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Research article

Distributed consensus of discrete time-varying linear multi-agent systems with event-triggered intermittent control

Mingxia Gu[1], **Zhiyong Yu**[1,*], **Haijun Jiang**[1,2] **and Da Huang**[3]

[1] College of Mathematics and System Sciences, Xinjiang University, Urumqi 830017, China
[2] School of Mathematics and Statistics, Yili Normal University, Yining 835000, China
[3] School of Mathematics and Physics, Xinjiang Institute of Engineering, Urumqi 830023, China

* **Correspondence:** Email: yzygsts@163.com.

Abstract: The consensus problem of discrete time-varying linear multi-agent systems (MASs) is studied in this paper. First, an event-triggered intermittent control (ETIC) protocol is designed, aided by a class of auxiliary functions. Under this protocol, some sufficient conditions for all agents to achieve consensus are established by constructing an error dynamical system and applying the Lyapunov function. Second, in order to further reduce the communication burden, an improved event triggered intermittent control (I-ETIC) strategy is presented, along with corresponding convergence analysis. Notably, the difference between the two control protocols lies in the fact that the former protocol only determines when to control or not based on the trigger conditions, while the latter, building upon this, designs new event trigger conditions for the update of the controller during the control stage. Finally, two numerical simulation examples are provided to demonstrate the effectiveness of the theoretical results.

Keywords: consensus; discrete-time; event-triggered intermittent control; multi-agent systems; time-varying matrix

1. Introduction

In recent years, cooperative control of multi-agent systems (MASs) has become a hot research topic in the field of control, with distributed consensus being one of the fundamental issues that attracts considerable attention from researchers. Various control methods have been proposed to achieve consensus, such as event-triggered control [1–5], intermittent control [6–8] and impulsive control [9–11]. These methodologies not only enable the system to achieve convergence but also reduce communication load to varying extents. Intermittent control refers to the strategy where the controller operates within certain time intervals and remains idle during other periods. Compared to continuous control, intermittent control can decrease information measurement and unnecessary communications. An advantage of intermittent control over impulsive control is that, in practical applications, it is more practical to assign a small time duration for each control task rather than controlling with zero control width. For instance, this approach has been leveraged in vehicle control [12, 13]. Hence, intermittent control methods have drawn extensive attention from scholars in recent years and have been widely applied in various fields, such as intermittent cluster synchronization of delayed coupling genetic regulatory networks [14], synchronization of complex networks [15] and consensus of MASs [16–21].

The concept of intermittent control was first introduced in [22], where it was shown that synchronization can be achieved under this control strategy for some systems. A theoretical analysis of nonlinear systems with periodic intermittent control was initially provided in [23]. Subsequently, the theory of intermittent control has been widely applied to the synchronization control of a large number of systems, such as complex-valued networks [24], memory neural networks [25] and delay-coupled systems [26,27]. Additionally, the stability issues of linear and nonlinear systems with intermittent control were considered in [28] and [29], respectively. The consensus problem of first-order and second-order MASs with intermittent communication was discussed in [30] and [31], respectively. Notably, the work in [32] derived a credible region for communication width given the coupling gain and network structure and proposed a time-delay protocol. However, the above intermittent control protocols are all periodic; that is, the working time intervals and resting time intervals are all fixed constants. As pointed out in [33], the approach of periodic intermittent control might have its shortcomings since it can limit the application range of intermittent control to some extent. By contrast, aperiodic intermittency relaxes the requirement on the control interval, allowing control strategies to be better aligned with practical situations. Therefore, the introduction of nonperiodic intermittent control methods bears significant importance. In [34–36], the distributed consensus of discrete-time and continuous-time systems were examined respectively using aperiodic intermittent control.

In the aforementioned literature, although the control interval is aperiodic, it is usually time-triggered, relying on the Lyapunov stability conditions or the design of the settling time [37]. Such control processes still generate some unnecessary information; a problem was depicted as the minimal activation rate of aperiodic intermittent control in [38]. While finding the least activation rate as a challenge when discussing the stability of nonperiodic intermittent control, designing a system with a relatively lower minimum activation rate is also significantly crucial. Results indicated that compared to time-triggered intermittent control, event-triggered intermittent control (ETIC) can achieve a smaller minimum activation rate. Therefore, nonperiodic and periodic ETIC schemes were designed in [39] and [40] to address synchronization problems in complex networks and distributed consensus in MASs, respectively. Note that in ETIC, the controller's working and resting switch are determined by trigger conditions, with the controller continually updating within each working interval. Subsequently, aperiodic control methods were proposed to address cooperative control problems in MASs. For the considered finite-time distributed optimization problem, the work in [41] proposed new aperiodic ETIC strategies under both undirected and directed network structures. The exponential stability of continuous-time systems with time-triggered and ETIC was investigated in [42]. Furthermore, a dynamic ETIC scheme with input delay was proposed in [43] to stabilize the delay dynamical systems.

Typically, intermittent control still requires continuous communication during a small execution time interval. To save more communication energy and reduce the number of control updates, introducing event-triggered control schemes into intermittent control is of great significance. In [44–46], intermittent event-triggered control methods were used to address the consensus problem in MASs, where the controller's information in each working interval was updated according to the event-triggering conditions.

Inspired by the aforementioned discussions, this paper proposes two distinct ETIC protocols to investigate the distributed consensus of MASs. Additionally, considering system uncertainties or discretization treatment methods in practical applications, system matrices may be time-varying. Therefore, studying the distributed consensus of MASs with time-varying linear matrices carries substantial significance. Notably, most of the above-stated research has been conducted within continuous systems. However, with the rapid development of computer technology, data sampling and transmission are primarily done through digital devices. This makes discrete-time algorithms more suitable for practical applications than continuous-time algorithms. To the best of our knowledge, no current studies have reported addressing the consensus issue in MASs within discrete time using a dual-triggered ETIC strategy. The main contributions of this paper can be summarized as follows:

1). In existing consensus research works [47–49], the linear time-invariant discrete systems are considered. However, in this paper, we consider the consensus of time-varying linear multi-agent systems. The system considered in this paper can be seen as an extension of existing ones, which has a wider range of application prospects.

2). Although the ETIC strategy has been used in [50], this work is focused on continuous-time systems. Our research is the first attempt to consider the consensus of discrete systems under event-triggered intermittent control.

3). Two kinds of discrete-time event-triggered control protocols are proposed, and some sufficient conditions are provided for MASs achieving consensus under the control protocols. The proposed control protocols can effectively reduce the communication frequency among agents, thereby effectively reducing control costs.

The structure of the paper is as follows. In Section 2, we introduce some preliminaries about graph theory and give the problem formulation. Two different ETIC protocols are proposed, and the convergence property for the proposed algorithms are analyzed in Section 3. Two numerical simulations are presented in Section 4. Section 5 summarizes the main results of this paper.

Notations: In this paper, $\mathcal{R}^n$ and $\mathcal{R}^{n\times n}$ represent the n-dimensional real space and the $n \times n$ dimensional set of real matrices, respectively. We denote $\mathbf{0}_N$ and $\mathbf{1}_N$ as N-dimensional column vectors with all elements being zero and one, respectively. I_n represents the n-dimensional identity matrix. For a vector $x \in \mathcal{R}^n$, denote x^T and x^{-1} as transpose and inversion of x and $\|x\|$ is defined as the standard Euclidean norm. The Kronecker product is given by $\otimes$. $\mathbb{N}$ denotes the set of natural numbers.

2. Preliminaries

2.1. Algebraic graph theory

Consider an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{1, \ldots, N\}$ represents the set of nodes and $\mathcal{E}$ indicates the set of interact links. For an edge $(i, j) \in \mathcal{E}$, i, $j \in \mathcal{V}$, if agent i and agent j can send messages to each other, let $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$ be defined as the set neighbors of agent i. The degree of agent i is the number of its neighbors and is given by $d_i = |\mathcal{N}_i|$. The adjacency matric associated to graph $\mathcal{G}$ is depicted by $\mathcal{A} = [a_{ij}] \in \mathcal{R}^{N\times N}$, which $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The Laplacian matrix $L = [l_{ij}]_{N\times N}$ is given by $l_{ij} = -a_{ij}$ for $i \neq j$, and $l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}$. i, $j \in \mathcal{V}$, which means $L\mathbf{1}_N = \mathbf{0}_N$. Notice that $\mathcal{A}$ and L are symmetric. Additionally, let $\tilde{L} = \text{diag}(\lambda_2, \lambda_3, \cdots, \lambda_N)$, where $\lambda_2, \lambda_3, \cdots, \lambda_N$ are positive eigenvalues of L, respectively, and satisfy $\lambda_2 \leq \lambda_3 \leq \cdots \leq \lambda_N$.

2.2. *Problem formulation*

Consider the following discrete time-varying linear MASs consisting of N agents, the dynamics of the i-th agent are described as

$$\begin{cases} x_i(t+1) = A(t)x_i(t) + u_i(t), & t \in \mathbb{N}, \\ x_i(t_0) = x_i(0), \end{cases} \tag{2.1}$$

where $x_i(t) \in \mathcal{R}^n$ represents the state variable and $u_i(t) \in \mathcal{R}^n$ is the distributed control input. $A(t) \in \mathcal{R}^{n\times n}$ is controllable system matrix of (2.1).

Assumption 1. Suppose the graph $\mathcal{G}$ is undirected and connected.

Assumption 2. Let $A(t)$ is a time-varying positive definite bounded matrix, and it satisfies $\alpha < \|A(t)\| < \beta$, where α, β are two constants.

Remark 1. A time-varying linear dynamic is considered in this paper, because it has some advantages such as simplified model, excellent scalability and strong robustness. Up to now, time-varying linear dynamics have been applied to many practical systems, such as ecosystems, transportation systems, financial markets, social networks, and control research. In [51], a fully distributed control strategy was considered to study a synchronization problem. In addition, undirected connected graphs are considered in this article. Therefore, how to study the distributed consensus of time-varying linear MASs under directed graphs or fully distributed control schemes by weakening conditions is a challenge that needs to be broken through in future research work.

Remark 2. Time-varying linear dynamics are considered in this paper, but we still need to consider many factors and overcome some difficulties to better simulate practice in practical applications. The work in [52] considered switching stochastic nonlinear large-scale systems with time delay and systematically proposed an adaptive neural fault-tolerant decentralized control scheme. When unknown system coefficient and time-varying delay occur at the same time, this control scheme can ensure the performance of the system and facilitate on-line adjustment. Therefore, we will try to combine time-varying time-delay systems with adaptive control strategies to achieve distributed consensus of MASs in future work.

3. Main results

3.1. *Consensus of linear time-varying systems with ETIC*

The following distributed control algorithm is proposed to achieve state consensus of agents.

$$u_i(t) = \begin{cases} -k_1 \sum_{j\in N_i} a_{ij}(x_i(t) - x_j(t)), & t \in [t_k, s_k), \\ 0, & t \in [s_k, t_{k+1}), \end{cases} \tag{3.1}$$

where $k_1 > 0$, t_k and s_k are control actuated and off time instants.

