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Learning by Doing, Productivity, and Growth: New Evidence on the Link between Micro and Macro Data

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Abstract

Research suggests athletic performances are well-measured proxies for technological progress. This paper uses a century of auto and foot racing data to analyze technological changes in microeconomic learning-by-doing (LBD), observed as declining elapsed times, and macroeconomic aggregates like total factor productivity (TFP). The pace of LBD in athletics also declined around the 1973 Productivity Slowdown and varies widely across time and athletes. Auto racing speeds mainly reflect technological changes in capital (cars) and share a common stochastic trend with TFP (cointegration). Speeds error correct to TFP, but not vice versa, affirming TFP diffusion assumed in basic macro growth models.

JEL Codes: O47, O33, E24, D24, C32, C22

Keywords: Technological progress; learning by doing; TFP; labor productivity; auto racing, track and field, RBC model, cointegration, error correction, Indianapolis 500; NHRA Winternationals

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1 Introduction

Prior research on productivity analyzed improvements in athletic performance over time to investigate the relationship between changes in output and factors driving technological progress like learning-by-doing (LBD). A large, rich theoretical literature on the relationship between LBD and economic growth (Jovanovic, 1996; Solow, 1997) motivated this analysis. Early studies of athletic performance in the Olympic Games (Fellner, 1969) and the Indianapolis 500 automobile race (Barzel, 1972) developed deterministic measures of the long-run impact of LBD on contest winning times (Fellner) and speeds (Barzel). Both argued that time and speed over constant contest distances and venues are more standardized and less prone to measurement error than alternative measures of technological progress or productivity, which are notoriously hard to measure.[1]

Despite this intriguing evidence of a link between a well-measured proxy for LBD and labor productivity, relatively little subsequent microeconomic research analyzed data from this setting or explored the theoretical underpinnings more deeply. Mantel Jr et al. (1995) extended the analysis of Indy 500 outcomes to include the roles of car technology (changes embodied in capital) and organizer rules and regulations. Munasinghe et al. (2001) and Preston and Johnson (2015) extended this research to an analysis of record-setting times instead of winning times in human foot races, drawing from work on statistical distributions of the record setting process.

The macroeconomic literature on economic growth emphasizes the importance of changes in technology, or technical progress in long-run growth. Macroeconomists measure aggregate technological change using either total factor productivity (TFP) constructed from the Solow (1956) growth model or labor productivity (LP) defined as real output per unit of labor input. Macroeconomists generally view TFP as driven by a long-run stochastic trend rather than a deterministic process (Wickens, 1996).

More recent work explores the role of microeconomic heterogeneity in aggregate productivity (Basu and Fernald, 1997; Foster et al., 2001) and its measurement (Syverson, 2017). Thus far, however, little research analyzed the theoretical or empirical linkages between long-run growth trends in micro-foundations, like LBD at the firm or industry level, and macro outcomes, like aggregate TFP or LP.

This paper makes two contributions toward linking micro LBD and macro TFP. First, it updates and expands the athletic performance data analyzed in previous studies. Relative to the literature, we update most time-series data by up to a half century, incorporating

[1] For examples, see Sichel (2019) and the *Journal of Economic Perspectives* symposia in fall 1988 and spring 2017.

recent decades when aggregate U.S. productivity growth exhibited significant medium-term fluctuations. We also expand the analysis to new types of athletic contests (e.g., NHRA drag racing) for robustness and additional insights.

A second contribution is to update, improve, and modernize the range of econometric methods used to analyze long-run trends in technology and economic growth present in athletic competitions and technology. Deterministic-trend models used in the early literature are re-estimated with new data and improved specifications for LBD and compared with growth in aggregate TFP. Finally, we estimate alternative stochastic-trend models from the contemporary literature, a new bi-variate model of cointegration between auto racing speeds and TFP with dynamic error correction.

Results from data on foot racing (track and field) and auto racing performances show that the pace of LBD slowed significantly after 1973, the traditional dating of the start of the U. S. Productivity Slowdown. This finding holds for both length-of-time and cumulative output specifications of LBD trends, but a preferred specification of these models (log-log versus semi-log) remains unclear despite analysis of an additional half century of data.

At first glance, the coincidence of a slowdown in LBD with aggregate productivity is not surprising because LBD represents an important component of productivity and athletic performances are an industry in the aggregate economy. However, many proposed explanations for the Productivity Slowdown—oil price shocks, energy inefficiency, capital obsolescence, declining educational quality, monetary policy mistakes, and such—do not apply directly to LBD in athletic performances. Even declining education quality does not obviously explain much of the improvement in elapsed times, especially in foot races.

The data also show the impact of LBD on auto racing varies significantly across time (eras) and drivers. Naturally, LBD in auto racing was faster early in our sample as the sport emerged and developed. Because auto racing drivers tend to have longer careers than foot racers, data on individual-specific LBD in auto racing verify that the most successful drivers exhibit faster LBD than the average driver. These micro data also reflect tremendous heterogeneity in LBD across successful drivers and even within their specific careers. After controlling for vintage technology, however, the vast majority of decline in elapsed auto racing times is explained by technological improvement in *capital* (race cars) rather than driver-specific LBD. Thus, trends in auto racing speeds may be better explained by trends in TFP than trends in LBD.

Econometric results confirm the empirical evidence that auto racing speeds and aggregate TFP share a common long-run trend in both deterministic and stochastic model specifications. Coefficients on full-sample deterministic trends of speed and TFP are statistically equal; coefficients on race-day controls have expected signs and magnitudes when significant. However, neither exogenous nor endogenous estimated subsample breakpoints of deterministic trend models align well among speeds or between speeds and TFP. Furthermore, estimated sub-sample deterministic trends in various auto racing speed do not match the major breakpoints in U.S. productivity growth well (1973, mid-1990s, and mid-2000s).[2]

Joint tests of deterministic and stochastic trends (Bai and Perron, 2003) support the latter, and a stochastic trend (cointegration) between speed and TFP cannot be rejected. Bivariate vector error-correction models (VECM) of TFP and speed estimated using the method of Johansen (1995) reveal the presence of one cointegrating vector with long-run coefficients that are similar across speeds (0.4-0.5) and consistent with the raw data.

The estimated VECM model produces an unusually clear econometric result of asymmetric dynamic adjustment in which auto race speeds adjust significantly to aggregate TFP but not vice versa. This result supports the basic stochastic growth RBC model that assumes aggregate TFP diffuses throughout the macroeconomy, rather than micro LBD or labor productivity bubbling up to the macroeconomy. Estimated adjustment speed coefficients vary across types of auto races, with Indy 500 and NHRA about four times faster than Indy 500 qualifying speeds ($-.4$ versus $-.1$).

The magnitude and heterogeneity of these estimates suggests that more detailed specification of idiosyncratic factors related to race rules and regulations or car technologies may be warranted. Empirically, TFP grows roughly twice as fast as auto racing speeds over the full sample, so not all of TFP diffuses into auto racing performances. Further formal modeling of the supply and demand functions in the auto racing industry in a general equilibrium macro model could yield additional insights into the connection between trends and dynamics in micro and macro technology and productivity.

2 Existing Literature

This paper extends and combines two distinct branches of literature. One focuses on the microeconomics of technical change manifested in athletic performances over time. The other focuses on the macroeconomic effects of aggregate technical progress on ouput.

2.1 Technological Change and Sports Outcomes

A body of literature analyzes outcomes from sports competitions to understand the rate of technological progress over time. These papers posit that outcomes in sports contests reflect

[2]See Romer (1987), Baily and Gordon (1988), and Hansen (2001), and Cette et al. (2016) for exposition on consensus trend breaks in US productivity as well as competing views on the sources of economic slowdown, particularly from 1973 through the present day.

both increases in athlete (worker) productivity and improvements in capital like cars and engines in motor racing, skis in skiing, and swimsuits in swimming contests. These papers all point out a number of advantages inherent in sports data relative to other more commonly used macroeconomic data. These advantages include clean and consistently measured output variables and implicit or explicit controls for factors that affect output unobservable in other settings. For example, the Indianapolis 500 car race has been held annually at the same time of year on the same track under almost identical conditions for more than a century.

Fellner (1969) performed the first empirical analysis of progress using data from sports contests. This paper focused on understanding the role of "learning by doing," (LBD) defined by decreasing production costs over time when capital and output remain unchanged, in driving technological progress. Fellner (1969) developed two competing empirical proxy variables for learning by doing in automobile racing. The first used cumulative production as a proxy for increased learning by doing, reflecting the idea that doing more of some productive activity generated more LBD. The second used the passage of time as a proxy for LBD, reflecting the idea that performing some productive activity for a longer period of time generated more LBD.

Fellner (1969) analyzed data on the winning performance in the modern Summer Olympic Games held from 1896 to 1964 to assess the role played by performing activities longer in driving technological progress. Fellner (1969) estimated log-log regression models with winning outcomes as the dependent variable and the number of Olympic Games where the event was held as the explanatory variable. He focused on 11 men's Olympic events where capital remained unchanged. The paper reported evidence that the winning outcomes in all these events changed significantly with the number of Olympic Games where each was contested. Rates of increase averaged 5% to 7% in most events although swimming (14.4%) and Discuss (28.7%) increased more quickly. The results supported the idea that performing a productive activity for a longer period of time, a form of LBD, can explain observed technological progress.

Barzel (1972) analyzed data from automobile racing, winning speeds and times at the Indianapolis (Indy) 500, an open wheeled car race conducted annually since 1911. This paper exploited exogenous variation in the amount of time over which this race was contested, generated by breaks in competition from the two World Wars, to revisit the length of time LBD proxy employed by Fellner (1969). Barzel (1972) estimated log-log regression models using both winning speed and winning race time ($\frac{miles}{speed}$) as dependent variables and the number of previous races contested, a time trend, as the main explanatory variable.

