Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 9(2): 3289–3302.
DOI:10.3934/math.2024160
Received: 21 November 2023
Revised: 11 December 2023
Accepted: 19 December 2023
Published: 04 January 2024

Research article

Soft nodec spaces

Mesfer H. Alqahtani[1] **and Zanyar A. Ameen**[2,*]

[1] Department of Mathematics, University College of Umluj, University of Tabuk, Tabuk 48322, Saudi Arabia

[2] Department of Mathematics, College of Science, University of Duhok, Duhok 42001, Iraq

* **Correspondence:** Email: zanyar@uod.ac, Tel: +9647504727411.

Abstract: Following van Douwen, we call a soft topological space soft nodec if every soft nowhere dense subset of it is soft closed. This paper considers soft nodec spaces, which contain soft submaximal and soft door spaces. We investigate the basic properties and characterizations of soft nodec spaces. More precisely, we show that a soft nodec space can be written as a union of two disjoint soft closed soft dense (or soft open) soft nodec subspaces. Then, we study the behavior of soft nodec spaces under various operations, including the following: taking soft subspaces, soft products, soft topological sums, and images under specific soft functions with the support of appropriate counterexamples. Additionally, we show that the Krull dimension of a soft nodec soft T_0-space is less than or equal to one. After that, we present some connections among soft nodec, soft strong nodec, and soft compact spaces. Finally, we successfully determine a condition under which the soft one-point compactification of a soft space is soft nodec if and only if the soft space is soft strong nodec.

Keywords: soft nodec space; soft submaximal space; soft one-point compactification; soft door space
Mathematics Subject Classification: 54E99, 54F65

1. Introduction

Diverse mathematical tools have been proposed to transact with vagueness and suspicion existing in practical difficulties in social science, medical science, engineering, and economics. Molodtsov [27] developed a new model called a soft set, whose parameterizations are appropriate to solve suspicions and are free from the inherent restrictions of previous tools. Molodtsov applied his theory to several mathematical disciplines. After him, various scholars utilized his soft set theory to other areas of mathematics.

One of these areas is soft topology, which particularly merges soft set theory with (general) topology. It is driven by the basic assumptions of classic topology and focuses on the set of all soft sets. Soft topology was established by Shabir and Naz [35]. Several subclasses of soft topological spaces were suggested, including soft separation axioms [17], soft separable [17], soft connected [25], soft compact [15], soft Lindelof [5], soft paracompact [25] and soft extremally disconnected spaces [13]. Furthermore, different generalized soft (open) sets in soft topological spaces were also proposed, such as soft sets of the first or second Baire category [10, 11].

The motivations for writing this paper are as follows. First, we anticipate that by creating a new class of soft topological spaces, namely soft nodec, we will simplify the path for many future articles on this topic. Second, by presenting a new framework that contains previous concepts such, as soft submaximal and soft door spaces, and linking them to new concepts that may contribute to many concepts developed in soft environments, substantial contributions can also be provided. Finally, it reinforces the importance of the concept of soft topology as a strong tribute to other modern concepts.

The structure of this article is designed as follows. In Section 2, we recall the main ideas and findings that make this work self-contained. In Section 3, we present the definition of a soft nodec space, followed by some properties and operations. In Section 4, we establish some connections between soft nodec spaces and other related known soft spaces; additionally, we state what it means to be the Krull dimension of a soft space to characterize soft nodec spaces. Section 5 ends the work with a short conclusion.

2. Preliminaries

This section presents definitions and results related to soft sets and soft topology, which help us prove our results in the subsequent sections.

Definition 2.1. *[27] Assume X and $\mathfrak{R}$ are the initial universal set and a set of parameters, respectively. Let $K : R \to \mathcal{P}(X)$ be a set-valued function, whereas $R \subseteq \mathfrak{R}$ and $\mathcal{P}(X)$ is the power set of X. An ordered pair $(K, R) = \{(r, K(r)) : r \in R\}$ is stated to be the soft set over X.*

$SS(X_R)$ refers to the class of all soft sets over X linked with R.

Remark 2.2. *We can extend a soft set (K, R) to the soft set $(K, \mathfrak{R})$ by assuming $K(r) = \emptyset$ for any $r \in \mathfrak{R} - R$.*

Definition 2.3. *[30] The soft complement $(K, R)^c$ of a soft set (K, R) is a soft set (K^c, R) such that $K^c : R \to \mathcal{P}(X)$ is a function that has the property $K^c(r) = X - K(r)$ for all $r \in R$.*

Definition 2.4. *[7, 19] A soft set (K, R) over X is called the following:*

1) A null soft set with respect to R, Φ_R, if $K(r) = \emptyset$ for all $r \in R$.
2) An absolute soft set with respect to R, X_R, if $K(r) = X$ for all $r \in R$.
3) Finite (resp. countable) if $K(r)$ is finite (resp. countable) for each $r \in R$. Otherwise, it is called infinite (resp. uncountable).

Note that $\Phi_R^c = X_R$ and $X_R^c = \Phi_R$.

Definition 2.5. *[7, 26] For any index set I, let $\{(K_i, R) : i \in I\}$ be a family of soft sets over X.*

1) For $i \in I$, the soft intersection of (K_i, R) is a soft set (K, R) such that $K(r) = \bigcap_{i\in I} K_i(r)$ for all $r \in R$ and is denoted by $(K, R) = \widetilde{\bigcap}_{i\in I}(K_i, R)$;
2) For $i \in I$, the soft union of (K_i, R) is a soft set (K, R) such that $K(r) = \bigcup_{i\in I} K_i(r)$ for all $r \in R$ and is denoted by $(K, R) = \widetilde{\bigcup}_{i\in I}(K_i, R)$.

Definition 2.6. *[7, 8] Let $(K, R), (L, R) \in SS(X_R)$. Then, the soft set difference between (K, R) and (L, R) is defined to be the soft set $(H, R) = (K, R) - (L, R)$ such that $H(r) = K(r) - L(r)$ for all $r \in R$.*

Definition 2.7. *[26, 30] Let $R_1, R_2 \subseteq \Re$. It is said that (K_1, R_1) is a soft subset of (K_2, R_2) (denoted by $(K_1, R_1)\widetilde{\subseteq}(K_2, R_2)$) if $R_1 \subseteq R_2$ and $K_1(r) \subseteq K_2(r)$ for all $r \in R_1$. Additionally, (K_1, R_1) is soft equal to (K_2, R_2), written by $(K_1, R_1) = (K_2, R_2)$, if $(K_1, R_1)\widetilde{\subseteq}(K_2, R_2)$ and $(K_2, R_2)\widetilde{\subseteq}(K_1, R_1)$.*

Definition 2.8. *[37] A soft point is a soft set (K, R) over X, denoted by x_r, provided that $K(r) = \{x\}$ for some $r \in R$ and $K(s) = \emptyset$ for all $s \in R$ with $r \neq s$, where $r \in R$ and $x \in X$. By a statement $x_r \in (K, R)$, we mean $x \in K(r)$. By $SP(X_R)$, we denote the set of all soft points over X along with R.*

In the following sections, by two different soft points x_r, y_s, we intend either $x \neq y$ or $r \neq s$, and by two disjoint soft sets $(K, R), (L, R)$ over X, we mean $(K, R)\widetilde{\cap}(L, R) = \Phi_R$.