We denote the average state of the agents as $\bar{x}(t) = \frac{1}{N}\sum_{i=1}^{N} x_i(t)$, and the consensus error as $\delta_i(t) = x_i(t) - \bar{x}(t)$, and one obtains $\delta_i(t) = x_i(t) - \frac{1}{N}\sum_{i=1}^{N} x_i(t)$. Letting $x(t) = [x_1^T(t), x_2^T(t), ..., x_N^T(t)]^T$, $\delta(t) =$

$[\delta_1^T(t), \delta_2^T(t), ..., \delta_N^T(t)]^T$, we further derive its compact form as

$$\delta(t) = x(t) - (\frac{1}{N}\mathbf{1}_N\mathbf{1}_N^T \otimes I_n)x(t) = [(I_N - J_N) \otimes I_n]x(t) = (\Psi \otimes I_n)x(t), \tag{3.2}$$

where $J_N = \frac{1}{N}\mathbf{1}_N\mathbf{1}_N^T, \Psi = I_N - J_N$.

Define the two positive definite decreasing to zero functions $\Im_1(t)$ and $\Im_2(t)$, which satisfy

$$\begin{cases} \Im_1(t+1) = \alpha_1 \Im_1(t), & \Im_1(0) = \theta_1 \|\delta(0)\|^2, \\ \Im_2(t+1) = \alpha_2 \Im_2(t), & \Im_2(0) = \theta_2 \|\delta(0)\|^2, \end{cases} \tag{3.3}$$

where $0 < \alpha_2 < \alpha_1 < 1$, $0 < \theta_2 < 1 < \theta_1$. Controller execution and off times are determined by the following event triggering mechanism

$$\begin{cases} s_k = \{\inf t | t > t_k, \delta^T(t)\delta(t) \le \Im_2(t)\}, \\ t_{k+1} = \{\inf t | t > s_k, \delta^T(t)\delta(t) \ge \Im_1(t)\}, \end{cases} \tag{3.4}$$

where $k \in \mathbb{N}$, $t_0 = 0$.

The idea of the ETIC method considered in this section can be further understood according to the following Algorithm 1 and Figure 1.

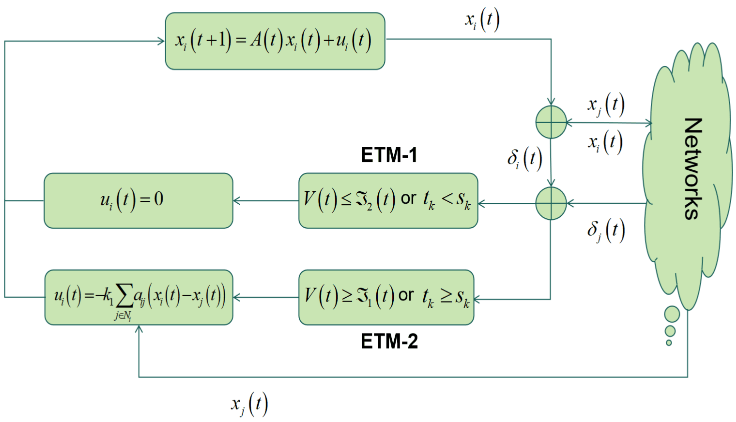

Figure 1. Structure diagram of ETIC method.

Algorithm 1 The control process under the ETIC protocol.

The related parameters and initial states of all agents are given.
for $t = 1$ to n **do**
    $\Im_1(t+1) = \alpha_1 \Im_1(t)$.
    $\Im_2(t+1) = \alpha_2 \Im_2(t)$.
    if $V(t) \geq \Im_1(t)$ **then**
        $x_i(t+1) = A(t)x_i(t) + u_i(t)$.
        if $V(t-1) < \Im_1(t-1)$ **then**
            $t_k = t_k + 1$.
    else if $V(t) \leq \Im_2(t)$ **then**
        $x_i(t+1) = A(t)x_i(t)$.
        if $V(t-1) > \Im_2(t-1)$ **then**
            $s_k = s_k + 1$.
    Get the value of $V(t+1)$.
end for

Remark 3. It is well known that intermittent control can reduce the communication between the agent and the neighbor agents compared with continuous control. Moreover, different from periodic and aperiodic intermittent control, this paper considers the ETIC protocol, which is a dual trigger control method, in the process of achieving consensus. Start control and stop control trigger points are dynamic and determined by two different event triggering conditions. This strategy of not limiting the control interval and stopping the control interval can simulate the actual scenario well and further reduce the communication load. As far as we know, no such dual-triggered consensus protocol with ETIC has been reported in discrete-time MASs. Therefore, the study of this paper is of great significance.

Remark 4. Observing (3.3) and (3.4), the function $V(t)$ generally fluctuates between $\Im_1(t)$ and $\Im_2(t)$, and it is certain that the controller starts to stop control if $V(t) \leq \Im_2(t)$; that is, to produce point s_k, the controller starts to perform control if $V(t) \geq \Im_1(t)$; that is, to produce point t_k.

Next, in Theorem 1, we give sufficient conditions for achieving consensus of discrete-time MASs with ETIC protocol in general.

Theorem 1. Suppose that Assumptions 1 and 2 hold, the positive constant k_1 satisfies $\|\varrho(t)\| \leq \sqrt{\alpha_2}$, where $\varrho(t) = I_{N-1} \otimes A(t) - k_1 \tilde{L} \otimes I_n$, $\tilde{L} = \mathrm{diag}(\lambda_2, \lambda_3, \cdots, \lambda_N)$, then the MASs (2.1) will achieve consensus through ETIC (3.1) with event triggering mechanism (3.4).

Proof. Consider the Lyapunov function of the following form

$$V(t) = \delta^T(t)\delta(t). \tag{3.5}$$

For $t \in [t_k, s_k)$, the controller is actuated. Combining (2.1) and (3.1), one has

$$\begin{aligned} x_i(t+1) =& A(t)x_i(t) + u_i(t) \\ =& A(t)x_i(t) - k_1 \sum_{j \in N_i} a_{ij}(x_i(t) - x_j(t)) \end{aligned}$$

$$=A(t)x_i(t) - k_1 \sum_{j=1}^{N} l_{ij}x_j(t). \tag{3.6}$$

Furthermore, one obtains

$$\begin{aligned} x(t+1) =&(I_N \otimes A(t))x(t) - k_1(L \otimes I_n)x(t) \\ =&(I_N \otimes A(t) - k_1 L \otimes I_n)x(t). \end{aligned} \tag{3.7}$$

According to (3), and $\Psi L = L\Psi = L$, one has

$$\begin{aligned} \delta(t+1) =&(\Psi \otimes I_n)x(t+1) \\ =&(\Psi \otimes I_n)(I_N \otimes A(t) - k_1 L \otimes I_n)x(t) \\ =&(\Psi \otimes A(t) - k_1 \Psi L \otimes I_n)x(t) \\ =&[(I_N \otimes A(t))(\Psi \otimes I_n) - (k_1 L \otimes I_n)(\Psi \otimes I_n)]x(t) \\ =&(I_N \otimes A(t) - k_1 L \otimes I_n)(\Psi \otimes I_n)x(t) \\ =&(I_N \otimes A(t) - k_1 L \otimes I_n)\delta(t). \end{aligned} \tag{3.8}$$

Let $\tilde{\delta}(t) = (T^{-1} \otimes I_n)\delta(t)$, where $T = [\xi, \phi_2, \ldots, \phi_N]$ is an orthogonal matrix obtained by the Schmidt's orthogonalization Method and $\xi, \phi_2, \ldots, \phi_N$ are the eigenvectors corresponding to the eigenvalues of the Laplacian matric L under the undirected connected graph $\mathcal{G}$. Based on (3.8), it follows that

$$\begin{aligned} \tilde{\delta}(t+1) &= (T^{-1} \otimes I_n)\delta(t+1) \\ &= (T^{-1} \otimes I_n)[(I_N \otimes A(t) - k_1 L \otimes I_n)]\delta(t) \\ &= (T^{-1} \otimes I_n)[(I_N \otimes A(t) - k_1 L \otimes I_n)](T \otimes I_n)\tilde{\delta}(t) \\ &= (I_N \otimes A(t) - k_1 T^{-1} L T \otimes I_n)\tilde{\delta}(t). \end{aligned} \tag{3.9}$$

Denote $\tilde{\delta}(t) = \begin{pmatrix} \tilde{\delta}_1(t) \\ \tilde{\delta}_2(t) \end{pmatrix}$. Due to $(T^{-1} \otimes I_n) = (T^T \otimes I_n) = \tilde{\delta}(t)$, it has

$$\begin{aligned} \tilde{\delta}_1(t) &= (\xi^T \otimes I_n)\delta(t) \\ &= (\xi^T \otimes I_n)[x(t) - (J_N \otimes I_n)x(t)] \\ &= (\frac{\mathbf{1}_N^T}{\sqrt{N}} \otimes I_n)x(t) - (\frac{\mathbf{1}_N^T}{\sqrt{N}} J_N \otimes I_n)x(t) \\ &= (\frac{\mathbf{1}_N^T}{\sqrt{N}} \otimes I_n)x(t) - (\frac{\mathbf{1}_N^T}{\sqrt{N}} \frac{1}{N}\mathbf{1}_N\mathbf{1}_N^T \otimes I_n)x(t) \\ &= (\frac{\mathbf{1}_N^T}{\sqrt{N}} \otimes I_n)x(t) - (\frac{\mathbf{1}_N^T}{\sqrt{N}} \otimes I_n)x(t) \\ &= 0. \end{aligned}$$

Define $\tilde{L} = \mathrm{diag}(\lambda_2, \lambda_3, \cdots, \lambda_N)$. According to (3.9), it can be seen that

$$\tilde{\delta}_2(t+1) = (I_{N-1} \otimes A(t) - k_1 \tilde{L} \otimes I_n)\tilde{\delta}_2(t). \tag{3.10}$$

According to $V(t) = \delta^T(t)\delta(t)$, one has

$$V(t+1) = ||\delta(t+1)||^2 = ||\tilde{\delta}(t+1)||^2 \tag{3.11}$$

Due to $\tilde{\delta}_1(t) = 0$, define $V_2(t) = \tilde{\delta}_2^T(t+1)\tilde{\delta}_2^T(t+1)$ and it yields that $V(t+1) = V_2(t+1)$. Therefore, it has

$$\begin{aligned} V(t+1) =& V_2(t+1) \\ =& \tilde{\delta}_2^T(t+1)\tilde{\delta}_2^T(t+1) \\ =& [(I_{N-1} \otimes A(t) - k_1 \tilde{L} \otimes I_n)\tilde{\delta}_2(t)]^T [(I_{N-1} \otimes A(t) - k_1 \tilde{L} \otimes I_n)\tilde{\delta}_2(t)] \\ =& \tilde{\delta}_2^T(t)(I_{N-1} \otimes A^T(t) - k_1 \tilde{L} \otimes I_n)(I_{N-1} \otimes A(t) - k_1 \tilde{L} \otimes I_n)\tilde{\delta}_2(t). \end{aligned} \tag{3.12}$$