These models also included separate intercepts and time trend slopes for the pre-WWI and post-WWII periods, making the interwar period the omitted category. Data spanned the 1911 to 1969 competitions. The results from the winning speed model showed a faster rate of technological progress in the pre-WWI era, about 3.1% per year, followed by a slower rate of increase in the post-WWII era. The model using winning race time as the dependent variable generated similar results.

Mantel Jr et al. (1995) explored the role that car characteristics and race-organizer imposed characteristics played in determining Indy 500, and thus proxy for the rate of technological progress. The paper estimated linear regression models with separate slopes and intercepts for three discrete time periods: 1920-1922 and 1930-1937 (front engine cars with driver and riding mechanic), 1923-1929 and 1938-1960 (front engine cars with one driver), and 1961-1992 (rear engine cars). Mantel Jr et al. (1995) found a result opposite to Barzel (1972) in that the rate of change in the average qualifying speed in the post-1961 period was roughly double that in the earlier two time periods.

Subsequent research in this area focused on record setting times in athletic events as a proxy for output. Munasinghe et al. (2001) analyzed the process describing record setting in track and field competitions. They compared record setting in two types of track and field competitions: competitions open to anyone in the world (world record times, Olympic record times, Milrose Games record times) and competitions open to a restricted group of athletes (the US record time and the New Jersey state track and field record times). Preston and Johnson (2015) analyzed the frequency of record setting in the context of competitive swimming. Section A.1.3 discusses these papers in detail.

We focus on the performance based measures analyzed by Fellner (1969) and Barzel (1972). While record breaking may be a better indicator of technological progress, annual performance in the Indy 500 generates a time series that can be easily compared to other macroeconomic measures of technological change, like Total Factor Productivity and labor productivity, used in the broad macroeconomic literature on technological change. This allows us to link our results more closely to this broader literature than Munasinghe et al. (2001) and Preston and Johnson (2015).

2.2 Technological Change and Productivity

Three overarching themes motivate this study: 1) the theoretical basis for productivity [or lack thereof]; 2) problems in measuring productivity; and 3) fluctuations in the trend of productivity since World War II. A comprehensive review of this vast is beyond the scope of this paper. Hulten (2001) offer a primer and history of aggregate technological progress through the 20th century and Sichel (2019) provides a recent extension. Syverson (2011) gives an analogous review of firm-level productivity within industries starting in the late

20th century.

Growth models embody a theory of the trend in TFP. Exogenous growth models (Solow, 1956) view TFP as the "Solow residual" with a deterministic or stochastic trend to be specified. Endogenous growth models (Romer, 1986; Lucas Jr, 1988) view TFP trends as the result of accumulation of human capital (e.g., education or LBD), technological changes embodied in capital, or intangible capital (e.g., research and development or other innovations). Prescott (1998) showed traditional passive definitions of TFP cannot fully explain cross-country data and called for deeper investigation of theoretical underpinnings of TFP. "New Growth Theory" (Hulten, 2001) adds non-constant returns to scale and imperfect competition.[3] It's unclear how these more complex extensions apply to athletic performances.

Measurement of TFP has been equally nettlesome. If output and factor inputs are measured exactly, TFP is zero; if not, the Solow residual is a "measure of ignorance" (Hulten, 2001) or a "productivity puzzle." Van Beveren (2012) surveys issues with measure of the Solow residual, which is especially challenging because it requires output and input data for all firms and industries; see Denison (2010) for details on this issue. More recent research emphasizes other important factors in productivity measurement: reallocation and heterogeneity across industries (Basu and Fernald, 1997; Foerster et al., 2019) or firms and establishments; cyclical utilization of inputs (Basu, 1996); and information and communication technology (ICT) (Syverson, 2017; Nordhaus, 2021). In contrast, elapsed times in athletic performances over constant distances and similar tracks are measured relatively accurately.

Since World War II, U.S. productivity has experienced three large, complex swings in trend growth. After a quarter century of relatively strong, steady growth, productivity growth fell by about half starting around 1973. Most of the Productivity Slowdown is attributed to changes in capital and labor input services that were unmeasured at the time due to complexities associated with oil price shocks, capital obsolecense, structural reallocation (away from energy intense sectors), booming labor force participation, education quality, and high inflation and unemployment (Baily and Schultze, 1990).

In the mid-1990s, productivity recovered to its pre-Slowdown growth rate. The leading "suspect" was information and communication technology (ITC), which Greenwood and Yorukoglu (1997) argue ignited a technological revolution in 1974 that ironically *caused* the Productivity Slowdown before eventually triggering the "New Economy" boom.[4] But productivity growth slowed again in the mid-2000s—before the Global Financial Crisis (GFC),

[3]Deeper progress on this challenging task has been modest. One recent proposal is Nordhaus (2021), which introduces a "singularity" in information technology and threshold where computation and artificial intelligence accelerate the pace of growth.

[4]For more details about the role of ICT, see Syverson (2017) and references in this paper.

which also may have restrained productivity.[5] Although medium-term fluctuations in productivity have been explained partly, Section 4.1 asks whether better-measured athletic performances also experienced similar swings.

3 Data

Auto racing data come from two sources. First, Indy 500 data for 1911–2022 are from the Indianapolis Motor Speedway's historical archives, which extend a half century beyond Barzel (1972), and three decades beyond Mantel Jr et al. (1995). The Indy 500 data include race results by driver and by year, plus additional data on race-day incidents, prize distribution, and so forth. Speeds are available for the Indy 500 race and pre-race qualifying trials that determine starting pole positions. Second, National Hot Rod Association (NHRA) Winternationals drag racing data for 1961–2022 are constructed from ProQuest news articles containing race results and the results archive maintained by the NHRA.[6] Appendix A provides more details about the auto racing data.

For foot racing, winning one-hundred meter dash (100M dash) times and one-mile run times are from the annual World Athletics Championships (WAC) occurring biannually since 1912 at essentially the same distances, except for a change from 100 meters from yards. Unlike the Indy 500, however, the WAC have been held at different geographic locations with different tracks.

Aggregate US productivity data come from two sources. Annual productivity data for 1890–2018 come from the Bank of France's Long-Term Productivity Database (LTPD), which includes total factor productivity (TFP) and labor productivity (LP), or output per hour.[7] A second source of annual (TFP) and quarterly (LP) productivity in the non-farm business sector for 1948–2021 is from Bureau of Labor Statistics (BLS) (also available in FRED).

Notable differences exist between statistical measures of data on athletic performances (time and speed) found in the literature. Most racing data are *extreme-values* (maximums, like the winner or a world record), which contrast with TFP and LP data that implicitly are *central moments* (i.e., average across agents). Thus, we construct Indy 500 race speeds

[5] Analyses of the contribution of the GFC to productivity include Byrne et al. (2016), Fernald (2015), and Fernald et al. (2017).

[6] Unlike the Indy 500 (and marathons or mile races), which have many simultaneous competitors and take hours to complete, drag races have two parallel, simultaneous competitors and take seconds to complete. Mantel Jr et al. (1995) emphasize the importance of extensive changes to rules and regulations governing the more than two-hour Indy 500, and the number and severity of crashes varies widely by year—both of which influence the winning speed.

[7] We thank Dan Sichel for referring us to these data, which extend farther back in time (1890) than the Bureau of Labor Statistics (BLS) multi-factor productivity estimates (1948) and Penn World Tables TFP estimates (1954). The correlation between TFP data from LTPD and PWT is 0.91.

as field *averages* to match TFP.[8] NHRA drag race data are limited to winning speeds due to the difficulty of data collection. Empirically, differences between the extreme- value and central-moment racing data are modest but have declined steadily over time, inducing trend differences. Trends in central-moment racing speeds clearly correspond more closely to trends in TFP and labor productivity. Appendix A.1.3 provides more details about differences in extreme and central moment measures of Indy 500 speeds. We also test for evidence of non-normality in estimates of deterministic trend models reported in Section 6.

4 Trend Breaks in Race Outcomes

The literature using outcomes in sporting events as proxy variables for TFP growth contains several issues related to output measurement and LBD that merit further discussion. Heterogeneity in outcomes and inputs *across* sports (track and field, swimming, car racing) represents one important issue. Although research in this area focuses on changes in outcomes over time, foot and swim racing depends on limited capital inputs (shoes, track surfaces, pools, swim suits) while auto racing depends more heavily on capital inputs (cars, engines, suspension systems). Auto racers also tend to have long careers, which provides rich variation in LBD across individuals.[9]

Heterogeneity also exists in outcomes and inputs *within* sports, where LBD may differ between short versus long distance events, or between events based on time versus distance (as shown in Fellner, 1969). Practical issues in productivity measurement arise because outcome options include elapsed time versus speed (inverses of each other) and winning times in regularly scheduled events versus irregularly occurring record times. Finally, different functional forms of trend growth in contest outcomes exist in the literature.

4.1 Race Outcomes and LBD Over Time

Fellner (1969) defined athletic performance as the inverse of labor productivity, $P_t = L_t/Y_t$, where L_t is labor input (time), Y_t is output (distance traveled), and P_t reflects the elapsed time to cover a certain distance during a race. Fellner (1969) argued that LBD manifests in

[8]Technically, the average speed of the Indy 500 race is a truncated mean because data are available only for the drivers still running at the finish of the race, which can be less than half of the initial field. The truncation bias may be small, however, if drivers who drop out due to crashes and mechanical failures do not have systematically different average speeds, which is a reasonable null hypothesis. In contrast, essentially all drivers post a qualifying time (their fastest single lap around the track in the weeks prior to the actual race) because these times are needed to determine the pole positions for the actual race.

[9]A few papers in the sports economics literature analyzes trend breaks in other outcomes over time like team winning percentages (Salaga and Fort, 2017; Groothuis et al., 2017; Jang et al., 2019), attendance (Fort and Lee, 2006; Mills and Fort, 2014), or team scoring (Depken et al., 2020).

declines in elapsed times (increases in labor productivity) over many years as competitors repeatedly train for, and participate in, the same race. For example, if the mile world record time drops from 4 minutes to 3 minutes and 50 seconds, the cost of producing output fell because fewer units of inputs (labor time) were required to produce the same unit of output (completion of the race).