Definition 2.9. *[35] A soft topology on X is a subfamily $\mathcal{F} \subseteq SS(X_R)$ containing Φ_R, X_R and satisfying the condition that the soft union of the arbitrary and the soft intersection of a finite number of soft sets in $\mathcal{F}$ is a part of $\mathcal{F}$.*

Remark 2.10. *1) The triplet $(X, \mathcal{F}, R)$ is called a soft topological space on X, where the members of $\mathcal{F}$ are called soft open sets; and*
2) A soft closed set in $(X, \mathcal{F}, R)$ is the soft complement of a soft open set. The set of all soft closed sets is denoted by $\mathcal{F}^c$.

Definition 2.11. *[18] A (countable) soft base for a soft topology $\mathcal{F}$ is a (countable) subfamily $\mathcal{L} \subseteq \mathcal{F}$ such that members of $\mathcal{F}$ are unions of members of $\mathcal{L}$.*

Definition 2.12. *[6] Let $C \subseteq SS(X_R)$. The soft intersection of all soft topologies on X containing C is called the soft topology generated by C.*

Definition 2.13. *[35] Let $(Y, R) \neq \Phi_R$ be a soft subset of $(X, \mathcal{F}, R)$. Then, $\mathcal{F}_{(Y,R)} = \{(L, R)\widetilde{\cap}(Y, R) : (L, R) \in \mathcal{F}\}$ is called a relative soft topology over Y and $(Y, \mathcal{F}_{(Y,R)}, R)$ is a soft subspace of $(X, \mathcal{F}, R)$.*

Lemma 2.14. *[35] Let $(Y, \mathcal{F}_{(Y,R)}, R)$ be a soft subspace of $(Y, \mathcal{F}, R)$ and let $(K, R)\widetilde{\subseteq}(Y, R) \in \mathcal{F}$. Then, $(K, R) \in \mathcal{F}_{(Y,R)}$ if and only if $(K, R) \in \mathcal{F}$.*

Definition 2.15. *[28] Let $(L, R)\widetilde{\subseteq}(X, \mathcal{F}, R)$. Then, (L, R) is called a soft neighborhood of $x_r \in SP(X_R)$ if there exists $(W, R) \in \mathcal{F}(x_r)$ such that $x_r \in (W, R)\widetilde{\subseteq}(L, R)$, where $\mathcal{F}(x_r)$ is the family of all elements of $\mathcal{F}$ that contains x_r.*

Definition 2.16. *[35] Let $(K, R)\widetilde{\subseteq}(X, \mathcal{F}, R)$.*

1) The soft closure of (K, R), denoted by $cl_X(K, R)$ (simply $cl(K, R)$), is the smallest soft closed set containing (K, R); and
2) The soft interior of (K, R), denoted by $int_X(K, R)$ (simply $int(K, R)$), is the largest soft open set that is contained in (K, R).

Lemma 2.17. *[22] Let $(K,R)\widetilde{\subseteq}(X,\mathcal{F},R)$. Then*

$$int((K,R)^c) = (cl(K,R))^c \text{ and } cl((K,R)^c) = (int(K,R))^c.$$

Definition 2.18. *[16, 22] Let $(K,R)\widetilde{\subseteq}(X,\mathcal{F},R)$. The soft boundary of (K,R) is given by $b(K,R) = cl(K,R) - int(K,R)$.*

Definition 2.19. *[18] Let $(L,R)\widetilde{\subseteq}(X,\mathcal{F},R)$. A soft point $x_r \in SP(X_R)$ is called a soft limit point of (L,R) if $(K,R)\widetilde{\cap}(L,R) - \{x_r\} \neq \Phi_R$ for all $(K,R) \in \mathcal{F}(x_r)$. The family of all soft limit points of (L,R) is denoted by $\mathcal{D}(L,R)$.*

Then, $cl(K,R) = (K,R)\widetilde{\cup}\mathcal{D}(K,R)$ (see [28]).

Definition 2.20. *[34] Let $(X,\mathcal{F},R)$ be a soft topology space on X and $x_r, y_s \in SP(X_R)$ such that $x_r \neq y_s$.*

1) If there exists at least one soft open set, either (K,R) or (L,R) such that $x_r \in (K,R)$, $y_s \notin (K,R)$, or $x_r \notin (L,R)$, $y_s \in (L,R)$, then $(X,\mathcal{F},R)$ is called a soft T_0-space;
2) If there exists at least one soft open set, either (K,R) or (L,R) such that $x_r \in (K,R)$, $y_s \notin (K,R)$, and $x_r \notin (L,R)$, $y_s \in (L,R)$, then $(X,\mathcal{F},R)$ is called a soft T_1-space.

Definition 2.21. *Let $(K,R), (L,R)\widetilde{\subseteq}(X,\mathcal{F},R)$. Then, (K,R) is called the following:*

1) soft regular open [38] if $int(cl(K,R)) = (K,R)$;
2) soft dense in (L,R) [12, 31] if $(L,R)\widetilde{\subseteq}cl(K,R)$;
3) soft codense [12] if $int(K,R) = \Phi_R$;
4) soft α-open set [4] if $(K,R)\widetilde{\subseteq}int(cl(int(K,R)))$; and
5) soft nowhere dense [11, 31] if $int(cl(K,R)) = \Phi_R$.

The collection of all soft nowhere dense sets in $(X,\mathcal{F},R)$ is denoted by $\mathcal{N}(\mathcal{F})$.

Definition 2.22. *[24, 39] Let $SS(X_{R_1}), SS(Y_{R_2})$ be collections of soft sets, and let $p: X \to Y, q: R_1 \to R_2$ be functions. The image of a soft set $(F,R_1) \in SS(X_{R_1})$ under $h: SS(X_{R_1}) \to SS(Y_{R_2})$ is a soft set $h(F,R_1) = (h(F), q(R_1))$ in $SS(Y_{R_2})$, which is given by the following:*

$$h(F)(r_2) = \begin{cases} \bigcup\limits_{r_1 \in q^{-1}(r_2)\cap R_1} p\left(F(r_1)\right), & q^{-1}(r_2) \cap A \neq \emptyset; \\ \emptyset, & \text{otherwise,} \end{cases}$$

for each $r_2 \in R_2$.