Due to $||\varrho(t)|| \leq \sqrt{\alpha_2}$, it has

$$\begin{aligned} V(t+1) - \alpha_2 V(t) =& V_2(t+1) - \alpha_2 V_2(t) \\ =& \tilde{\delta}_2^T(t)\varrho^T(t)\varrho(t)\tilde{\delta}_2^T(t) - \alpha_2 V_2(t) \\ =& ||\varrho(t)||^2||V_2(t) - \alpha_2 V_2(t) \\ =& (||\varrho(t)||^2 - \alpha_2)V_2(t) \\ =& (||\varrho(t)||^2 - \alpha_2)V(t) \\ \leq& 0. \end{aligned} \tag{3.13}$$

According to (3.3) and (3.4), $\Im_2(t_k) \leq V(t_k) = \Im_1(t_k)$. Combined with (3.13), we can obtain that $V(t+1) \leq \alpha_2 V(t)$ for $t \in [t_k, s_k)$, and since $\Im_2(t+1) = \alpha_2 \Im_2(t)$, one has $\frac{V(t+1)}{V(t)} \leq \alpha_2 = \frac{\Im_2(t+1)}{\Im_2(t)}$; that is, $\frac{V(t+1)}{V(t)} \leq \frac{\Im_2(t+1)}{\Im_2(t)}$, which yields

$$|V(t+1) - V(t)| \geq |\Im_2(t+1) - \Im_2(t)|.$$

Next, based on (3.3), $\frac{\Im_2(t+1)}{\Im_2(t)} = \alpha_2 < \alpha_1 = \frac{\Im_1(t+1)}{\Im_1(t)}$; that is, $\frac{\Im_2(t+1)}{\Im_2(t)} < \frac{\Im_1(t+1)}{\Im_1(t)}$, and it follows that

$$|\Im_2(t+1) - \Im_2(t)| \geq |\Im_1(t+1) - \Im_1(t)|.$$

Thus $|V(t+1) - V(t)| \geq |\Im_2(t+1) - \Im_2(t)| \geq |\Im_1(t+1) - \Im_1(t)|$ and $\Im_2(t) < V(t) < \Im_1(t)$ for $t \in [t_k, s_k)$. Therefore, there exists an instant s_k such that $V(s_k) \leq \Im_2(s_k)$ for $t > t_k$.

For $t \in [s_k, t_{k+1})$, the controller is closed, one has

$$x_i(t+1) = A(t)x_i(t). \tag{3.14}$$

Moreover,

$$x(t+1) = (I_N \otimes A(t))x(t). \tag{3.15}$$

Since $\delta(t) = (\gamma \otimes I_n)x(t)$, it has

$$\begin{aligned} \delta(t+1) =& (\Psi \otimes I_n)x(t+1) \\ =& (\Psi \otimes I_n)(I_N \otimes A(t))x(t) \end{aligned}$$

$$
\begin{aligned}
&=(\Psi \otimes A(t))x(t) \\
&=(I_N \otimes A(t))(\Psi \otimes I_n)x(t) \\
&=(I_N \otimes A(t))\delta(t).
\end{aligned} \tag{3.16}
$$

According to $V(t) = \delta^T(t)\delta(t)$, one obtains

$$
\begin{aligned}
V(t+1) =&\delta^T(t+1)\delta(t+1) \\
=&[(I_N \otimes A(t))\delta(t)]^T[(I_N \otimes A(t))\delta(t)] \\
=&\delta^T(t)(I_N \otimes A^T(t))(I_N \otimes A(t))\delta(t) \\
=&\delta^T(t)(I_N \otimes A^T(t)A(t))\delta(t) \\
\leq&\beta^2 V(t).
\end{aligned} \tag{3.17}
$$

Therefore, $V(t)$ may be increasing, but $V(t) < \Im_1(t)$. In general, $0 \leq V(t) < \Im_1(t)$ for $\forall t > 0$. Let $0 < \alpha_1 = \alpha_2 + \varepsilon < 1$, where ε is an arbitrarily small positive constant. $V(t) \to 0$ because of $\Im_1(t) \to 0$ as $t \to \infty$.

On the basis of achieving consensus under the ETIC protocol, in order to narrow the value range of the control gain in order to further reduce the energy consumption, the following Theorem 2 is considered.

Theorem 2. Suppose that Assumptions 1 and 2 hold. If

$$
0 < \frac{\beta^2}{\alpha_2} - \Big(\frac{\lambda_2}{\lambda_{max}}\Big)^2 \frac{\alpha^2}{\alpha_2} < 1
$$

and the positive constant k_1 satisfies

$$
k_{11}^{'} < k_1 < k_{12},
$$

where $k_{11}^{'} = \max\{0, k_{11}\}$, $k_{11} = \frac{\alpha\lambda_2 - \sqrt{\alpha^2\lambda_2^2 - \lambda_{max}^2\beta^2 + \alpha_2\lambda_{max}^2}}{\lambda_{max}^2}$ and $k_{12} = \frac{\alpha\lambda_2 + \sqrt{\alpha^2\lambda_2^2 - \lambda_{max}^2\beta^2 + \alpha_2\lambda_{max}^2}}{\lambda_{max}^2}$, $0 < \alpha_2 < 1$, λ_2 and λ_{max} are the second smallest and largest eigenvalues of the Laplacian matrix L of the graph G, respectively. Then the MASs (2.1) will achieve consensus through ETIC (3.1) with the event triggering mechanism (3.4).

Proof. Consider the Lyapunov function of the following form

$$
V(t) = \delta^T(t)\delta(t). \tag{3.18}
$$

For $t \in [t_k, s_k)$, the controller is actuated. According to Theorem 1, one has

$$
\delta(t+1) = (I_N \otimes A(t) - k_1 L \otimes I_n)\delta(t). \tag{3.19}
$$

Furthermore, one obtains

$$
\begin{aligned}
&V(t+1) \\
=&\delta^T(t+1)\delta(t+1) \\
=&[(I_N \otimes A(t) - k_1 L \otimes I_n)\delta(t)]^T[(I_N \otimes A(t) - k_1 L \otimes I_n)\delta(t)]
\end{aligned}
$$

$$
\begin{aligned}
&=\delta^T(t)(I_N \otimes A^T(t) - k_1 L \otimes I_n)(I_N \otimes A(t) - k_1 L \otimes I_n)\delta(t)\\
&=\delta^T(t)(I_N \otimes A^T(t)A(t) - k_1 L \otimes A^T(t) - k_1 L \otimes A(t) + k_1^2 L^2 \otimes I_n)\delta(t)\\
&=\delta^T(t)(I_N \otimes A^T(t)A(t) - 2k_1 L \otimes A(t) + k_1^2 L^2 \otimes I_n)\delta(t)\\
&=\delta^T(t)(I_N \otimes A^T(t)A(t))\delta(t) - 2k_1\delta^T(t)(L \otimes A(t))\delta(t) + k_1^2\delta^T(t)(L^2 \otimes I_n)\delta(t)\\
&\leq\beta^2||\delta(t)||^2 - 2\alpha\lambda_2 k_1||\delta(t)||^2 + \lambda_{max}^2 k_1^2||\delta(t)||^2\\
&=(\beta^2 - 2\alpha\lambda_2 k_1 + \lambda_{max}^2 k_1^2)V(t)\\
&\leq\alpha_2 V(t). \qquad (3.20)
\end{aligned}
$$

Let $h(k_1) = \lambda_{max}^2 k_1^2 - 2\alpha\lambda_2 k_1 + \beta^2$. On the one hand, assuming $h(k_1) = 0$,

$$\lambda_{max}^2 k_1^2 - 2\alpha\lambda_2 k_1 + \beta^2 = 0. \qquad (3.21)$$

Obviously, $\Delta = 4\alpha^2\lambda_2^2 - 4\lambda_{max}^2\beta^2 < 0$; thus, (3.21) has no solution. Due to $0 < \min\{h(k_1)\} = \frac{4ac-b^2}{4a} = \frac{4\lambda_{max}^2\beta^2-4\alpha^2\lambda_2^2}{4\lambda_{max}^2}$ and $0 < \frac{\beta^2}{\alpha_2} - \left(\frac{\lambda_2}{\lambda_{max}}\right)^2\frac{\alpha^2}{\alpha_2} < 1$, one has $0 < \min\{h(k_1)\} < \alpha_2$.

On the other hand, assuming $h(k_1) = \alpha_2$,

$$\lambda_{max}^2 k_1^2 - 2\alpha\lambda_2 k_1 + \beta^2 - \alpha_2 = 0. \qquad (3.22)$$

Furthermore, it has $\Delta_1 = 4(\alpha^2\lambda_2^2 - \lambda_{max}^2\beta^2 + \alpha_2\lambda_{max}^2)$. Based on Assumption 1, $\Delta_1 \geq 0$. Thus, (3.22) has two roots

$$k_{11} = \frac{\alpha\lambda_2 - \sqrt{\alpha^2\lambda_2^2 - \lambda_{max}^2\beta^2 + \alpha_2\lambda_{max}^2}}{\lambda_{max}^2}, \qquad (3.23)$$

$$k_{12} = \frac{\alpha\lambda_2 + \sqrt{\alpha^2\lambda_2^2 - \lambda_{max}^2\beta^2 + \alpha_2\lambda_{max}^2}}{\lambda_{max}^2}. \qquad (3.24)$$

Case 1: When $\alpha\lambda_2 < \sqrt{\alpha^2\lambda_2^2 - \lambda_{max}^2\beta^2 + \alpha_2\lambda_{max}^2}$; that is, $\alpha_2 > \beta^2$, let $k_{11}^{'} = 0$.

Case 2: When $\alpha\lambda_2 \geq \sqrt{\alpha^2\lambda_2^2 - \lambda_{max}^2\beta^2 + \alpha_2\lambda_{max}^2}$; that is, $\alpha_2 \leq \beta^2$, let $k_{11}^{'} = k_{11}$.

Thus when $k_{11}^{'} < k_1 < k_{12}$, one has $\lambda_{max}^2 k_1^2 - 2\alpha\lambda_2 k_1 + \beta^2 \leq \alpha_2$; that is, $h(k_1) \leq \alpha_2$.