Figure 1 plots the logs of annual winning elapsed times (black lines) for two long-distance competitions (Indy 500 mile auto race and World Championships one-mile foot races) and two short-distance competitions (NHRA Winternationals quarter-mile drag race and World Championships 100M dash). These data extend the samples analyzed in Fellner (1969) and Barzel (1972) by a half century, offering new insights into LBD trends. The figure also includes estimated trends (colored lines) using common functional forms in the literature, which all fit the data reasonably well, and evidence from one model of a trend break in 1973.[10]

Two of the estimated trends are from Fellner (1969), who hypothesized that competitors acquired productivity-enhancing human capital (LBD) from past race experience by repeatedly performing the same task. He developed econometric models for trends in athletic performances of the following form:

$$\ln P_t = \beta_0 + \beta_1 f_i(T), \quad i = \{1, 2, 3\}, \quad \beta_1 < 0 \tag{1}$$

where $f_i(T)$ are functions of a deterministic linear time trend, T. Fellner's specifications are $f_1(T) = \ln(T)$ and $f_2(T) = T$, labeled "log-log" and "semi-log" respectively. He hypothesized the log-log model would better capture changes in LBD in events with very few margins for improvement and heavily dependent on existing facilities and equipment, while the semi-log model would better fit events with more margins for improvement, including technological progress generated by macroeconomic growth. His results indicated that outcomes in short events were better represented by log-log models and outcomes in long running events (5,000 meters or more) by semi-log models.

[10] All elapsed times are expressed in hours for comparability. See below and Appendix B.1 for details of the estimation of the trends.

Figure 1: Actual and Estimated Elapsed Times in Auto and Foot Races

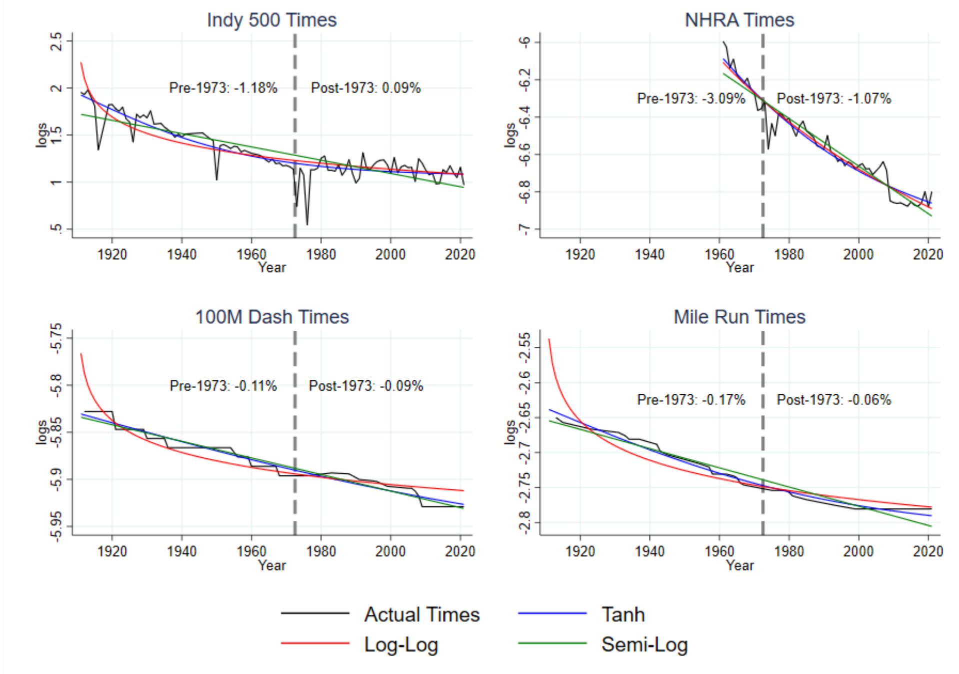


Figure 1 shows the fitted values of Fellner's semi-log (green lines) and log-log (red lines) models for the full extended sample. Based on his proposed criteria of R^2, the semi-log model fits the data from the full sample slightly better for all but the 100M dash, but the differences in R^2 between models are economically modest and smaller than in the literature before 1973. Perhaps the most striking feature of Figure 1 is an economically significant decline (flattening) in the LBD trends of all four contests after 1973 (vertical lines), commonly identified as the start of the US Productivity Slowdown. After 1973, the trends in auto racing fell (increased in absolute value) about 1 to 2 percentage points (Indy 500 and NHRA, respectively)—roughly similar magnitudes to the estimated decline in US labor productivity growth. The trends in foot races also declined (flattened) after 1973 but by less in percentage points.[11]

For logical reasons, neither of Fellner's applied specifications likely represents the best fit over the all periods of time. Semi-log models predict that future elapsed times eventually will reach zero and turn negative, which is infeasible. On the other hand, log-log models backcast that elapsed times in the distant past before 1911 would have been implausibly slow This result is analogous to the debate between Lucas (2000) and Ireland (2009) over

[11]Data Appendix B.1 reports the estimation results for subsamples split at 1973. The results show a steady decline in auto racing elapsed times since 1973 for the log-log specification, but little change in the pace for auto racing semi-log models or in any track and field models. R^2 declines modestly (up to 6 percentage points) after 1973 for six of the eight models and the relative performance across models after 1973 differs from before 1973.

the functional form of money demand. As with the money demand, more data could make each LBD specifications look more sensible within sample, as estimated curvatures and slopes diminish in absolute, or make one specification fit significantly better. However, one of the asymptotic predictions in each model would remain theoretically implausible. Thus, deespite a half century of additional data, which of the two traditional functional forms for trends in elapsed times (LBD) remains uncertain.

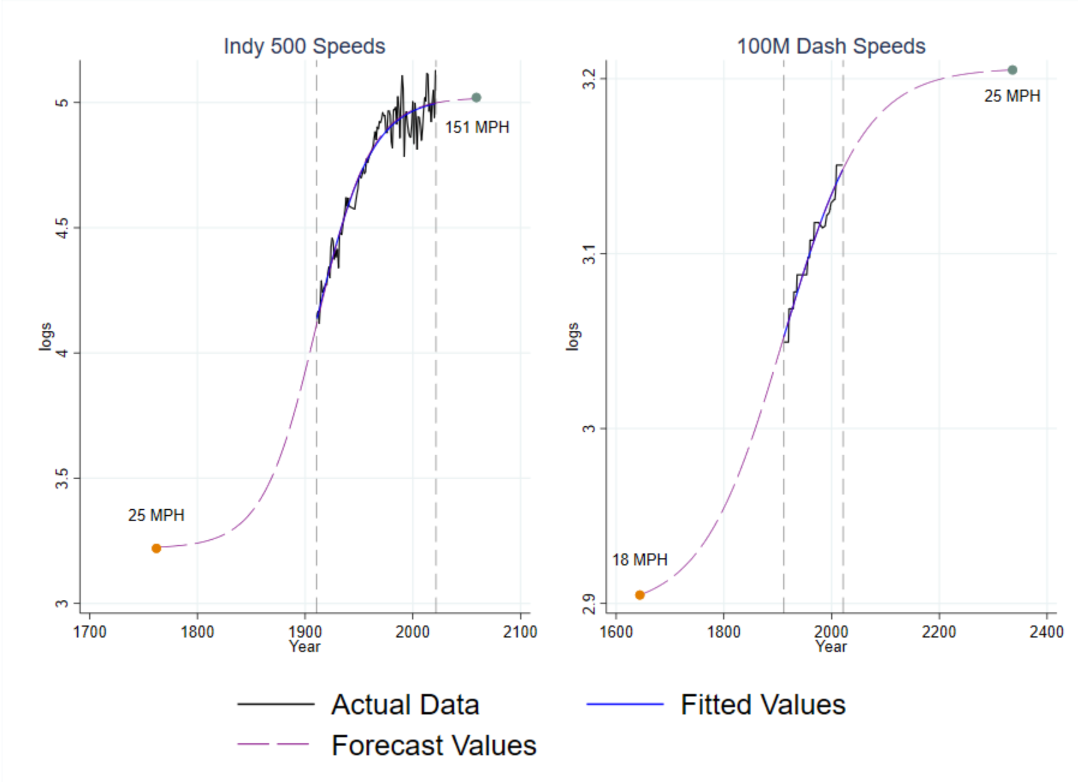
To redress asymptotic infeasibility associated with log-log and semi-log models of LBD, we propose an alternative model: $f_3(T) = -\frac{e^T}{1+e^T} = -tanh(T)$, or hyperbolic tangent function.[12] Figure 1 also plots the fitted values of the *tanh* model of elapsed times (blue lines); see Appendix B.1 for estimation details. An important advantage of the *tanh* model is that it exhibits finite asymptotes in both directions of time ($\lim_{T\to\pm\infty} f_3(T) = \pm\alpha > 0$), which provides more theoretically sensible estimates of the maximum and minimum elapsed race times over all time. A finite maximum elapsed time is plausible for foot races in early recorded human history but may be less so for auto racing given that cars were invented in the late 1800s. The *tanh* model's assumption of a *fixed*, non-zero asymptotic minimum elapsed time may be too strong for both types of races.

To evaluate implications of the *tanh* model and connect these results to the rest of the paper, Figure 2 plots the inverse of estimated log elapsed times (i.e., $p_t^{-1} = s_t$, or speed in miles per hour, MPH) for the Indy 500 and 100M dash. The *tanh* model produces more feasible and sensible estimates of LBD over all years. Estimated MPH for the 100M dash (right panel) increased from 18.4 near 1600 and is predicted to peak at 24.8 after 2300. It's hard to assess whether the minimum 100M dash speed is plausible for pre-historic times. Estimated MPH for the Indy 500 increased from 25 prior to 1800 and will peak at only 151.4 by about 2055.[13] The *tanh* model clearly is less well-suited to auto racing because cars did not exist prior to 1800 (i.e., MPH $\equiv 0$). For both speeds, the *tanh* model's assumption of a hard physical limit on top speeds may be too strong, especially for the Indy 500 where further changes in technology, rules, and regulations may enable much higher speeds.