The inverse image of a soft set $(G,R_2) \in SS(Y_{R_2})$ under h is a soft subset $h^{-1}(G,R_2) = (h^{-1}(G), q^{-1}(R_2))$, such that

$$(h^{-1}(G)(r_1) = \begin{cases} p^{-1}\left(G(q(r_1))\right), & q(r_1) \in R_2; \\ \emptyset, & \text{otherwise,} \end{cases}$$

for each $r_1 \in R_1$.

The soft function h is injective (resp. surjective, bijective) if both p and q are injective (resp. surjective, bijective).

Definition 2.23. *A soft function $h : (X, \mathcal{F}_1, R_1) \to (Y, \mathcal{F}_2, R_2)$ is said to be the following:*

1) soft continuous [39] if $h^{-1}(K, R_2) \in \mathcal{F}_1$ for each $(K, R_2) \in \mathcal{F}_2$;
2) soft $\mathcal{N}$-open [12] if $h^{-1}(K, R_2) \in \mathcal{N}(\mathcal{F}_1)$ for every $(K, R_2) \in \mathcal{N}(\mathcal{F}_2)$;
3) soft open [39] if $h(K, R_1) \in \mathcal{F}_2$ for each $(K, R_1) \in \mathcal{F}_1$; and
4) soft closed [39] if $h(K, R_1) \in \mathcal{F}_2^c$ for each $(K, R_1) \in \mathcal{F}_1^c$.

Lemma 2.24. *[15] The soft projection function $\pi_i : (\prod X_i, \prod \mathcal{F}_i, R)_{i \in I} \to (X_i, \mathcal{F}_i, R)$ is soft open for each i.*

Definition 2.25. *[15] Let $\{(X_i, \mathcal{F}_i, R) : i \in I\}$ be a family of soft topological spaces with a fixed parametric set R. The product soft topology $\mathcal{F}$ on $X = \prod_{i \in I} X_i$ is the initial soft topology on X generated by the family $\{\pi_i : i \in I\}$, where π_i is the soft projection function from $(X, \mathcal{F}, R)$ to $(X_i, \mathcal{F}_i, R)$ for each $i \in I$.*

Definition 2.26. *[29] Let $\{(X_i, \mathcal{F}_i, R) : i \in I\}$ be a family of soft topological spaces such that $X_i \widetilde{\cap} X_j = \Phi_R$ for each $i \neq j$. The soft topology $\mathcal{F}$ on $\widetilde{\bigcup}_{i \in I}(X_i, R)$ generated by the soft base $\mathcal{A} = \{(K, R) \widetilde{\subseteq} \widetilde{\bigcup}_{i \in I}(X_i, R) : (K, R) \in \mathcal{F}_i \text{ for some } i\}$ is called the sum of soft topological spaces and denoted by $(\bigoplus_{i \in I} X_i, \mathcal{F}, R)$.*

Definition 2.27. *[14] A soft topological space $(X, \mathcal{F}, R)$ is called soft compact if every cover of X_R by the soft open sets has a finite subcover.*

Theorem 2.28. *[14] A soft closed set of a soft compact topological space is soft compact.*

Definition 2.29. *[32] Let $(X, \mathcal{F}, R)$ be a soft non-compact topological space and let $y_r \notin SP(X_R)$. A soft topology $\mathcal{F}^*$ on $X^* = X \cup \{y\}$ is defined by*

$$\mathcal{F}^* = \mathcal{F} \widetilde{\cup} \{(K, R) \in SS(X_R^*) : y_r \in (K, R), (K, R)^c \in CC_{\mathcal{F}}(X_R)\}, \tag{2.1}$$

where $CC_{\mathcal{F}}(X_R)$ is the collection of all soft closed and soft compact subsets of $(X, \mathcal{F}, R)$. Then, $(X^*, \mathcal{F}^*, R)$ is either called an Alexandroff soft compactification or a soft one-point compactification of $(X, \mathcal{F}, R)$.

3. Soft nodec spaces

In this section, we introduce the concept of soft nodec spaces, followed by their essential characterizations and properties. Moreover, the fundamental operations on soft nodec spaces are discussed.

Definition 3.1. *A soft topological space $(X, \mathcal{F}, R)$ is called soft nodec if each soft nowhere dense subset of $(X, \mathcal{F}, R)$ is soft closed.*

Notice that the definition of nodec spaces in classical topology was given by van Douwen [36].

Theorem 3.2. *For a soft topological space $(X, \mathcal{F}, R)$, the following conditions are equivalent:*

1) $(X, \mathcal{F}, R)$ is a soft nodec space;

2) Every soft nowhere dense subset of $(X, \mathcal{F}, R)$ is both soft closed and soft discrete; and
3) Every soft subset of $(X, \mathcal{F}, R)$ containing a soft dense soft open set is soft open.

Proof. ($1 \Longrightarrow 2$) Let $(X, \mathcal{F}, R)$ be a soft nodec space and $x_r \in SP(X_R)$. Let $(K, R)\widetilde{\subseteq}(X, \mathcal{F}, R)$ be a soft nowhere dense set. By soft nodecness of $(X, \mathcal{F}, R)$, (K, R) is soft closed. Since $(K, R) - \{x_r\}$ is a soft nowhere dense subset of $(X, \mathcal{F}, R)$, $(K, R) - \{x_r\}$ is a soft closed subset of $(X, \mathcal{F}, R)$. Then, $(X_R - (K, R))\widetilde{\cup}\{x_r\}$ is soft open in $(X, \mathcal{F}, R)$; hence, $\{x_r\}$ is a relatively soft open subset of (K, R). Thus, (K, R) is soft discrete.

($2 \Longrightarrow 3$) Let (K, R) be a soft subset of $(X, \mathcal{F}, R)$ containing a soft dense soft open (L, R). Since $(L, R) = int(L, R)\widetilde{\subseteq}int(K, R)$, then

$$X_R = cl(L, R)\widetilde{\subseteq}cl(int(K, R));$$

hence,

$$\begin{aligned} cl(int(K, R)) = X_R &\Rightarrow X_R - cl(int(K, R)) = \Phi_R \\ &\Rightarrow int[X_R - int(K, R)] = \Phi_R \\ &\Rightarrow int[cl(X_R - (K, R))] = \Phi_R. \end{aligned}$$

This means that $X_R - (K, R)$ is soft nowhere dense, and since every nowhere dense is soft closed, $X_R - (K, R)$ is soft closed. Thus, (K, R) is soft open.