According to (3.3) and (3.4), $\Im_2(t_k) \leq V(t_k) = \Im_1(t_k)$. Combined with (3.20), we can obtain that $V(t+1) \leq \alpha_2 V(t)$ for $t \in [t_k, s_k)$. Since $\Im_2(t+1) = \alpha_2\Im_2(t)$, one obtains $\frac{V(t+1)}{V(t)} \leq \alpha_2 = \frac{\Im_2(t+1)}{\Im_2(t)}$; that is, $\frac{V(t+1)}{V(t)} \leq \frac{\Im_2(t+1)}{\Im_2(t)}$, and

$$|V(t+1) - V(t)| \geq |\Im_2(t+1) - \Im_2(t)|.$$

Next, according to (3.3), $\frac{\Im_2(t+1)}{\Im_2(t)} = \alpha_2 < \alpha_1 = \frac{\Im_1(t+1)}{\Im_1(t)}$; that is, $\frac{\Im_2(t+1)}{\Im_2(t)} < \frac{\Im_1(t+1)}{\Im_1(t)}$, which yields

$$|\Im_2(t+1) - \Im_2(t)| \geq |\Im_1(t+1) - \Im_1(t)|.$$

Therefore, $|V(t+1) - V(t)| \geq |\Im_2(t+1) - \Im_2(t)| \geq |\Im_1(t+1) - \Im_1(t)|$ and $\Im_2(t) < V(t) < \Im_1(t)$ for $t \in [t_k, s_k)$. Thus, there exists a instant s_k such that $V(s_k) \leq \Im_2(s_k)$ for $t > t_k$.

For $t \in [s_k, t_{k+1})$, the controller is closed, and the proof is similar to Theorem 1. Thus, it is omitted here.

Remark 5. In continuous-time systems, there is $V(t) < \mathfrak{I}_1(t)$ for $t \in [t_k, s_k)$, but this paper considers discrete-time MASs; thus, the equal sign may not be valid. For instance, there exists a point $s \in \mathbb{N}$ such that $V(s) < \mathfrak{I}_1(s)$, but $V(s+1) > \mathfrak{I}_1(s+1)$. That is to say, the point $s+1$ is the next trigger point t_{k+1} after the previous stop time s_k. In other words, since the $s+1$, the controller begins to perform the control function, and the function $V(t)$ gradually decreases; that is, there exists $V(s+y) \leq \mathfrak{I}_1(s+y)$, $y \in \mathbb{N}$. Similarly, the case of $V(s) > \mathfrak{I}_2(s)$ as $t \in [s_k, t_{k+1})$.

3.2. Consensus of linear time-varying systems with I-ETIC

For the discrete-time MASs, the ETIC protocol adopted in the above section of this paper has been optimized and improved to a great extent compared with the previous control strategies. However, the control phase of the protocol is still continuous control. Thus, in order to further reduce communication, this section considers the improved ETIC protocol whose control phase is event-triggered control, i.e., I-ETIC.

Next, the following form of distributed control input is given to achieve state consensus of agents.

$$u_i(t) = \begin{cases} -k_2 \sum_{j \in N_i} a_{ij}(\hat{x}_i(t) - \hat{x}_j(t)), & t \in [t_k, s_k), \\ 0, & t \in [s_k, t_{k+1}), \end{cases} \tag{3.25}$$

where $k_2 > 0$. $\hat{x}_i(t)$ implies the last broadcast state of agent i at time step t, which can be described as

$$\hat{x}_i(t) = x_i(t_h^i), \quad t \in [t_h^i, t_{h+1}^i), \tag{3.26}$$

where $t_0^i, t_1^i, \cdots, t_h^i, \cdots$ refers to the event-triggered time series of each agent i, $t_h^i \in \mathbb{N}$.

Denote t_{h+1}^i as the next event-triggered time instant after t_h^i, which is described as

$$t_{h+1}^i = \inf\Big\{t > t_h^i : \|e_i(t)\|^2 > \varsigma \|\delta_i(t)\|^2 \hat{q}_i(t) + \varpi \|\delta_i(t)\|^2\Big\}, \tag{3.27}$$

where the measurement error is defined as $e_i(t) = \hat{x}_i(t) - x_i(t)$ and the threshold is the sum of two variables with respect to t. The control parameters are $\varsigma > 0$ and $\varpi > 0$ and $\hat{q}_i(t)$ is represented by

$$\hat{q}_i(t) = \min\Big\{\sum_{j \in N_i} a_{ij} \|\hat{x}_i(t) - \hat{x}_j(t)\|^2, M\Big\}, \tag{3.28}$$

where M is a positive constant, which guarantees that $\hat{q}_i(t)$ is limited by M. It is not difficult to find that the threshold value in (3.27) will become larger when the consensus error and the state errors of the agent and the neighbors are larger, which subtly reduces the event-triggered instants while ensuring the convergence performance.

Let the two positive definite decreasing to zero functions $\mathfrak{I}_1(t)$ and $\mathfrak{I}_2(t)$, which satisfy

$$\begin{cases} \mathfrak{I}_3(t+1) = \alpha_1 \mathfrak{I}_3(t) + \Xi_3(t), & \mathfrak{I}_3(0) = \theta_1 \|\delta(0)\|^2 + \Xi_3(0), \\ \mathfrak{I}_4(t+1) = \alpha_2 \mathfrak{I}_4(t) + \Xi_4(t), & \mathfrak{I}_4(0) = \theta_2 \|\delta(0)\|^2 + \Xi_4(0), \end{cases} \tag{3.29}$$

where $0 < \alpha_2 < \alpha_1 < 1$, $0 < \theta_2 < 1 < \theta_1$, $\Xi_3(t) < (1-\alpha_1)\mathfrak{I}_3(t)$, $\Xi_4(t) < (1-\alpha_1)\mathfrak{I}_4(t)$.

$$\begin{cases} s_k = \{\inf t | t > t_k, \delta^T(t)\delta(t) \leq \mathfrak{I}_4(t)\}, \\ t_{k+1} = \{\inf t | t > s_k, \delta^T(t)\delta(t) \geq \mathfrak{I}_3(t)\}, \end{cases} \tag{3.30}$$

where $k \in \mathbb{N}$ and $t_0 = 0$.

The following shows the structure of the I-ETIC strategy as shown in Figure 2 and Algorithm 2.

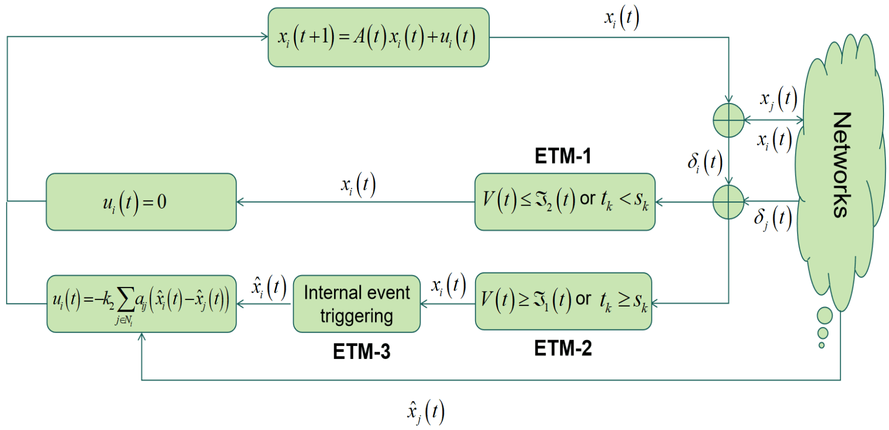

Figure 2. Structure diagram of I-ETIC method.

Algorithm 2 The control process under the I-ETIC protocol.

```
The related parameters and initial states of all agents are given.
for t = 1 to n do
    ℑ3(t + 1) = α1ℑ3(t) + Ξ3(t).
    ℑ4(t + 1) = α2ℑ4(t) + Ξ4(t).
    if V(t) ≥ ℑ3(t) then
        xi(t + 1) = A(t)xi(t) + ui(t).
        if the trigger condition is satisfied for the i th agent, then
            x̂i(t + 1) = xi(t + 1).
        else
            x̂i(t + 1) = x̂i(t).
        if V(t − 1) < ℑ3(t − 1) then
            tk = tk + 1.
    else if V(t) ≤ ℑ4(t) then
        xi(t + 1) = A(t)xi(t).
        if V(t − 1) > ℑ4(t − 1) then
            sk = sk + 1.
    Get the value of V(t + 1).
end for
```

Remark 6. It is important to note that the ETIC protocol proposed in this section is different from that in the previous section. The event trigger conditions in the above section determine when intermittent control is exercised and for how long. On the basis of the above control, this section introduces another event-triggered control, which will determine how and when to update the controller during the control phase of intermittent control.

Theorem 3. Suppose that Assumptions 1 and 2 hold, then the MASs (2.1) will achieve consensus through I-ETIC (3.25) with event triggering mechanism (3.30) if the control parameters satisfy

$$\varepsilon > 0, \varpi > 0, 0 < \varsigma < \frac{1-\alpha_1}{M(\lambda_{max}^2 k_2^2 + \frac{k_2^2+k_2}{\varepsilon})}$$

and k_2 satisfies one of the following conditions.

1) If $\beta^2 < \alpha_2$, $\wp > 0$ and $\min h(k_2) > 0$, then $k_2 \in (0, k_{24})$,

2) If $\beta^2 < \alpha_2$, $\wp > 0$ and $\min h(k_2) < 0$, then $k_2 \in (0, k_{21}) \cup (k_{22}, k_{24})$,

3) If $\beta^2 < \alpha_2$ and $\wp < 0$, then $k_2 \in (0, k_{24})$,

4) If $\beta^2 > \alpha_2$, $\wp > 0$ and $\min h(k_2) > 0$, then $k_2 \in (k_{23}, k_{24})$,

5) If $\beta^2 > \alpha_2$, $\wp > 0$ and $\min h(k_2) < 0$, then $k_2 \in (k_{23}, k_{21}) \cup (k_{22}, k_{24})$,

where $\wp = \frac{2\alpha\lambda_2-\varepsilon\lambda_{max}^2\beta^2-\frac{\varpi}{\varepsilon}}{2(\varepsilon\lambda_{max}^4+(\varpi+1)\lambda_{max}^2+\frac{\varpi}{\varepsilon})}$, $k_{21} = \frac{2\alpha\lambda_2-\varepsilon\lambda_{max}^2\beta^2-\frac{\varpi}{\varepsilon}-\sqrt{\Delta_2}}{2(\varepsilon\lambda_{max}^4+(\varpi+1)\lambda_{max}^2+\frac{\varpi}{\varepsilon})}$, $k_{22} = \frac{2\alpha\lambda_2-\varepsilon\lambda_{max}^2\beta^2-\frac{\varpi}{\varepsilon}+\sqrt{\Delta_2}}{2(\varepsilon\lambda_{max}^4+(\varpi+1)\lambda_{max}^2+\frac{\varpi}{\varepsilon})}$,, $k_{23} = \frac{2\alpha\lambda_2-\varepsilon\lambda_{max}^2\beta^2-\frac{\varpi}{\varepsilon}-\sqrt{\Delta_3}}{2(\varepsilon\lambda_{max}^4+(\varpi+1)\lambda_{max}^2+\frac{\varpi}{\varepsilon})}$, and $k_{24} = \frac{2\alpha\lambda_2-\varepsilon\lambda_{max}^2\beta^2-\frac{\varpi}{\varepsilon}+\sqrt{\Delta_3}}{2(\varepsilon\lambda_{max}^4+(\varpi+1)\lambda_{max}^2+\frac{\varpi}{\varepsilon})}$.