[12] We thank Susanto Basu for suggesting this specification. The *tanh* function is equivalent to a re-scaled logistic sigmoid transformation, or inverse logit.

[13] By comparison, the first recognized auto race from Paris to Rouen, France, in 1894 was won at an average speed of 10.2 MPH. A similar race one year later reached 15.0 MPH, and other races reached 50.0 MPH by 1900. See https://www.britannica.com/sports/automobile-racing. Although these race courses and cars differed from those in the Indy 500, early recorded speeds suggest even the *tanh* deterministic model should be interpreted with caution when evaluating estimated asymptotes.

Figure 2: Fitted and Forecasted Speeds from *tanh* Models



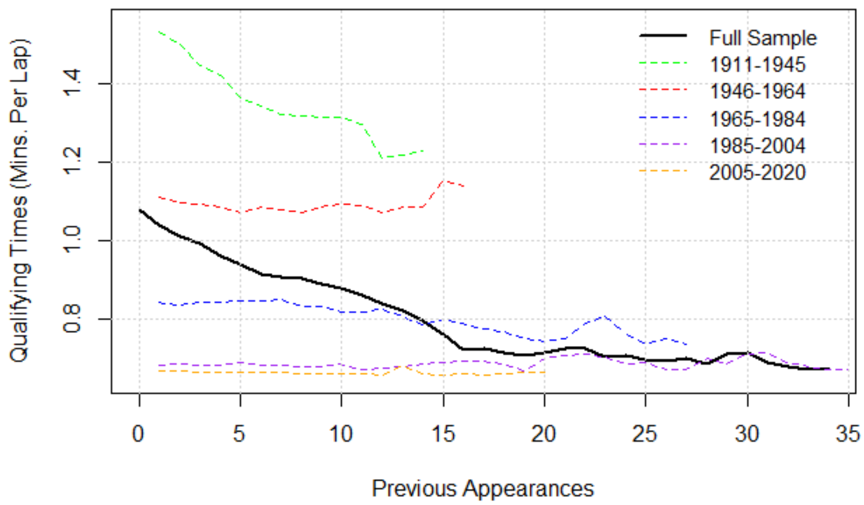
An alternative to Fellner's time-trend specification is that LBD accrues from repeatedly performing an activity independently the time passed (Alchian, 1963). In this case LBD increases directly with cumulative output over any time period, short or long, rather than as a direct function of elapsed time. Levitt et al. (2013) apply this approach to auto assemblies in an individual plant and found that the average amount of time needed to assemble a car dropped with the number of cars assembled during the first eight weeks of production but did not change much after that. In the context of the Indy 500, we define cumulative output as individual driver $j's$ experience as E_{jt}, the number of races in which driver j participated before year t.[14]

Figure 3 gives a snapshot of trends in cumulative LBD with Indy 500 qualifying times plotted by driver experience. Full-sample data show LBD increases (elapsed time decreases) with experience for at least the first 15 years. To isolate improvements in car technology, the figure plots LBD experience curves for different eras; earlier curves are higher because cars were slower. However, changes in the slopes of experience curves reveal the pace of LBD varied across eras, which is another type of trend break. From 1911-1945, LBD grew fast

[14]This measure could be refined. One way is to weight participation by the number of laps or miles completed. Full completion of the race (all 200 laps and 500 miles) yields maximum learning but completing part of the race yields less. If successful drivers learn more, participation could be skill-weighted by finishing position or average speed. Explorations of these refinements (unreported) produced similar results.

then flattened from 1946-1964. LBD growth resumed in 1965-1984 but much more slowly, and has been essentially flat since. For more detailed analysis of cumulative measures of LBD at the microeconomic level of individual drivers in auto racing, see Humphreys, Schuh, and Williams (forthcoming).

Figure 3: Indy 500 Qualifying Times by Driver Experience and Era



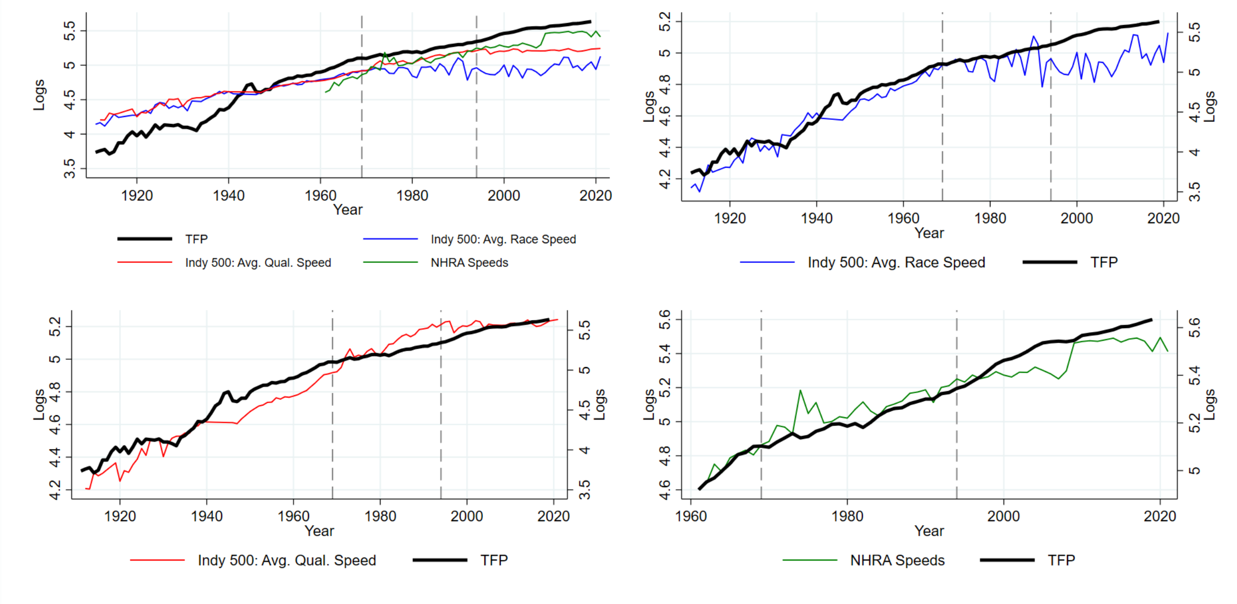
5 Race Outcomes and Aggregate Productivity

This section generates new evidence on the long-run relationship between auto racing outcomes and aggregate productivity. For this analysis, we shift our focus from elapsed race times, $P_t = L_t/Y_t$, to race speed, $S_t = P_1^{-1}$, the distance driven divided by elapsed time. Speed can be interpreted as either labor productivity (LP), or output per hour, of race drivers given a car (capital stock) and track. TFP represents a broader estimate of technological changes that are unmeasured. These might include the productivity of complementary services (e.g., pit crews and racing regulations) or unobserved technological changes embodied in capital (e.g., race cars).

Figure 4 plots data on three auto racing speeds and U.S. TFP since 1911. All variables are converted to a common index (1947=100) and plotted in natural logs to facilitate growth comparisons. The dashed vertical lines denote the ends of the data samples in Barzel (1972) and Mantel Jr et al. (1995) (1969 and 1992, respectively).[15]

[15]Long historical time series of TFP were unavailable at the time of earlier research in the literature.

Figure 4: TFP and Auto Racing Speeds, 1911-2021



Several basic facts emerge from this graph. TFP grew about twice as fast (1.8 percent per year) as Indy 500 race and qualifying speeds (0.8 and 1.0 percent per year, respectively) over the full sample (upper left panel, common y-axis). The fact that aggregate technology diffuses less than one-for-one into speeds suggests that either some components of TFP are irrelevant for auto racing outcomes or an idiosyncratic component of technological change in auto racing offsets some TFP growth. Both Indy 500 speeds grew at a similar constant rate until the 1970s, when growth in race speeds slowed relative to qualifying speeds.

Race outcomes can reflect additional complications (weather, in-race events like caution flags, strategic driving interaction, etc.) or race-specific rules and regulations (for which we do not have data) that would influence their trends relative to qualifying speeds. Since the 1960s, drag racing speeds grew about the same rate as Indy qualifying speeds (0.7 versus 0.6 percent, respectively). The remaining panels plot TFP pairwise with each speed outcome using two y-axis scales to highlight common trends, which are evident in the econometric models.

Two major developments occurred after the sample periods analyzed in previous research. First, no more World Wars occurred, which Barzel (1972) had interpreted as a major disruptions to trends in technology and production. Second, U.S. labor productivity growth fluctuated substantially since World War II, as shown in Table 1. Growth rates of speed and productivity fell more than half after the Productivity Slowdown.[16] Productivity growth increased from the mid-1990s to mid-2000s by roughly half, but it was transitory and growth

[16]See Griliches (1980) and Romer (1987) for more details on the Productivity Slowdown.

declined to its average during 1973-1994 (or less) in the mid-2000s.

Many researchers attributed the productivity boom-bust to surging investment in information and communication technology (ITC) (for examples, Oliner and Sichel, 2000; Brynjolfsson and McAfee, 2014). However, speeds did not increase commensurately, perhaps because ITC impacted auto racing less than the rest of the economy. [17] Whether the mid-2000s decline reflected a second slowdown from the GFC (Andrews et al., 2016), increased measurement error, or complications from ICT is unresolved.

Table 1: Annual Growth Rates of Speeds and Productivity by Era

	Speeds		Productivity	
Era	Race	Qualifying	LP (BLS)	TFP
1911–1973	**1.42**	**1.56**	**na**	**2.28**
1911–1916	1.68	1.89	na	2.32
1919–1941	1.36	1.09	na	1.88
1946–1973	1.34	1.63	2.78	1.95
1974–2021	**0.38**	**0.48**	**1.93**	**1.09**
1974–1994	0.07	0.96	1.61	0.99
1995–2006	0.19	-0.16	2.72	1.56
2007–2021	1.52	0.23	1.73	0.70

6 Deterministic Trend Models

This section describes the specification and estimation of single-equation models of long-run growth in auto racing speeds and technology based on deterministic trends that shift exogenously and discretely over time. This econometric approach was common in the literature prior to the 1980s for variables like athletic performances and productivity. This section extends the literature by formally testing whether deterministic trends in auto racing speeds coincide with those in TFP over the long run and during subsamples.