($3 \Longrightarrow 1$) Let (K, R) be a soft nowhere dense subset of $(X, \mathcal{F}, R)$. Then,

$$\begin{aligned} int(cl(K, R)) = \Phi_R &\Rightarrow X_R - int(cl(K, R)) = X_R \\ &\Rightarrow cl[X_R - cl(K, R)] = X_R \\ &\Rightarrow cl[int(X_R - (K, R))] = X_R. \end{aligned}$$

This means that $int(X_R - (K, R))$ is a soft dense soft open set, and

$$int(X_R - (K, R))\widetilde{\subseteq}X_R - (K, R);$$

hence, $X_R - (K, R)$ is soft open. Thus, (K, R) is soft closed. Therefore, $(X, \mathcal{F}, R)$ is soft nodec. □

Theorem 3.3. *For a soft topological space $(X, \mathcal{F}, R)$, the following conditions are equivalent:*

1) $(X, \mathcal{F}, R)$ is a soft nodec space;
2) $\mathcal{D}(K, R)\widetilde{\subseteq}cl(int(cl(K, R)))$ for each $(K, R)\widetilde{\subseteq}(X, \mathcal{F}, R)$; and
3) $cl(K, R) = (K, R)\widetilde{\cup}cl(int(cl(K, R)))$ for each $(K, R)\widetilde{\subseteq}(X, \mathcal{F}, R)$.

Proof. ($1 \Longrightarrow 2$) Let $(K, R)\widetilde{\subseteq}(X, \mathcal{F}, R)$. Since

$$\begin{aligned} int(cl[(K, R) - int(cl(K, R))]) &= int(cl[(K, R)\widetilde{\cap}(int(cl(K, R)))^c]) \\ &= int(cl[(K, R)\widetilde{\cap}cl(int((K, R)^c))]) \\ &\widetilde{\subseteq} int[cl(K, R)\widetilde{\cap}cl(int((K, R)^c))] \\ &= int(cl(K, R))\widetilde{\cap}int(cl(int((K, R)^c))) \end{aligned}$$

$$\begin{aligned} &= int(cl(K,R))\widetilde{\cap}(cl(int(cl(K,R))))^c \\ &= int(cl(K,R)) - (cl(int(cl(K,R)))) \\ &= \Phi_R, \end{aligned}$$

we have that $(K,R) - cl(int(cl(K,R)))$ is a soft nowhere dense set, and by Theorem 3.2, $(K,R) - cl(int(cl(K,R)))$ is both soft closed and soft discrete. Thus, $\mathcal{D}((K,R) - cl(int(cl(K,R)))) = \Phi_R$, and as $\mathcal{D}(K,R)\widetilde{\subseteq}cl(int(cl(K,R)))$ and $\mathcal{D}(K,R) - \mathcal{D}(L,R)\widetilde{\subseteq}\mathcal{D}((K,R) - (L,R))$ (from Lemma 6 part 5 in [16]), we have that

$$\begin{aligned} \mathcal{D}(K,R) - cl(int(cl(K,R))) &\widetilde{\subseteq} \mathcal{D}(K,R) - \mathcal{D}(int(cl(K,R))) \\ &\widetilde{\subseteq} \mathcal{D}((K,R) - (int(cl(K,R)))) \\ &= \Phi_R. \end{aligned}$$

It follows that $\mathcal{D}(K,R)\widetilde{\subseteq}cl(int(cl(K,R)))$.

(2 $\Longrightarrow$ 3) For (K,R), since $cl(int(cl(K,R)))\widetilde{\subseteq}cl(K,R)$, we have that

$$(K,R)\widetilde{\cup}cl(int(cl(K,R)))\widetilde{\subseteq}cl(K,R).$$

Conversely, $cl(K,R) = (K,R)\widetilde{\cup}\mathcal{D}(K,R)\widetilde{\subseteq}(K,R)\widetilde{\cup}cl(int(cl(K,R)))$. Thus, the equality obtained.

(3 $\Longrightarrow$ 1) If $(K,R)\widetilde{\subseteq}(X,\mathcal{F},R)$ is soft nowhere dense, then

$$\begin{aligned} cl(K,R) &= (K,R)\widetilde{\cup}cl(int(cl(K,R))) \\ &= (K,R)\widetilde{\cup}\Phi_R \\ &= (K,R). \end{aligned}$$

Therefore, (K,R) is soft closed. □

Theorem 3.4. *For a soft topological space $(X,\mathcal{F},R)$, the following conditions are equivalent:*

1) $(X,\mathcal{F},R)$ is a soft nodec space;
2) For every $(K,R)\widetilde{\subseteq}(X,\mathcal{F},R)$, if (K,R) is a soft α-open set, then (K,R) is soft open;
3) For every $(K,R)\widetilde{\subseteq}(X,\mathcal{F},R)$, if $cl(int(cl(K,R)))\widetilde{\subseteq}(K,R)$, then (K,R) is soft closed; and
4) $int(K,R) = (K,R)\widetilde{\cap}int(cl(int(K,R)))$ for every $(K,R)\widetilde{\subseteq}(X,\mathcal{F},R)$.

Proof. (1 $\Longrightarrow$ 2) Let $(X,\mathcal{F},R)$ be a soft nodec space and let $(K,R)\widetilde{\subseteq}(X,\mathcal{F},R)$ be a soft α-open set. By [9, Lemma 13], if (K,R) is a soft α-open set, then $(K,R) = (L,R) - (N,R)$, where $(L,R) \in \mathcal{F}$ and $(N,R) \in \mathcal{N}(\mathcal{F})$. Hence,

$$\begin{aligned} int(K,R) &= int[(L,R) - (N,R)] \\ &= int[(L,R)\widetilde{\cap}(N,R)^c] \\ &= int(L,R)\widetilde{\cap}int((N,R)^c) \\ &= (L,R)\widetilde{\cap}(cl((N,R)))^c \\ &= (L,R)\widetilde{\cap}(N,R)^c \\ &= (L,R) - (N,R) = (K,R). \end{aligned}$$

Therefore, (K,R) is soft open.

(2 $\Longleftrightarrow$ 3) Is obvious.

(2 $\Longrightarrow$ 4) Let $(K,R)\widetilde{\subseteq}(X,\mathcal{F},R)$ be a soft α-open set. Then, $(K,R)\widetilde{\subseteq}int(cl(int(K,R)))$. Since (K,R) is soft open, then $int(K,R)\widetilde{\subseteq}int(cl(int(K,R)))$. Hence, $int(K,R) = (K,R)\widetilde{\cap}\, int(cl(int(K,R)))$.