Proof. Consider the Lyapunov function of the following form

$$V(t) = \delta^T(t)\delta(t). \tag{3.31}$$

For $t \in [t_k, s_k)$, the controller is actuated. Combining (2.1) and (3.25), one has

$$\begin{aligned}
x_i(t+1) =& A(t)x_i(t) + u_i(t)\\
=& A(t)x_i(t) - k_2 \sum_{j\in N_i} a_{ij}(\hat{x}_i(t) - \hat{x}_j(t))\\
=& A(t)x_i(t) - k_2 \sum_{j=1}^{N} a_{ij}(e_i(t) + x_i(t) - e_j(t) - x_j(t))\\
=& A(t)x_i(t) - k_2 \sum_{j=1}^{N} a_{ij}(e_i(t) - e_j(t)) - k_2 \sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t))\\
=& A(t)x_i(t) - k_2 \sum_{j=1}^{N} l_{ij}x_j(t) - k_2 \sum_{j=1}^{N} l_{ij}e_j(t).
\end{aligned} \tag{3.32}$$

Furthermore, one obtains

$$x(t+1) = (I_N \otimes A(t) - k_2(L \otimes I_n))x(t) - k_2(L \otimes I_n)e(t). \tag{3.33}$$

According to $\delta(t) = (\Psi \otimes I_n)x(t)$, it yields

$$\begin{aligned}
\delta(t+1) =& (\Psi \otimes I_n)x(t+1)\\
=& (\Psi \otimes I_n)\big[(I_N \otimes A(t) - k_2(L \otimes I_n))x(t) - k_2(L \otimes I_n)e(t)\big]\\
=& (\Psi \otimes A(t) - k_2\Psi L \otimes I_n)x(t) - k_2(\Psi L \otimes I_n)e(t)\\
=& \big[(I_N \otimes A(t))(\Psi \otimes I_n) - k_2(L \otimes I_n)(\Psi \otimes I_n)\big]x(t) - k_2(L \otimes I_n)e(t)\\
=& (I_N \otimes A(t) - k_2 L \otimes I_n)(\Psi \otimes I_n)x(t) - k_2(L \otimes I_n)e(t)\\
=& (I_N \otimes A(t) - k_2 L \otimes I_n)\delta(t) - k_2(L \otimes I_n)e(t).
\end{aligned} \tag{3.34}$$

Since $V(t) = \delta^T(t)\delta(t)$, it follows that

$$\begin{aligned}
V(t+1) =&\delta^T(t+1)\delta(t+1)\\
=&[(I_N \otimes A(t) - k_2 L \otimes I_n)\delta(t) - k_2(L \otimes I_n)e(t)]^T[(I_N \otimes A(t)\\
&- k_2(L \otimes I_n))\delta(t) - k_2(L \otimes I_n)e(t)]\\
=&[\delta^T(t)(I_N \otimes A^T(t) - k_2 L \otimes I_n) - k_2 e^T(t)(L \otimes I_n)][(I_N \otimes A(t)\\
&- k_2(L \otimes I_n))\delta(t) - k_2(L \otimes I_n)e(t)]\\
=&\delta^T(t)(I_N \otimes A^T(t) - k_2 L \otimes I_n)(I_N \otimes A(t) - k_2(L \otimes I_n))\delta(t)\\
&- k_2\delta^T(t)(I_N \otimes A^T(t) - k_2 L \otimes I_n)(L \otimes I_n)e(t)\\
&- k_2 e^T(t)(L \otimes I_n)(I_N \otimes A(t) - k_2(L \otimes I_n))\delta(t)\\
&+ k_2^2 e^T(t)(L^2 \otimes I_n)e(t).
\end{aligned} \tag{3.35}$$

Based on Assumption 1, the first term on the righthand side of (3.35) can be written as

$$\begin{aligned}
&\delta^T(t)(I_N \otimes A^T(t) - k_2 L \otimes I_n)(I_N \otimes A(t) - k_2(L \otimes I_n))\delta(t)\\
\leq&(\beta^2 - 2\alpha\lambda_2 k_2 + \lambda_{max}^2 k_2^2)||\delta(t)||^2.
\end{aligned} \tag{3.36}$$

The following equation can be obtained by further sorting out the second and third items at the right of (3.35)

$$\begin{aligned}
&- k_2\delta^T(t)(I_N \otimes A^T(t) - k_2 L \otimes I_n)(L \otimes I_n)e(t) - k_2 e^T(t)(L \otimes I_n)\\
&(I_N \otimes A(t) - k_2 L \otimes I_n)\delta(t)\\
=&2k_2^2 e^T(t)(L^2 \otimes I_n)\delta(t) - 2k_2 e^T(t)(L \otimes A(t))\delta(t).
\end{aligned} \tag{3.37}$$

According to Young's inequality: $a^T b \leq \frac{1}{2\varepsilon}||a||^2 + \frac{\varepsilon}{2}||b||^2$, it has

$$\begin{aligned}
2k_2^2 e^T(t)(L^2 \otimes I_n)\delta(t) \leq&2k_2^2\Big(\frac{1}{2\varepsilon}\sum_{i=1}^N ||e_i(t)||^2 + \frac{\varepsilon}{2}\sum_{i=1}^N \lambda_i^4||\delta_i(t)||^2\Big)\\
\leq&\frac{k_2^2}{\varepsilon}\sum_{i=1}^N ||e_i(t)||^2 + \varepsilon k_2^2\lambda_{max}^4\sum_{i=1}^N ||\delta_i(t)||^2.
\end{aligned} \tag{3.38}$$

Similarly,

$$\begin{aligned}
-2k_2 e^T(t)(L \otimes A(t))\delta(t) \leq&2k_2\Big(\frac{1}{2\varepsilon}\sum_{i=1}^N ||e_i(t)||^2 + \frac{\varepsilon}{2}||A(t)||_{max}^2\sum_{i=1}^N \lambda_i^2||\delta_i(t)||^2\Big)\\
\leq&\frac{k_2}{\varepsilon}\sum_{i=1}^N ||e_i(t)||^2 + \varepsilon k_2\beta^2\lambda_{max}^2\sum_{i=1}^N ||\delta_i(t)||^2.
\end{aligned} \tag{3.39}$$

By substituting (3.38) and (3.39) into (3.37), one has

$$\begin{aligned}
&- k_2\delta^T(t)(I_N \otimes A^T(t) - k_2 L \otimes I_n)(L \otimes I_n)e(t) - k_2 e^T(t)(L \otimes I_n)\\
&(I_N \otimes A(t) - k_2 L \otimes I_n)\delta(t)
\end{aligned}$$

$$
\begin{aligned}
&=2k_2^2e^T(t)(L^2\otimes I_n)\delta(t)-2k_2e^T(t)(L\otimes A(t))\delta(t)\\
&\leq\frac{k_2^2}{\varepsilon}\sum_{i=1}^{N}\|e_i(t)\|^2+\varepsilon k_2^2\lambda_{max}^4\sum_{i=1}^{N}\|\delta_i(t)\|^2\\
&\quad+\frac{k_2}{\varepsilon}\sum_{i=1}^{N}\|e_i(t)\|^2+\varepsilon k_2\beta^2\lambda_{max}^2\sum_{i=1}^{N}\|\delta_i(t)\|^2\\
&=\Big(\frac{k_2+k_2^2}{\varepsilon}\Big)\sum_{i=1}^{N}\|e_i(t)\|^2+\varepsilon\lambda_{max}^2k_2(\beta^2+k_2\lambda_{max}^2)\sum_{i=1}^{N}\|\delta_i(t)\|^2. \qquad (3.40)
\end{aligned}
$$

For the last term to the right of (3.35), one obtains

$$
k_2^2e^T(t)(L^2\otimes I_n)e(t)\leq k_2^2\lambda_{max}^2\sum_{i=1}^{N}\|e_i(t)\|^2. \qquad (3.41)
$$

Combined with (3.36), (3.40) and (3.41), (3.35) can be further written as

$$
\begin{aligned}
V(t+1)\leq&(\beta^2-2\alpha\lambda_2k_2+\lambda_{max}^2k_2^2)\|\delta(t)\|^2+k_2^2\lambda_{max}^2\sum_{i=1}^{N}\|e_i(t)\|^2\\
&+\Big(\frac{k_2+k_2^2}{\varepsilon}\Big)\sum_{i=1}^{N}\|e_i(t)\|^2+\varepsilon\lambda_{max}^2k_2(\beta^2+k_2\lambda_{max}^2)\sum_{i=1}^{N}\|\delta_i(t)\|^2\\
=&(\beta^2-2\alpha\lambda_2k_2+\lambda_{max}^2k_2^2+\varepsilon\lambda_{max}^2\beta^2k_2+\varepsilon k_2^2\lambda_{max}^4)\sum_{i=1}^{N}\|\delta_i(t)\|^2\\
&+(\lambda_{max}^2k_2^2+\frac{k_2^2+k_2}{\varepsilon})\sum_{i=1}^{N}\|e_i(t)\|^2. \qquad (3.42)
\end{aligned}
$$

From (3.27) and (3.28), one has

$$
\begin{aligned}
V(t+1)\leq&(\beta^2-2\alpha\lambda_2k_2+\lambda_{max}^2k_2^2+\varepsilon\lambda_{max}^2\beta^2k_2+\varepsilon k_2^2\lambda_{max}^4)\sum_{i=1}^{N}\|\delta_i(t)\|^2\\
&+(\lambda_{max}^2k_2^2+\frac{k_2^2+k_2}{\varepsilon})\Big[M\varsigma\sum_{i=1}^{N}\|\delta_i(t)\|^2+\varpi\sum_{i=1}^{N}\|\delta_i(t)\|^2\Big]\\
=&[(\varepsilon\lambda_{max}^4+(\varpi+1)\lambda_{max}^2+\frac{\varpi}{\varepsilon})k_2^2+(\varepsilon\lambda_{max}^2\beta^2-2\alpha\lambda_2+\frac{\varpi}{\varepsilon})k_2+\beta^2]V(t)+\Xi(t)\\
\leq&\alpha_2V(t)+\Xi(t), \qquad (3.43)
\end{aligned}
$$

where $\varpi>0$ and $\Xi(t)=(\lambda_{max}^2k_2^2+\frac{k_2^2+k_2}{\varepsilon})M\varsigma\sum_{i=1}^{N}\|\delta_i(t)\|^2$ with $k_2>0$, $\varsigma>0$.