6.1 Econometric Models

For simplicity, let S_t^k denote speeds for $k = \{r, q\}$ where r indexes annual Indy 500 winning speeds and q is Indy 500 qualfying times. Lowercase variables s_t^k denote log levels. The

[17] See Jorgenson et al. (2007) and Jorgenson et al. (2008) for more details on the New Economy Boom/Bust, and the Vu et al. (2020) review of ICT in growth. Ireland and Schuh (2008) argue these low-frequency TFP dynamics are best interpreted as a one-time shift in the level of investment-sector TFP in a two-sector RBC model.

econometric models for race speeds,

$$s_t^k = \alpha_{s0} + \beta_{s0} T + \left[\sum_{j=1}^{N^D} \alpha_{sj} D_j + \beta_j (D_j \times T) \right] + \gamma_s CONTROLS_t + \varepsilon_{st}^k \,, \tag{2}$$

are extended versions of the semi-log transformation in Barzel (1972).[18] T is a deterministic time trend; D_j are N^D dummy variables that capture exogenous shifts in the time trend T; and $CONTROLS$ is a set of variables that might influence estimation of the trend coefficients.

As explained in Section 4.1, the data on race speeds are averages of the field so the regression error ε_{st} is assumed to be distributed i.i.d. $N(0, \sigma^2_{\varepsilon_s^k})$ rather than an Extreme Value distribution, which might be warranted with data based on winning speeds or world records. Omitted variables that affect the trend in race speed are captured in the regression error.

We estimate Equation (2) using data from 1911-2022, extending the time period in earlier papers. Barzel (1972) (1911-1969) examined the impact of World Wars I and II impacted technical change through spillover improvements to automobile production. Recent data (1994-2022) incorporate the New Economy and Dot Com boom-bust dynamics around the turn of the 21st century, which influenced technical change and productivity growth in new ways. Equation (2) incorporates medium-term fluctuations in technical change and productivity growth using deterministic trends from the literature listed in Table 2. The inter-War period (1917-1945) is the omitted trend variable. Where applicable, $D_3' = D_3 + D_4 + D_5$ a dummy variable for the Productivity Slowdown starting in 1974.

Equation (2) also includes a vector of $CONTROLS$ containing two types of variables that might influence trends in race speed. One type captures variation in race-day conditions, including: precipitation ($Prec_t$), which may reduce race speed even with improvements in tire technologies; the number of incidents per race lap, (Inc_t), which reflects caution laps run at lower speeds; $Spread_t$, which measures dispersion in the dollar value of driver winnings that induces greater competition among drivers (hence higher speeds); and $Temp$, the ambient temperature during the race.

The other type of explanatory variables capture trends in rules set by race organizers pertaining to improvements in car technologies and governing the running of the Indy 500 race, including: experience, Exp, which measures the total number of prior Indy 500 races run by field of drivers in year t; size of the field, $Field_t$, which is the total number of drivers;

[18] Equation (2) is $f_2(S^k) = \ln(S_t^k) = s_t^k$ from Barzel (1972). He also estimated transformations $f_1(S^k) = S^k$ and $f_3(S^k) = 500/S^k$; the latter is equivalent to elapsed time for the 500-mile race (or the number of miles actually completed). Results were not qualitatively significantly different across transformations.

Table 2: Variable Definitions

Variable	Description	Units
T	Time Trend	[1, ..., 112]
D_1	1911-1916 (pre-WWI)	1/0 indicator
D_2	1946-1973 (post-WWII)	1/0 indicator
D_3	1974-1995 (Productivity Slowdown)	1/0 indicator
D_4	1996-2005 (New Economy Boom)	1/0 indicator
D_5	2006-2022 (Productivity Bust)	1/0 indicator
$Prec_t$	Precipitation	Inches
Inc_t	Incidents Per Lap	$(0, 1]$ interval
$Spread_t$	Real Prize Spread	log \$s
$Temp_t$	High Temperature	logs
Exp_t	Average Field Experience	logs
$Field_t$	Field Size	logs
$Rider_t$	Number of Vehicle Occupants	$\{1, 2\}$
$Pole_t$	Pole Position	logs

$Rider_t$, which measures the number of vehicle occupants; and pole position, $Pole$, which measures the extent to which the winning driver had to negotiate past other drivers.

Analogous econometric models for total factor productivity (TFP), A_t, are:

$$\ln(A_t) = \alpha_{a0} + \beta_{a0}T + \left[\sum_{j=1}^{N^D} \alpha_{aj} D_j + \beta_j (D_j \times T)\right] + \varepsilon_{at} \tag{3}$$

the deterministic trends (T and D_j) are the same as in the models for race speeds discussed above. $CONTROLS$ for auto racing presumably do not influence aggregate TFP (at least not directly) and thus are excluded.

Equations (2) and (3) are estimated with ordinary least squares (OLS). In addition to coefficient estimates for each equation, tests of the null hypothesis, $H_0 : \alpha_{sj} = \alpha_{aj} \; \forall \; j = \{1, \ldots, N^D\}$, are of central interest. These tests are conducted by jointly estimating bivariate speed and TFP models using seemingly unrelated regression estimates (SURE) of the common deterministic trends. Failure to reject these null hypotheses provides evidence that race speeds and technical change share common deterministic trends. Residuals are tested for normality using the Jarque-Bera test to assess whether the coefficient estimates might be drawn from a non-normal distribution, perhaps due to imperfect averaging of the race speed data.

6.2 Results

Table 3 contains estimation results for deterministic trend models explaining Indy 500 winning speeds, qualifying speeds, and TFP beginning in 1911; NHRA drag race speeds are excluded from this analysis because the data begin much later, in 1961. All coefficient estimates from these semi-log models are multiplied by 100 to convert them to annual percent changes. For each dependent variable, the table reports four sets of model estimates using samples of increasing length from left to right. Qualifying speed data begin in 1912. No qualifying heats occurred in the 1911 race. The first sample (1911-1973) shows the effects of including a vector of race control variables, $CONTROLS$, which were not originally in Barzel (1972).[19] The second sample (1911-1994) is similar to that analyzed by Mantel Jr et al. (1995) and this sample period includes the influence of the Productivity Slowdown. The final two samples (through 2005 and 2022) incorporate the effects of the boom-bust cycle in productivity occurring around the beginning of the 21st century.

The results in Table 3 reveal broad similarity and general statistical significance in the deterministic trend parameter estimates across dependent variables and sample periods.[20] Several key conclusions can be drawn from these results.

Estimated coefficients on the full-sample time trend (T) are economically and statistically similar. Both race speed outcomes and TFP grew about 1.6 to 1.7 percent per year on average after accounting for trend breaks and controls. This result suggests that race speeds and TFP share a common trend, at least in the very long run. Estimated coefficients on subsamples with separate time trends (D_i) generally are stable across sample periods as more time series data are added to the sample. However, about half of the subsample trend estimates are statistically insignificant, especially during the earlier periods. This result suggests that adding considerably more time series data does not alter the results in an economically significant way.

Estimated coefficients on subsample time trends (D_i) and periods are qualitatively similar between race and qualifying speeds, although the difference in quantitative magnitudes is sometimes economically significant. This indicates that trend breaks are not the same for race and qualifying speeds, so the choice of speed measure will likely influence the analysis of the relationship between speed and TFP. Estimated coefficients on subsample time trends (D_i) for TFP exhibit economically large differences with those for speeds, although the degree of statistical significance in the speed and TFP estimates for subsample trends is similar in

[19] Estimates of the Barzel model over the original sample (through 1969) with *winning* race speeds but no controls are replicated exactly. Estimates using *average* race speed data are not economically or statistically significantly different from those using *winning* race speeds in any sample.

[20] Results for TFP in Table 3 are robust to the use of alternative labor and mutifactor measures of productivity. See Appendix B.4 for more details.

general, suggesting that, although speed and TFP share a common trend in the long run, the two variables exhibit quite difference trend break properties in the subsamples.

Most estimated coefficients on race-level control variables are not statistically significant except for precipitation and number of incidents per race, which have the expected negative sign. Only some of these control variables are available and relevant for qualifying speeds; most notably, coefficients on the number of riders are economically large and statistically significant—more so than for race speed. Although most control coefficients are not statistically significant, many of the controls contain statistically significant time trends of their own and thus contribute to the estimated deterministic trends in speeds. Finally, as typically found in time series trend models, the adjusted R^2 statistics are uniformly high and thus not surprising.

The Durbin-Watson statistics reveal some autocorrelation in the residuals, especially for TFP. The Jarque-Bera test (of skewness and kurtosis matching a Normal distribution) suggests the hypothesis of Normality can be rejected at conventional levels except for the full-sample models of average speeds, but the chi-squared approximation is sensitive in small samples like this (about 110 observations) and susceptible to high rates of Type I errors. These results suggest deterministic time trend models may reflect serial-correlation and lack of sufficient observations to draw firm conclusions. Correcting for these issues in estimation or using more advanced diagnostics may generate different results. We explore the robustness of the results to these alternative methods below.

Overall, the deterministic trend regression results offer a mixed view on the central research question of the paper. Estimated coefficients on the full-sample time trend (T) are similar across all model specifications. This indicates that speeds and TFP share a common trend in the long run. However, differences in the parameters on the subsample time trend variables, the D_i variables on Table 3, casts some doubt on that conclusion because the speeds and TFP do not appear to clearly and consistently share common trends over the five shorter subsamples. These conclusions are formally confirmed with hypothesis tests using seeming unrelated regression estimation (SURE) of the speed and TFP variables that are reported in Appendix B.2.