(4 $\Longrightarrow$ 1) Let $(K,R)\widetilde{\subseteq}(X,\mathcal{F},R)$ be a soft nowhere dense set. Since $int(K,R) = (K,R)\widetilde{\cap}\, int(cl(int(K,R)))$, then $int((K,R)^c) = (K,R)^c\widetilde{\cap} int(cl(int((K,R)^c)))$, and is equivalent to $cl(K,R) = (K,R)\widetilde{\cup}cl(int(cl(K,R)))$; hence,

$$\begin{aligned} cl(K,R) &= (K,R)\widetilde{\cup}cl(int(cl(K,R))) \\ &= (K,R)\widetilde{\cup}\Phi_R \\ &= (K,R). \end{aligned}$$

Therefore, (K,R) is soft closed. □

Theorem 3.5. *Soft nodecness is hereditary.*

Proof. Let $(Y,\mathcal{F}_Y,R)$ be a soft subspace of a soft nodec space $(X,\mathcal{F},R)$. Let (K,R) be a soft nowhere dense subset of $(Y,\mathcal{F}_Y,R)$. Then, $int_Y(cl_Y(K,R)) = \Phi_R$; hence, $int_X(cl_X(K,R)) = \Phi_R$, and by nodecness of $(X,\mathcal{F},R)$, (K,R) is soft closed in $(X,\mathcal{F},R)$. Thus, (K,R) is soft closed in $(Y,\mathcal{F}_Y,R)$. Therefore, $(Y,\mathcal{F}_{(Y,R)},R)$ is a soft nodec space. □

Theorem 3.6. *Let $(X,\mathcal{F},R)$ be a soft space and $X_R = (K,R)\widetilde{\cup}(L,R)$, where $(K,R)\widetilde{\cap}(L,R) = \Phi_R$ and $(K,R),(L,R)$ are soft closed soft dense soft nodec subspaces. Then, $(X,\mathcal{F},R)$ is soft nodec.*

Proof. Let (H,R) be a soft nowhere dense subset of $(X,\mathcal{F},R)$. If $(H,R)\widetilde{\subseteq}(K,R)$ or $(H,R)\widetilde{\subseteq}(L,R)$, then by soft density of (K,R) and (L,R), we have that (H,R) is soft nowhere dense in (K,R) or (L,R); by the soft nodecness of (K,R) and (L,R), then (H,R) is soft closed in (K,R) or (L,R). Since (K,R) and (L,R) are soft closed, then (H,R) is soft closed in $(X,\mathcal{F},R)$. Suppose that $(H,R)\widetilde{\cap}(K,R) \neq \Phi_R$ and $(H,R)\widetilde{\cap}(L,R) \neq \Phi_R$. Then, $(H,R)\widetilde{\cap}(K,R)$ and $(H,R)\widetilde{\cap}(L,R)$ are nowhere dense in (K,R) and (L,R), respectively, and by the soft nodecness of (K,R) and (L,R), we have $(H,R)\widetilde{\cap}(K,R)$ and $(H,R)\widetilde{\cap}(L,R)$ are soft closed in (K,R) and (L,R), respectively. Since (K,R) and (L,R) are soft closed, then $(H,R)\widetilde{\cap}(K,R)$ and $(H,R)\widetilde{\cap}(L,R)$ are soft closed subsets of $(X,\mathcal{F},R)$. Hence, $(H,R) = ((H,R)\widetilde{\cap}(K,R))\widetilde{\cup}((H,R)\widetilde{\cap}(L,R))$ is soft closed in $(X,\mathcal{F},R)$. □

Theorem 3.7. *Let $(X,\mathcal{F},R)$ be a soft space and $X_R = (K,R)\widetilde{\cup}(L,R)$, where $(K,R)\widetilde{\cap}(L,R) = \Phi_R$ and $(K,R),(L,R)$ are soft closed soft open soft nodec subspaces. Then, $(X,\mathcal{F},R)$ is soft nodec.*

Proof. Use the same proof of Theorem 3.6. □

Theorem 3.8. *The soft nodecness is additive.*

Proof. Let (K,R) be a soft nowhere dense subset of $\widetilde{\bigcup}_{i\in I}(X_i,R)$. Then, $(K,R)\widetilde{\cap}(X_i,R)$ is soft nowhere dense of (X_i,R) for each i. Since $(X_i,\mathcal{F}_i,R)$ is soft nodec for each i, then $(K,R)\widetilde{\cap}(X_i,R)$ is soft closed for each i, which implies that $(K,R)\widetilde{\cap}(X_i,R)$ is soft closed in $\widetilde{\bigcup}_{i\in I}(X_i,R)$. Hence, $(K,R) = \widetilde{\bigcup}_{i\in I}[(K,R)\widetilde{\cap}(X_i,R)]$ is a soft closed set in $\widetilde{\bigcup}_{i\in I}(X_i,R)$. Therefore, $(\bigoplus_{i\in I} X_i,\mathcal{F},R)$ is a soft nodec space. □

Theorem 3.9. *If $(\bigoplus_{i\in I} X_i,\mathcal{F},R)$ is a soft nodec space, then $(X_i,\mathcal{F}_i,R)$ is soft nodec for each i.*

Proof. Follows from Theorem 3.5. □

Theorem 3.10. *The soft $\mathcal{N}$-open soft closed surjective image of a soft nodec space is soft nodec.*

Proof. Let $h : (X, \mathcal{F}, R) \to (Y, \mathcal{F}', R')$ be a soft $\mathcal{N}$-open soft closed surjection and $(X, \mathcal{F}, R)$ be a soft nodec space. Let (K, R') be a soft nowhere dense subset of $(Y, \mathcal{F}', R')$. By the $\mathcal{N}$-openness of h, $h^{-1}(K, R')$ is a soft nowhere dense subset of $(X, \mathcal{F}, R)$. Since $(X, \mathcal{F}, R)$ is soft nodec, $h^{-1}(K, R')$ is a soft closed subset of $(X, \mathcal{F}, R)$. By the soft closedness and surjectivity of h, we have that $h(h^{-1}(K, R')) = (K, R')$ is soft closed in $(Y, \mathcal{F}', R')$; thus, $(Y, \mathcal{F}', R')$ is a soft nodec space. □

As an immediate consequence of the above result, we have the following corollary.

Corollary 3.11. *The soft continuous soft open bijective image of a soft nodec space is soft nodec.*

Theorem 3.12. *If the product soft space $(\prod X_i, \prod \mathcal{F}_i, R)_{i \in I}$ is a soft nodec space for each i, then $(X_i, \mathcal{F}_i, R)$ is a soft nodec space for each i.*

Proof. The proof can be followed from Theorem 3.5. □

The converse of Theorem 3.12 is not true.

Example 3.13. *Let $X = \{x, y\}$ and let R be any set parameters. Define a soft topology on X by $\mathcal{F} = \{\Phi_R, X_R, (\{x\}, R)\}$. The soft nodecness of $(X, \mathcal{F}, R)$ can be easily followed but not of $(X \times X, \mathcal{F} \times \mathcal{F}, R)$.*

4. Comparisons and connections

This section studies some relationships between soft nodec spaces and some other known soft spaces from the literature and gives some equivalent statements for such soft spaces.