Let $h(k_2)=(\varepsilon\lambda_{max}^4+(\varpi+1)\lambda_{max}^2+\frac{\varpi}{\varepsilon})k_2^2+(\varepsilon\lambda_{max}^2\beta^2-2\alpha\lambda_2+\frac{\varpi}{\varepsilon})k_2+\beta^2$. On the one hand, assuming $h(k_2)=0$,

$$
(\varepsilon\lambda_{max}^4+(\varpi+1)\lambda_{max}^2+\frac{\varpi}{\varepsilon})k_2^2+(\varepsilon\lambda_{max}^2\beta^2-2\alpha\lambda_2+\frac{\varpi}{\varepsilon})k_2+\beta^2=0. \qquad (3.44)
$$

To solve (3.44), one has

$$k_{21}=\frac{2\alpha\lambda_2-\varepsilon\lambda_{max}^2\beta^2-\frac{\varpi}{\varepsilon}-\sqrt{\Delta_2}}{2(\varepsilon\lambda_{max}^4+(\varpi+1)\lambda_{max}^2+\frac{\varpi}{\varepsilon})},k_{22}=\frac{2\alpha\lambda_2-\varepsilon\lambda_{max}^2\beta^2-\frac{\varpi}{\varepsilon}+\sqrt{\Delta_2}}{2(\varepsilon\lambda_{max}^4+(\varpi+1)\lambda_{max}^2+\frac{\varpi}{\varepsilon})},$$

where $\Delta_2=(\varepsilon\lambda_{max}^2\beta^2-2\alpha\lambda_2+\frac{\varpi}{\varepsilon})^2-4\beta^2(\varepsilon\lambda_{max}^4+(\varpi+1)\lambda_{max}^2+\frac{\varpi}{\varepsilon})$.

On the one hand, assuming $h(k_2)=\alpha_2$,

$$(\varepsilon\lambda_{max}^4+(\varpi+1)\lambda_{max}^2+\frac{\varpi}{\varepsilon})k_2^2+(\varepsilon\lambda_{max}^2\beta^2-2\alpha\lambda_2+\frac{\varpi}{\varepsilon})k_2+\beta^2-\alpha_2=0. \tag{3.45}$$

To solve (3.45), one has

$$k_{23}=\frac{2\alpha\lambda_2-\varepsilon\lambda_{max}^2\beta^2-\frac{\varpi}{\varepsilon}-\sqrt{\Delta_3}}{2(\varepsilon\lambda_{max}^4+(\varpi+1)\lambda_{max}^2+\frac{\varpi}{\varepsilon})},k_{24}=\frac{2\alpha\lambda_2-\varepsilon\lambda_{max}^2\beta^2-\frac{\varpi}{\varepsilon}+\sqrt{\Delta_3}}{2(\varepsilon\lambda_{max}^4+(\varpi+1)\lambda_{max}^2+\frac{\varpi}{\varepsilon})},$$

where $\Delta_3=(\varepsilon\lambda_{max}^2\beta^2-2\alpha\lambda_2+\frac{\varpi}{\varepsilon})^2-4(\beta^2-\alpha_2)(\varepsilon\lambda_{max}^4+(\varpi+1)\lambda_{max}^2+\frac{\varpi}{\varepsilon})$.

Next, the discussion is divided into several scenarios. To simplify writing, write $\wp=\frac{2\alpha\lambda_2-\varepsilon\lambda_{max}^2\beta^2-\frac{\varpi}{\varepsilon}}{2(\varepsilon\lambda_{max}^4+(\varpi+1)\lambda_{max}^2+\frac{\varpi}{\varepsilon})}$.

1) $\beta^2<\alpha_2$,

a) If $\wp>0$,

Case 1: If $\min h(k_2)>0$, that is $4\beta^2(\varepsilon\lambda_{max}^4+(\varpi+1)\lambda_{max}^2+\frac{\varpi}{\varepsilon})>(\varepsilon\lambda_{max}^2\beta^2-2\alpha\lambda_2+\frac{\varpi}{\varepsilon})^2$.

When $0<k_2<k_{24}$, then $V(t+1)<\alpha_2V(t)$.

Case 2: If $\min h(k_2)<0$, that is $4\beta^2(\varepsilon\lambda_{max}^4+(\varpi+1)\lambda_{max}^2+\frac{\varpi}{\varepsilon})<(\varepsilon\lambda_{max}^2\beta^2-2\alpha\lambda_2+\frac{\varpi}{\varepsilon})^2$.

When $0<k_2<k_{21}$ or $k_{22}<k_2<k_{24}$, then $V(t+1)<\alpha_2V(t)$.

b) If $\wp<0$,

Case 3: When $0<k_2<k_{24}$, then $V(t+1)<\alpha_2V(t)$.

2) $\beta^2>\alpha_2$,

A) If $\wp>0$,

Case 4: If $\min h(k_2)>0$, that is $4\beta^2(\varepsilon\lambda_{max}^4+(\varpi+1)\lambda_{max}^2+\frac{\varpi}{\varepsilon})>(\varepsilon\lambda_{max}^2\beta^2-2\alpha\lambda_2+\frac{\varpi}{\varepsilon})^2$.

When $k_{23}<k_2<k_{24}$, then $V(t+1)<\alpha_2V(t)$.

Case 5: If $\min h(k_2)<0$, that is $4\beta^2(\varepsilon\lambda_{max}^4+(\varpi+1)\lambda_{max}^2+\frac{\varpi}{\varepsilon})<(\varepsilon\lambda_{max}^2\beta^2-2\alpha\lambda_2+\frac{\varpi}{\varepsilon})^2$.

When $k_{23}<k_2<k_{21}$ or $k_{22}<k_2<k_{24}$ then $V(t+1)<\alpha_2V(t)$.

B) If $\wp<0$,

In this case, no matter what the value of k_2 is, the condition $V(t+1)<\alpha_2V(t)$ cannot be satisfied.

Note that $\Xi(t)$ has little effect on the rate of decline of $V(t)$, similarly $\Xi_3(t)$ and $\Xi_4(t)$. $\Xi(t)$, $\Xi_3(t)$ and $\Xi_4(t)$ have almost the same rate of decline for $V(t)$, $\Im_3(t)$ and $\Im_4(t)$, respectively; thus, $V(t+1)<\alpha_2V(t)$, $\Im_3(t+1)<\alpha_1\Im_3(t)$ and $\Im_4(t+1)<\alpha_2\Im_4(t)$ are mainly considered next. Based on the above analysis, $V(t+1)<\alpha_2V(t)$ when k_2 meets the above conditions.

According to (3.29) and (3.30), $\Im_4(t_k) \le V(t_k) = \Im_3(t_k)$. Combined with (3.35), we can obtain that $V(t+1) \le \alpha_2 V(t)$ for $t \in [t_k, s_k)$, and because $\Im_4(t+1) = \alpha_2 \Im_4(t)$, one has $\frac{V(t+1)}{V(t)} \le \alpha_2 = \frac{\Im_4(t+1)}{\Im_4(t)}$; that is, $\frac{V(t+1)}{V(t)} \le \frac{\Im_4(t+1)}{\Im_4(t)}$, and it follows that

$$|V(t+1) - V(t)| \ge |\Im_4(t+1) - \Im_4(t)|.$$

Next, according to (3.29), $\frac{\Im_4(t+1)}{\Im_4(t)} = \alpha_2 < \alpha_1 = \frac{\Im_3(t+1)}{\Im_3(t)}$; that is, $\frac{\Im_4(t+1)}{\Im_4(t)} < \frac{\Im_3(t+1)}{\Im_3(t)}$, and

$$|\Im_4(t+1) - \Im_4(t)| \ge |\Im_3(t+1) - \Im_3(t)|.$$

Thus $|V(t+1) - V(t)| \ge |\Im_4(t+1) - \Im_4(t)| \ge |\Im_3(t+1) - \Im_3(t)|$ and $\Im_4(t) < V(t) < \Im_3(t)$ for $t \in [t_k, s_k)$. Therefore, there exists an instant s_k such that $V(s_k) \le \Im_4(s_k)$ for $t > t_k$.

For $t \in [s_k, t_{k+1})$, the controller is closed, and the proof is similar to Theorem 1. Thus, it is omitted here.

Remark 7. This section introduces the event-triggered control strategy in the control phase of intermittent control. Compared with the previous section, it is obvious that some more conservative conditions for achieving consensus have been obtained. This shows that with the improvement of the control strategy, it also increases the complexity of some theoretical analysis. How to balance the relationship between them is worth further discussion.

Remark 8. It is worth noting that the exclusion of zero behavior is not proved in the analysis in this paper, because it does not exist in discrete-time systems. In addition, from the practical point of view, although the system itself is a continuous process in a large number of practical application scenarios, due to the limited bandwidth of the channel in the communication system, the controller can only apply the sampled data obtained at discrete moments. From the theoretical point of view, compared with the continuous system, the discrete-time system does not have to exclude zero behavior in theoretical analysis, which simplifies many complexities. Therefore, it is of great practical value to study the consensus of discrete-time MASs.

Remark 9. The problem of asymptotic consensus in discrete-time MASs is considered in this paper. From the perspective of time, asymptotic convergence takes a long time, which will result in a waste of some resources. Therefore, how to achieve consensus in discrete-time MASs within preassigned time is a difficult problem to be overcome, and for consideration of this challenge, see [53] and [54].

4. Simulation example

In this section, some numerical simulation examples are given to demonstrate the feasibility of the two proposed algorithms and the validity of the theoretical analysis.

Example 1. First, consider an undirected connected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, in which the four agents in the network are shown in Figure 3, and their initial states are given as

$$x_1(0) = \begin{bmatrix} -0.6 \\ 0.8 \end{bmatrix},\ x_2(0) = \begin{bmatrix} 0.4 \\ 1.7 \end{bmatrix},\ x_3(0) = \begin{bmatrix} -0.2 \\ 0.3 \end{bmatrix},\ x_4(0) = \begin{bmatrix} 0.2 \\ -0.9 \end{bmatrix}.$$

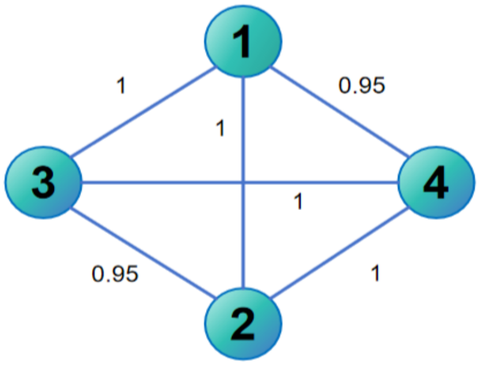

Figure 3. Network topology.

Second, consider the system matrice of each agent has the form

$$A(t) = \begin{pmatrix} 1 - e^{-2t} & -e^{-2t} \\ 1 - e^{-3t} & 1 - e^{-3t} \end{pmatrix}.$$

In the simulation, choose design parameters $\alpha = 1.3, \beta = 1.5$. It should be noted that the values of the above parameters and the initial states of the agents are randomly selected under the conditions of satisfying Theorems 1–3, which has certain practicability from a practical point of view.

