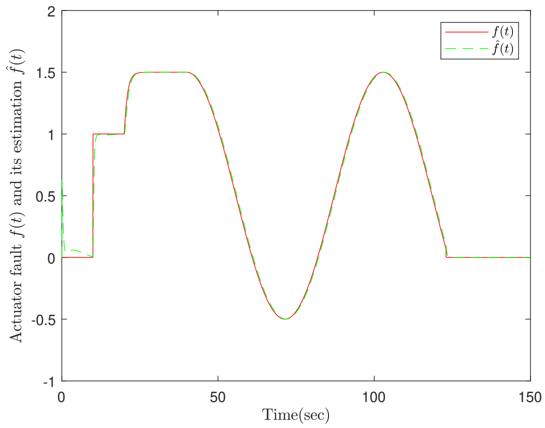

Fig. 4. Time response of actuator fault $f(t)$ and its estimation $\hat{f}(t)$

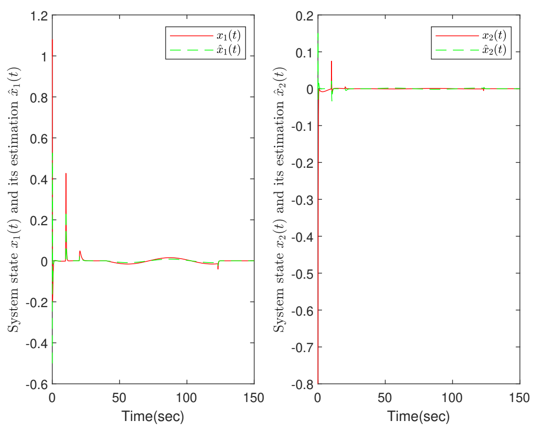

Fig. 5. Time response of system state $x(t)$ and its estimation $\hat{x}(t)$

tion, and Fig. 5 exhibits the trajectories of system states and their estimations. It indicates that the actuator fault and system states can be effectively estimated by the designed fuzzy observer, and the system states converge to zero gradually based on the obtained fuzzy fault-tolerant controller. In light of the simulation results above, we can draw a conclusion that the fault compensation based fault-tolerant controller is well built to stabilize the nonlinear system suffering from uncertainties, actuator faults and external disturbance, which demonstrates the validity of the proposed sampled-data observer-based FTC approach.

Remark 6. *To validate the merits of the proposed MFD FTC technique, the elements related to the information on membership functions in Theorem 2 are removed. The stability conditions will reduce to the membership-function-independent (MFI) form, which are concluded by a minimization problem* ϱ *subject to the LMI constraints of (24), (33), and* $\breve{\Xi}_{ijl} < 0$ *for* $i, j = 1, 2, \ldots, r,\ l = 1, 2, \ldots, c$*. The remaining parameters are under the same setting above-mentioned, and the MFI approach does not provide a feasible solution, which manifests that our proposed MFD FTC strategy excels the MFI one with the results relaxed.*

V. Conclusion

In this paper, we propose an integrated design strategy of the FE and FTC for IT2 fuzzy systems subject to uncertainties, actuator faults, and external disturbance. Using only sampled-output measurements, an IT2 fuzzy observer on jointly estimating actuator faults and system states is presented. Upon the estimated information, an IT2 fuzzy fault-tolerant controller is designed to compensate for the impact of faults on the plant and stabilize the closed-loop system with a satisfactory performance. To make the results less conservative, the MFD approach is employed in the stability analysis. This approach considers information from the sampling process to narrow down the boundaries of membership functions. Finally, the simulation example demonstrates the efficacy of the integrated FTC approach with sampled-output measurements provided in this work. The future research topics could include the extension of the main results in the IT2 fuzzy framework, such as fault-tolerant tracking control and FTC for simultaneous actuator and sensor faults.