Definition 4.1. *[2, 23] A soft topological space $(X, \mathcal{F}, R)$ is called soft submaximal if every soft dense subset of $(X, \mathcal{F}, R)$ is soft open.*

Theorem 4.2. *Every soft submaximal space is soft nodec.*

Proof. Let $(X, \mathcal{F}, R)$ be a soft submaximal space. Let $(K, R) \in \mathcal{N}(\mathcal{F})$. If (K, R) is soft closed, we are done. Suppose that (K, R) is not soft closed. Then, $(K, R)^c$ is not soft open and, by soft submaximality of $(X, \mathcal{F}, R)$, $(K, R)^c$ is not soft dense in $(X, \mathcal{F}, R)$. Therefore, $cl((K, R)^c) \neq X_R$. This implies that there exists $x_r \in SP(X_R)$ such that $x_r \notin cl((K, R)^c)$; moreover, there exists a soft open set (L, R) that contains x_r and $(L, R) \widetilde{\cap} (K, R)^c = \Phi_R$. Hence, $(L, R) \widetilde{\subseteq} (K, R)$, which means $int(K, R) \neq \Phi_R$. This contradicts the assumption that $(K, R) \in \mathcal{N}(\mathcal{F})$. Thus, (K, R) must be soft closed. Therefore, $(X, \mathcal{F}, R)$ is soft nodec. □

In general, the converse of Theorem 4.2 is not true, as shown in the following example.

Example 4.3. *Let $X = \{x\}$ and $R = \{r_1, r_2\}$. Consider the soft indiscrete topology $\mathcal{F} = \{X_R, \Phi_R\}$ on X. Then $(X, \mathcal{F}, R)$ is a soft nodec space, but not soft submaximal.*

Definition 4.4. *[33] A soft topological space $(X, \mathcal{F}, R)$ is called soft door if every soft subset of $(X, \mathcal{F}, R)$ is either soft open or soft closed.*

From the aforementioned results, the following diagram is obtained:

soft door space $\Longrightarrow$ soft submaximal space $\Longrightarrow$ soft nodec space.

None of these implications are reversible, as can be concluded from Examples 4.3 and [3, Example 48].

Definition 4.5. *Let* $(X, \mathcal{F}, R)$ *be a soft* T_0*-space,* $x_r, y_s \in SP(X_R)$*, and* $\leq_{\mathcal{F}}$ *be an ordering defined on* $SP(X_R)$ *by the following:*

$$x_r \leq_{\mathcal{F}} y_s \text{ if and only if } y_s \in cl(\{x_r\}). \tag{4.1}$$

The order $\leq_{\mathcal{F}}$ will be called $\mathcal{F}$-ordering induced by the soft topology $\mathcal{F}$. By a chain of soft points of $(X, \mathcal{F}, R)$, we mean a chain of the following form:

$$x_r^1 \leq_{\mathcal{F}} x_s^2 \leq_{\mathcal{F}} x_t^3 \leq_{\mathcal{F}} \ldots \leq_{\mathcal{F}} x_u^n. \tag{4.2}$$

The positive integer n is called the length of the chain, and the Krull dimension of the soft space $(X, \mathcal{F}, R)$ is the supremum of the lengths and is denoted by $K\text{-}dim(X, \mathcal{F}, R)$.

Remark 4.6. *In [3], it is proven that if a soft* T_0*-space* $(X, \mathcal{F}, R)$ *is soft submaximal, then* $K\text{-}dim(X, \mathcal{F}, R) \leq 1$.

Nevertheless, we improve the result stated in the previous remark as follows.

Theorem 4.7. *Let* $(X, \mathcal{F}, R)$ *be a soft nodec soft* T_0*-space. Then,* $K\text{-}dim(X, \mathcal{F}, R) \leq 1$.

Proof. Let $(X, \mathcal{F}, R)$ be a soft nodec soft T_0-space and $x_{r_1}, y_{r_2} \in SP(X_R)$. If $y_{r_2} \in cl(\{x_{r_1}\}) - \{x_{r_1}\}$, then $\{y_{r_2}\}$ is a soft nowhere dense subset of $(X, \mathcal{F}, R)$. Hence, $\{y_{r_2}\}$ is soft closed. Thus, $\{y_{r_2}\}$ is a maximal soft point (for the soft order induced by the soft topology $\mathcal{F}$) of $(X, \mathcal{F}, R)$. Therefore, $K\text{-}dim(X, \mathcal{F}, R) \leq 1$. □

The following example justifies that Theorem 4.7 is a natural extension of the result mentioned in Remark 4.6.

Example 4.8. *Let* $X = \mathbb{Z}$*, where* $\mathbb{Z}$ *is the set of all integers, and* R *be a set of parameters. Let* $\mathcal{F}$ *be the soft topology on* X *defined by* $\mathcal{F} = \{(K, R) \in SS(X_R) : 0_r \in (K, R) \text{ and } X_R - (K, R) \text{ is finite}\}\widetilde{\cup}\{\Phi_R\}$*. We can easily check that* $(X, \mathcal{F}, R)$ *is a soft nodec space with* $K\text{-}dim(X, \mathcal{F}, R) = 1$*, since every soft nowhere dense set is finite and consequently is soft closed. Furthermore, for any soft point* $x_s \in SP(X_R)$ *with* $x_s \neq 0_r$*,* $X_R - \{x_s\}$ *is a soft open set containing* 0_r*, though not* x_s *implies that* $(X, \mathcal{F}, R)$ *is a soft* T_0*-space. On the other hand,* $(X, \mathcal{F}, R)$ *is not soft submaximal, since* $\{0_r\}$ *is a soft dense set, but not soft open.*

Theorem 4.9. *Let* $(X, \mathcal{F}, R)$ *be a soft nodec space and* $(K, R) \in SS(X_R)$*. If* (K, R) *is a soft compact soft nowhere dense set, then* (K, R) *is finite.*

Proof. Since (K, R) is a soft nowhere dense subset of a soft nodec space, then (K, R) is soft discrete by Theorem 3.2. Thus, (K, R) is finite, since $(K, R))$ is soft compact. □

We are in need of the following definition to characterize soft nodec spaces with respect to soft compact spaces.

Definition 4.10. *A soft topological space $(X, \mathcal{F}, R)$ is called strongly soft nodec if each soft nowhere dense set $(K, R)\widetilde{\subseteq}(X, \mathcal{F}, R)$ is finite and soft closed.*

Theorem 4.11. *Let $(X, \mathcal{F}, R)$ be a soft compact space. Then, the following statements are equivalent:*

1) $(X, \mathcal{F}, R)$ is a soft nodec space; and
2) $(X, \mathcal{F}, R)$ is a strongly soft nodec space.

Proof. ($1 \implies 2$) Let $(K, R)\widetilde{\subseteq}(X, \mathcal{F}, R)$ be a soft nowhere dense set. Since $(X, \mathcal{F}, R)$ is a soft nodec space, then (K, R) is soft closed; thus, (K, R) is soft compact. From Theorem 4.9, (K, R) is finite. Therefore, $(X, \mathcal{F}, R)$ is a strongly soft nodec space.