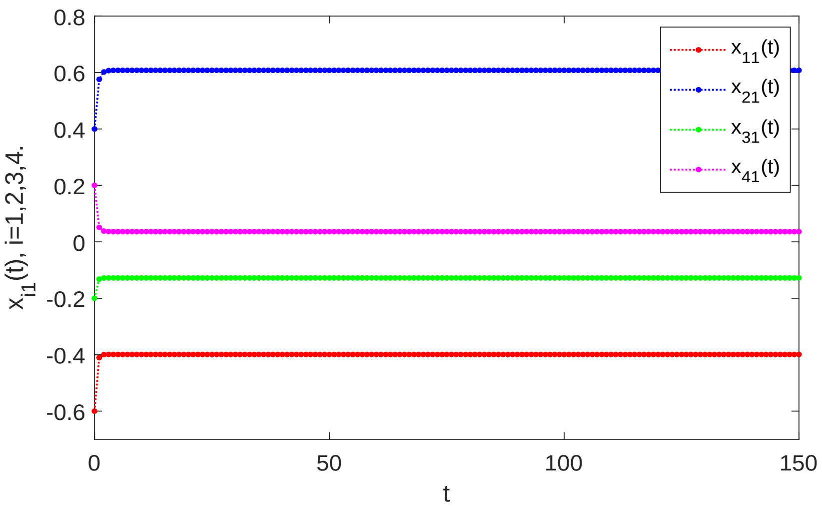

Figure 4. Evolution of the first component of $x_i(t), i = 1, 2, 3, 4$.

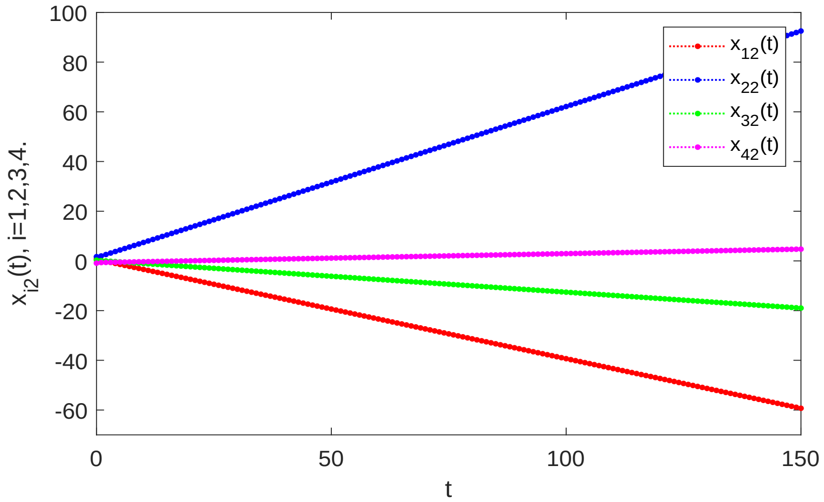

Figure 5. Evolution of the second component of $x_i(t), i = 1, 2, 3, 4$.

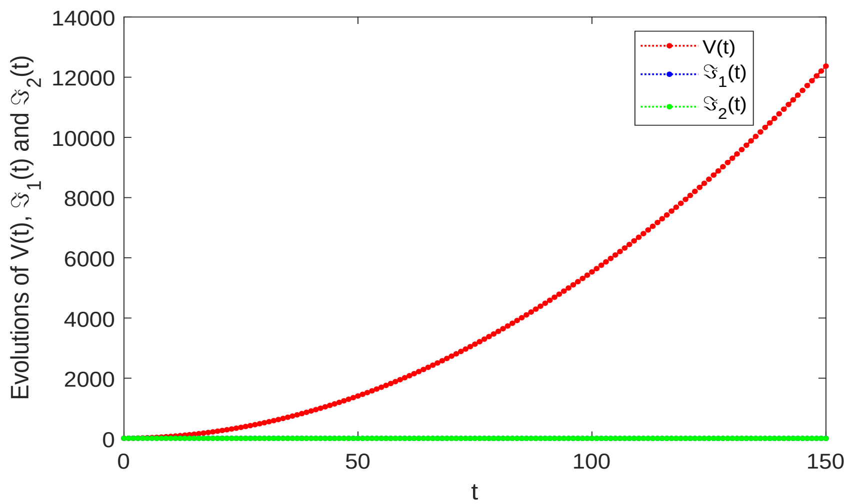

Figure 6. Evolutions of $V(t)$, $\Im_1(t)$ and $\Im_2(t)$.

Consider the case where no control is performed. According to Figures 4 and 5, it can be found that the state trajectories of these agents cannot agree as time t increases when control is not performed. Figure 6 shows that the function $V(t)$ is also divergent, i.e., the consensus error is also nonzero.

Case 1. Consider the dynamics with the general **ETIC**, i.e., the controller is in the form of (3.1) where $i = 1, 2, 3, 4$. The control gain in (3.1) is $k_1 = 0.3$. For the two auxiliary functions in (3.3), take the corresponding parameters $\alpha_1 = 0.98$, $\alpha_2 = 0.93$, $\theta_1 = 1.2$ and $\theta_2 = 0.8$. Next, scatter plots of the first and second dimensional state trajectes of the four agents can be obtained according to (2.1) and (3.1), as shown in Figures 7 and 8, respectively.

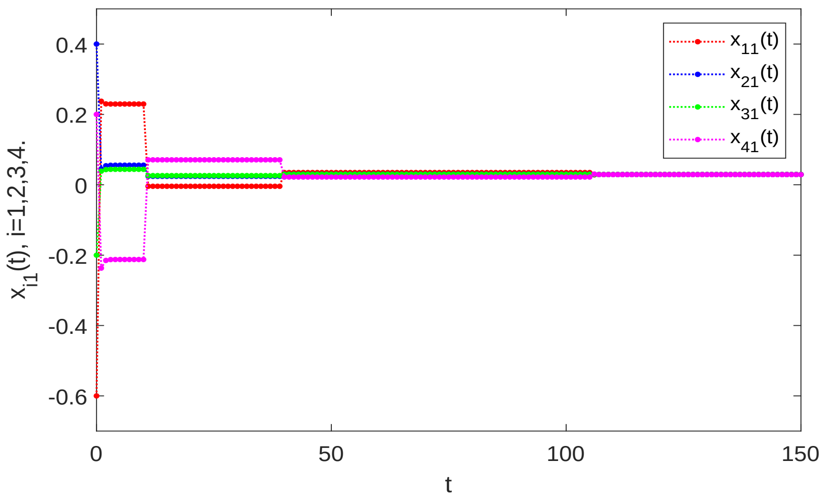

Figure 7. Evolution of the first component of $x_i(t), i = 1, 2, 3, 4$.

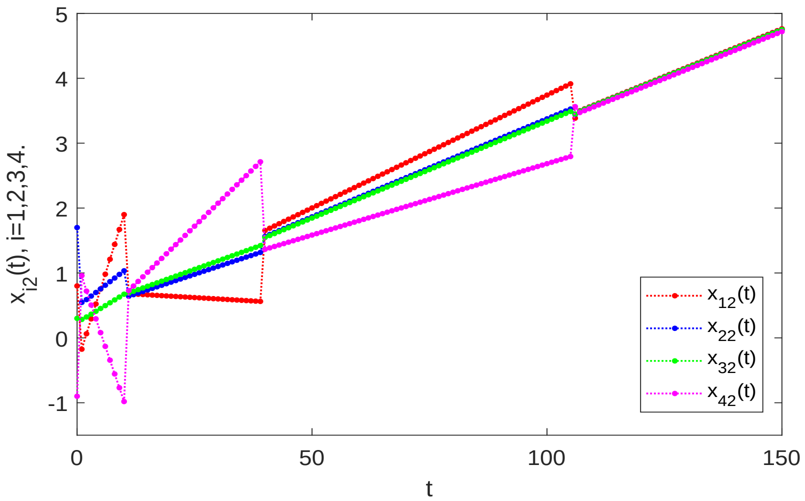

Figure 8. Evolution of the second component of $x_i(t), i = 1, 2, 3, 4$.

It can be seen from the above figure that all agents in the network can achieve consensus. Meanwhile, as shown in Figure 9, the function $V(t)$ related to consensus error is limited by the monotone decreasing function $\Im_1(t)$ under the intermittent control of event triggering; thus, when t approaches infinity, the function $V(t)$ asymptotically converges to zero. Through the analysis of Figure 10, it can be seen that the k th control trigger point and the k th stop control trigger point are very close, indicating that the controller proposed in this paper has a strong control function. In addition, the distance between the k th stop control trigger point and the $k + 1$ th control trigger point is far, which indicates that the control method proposed in this paper can effectively reduce the communication load in the whole process of achieving consensus.

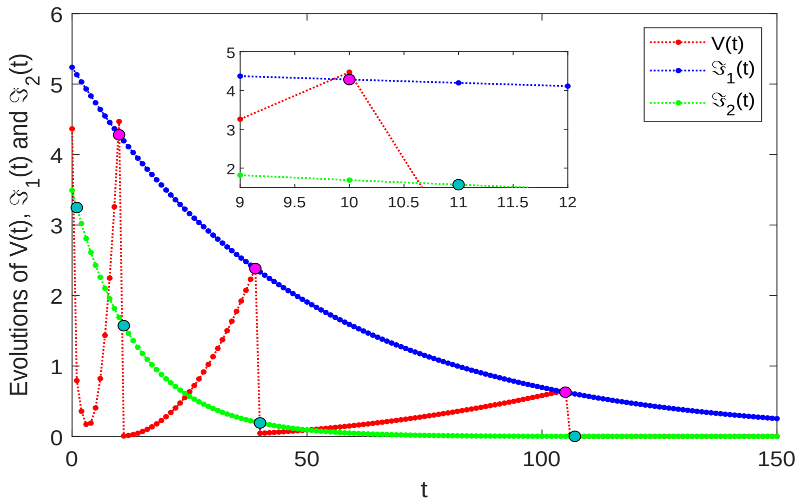

Figure 9. Evolutions of $V(t)$, $\mathfrak{I}_1(t)$ and $\mathfrak{I}_2(t)$.

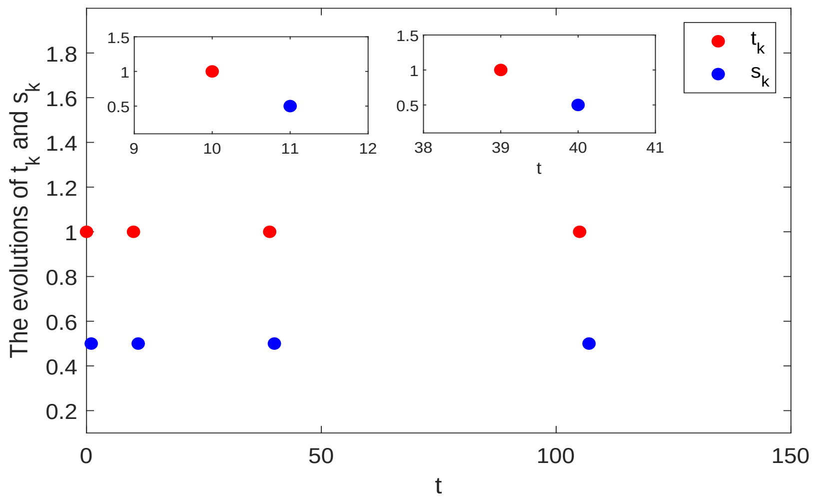

Figure 10. Evolutions of t_k and s_k.