The estimates in Table 3 reflect a distinct temporal pattern in deterministic trend breaks shown in Figure 5, which plots trend estimates (star symbols) and standards errors (horizontal lines) for the full sample and subsamples. The first row depicts the clear equality of the long-run trend coefficients. The remaining rows show the volatility and heterogeneity of deterministic trend estimates over different time periods. The point estimates over time show little consistency, even when accounting for sampling error. Instead, each subsample contains a different relationship between the deterministic trends. Notably, average speeds

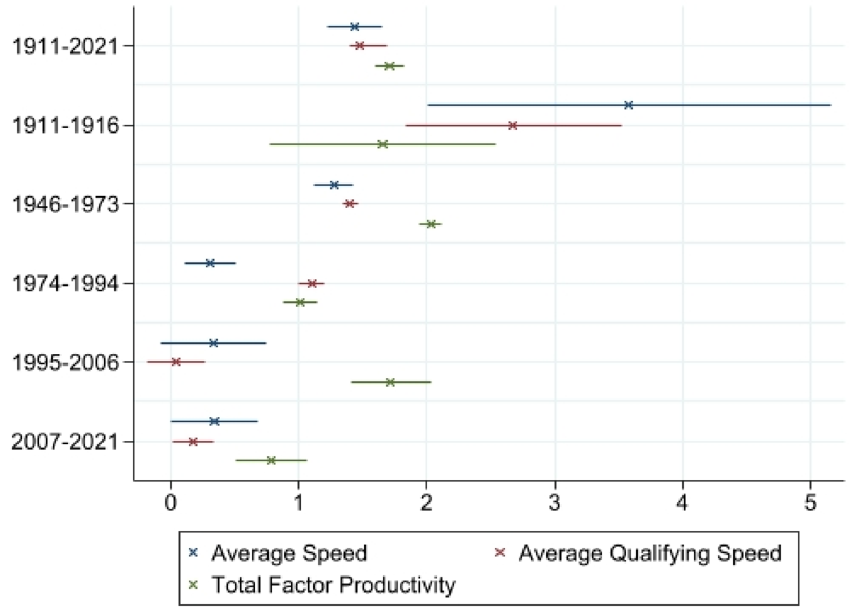
Table 3: Deterministic Trend Model Estimates

	Avg. Race Speed				Avg. Qual Speed				Total Factor Productivity			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
T	1.67***	1.73 ***	1.70***	1.67***	1.71***	1.6***	1.55***	1.54***	1.71***	1.71***	1.71***	1.71***
	(0.22)	(0.27)	(0.27)	(0.29)	(0.11)	(0.11)	(0.11)	(0.10)	(0.16)	(0.13)	(0.12)	(0.12)
$D_1 \times T$	1.63	1.79	1.70	1.88	1.63*	1.41	1.32	1.31	-0.05	-0.05	-0.05	-0.05
	(1.04)	(1.40)	(1.47)	(1.59)	(0.80)	(0.85)	(0.85)	(0.80)	(1.21)	(1.01)	(0.95)	(0.89)
$D_2 \times T$	-0.48*	-0.45	-0.40	-0.38	-0.43**	-0.17	-0.14	-0.13	0.42	0.32	0.32*	0.32*
	(0.23)	(0.26)	(0.27)	(0.29)	(0.13)	(0.12)	(0.12)	(0.11)	(0.22)	(0.16)	(0.16)	(0.15)
$D_3 \times T$		-1.45***	-1.41***	-1.37***		-0.50***	-0.44**	-0.44**		-0.69***	-0.70***	-0.70***
		(0.30)	(0.31)	(0.33)		(0.14)	(0.14)	(0.13)		(0.19)	(0.19)	(0.18)
$D_4 \times T$			-1.38**	-1.3152**			-1.48***	-1.48***			0.01	0.01
			(0.45)	(0.00)			(0.24)	(0.22)			(0.35)	(0.33)
$D_5 \times T$				-1.26**				-1.32***				-0.92**
				(0.39)				(0.17)				(0.30)
$Prec_t$	-1.43	-5.56**	-5.60***	-5.28**								
	(3.92)	(1.91)	(1.55)	(1.68)								
Inc_t	-68.09	-172.98***	-157.45***	-175.145***								
	(37.92)	(38.71)	(38.12)	(38.36)								
$Spread_t$	2.79*	1.78	1.64	1.97								
	(1.28)	(1.39)	(1.44)	(1.51)								
$Temp_t$	-5.31	-4.92	-0.97	-2.35								
	(4.28)	(4.37)	(4.29)	(4.43)								
Exp_t	-0.71	-2.34	-1.45	-1.41	-4.01**	-2.02	-1.18	-1.10				
	(2.27)	(2.63)	(2.65)	(2.70)	(1.42)	(1.35)	(1.30)	(1.18)				
$Field_t$	-12.77*	-10.44	-9.76	-9.83								
	(5.42)	(7.00)	(7.29)	(7.84)								
$Rider_t$	-1.50	-1.20	-1.32	-1.51	-4.03***	-4.11***	-4.14***	-4.14***				
	(1.52)	(1.96)	(2.04)	(2.21)	(1.03)	(1.10)	(1.10)	(1.03)				
$Pole_t$	-0.21	-0.26	-0.15	0.31								
	(0.45)	(0.50)	(0.49)	(0.49)								
Adj. R^2	0.98	0.97	0.97	0.96	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99
End Date	1969	1995	2006	2021	1969	1995	2006	2021	1969	1995	2006	2021
Durbin-Watson	1.67	1.85	2.14	1.97	1.97	1.44	1.47	1.46	0.75	0.75	0.75	0.75
Jarque-Bera	0.00	0.00	0.00	0.12	0.00	0.00	0.00	0.13	0.00	0.00	0.00	0.00

Note: $^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$. All coefficients are multiplied ×100.

clearly reflect the 1973 Productivity Slowdown but only TFP reflects the New Economy and Dot Com productivity booms.

Figure 5: Comparison of Subsample Deterministic Trends



In summary, these results approximately reflect the five predetermined, distinct trend periods identified in the literature on labor productivity since World War II. However, this approach is appropriate only under two strong assumptions: 1) TFP break points are known *a priori* with certainty; and 2) if auto racing speeds have their own trend breaks, they occur at exactly the same as those in TFP. The second condition could be true for deterministic trends by random chance (highly unlikely) or if TFP and speeds share a common *stochastic* trend. The latter is likely to appear in the data as if TFP and speeds have approximately the same deterministic trends.

Appendix B.2 explores the implications of relaxing the strong *a priori* assumptions that TFP and speed breakpoints are known and exogenously fixed. Allowing breakpoints for both variables to be estimated endogenously provides modest additional evidence that auto race speeds *may* share a common trend with TFP. When allowed to "speak freely," the data suggest *roughly* similar numbers and dates of trend breaks within the full sample. However, the large magnitude of discrepancies in the number and date of breaks firmly suggests that trends may be stochastic rather than deterministic.

7 Stochastic Trend Models

In the 1980s, macroeconomists questioned the validity of deterministic trend models for aggregate variables, such as GDP and TFP, and began specifying models with stochastic trends stemming from a unit root(s).[21] Variables that share a common stochastic trend are co-integrated (CI) with dynamic error correction to the common trend, as in Engle and Granger (1987). This section implements a contemporary vector error correction model (VECM) that assumes co-integration between auto racing speed and TFP that is consistent with a benchmark stochastic growth real business cycle (RBC) model of auto racing. While a "deeper" theoretical model may well be needed, it is informative to first assess whether speed and TFP share a common stochastic trend and error correction.[22]

7.1 A Motivating Theoretical Model

Consider the following partial-equilibrium stochastic growth real business cycle (RBC) model (Kydland and Prescott, 1982; Long Jr and Plosser, 1983) applied to the auto racing industry (subscript i).[23] The industry Planner chooses consumption of auto racing, c_{it}, and beginning-of-period effective capital stock, $k_{it} = K_{it}/L_{it}$, to solve

$$\max_{c_{it}, k_{i,t+1}} \sum_{t=0}^{\infty} \beta^t U(c_{it}) \tag{4}$$

subject to

$$\begin{aligned} y_{it} &= A_{it} f(k_{it}) & (5) \\ y_{it} &= c_{it} + \Delta k_{i,t+1} - \delta k_{it} & (6) \\ A_{it} &= \gamma_0 + \gamma_i A_t & (7) \\ \Delta A_t &= \varepsilon_t & (8) \end{aligned}$$

where β is the discount factor and $0 < \delta < 1$ is the depreciation rate. The representative auto racing team produces speed as defined earlier, $y_{it} = Y_{it}/L_{it} = P_{it}^{-1}$, where Y_{it} is the distance traveled by the representative race car and $L_{it} = (1/\tau_{it})$ is the inverse of elapsed time of the representative driver (τ_{it}).[24] Output of speed also includes industry TFP, A_{it}. Based on the deterministic trend results linking y_{it} and A_t in the long run, equation (7)

[21] See Nelson and Plosser (1982) and Durlauf and Phillips (1988).

[22] The central hypothesis in the literature is that technological progress somehow is manifest in the ongoing improvements of athletic outcomes measured by speed (or time). Knowing the precise theoretical mechanism by which technological progress diffuses into auto racing speeds is a crucial step in testing this hypothesis. Although basic, the structural model is a conventional macroeconomic rationalization of a common stochastic trend between speed and TFP.

[23] This subsection also is guided by the surveys of Fagerberg (1994) and Jorgenson (1991).

[24] Y_{it} and L_{it} vary by event: the Indy race is 500 miles and Indy qualifier is 2.5 miles; the NHRA race is .25 miles.

specifies industry TFP as linear function of aggregate TFP, A_t, with expected $\gamma_i \approx 1/2$.[25] The unit root in aggregate TFP in equation (8) introduces a stochastic trend.