($2 \implies 1$) Straightforward. □

Theorem 4.12. *Let $(X, \mathcal{F}, R)$ be a non-soft compact space. If $(X^*, \mathcal{F}^*, R)$ is a soft nodec space, then $(X, \mathcal{F}, R)$ is strongly soft nodec.*

Proof. Let $(K, R) \in \mathcal{N}(\mathcal{F})$ and let y_r be a soft point not in X_R and $X_R^* = X_R\widetilde{\cup}\{y_r\}$. Then, $int_X(cl_X(K, R)) = \Phi_R$. Let $(L, R) \in \mathcal{F}^*$ such that $(L, R)\widetilde{\subseteq}cl_{X^*}(K, R)$. Hence, $((L, R)\widetilde{\cap}X_R)\widetilde{\subseteq}$ $(cl_{X^*}(K, R)\widetilde{\cap}X_R)$. Since $cl_X(K, R) = cl_{X^*}(K, R)\widetilde{\cap}X_R$, $(L, R)\widetilde{\cap}X_R = \Phi_R$; so $(L, R) = \Phi_R$. Thus, (K, R) is a soft nowhere dense subset of $(X^*, \mathcal{F}^*, R)$. Since $(X^*, \mathcal{F}^*, R)$ is soft nodec and soft compact, then (K, R) is finite. Therefore, $(X, \mathcal{F}, R)$ is strongly soft nodec. □

Theorem 4.13. *Let $(X, \mathcal{F}, R)$ be a non-soft compact strongly soft nodec space. Then, $(X^*, \mathcal{F}^*, R)$ is soft nodec.*

Proof. Let $(K, R) \in \mathcal{N}(\mathcal{F}^*)$. If $(K, R) = \{x_r\}$, then clearly (K, R) is soft closed and we are done. On the other hand, if $(K, R)\widetilde{\cap}X_R \neq \Phi_R$, then $(K, R)\widetilde{\cap}X_R$ is a soft nowhere dense subset of $(X, \mathcal{F}, R)$. Since $(X, \mathcal{F}, R)$ is strongly soft nodec, $(K, R)\widetilde{\cap}X_R$ is a finite soft closed subset of $(X, \mathcal{F}, R)$; therefore, $(K, R)\widetilde{\cap}X_R$ is a soft compact soft closed subset of $(X, \mathcal{F}, R)$. Hence, $(K, R)\widetilde{\cap}X_R$ is a soft closed set in $(X^*, \mathcal{F}^*, R)$. Since (K, R) is equal to either $(K, R)\widetilde{\cap}X_R$ or $((K, R)\widetilde{\cap}X_R)\widetilde{\cup}\{x_r\}$, both cases imply that (K, R) is a soft closed subset of $(X^*, \mathcal{F}^*, R)$. Therefore, $(X^*, \mathcal{F}^*, R)$ is soft nodec. □

From the above two theorems, we have the following:

Corollary 4.14. *Let $(X, \mathcal{F}, R)$ be a non-soft compact space. Then, the following statements are equivalent:*

1) $(X^*, \mathcal{F}^*, R)$ is a soft nodec space; and
2) $(X, \mathcal{F}, R)$ is a strongly soft nodec space.

We present an example of a soft nodec space in such a way that its a soft one-point compactification is not soft nodec.

Example 4.15. *Let $X = \mathbb{Z}\cup\{y\}$, $y \notin \mathbb{Z}$, and R be a set of parameters, where $\mathbb{Z}$ is the set of integers. Let $\mathcal{F}$ be the soft topology on X defined by $\mathcal{F} = \{(K, R) \in SS(X_R) : y_r \in (K, R)\}\widetilde{\cup}\{\Phi_R\}$, where $y_r \in SP(X_R)$. We claim that $(X, \mathcal{F}, R)$ is a soft nodec space; however, the soft one-point compactification $(X^*, \mathcal{F}^*, R)$ of $(X, \mathcal{F}, R)$ is not soft nodec. All $(L, R) \in SS(X_R)$ with $y_r \notin (L, R)$ are soft nowhere dense sets, which are evidently soft closed in $(X, \mathcal{F}, R)$. On the other hand, since $(\mathbb{Z}, R)$ is soft nowhere dense in $(X^*, \mathcal{F}^*, R)$, which is not finite; thus, according to Theorem 4.11, $(X^*, \mathcal{F}^*, R)$ cannot be a soft nodec space.*

5. Conclusions

In recent years, several kinds of soft topological spaces have been analyzed, such as soft compact, soft one-point compactification, soft paracompact, soft connected, soft separable, soft separation axioms, and so on. We have continued working in the same direction by studying the class of soft nodec spaces, which is a wide class that contains soft submaximal spaces as well as soft door spaces. We have investigated the basic properties of nodec spaces and shown that a soft nodec space can be represented in terms of soft closed soft dense (or soft open) soft nodec sets as subspaces. The soft product of two soft nodec spaces need not be soft nodec. The soft sum of any collection of soft nodec spaces is a soft nodec space. The soft nodec spaces are preserved under surjective soft open soft continuous functions. Furthermore, we have proved that the Krull dimension of a soft nodec soft T_0-space is less than or equal to one. Finally, we have introduced some relationships between soft nodec, soft strong nodec and soft compact spaces. Then, we succeeded in establishing a criterion that determines the equivalence of the soft strong nodecness of a soft space and the soft nodecness of its soft one-point compactification.

One of the limitations of this topic is that we were not able to find a nice relationship between soft nodec spaces and ordinary nodec spaces. However, one can generate a soft nodec space from a collection of non-nodec spaces.

As a piece of future work, one can study this topic in different topological structures, such as fuzzy topology, rough topology, nano-topology, and so on. It is worth noting that these structures have applications in real-life problems (see, [1, 20, 21]).

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare that they have no conflicts of interest.