Case 2. Consider the dynamics with the **I-ETIC**, i.e., the controller is in the form of (3.25) where $i = 1, 2, 3, 4$. Similarly, the control gain and design parameters in the numerical simulation are $k_2 = 0.045$, $\varsigma = 0.6$, $\varpi = 0.298$, $\varepsilon = 0.2$ and $M = 0.2$, respectively. For the two auxiliary functions in (3.29), take the corresponding parameters $\alpha_1 = 0.95$, $\alpha_2 = 0.93$, $\theta_1 = 1.5$ and $\theta_2 = 0.6$. Observing Figures 11 and 12, it is found that the states of all agents can still achieve consensus but are different from Figures 7 and 8, which is because the controller is event-triggered controlled rather than continuous controlled in the control phase.

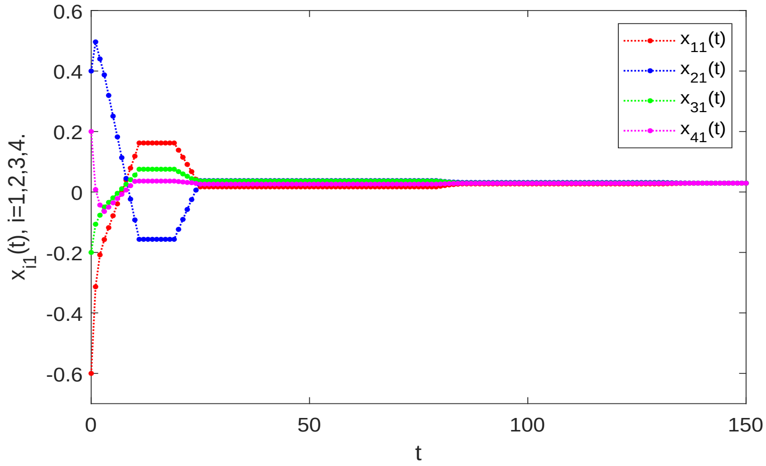

Figure 11. Evolution of the first component of $x_i(t), i = 1, 2, 3, 4$.

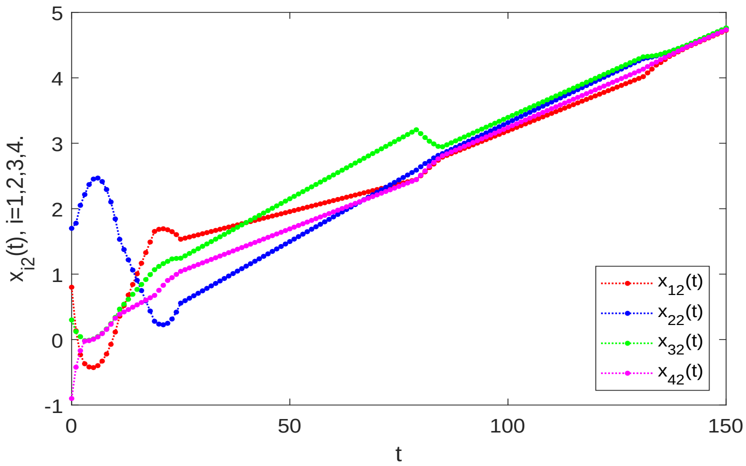

Figure 12. Evolution of the second component of $x_i(t), i = 1, 2, 3, 4$.

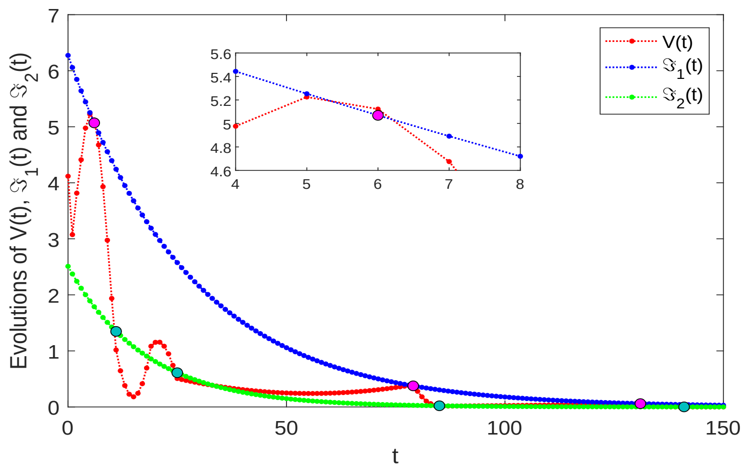

Figure 13. Evolutions of $V(t)$, $\Im_1(t)$ and $\Im_2(t)$.

Moreover, according to Figure 13, in general, the trajectory of function $V(t)$ is very close to that of

Case 1, which indicates that the introduction of event triggering mechanism in the control stage of the controller of Example 1 does not affect the convergence of function $V(t)$; that is, the state trajectories of these agents can still achieve consensus. Similarly, Figure 14 shows the trajectory diagram for the stop control point s_k and the start control point t_k under controller (3.25). Combined with Figures 10 and 14, it can be seen that compared with the control method in Example 1, the k th start control point t_k and the k th stop control point s_k appear more slowly under the I-ETIC action in the same time, indicating that this control strategy can further reduce communication.

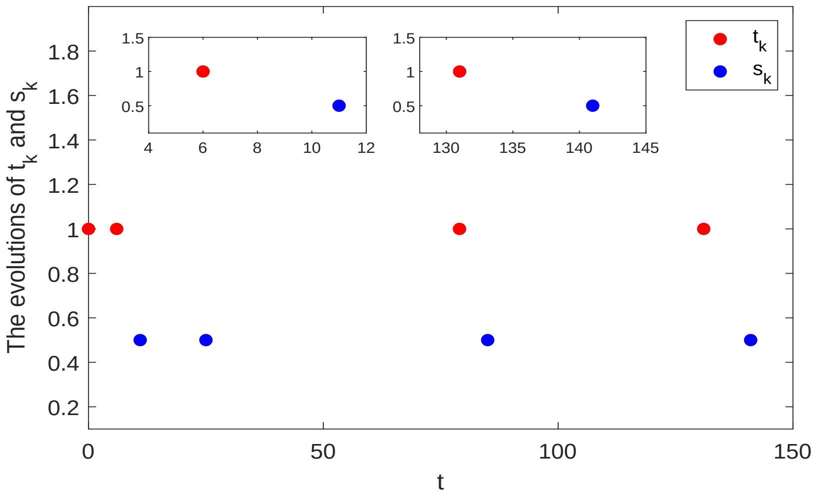

Figure 14. Evolutions of t_k and s_k.

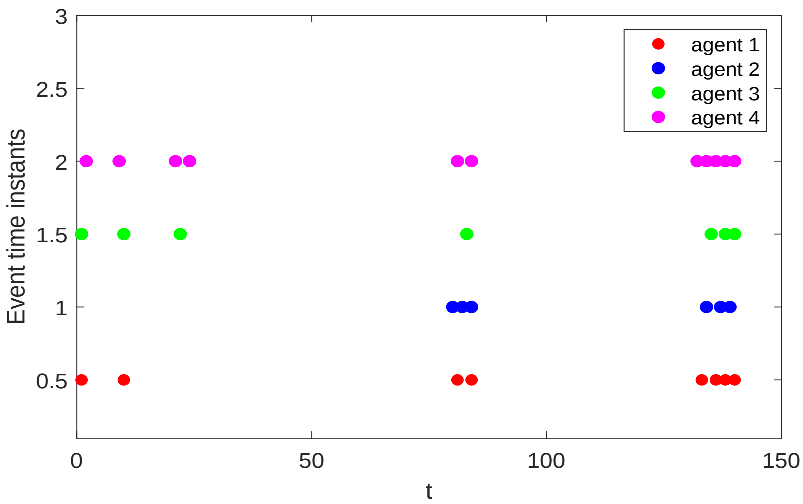

Figure 15. Event-triggering sampling time instants of all agents.

Finally, the corresponding event-triggered sampling time series of each agent is given in Figure 15. Through observation and analysis, the controller update of each agent is asynchronous, and it is obvious that the event-triggered control strategy can effectively reduce the communication between the agent and its neighbors. Moreover, according to Figures 11 and 16, it can be found that when all agent states reach consensus, the distributed control input $u_{i1}(t)$ gradually approaches zero.

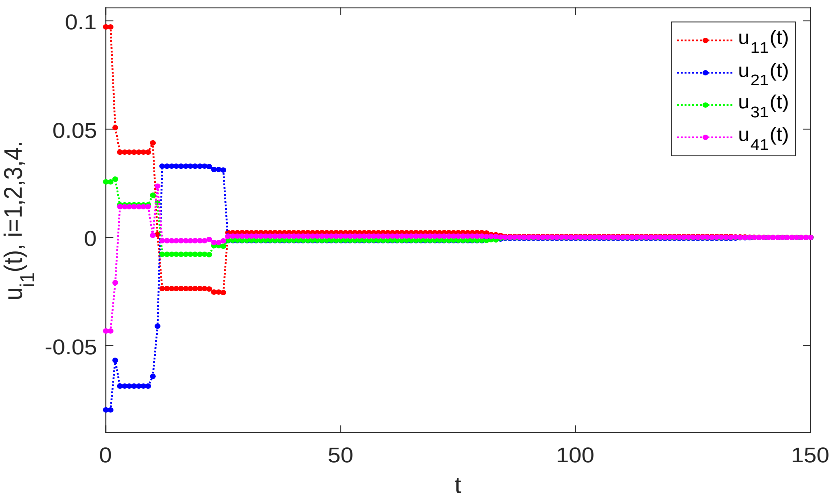

Figure 16. Evolutions of all agents' control inputs $u_{i1}(t)$.

5. Conclusions

This paper discussed the consensus problem of time-varying linear MASs within discrete-time using ETIC strategies. First, utilizing a class of auxiliary functions, two different ETIC protocols were designed. It was found that these two ETIC protocols are distinct and the latter has the advantage of further reducing the communication load compared with the former. Second, by constructing an error dynamical system and applying stability theory, we obtained some sufficient conditions for all agents to achieve consensus. Finally, two numerical simulation examples were provided to validate the effectiveness of the proposed algorithms and the feasibility of the theoretical analysis. It is an interesting and challenging topic to study distributed optimization problems with ETIC strategies within discrete time-varying linear MASs; thus, this will be a problem for further consideration in our future work.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

All authors declare that there is no conflict of interest in this article.

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