Although the simple benchmark model motivates the VECM estimation, two limitations merit brief discussion. First, the full model requires a more complete specification of demand for consumption of auto racing events. Most likely, consumers get utility from more than just the pure speed of the race cars and value other elements of auto racing entertainment more generally.[26] TFP combines these distinct effects in a multiplicative catchall term, A_t. Expanding the model to include them separately could be a potentially fruitful direction for future research.

7.2 VECM Specification

Let $Z_t^k = [s_t^k, a_t]'$ denote the bivariate vector motivated by the RBC model, now with $k = \{r, q, d\}$ where d is NHRA drag race. Lowercase variables continue to denote log levels. The stochastic trend makes $s_t^k, a_t \sim I(1)$. If speed and TFP share a common trend, they are cointegrated:

$$s_t^k = \alpha + \beta a_t + \varepsilon_t^k \ , \tag{9}$$

where the cointegrating residual is $\varepsilon_t \sim I(0)$. The stationary first differences (growth rates) are jointly determined by a VECM,

$$\Delta Z_t^k = K_k + \Pi_k Z_{t-1}^k + \sum_{i=l}^{L} \Gamma_{k,i} \Delta Z_{t-l}^k + \eta_t^k \tag{10}$$

for $k = \{r, q, d\}$. Equation (9) can be estimated in a separate first-step regression with OLS or as the simultaneous system described in equation (10) using the more efficient method of Johansen (1995).

The reduced-rank (r) matrix $\Pi_k = \alpha\beta'$ contains the cointegrating vector(s) β_k and adjustment speed(s) α_k and defines the $I(1)$ model $H(r)$. The rank of Π_k is the number of

[25] This assumption may be strong but is simple and tractable. See Foster et al. (2001), Foerster et al. (2019), Dosi and Nelson (2010) for discussions of the relationship between aggregate versus industry-specific technological change. See Basu et al. (2006) for an alternative approach to jointly modeling aggregate and industry-specific productivity. Note, however, the consumption share of spectator sports was only 0.2 percent in 2019. Thus, the share of auto racing (and especially one Indy 500 race) is a tiny fraction of GDP and A_{it} probably has little or no aggregate implications.

[26] See García and Rodríguez (2002) for exposition on determinants of sports viewership, attendance and demand for competitiveness of sports outcomes.

cointegrating vectors, which the structural model predicts is $r = 1$. If so,

$$\alpha_k \beta_k' Z_t^k = \begin{bmatrix} \alpha_{s^k} \\ \alpha_a \end{bmatrix} [s_t^k - \beta_k a_t] = \alpha \varepsilon_t^k$$

and ε_t^k is the CI error from equation (9). If the system exhibits dynamic correction to a common trend, $\alpha_i \leq 0$ for $i = \{s, a\}$; at least one adjustment speed must be strictly negative.

7.3 Stochastic Trend Results

Standard pre-estimation tests are reported in Appendix B.5 and support the VECM specification reasonably well. Tests for stationarity confirm that all variables are $I(1)$ in log levels and $I(0)$ in growth rates at conventional levels of significance, as required. Information criteria tests for the VAR representation of the model with annual data suggest an optimal lag length of one in log levels ($L = 1$), hence no lagged growth rates. On balance, the data reject the hypothesis of no cointegration between speeds and TFP, thus generally support the existence of a single cointegrating vector.

The VECM coefficient estimates for all three models are correctly signed and mostly statistically significant, as shown in Table 4. As expected, the cointegrating coefficients ($\widehat{\beta}^k$) are highly significant and range from $.38 - .53$, consistent with the relative magnitudes of the long-run growth rates of speeds and TFP essentially the same as the OLS estimates. Estimated adjustment speeds ($\widehat{\alpha}^k$) reveal an interesting asymmetry—those for the auto races are negative (as expected) and statistically significant but those for TFP are close to zero and statistically insignificant. Thus, the levels of auto racing speeds adjust to the level of TFP in the long run but not vice versa. Absolute values of the adjustment speeds coefficients are economically large as well. Race adjustment speeds (Indy and NHRA) are about .4 per year, but the qualifying adjustment speed is only one-fourth as large (.1).

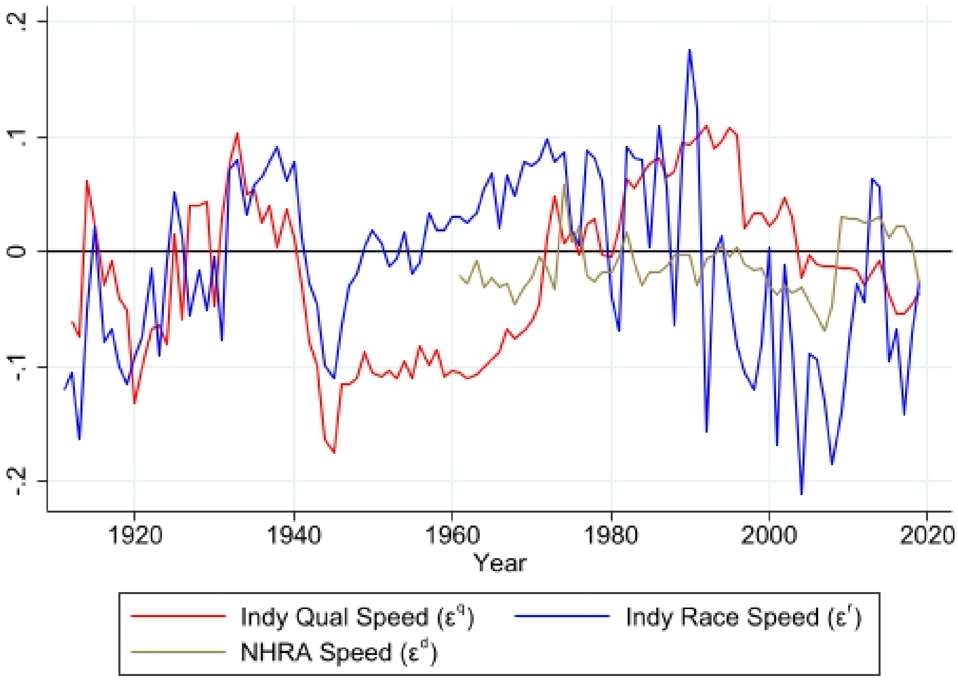
Table 4: VECM Model Coefficient Estimates ($L = 0$)

	Δs_t^r	Δa_t	Δs_t^q	Δa_t	Δs_t^d	Δa_t
K	.004	.019***	.008**	.018***	.000	.012***
	(.006)	(.003)	(.003)	(.004)	(.003)	(.002)
α	−.360***	.082	−.099*	.042	−.398***	.008
	(.075)	(.044)	(.041)	(.050)	(.102)	(.061)
β_{VECM}	.428***		.534***		.380***	
	(.000)		(.000)***		(.000)***	
β_{OLS}	.427***		.556***		.404***	
	(.000)		(.000)***		(.000)***	
R^2	.193	.219	.140	.196	.249	.545
$D.W.$	2.12		2.43		2.13	
N	108		107		58	

NOTE: $^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Figure 6 plots the estimated cointegrating errors from the VECMs. The race speeds (Indy 500 and NHRA drag) fluctuate around zero relatively frequently with few persistent deviations. Although Indy 500 qualifying speeds may be standard pre-estimation tests cointegrated with TFP, they exhibit two lengthy trend deviations in the 1940s-60s and 1980s-1990s. Indy 500 race speeds exhibit similar but less persistent deviations, such as the 1960s-70s and 1990s-2010s. Persistent deviations likely reflect periods when the bivariate VECMs are not capturing important factors affecting either the trends or adjustment to them. For examples, rule changes affecting adoption of embodied technology or governing the conduct of actual races and qualifying events may help explain these periods, as argued in Mantel Jr et al. (1995).

Figure 6: Estimated Cointegrating Errors from VECM Models



This section provides thought-provoking support for the stochastic trend model of auto racing speeds and TFP. Consistent with a basic stochastic RBC growth model, there is one common trend driven by a unit root in TFP. The asymmetric adjustment patterns in the estimated VECMs (speeds adjust to TFP but not vice versa) also support the simplifing model assumption that industry TFP (speeds) are linearly proportional to aggregate TFP in the long run.

8 Summary and Conclusions

This paper uses long time series data on auto and foot racing productivity (LBD), an accurately measured productivity proxy variable, to estimate long-run trends and compare them with trends in aggregate U.S. productivity data, which are measured less precisely. Trend changes in athletic performance over time exhibits two key features: 1) it shares a common long-run trend (growth) with aggregate productivity, including a large, persistent decline (trend break) around 1973—but not all subsequent breaks; and 2) trends in racing outcomes, reflecting micro technology changes adjust to the *stochastic* trend in aggregate productivity, which reflects macro technology. While the findings apply to a subset of a small service-producing industry (spectator sports, NAICS 71121), the consistency and clarity of the results offer thought-provoking implications.

First, causation appears to run from macro technology growth to micro technology growth in the long run. This result is not a foregone conclusion *a priori*. In principle, a comprehensive general equilibrium growth model with micro (industry) and macro technology that includes all industries and perfect aggregation could alternatively predict causality running from micro technology to the exactly aggregated macro technology or in both directions simultaneously. Because aggregate TFP does not diffuse one-for-one into auto racing technology, other causal factors (idiosyncratic to auto racing or non-transferable from aggregate TFP) likely exist. The results highlight the inadequacy of deterministic time-series models of technological change. Relatively simple stochastic RBC models with unit root shocks perform better.

Second, the strikingly synchronous decline in micro technology (racing LBD) and macro technology (TFP or labor productivity) around 1973 raises new questions about the nature of the Productivity Slowdown, one of the most important events in U.S. economic history that has not yet been fully explained. Although LBD is only part of firm TFP, and productivity in the spectator sports industry is only a small part of aggregate TFP, LBD in racing exhibits the same large one-time trend slowdown as virtually all other measures of productivity. It seems unlikely that some prominent explanations for the general Productivity Slowdown—capital obsolescence, energy prices, information technology, and such—can explain much of the slowdown in racing LBD. Another prominent explanation—education quality—might play a role, but formal human capital accumulation (other than LBD) appears unlikely to affect much of the output of speed in auto racing, much less human foot racing (especially at short distances).