References

1. R. Abu-Gdairi, A. A. El-Atik, M. K. El-Bably, Topological visualization and graph analysis of rough sets via neighborhoods: A medical application using human heart data, *AIMS Mathematics*, **8** (2023), 26945–26967. https://doi.org/10.3934/math.20231379
2. S. Al Ghour, Z. A. Ameen, On soft submaximal spaces, *Heliyon*, **8** (2022), E10574. https://doi.org/10.1016/j.heliyon.2022.e10574
3. M. H. Alqahtani, Z. A. Ameen, On soft door and soft submaximal spaces, submitted for publication.
4. M. Aktas, A. Ozkan, Soft α-open sets and soft α-continuous functions, *Abstr. Appl. Anal.*, **2014** (2014), 891341. https://doi.org/10.1155/2014/891341
5. S. Al Ghour, Soft-openness and soft-Lindelofness, *Int. J. Fuzzy Log. Inte.*, **23** (2023), 181–191. https://doi.org/10.5391/IJFIS.2023.23.2.181

6. S. Al Ghour, Z. A. Ameen, Maximal soft compact and maximal soft connected topologies, *Appl. Comput. Intell. S.*, **2022** (2022), 9860015. https://doi.org/10.1155/2022/9860015
7. M. I. Ali, F. Feng, X. Liu, W. K. Min, M. Shabir, On some new operations in soft set theory, *Comput. Math. Appl.*, **57** (2009), 1547–1553. https://doi.org/10.1016/j.camwa.2008.11.009
8. Z. A. Ameen, S. Al Ghour, Cluster soft sets and cluster soft topologies, *Comp. Appl. Math.*, **42** (2023), 337. https://doi.org/10.1007/s40314-023-02476-7
9. Z. A. Ameen, M. H. Alqahtani, Some classes of soft functions defined by soft open sets modulo soft sets of the first category, *Mathematics*, **11** (2023), 4368. https://doi.org/10.3390/math11204368
10. Z. A. Ameen, M. H. Alqahtani, Baire category soft sets and their symmetric local properties, *Symmetry*, **15** (2023), 1810. https://doi.org/10.3390/sym15101810
11. Z. A. Ameen, M. H. Alqahtani, Congruence representations via soft ideals in soft topological spaces, *Axioms*, **12** (2023), 1015. https://doi.org/10.3390/axioms12111015
12. Z. A. Ameen, A. B. Khalaf, The invariance of soft Baire spaces under soft weak functions, *J. Interdiscip. Math.*, **25** (2022), 1295–1306. https://doi.org/10.1080/09720502.2021.1978999
13. B. A. Asaad, Results on soft extremally disconnectedness of soft topological spaces, *J. Math. Comput. Sci.*, **17** (2017), 448–464. http://doi.org/10.22436/jmcs.017.04.02
14. S. Atmaca, Compactification of soft topological spaces, *Journal of New Theory*, **12** (2016), 23–28.
15. A. Aygünoglu, H. Aygün, Some notes on soft topological spaces, *Neural Comput. Applic.*, **21** (2012), 113–119. https://doi.org/10.1007/s00521-011-0722-3
16. A. A. Azzam, Z. A. Ameen, T. M. Al-shami, M. E. El-Shafei, Generating soft topologies via soft set operators, *Symmetry*, **14** (2022), 914. https://doi.org/10.3390/sym14050914
17. S. Bayramov, C. G. Aras, A new approach to separability and compactness in soft topological spaces, *TWMS J. Pure Appl. Math.*, **9** (2018), 82–93.
18. N. Çağman, S. Karataş, S. Enginoglu, Soft topology, *Comput. Math. Appl.*, **62** (2011), 351–358. https://doi.org/10.1016/j.camwa.2011.05.016
19. S. Das, S. Samanta, Soft metric, *Annals of Fuzzy Mathematics and Informatics*, **6** (2013), 77–94.
20. M. A. El-Gayar, R. Abu-Gdairi, M. K. El-Bably, D. I. Taher, Economic decision-making using rough topological structures, *J. Math.-UK*, **2023** (2023), 4723233. https://doi.org/10.1155/2023/4723233
21. M. El Sayed, M. A. El Safty, M. K. El-Bably, Topological approach for decision-making of COVID-19 infection via a nano-topology model, *AIMS Mathematics*, **6** (2021), 7872–7894. https://doi.org/10.3934/math.2021457
22. S. Hussain, B. Ahmad, Some properties of soft topological spaces, *Comput. Math. Appl.*, **62** (2011), 4058–4067. https://doi.org/10.1016/j.camwa.2011.09.051
23. G. Ilango, M. Ravindran, On soft preopen sets in soft topological spaces, *International Journal of Mathematics Research*, **5** (2013), 399–409.
24. A. Kharal, B. Ahmad, Mappings on soft classes, *New Math. Nat. Comput.*, **7** (2011), 471–481. https://doi.org/10.1142/S1793005711002025

25. F. Lin, Soft connected spaces and soft paracompact spaces, *International Journal of Mathematical and Computational Sciences*, **7** (2013), 277–283. https://doi.org/10.5281/zenodo.1335680
26. P. K. Maji, R. Biswas, A. R. Roy, Soft set theory, *Comput. Math. Appl.*, **45** (2003), 555–562. https://doi.org/10.1016/S0898-1221(03)00016-6
27. D. Molodtsov, Soft set theory-first results, *Comput. Math. Appl.*, **37** (1999), 19–31. https://doi.org/10.1016/S0898-1221(99)00056-5
28. S. Nazmul, S. K. Samanta, Neighbourhood properties of soft topological spaces, *Annals of Fuzzy Mathematics and Informatics*, **6** (2013), 1–15.
29. T. Y. Öztürk, S. Bayramov, Topology on soft continuous function spaces, *Math. Comput. Appl.*, **22** (2017), 32. https://doi.org/10.3390/mca22020032
30. D. Pei, D. Miao, From soft sets to information systems, *2005 IEEE International Conference on Granular Computing*, Beijing, China, 2005, 617–621. https://doi.org/10.1109/GRC.2005.1547365
31. M. Riaz, Z. Fatima, Certain properties of soft metric spaces, *The Journal of Fuzzy Mathematics*, **25** (2017), 543–560.
32. R. Şahin, Soft compactification of soft topological spaces: soft star topological spaces, *Annals of Fuzzy Mathematics and Informatics*, **10** (2015), 447–464.
33. I. M. Sabiha, On weak soft N-open sets and weak soft $\widetilde{D}_N$-sets in soft topological spaces, *Journal of Al-Nahrain University*, **20** (2017), 131–141.
34. S. Hussain, B. Ahmad, Soft separation axioms in soft topological spaces, *Hacet. J. Math. Stat.*, **44** (2015), 559–568.
35. M. Shabir, M. Naz, On soft topological spaces, *Comput. Math. Appl.*, **61** (2011), 1786–1799. https://doi.org/10.1016/j.camwa.2011.02.006
36. E. K. van Douwen, Applications of maximal topologies, *Topol. Appl.*, **51** (1993), 125–139. https://doi.org/10.1016/0166-8641(93)90145-4
37. N. Xie, Soft points and the structure of soft topological spaces, *Annals of Fuzzy Mathematics and Informatics*, **10** (2015), 309–322.
38. S. Yüksel, N. Tozlu, Z. G. Ergül, Soft regular generalized closed sets in soft topological spaces, *International Journal of Mathematical Analysis*, **8** (2014), 355–367. http://doi.org/10.12988/ijma.2014.4125
39. I. Zorlutuna, M. Akdag, W. Min, S. Atmaca, Remarks on soft topological spaces, *Annals of Fuzzy Mathematics and Informatics*, **3** (2012), 171–185.

AIMS Press