Third, the results affirm in a novel way the presence of substantial heterogeneity in technological change. Even within a narrowly defined industry, the driving forces of technology, for example such the contribution of capital, differ between auto racing and foot racing. Idiosyncratic factors apparently can be quite important in each type of firm or industry. In auto racing, organization of the sport (rules, regulations, field size, etc.) influences the growth of output and differs across events within the auto racing industry; for example between Indy 500 and NHRA Winternationals. However, this is not a classical form of TFP and thus requires distinctly separate modeling. This paper's application represents a unique setting with accurate measurement of productivity in a service-producing industry, but measurement in other service industries is more complex.

Finally, new insights about technological change gained by combining micro and macro data motivate additional research on measurement and structural modeling of LBD, productivity, and economic growth. The paper's insights stem from measuring these concepts in a unique industry with a clear production process. Examining the relationship between outcomes in this industry and TFP reveals gaps between standard theoretical modeling approaches and actual industry outcomes.

The literature in sports economics focuses on tournament theory and closeness of competition, one determinant of demand for spectator sports,. However, that approach does not incorporate a role for supply-side factors like speed, an input to production of spectator sport services influenced uniquely by LBD, technology, and productivity. From the macro perspective, reliance on a portmanteau measure like TFP misses the rich complexity of technology and growth, which arise more naturally in micro applications. Deeper understanding of these ideas and the micro-macro link may be a fruitful avenue for future research.

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For Online Publication

"Learning by Doing, Productivity, and Growth: New Evidence on the Link between Micro and Macro Data"

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A Auto Racing Data

A.1 Indianapolis 500 Data

The Indianapolis 500 is a world renowned car race. The organization IndyCar, an open-wheel automobile race sanctioning body, operates the Indianapolis 500. The vehicles competing in races sanctioned by IndyCar, called Indy cars, not surprisingly, have open wheels and a single seat. As one of earliest and most prestigious car races in the world, it attracts top teams and drivers from around the world. The race takes place annually at the Indianapolis Motor Speedway (IMS) in Speedway, Indiana, a suburb of Indianapolis.

The race traditionally occurs on Memorial Day weekend. The track is a 2.5-mile oval-shaped rounded rectangle with four turns of identical dimension, essentially the same track that has been in use since the first race in 1911. The track surface changed once, from brick in 1909-1961 to various forms of asphalt used since then.[27] Vehicles must conform to certain technical specifications pertaining to the engine, drive train, and body, and the race is governed by a set of rules; both are determined by IndyCar each year. Thus, neither the technology of Indy cars nor the race regulations is constant over time, and these changes affect measurent of LBD. The constancy and regularity of the Indy 500 race is a key advantage to it's measurement of learning by doing (LBD).

A.1.1 Data Sources and Construction

Annual race–day data are available from the Indianapolis Motor Speedway's historical data archives beginning in 1911. These data include the average speed of the winning car, qualifying results by driver by race for all drivers who qualified for the Indy 500, as well as tertiary data on race conditions such as on–track collisions, caution flags, and driver-specific statuses during each race. Finally, the Indianapolis Motor Speedway also publishes data on the prize purse in dollars for all races. We collected the full series for both the Indy 500 race (1911–2021) and its qualifying events (1912–2021).

Race–day and qualifying–event data are retrieved from the Indianapolis Motor Speedway website under their "Indianapolis 500 Historical Stats" page. This page contains the data of

[27] See Indy Star article for the history of the raceway itself.

interest utilized in our deterministic trend models and outlined in Table 2. Table 5 expands upon race day variables obtained from the Indianapolis Motor Speed historical stats archives. We specifically scrape an initial panel data set of all drivers by year from the "Race Results" section of the "Indianapolis 500 Historical Stats" page from 1911 through 2021.

Table 5: Variable Definitions

Variable	Definition	Construction	Archive Page
Inc_t	Incidents Per Lap	$(Accidents_t + Contact_t)/Laps_t$	Race Results
$Spread_t$	Real Prize Spread	$Winnings_{1p,t} - Winnings_{2p,t}$	Race Results
Exp_t	Average Field Experience	$\frac{\sum_{i=1}^{I} PreviousApp_{it}}{I}$	Race Results
$Field_t$	Field Size	–	Race Results
$Pole_t$	Pole Position	–	Race Results

Going variable-by-variable, "Incidents Per Lap" captures in any given Indy 500 race in year t how many drivers were involved in a collision or accident that rendered them unable to complete the race. The Indianapolis Motor Speedway codes drivers with a "Status" corresponding either to their average speed conditioned on finishing the race or with a string of text indicating why driver i in year t failed to finish. Any "Status" denoted as "Accident" or "Contact" are aggregated in each year t to form a measurement we call "Incidents." We normalize this variable to account for the reality that not all Indy 500 races finish with 200 laps completed, thus we adjust our "Incidents" variable by dividing it by the number of "Laps" completed in the race in year t. This forms our "Incidents Per Lap" regression variable, otherwise shortened to Inc_t.

Our price spread variable is also pulled from the "Race Results" page. Each driver i in year t receives a prize purse (denoted as "Winnings" within the historical archives) based on their "Finish" in the race. The race winner has a "Finish" variable coded as "1," while second place has a "Finish" coded as "2," and so on. The difference between the "Winnings" of the first place driver and second place driver forms our nominal prize spread variable. This nominal variable is deflated using the CPI index to produce the prize spread in real terms. Only after this prize spread is deflated is it logged for use in our econometric models as $Spread_t$.

Exp_t encompasses the average experience of the field of drivers in year t. This time series variable is created from the panel of all drivers, $i \ldots I$ in year t. In any given year t, each driver i is assigned a variable called $PreviousApp_{it}$, which counts the amount of previous Indy 500 races each driver i in t has participated in prior to year t. The average of this variable across all drivers, I, in year t returns to us our Exp_t measure used as a model control.

Finally, $Field_t$, and $Pole_t$ capture the field size (number of race participants), and pole position of the winning car, respectively. This variables are trivial and require no additional manipulation. Pole position is denoted as "Start" in the historical archives. The field size is simply the count of all participants in the race in year t or the maximum value associated with the "Start" variable in any given year.

The "Starting Grid" section of "Indianapolis 500 Historical Stats" page contains information on the *top* qualifying speed recorded for each driver at each pole position. The organization of the data within this section of the historical archives allows us to identify qualifying speeds in order of magnitude for each driver i in year t. The average qualifying speed used in our econometric models is simply the average of the variable "Qual Speed" in the historical archives across all drivers in year t.

A.1.2 Other Race-Day Data

The Indianapolis Motor Speedway does not collect race-day weather conditions. To obtain weather-related controls for each Indy 500 race in year t, we use data from the National Oceanic and Atmospheric Administration (NOAA). More specifically, we use reports available in NOAA from reporting entities in the Indianapolis area for days the race is run. The NOAA reporting entity we collect precipitation and temperature data from is the Anderson Sewage Plant monitoring station, which contains reports dating back to 1911.[28] Precipitation levels are coded for the specific race day and measured in terms of inches. "Trace" levels of precipitation are interpolated as 0.05 inches, which is the midpoint between zero and the lowest reported levels of 0.10 inches.

A.1.3 Measures of Race Outcomes

Elapsed time variables constitute the main outcome variable from car races analyzed in the literature. The literature focused mainly on two outcomes: 1) the elapsed time for the winner of a race or 2) the time of new world records in events (the fastest time ever). Both are maximum statistics that call for econometric specifications designed to fit data generated by extreme-value distributions. In contrast, aggregate data like TFP represent *average* values (or totals) across all agents that are typically specified to fit data generated by the normal distribution, like OLS estimation of basic linear models. We constructed Indy 500 speeds (race and qualifying) reflecting the average across all participating drivers in each year to

[28]The use of the Anderson Sewage Plant's reports on NOAA specifically are due to the length of its data relative to other reporting units in the Greater Indianapolis Area as well as the breadth of available data, namely temperature, and precipitation.

better match TFP and labor productivity.[29]

Two papers analyzed the frequency of record setting in athletic events as a proxy for output. Munasinghe et al. (2001) analyzed the process describing record setting in track and field competitions. They compared record setting in two types of track and field competitions: competitions open to anyone in the world (world record times, Olympic record times, Milrose Games record times) and competitions open to a restricted group of athletes (the US record time and the New Jersey state track and field record times). Munasinghe et al. (2001) argued that analyzing record breaks, and not actual performance offers advantages. In terms of methods, these outcomes can be analyzed nonparametrically, avoiding any strong assumptions about the underlying distribution of performance or changes in this distribution over time. In terms of measurement, Munasinghe et al. (2001) argued that records better reflect discrete changes by optimizing agents.

The results in Munasinghe et al. (2001) showed that the rate of technological progress, as reflected in record setting, remained constant over the 1900 to 1992 period, and that globalization did not affect the rate of change. The results also indicated that LBD played a role in explaining the rate of change, since the rate of record setting times for less experienced high school athletes fell below the rate in contests involving more experienced athletes in the world level competitions.

Preston and Johnson (2015) analyzed the frequency of record setting in the context of competitive swimming. This paper focused on the impact of innovations in swimsuit technology. They analyzed variation in the number of records set in a calendar year over the period 1969-2009. This variable contains many zeros (mean 1.1, min 0, max 5). The paper found that the number of new swim suit innovations introduced in a year was correlated with the number of swimming records set in that year. A new swimsuit innovation was associated with an increase of about 1/3 of a record in that year. But this sort of "counting" of innovations assumes homogeneity of the impact of each innovation on performance.

Extreme-value (winning car) and central-moment (field average) measures of Indy 500 auto race speeds have similar long-run trends but their levels converge over time, as shown in Figure 7. The top row shows that the levels of winning and average speeds are similar overall, but winning speeds were visibly higher until the second half of the sample. The bottom row of shows the differences between winning and average speeds expressed as a percentage of the average speed for comparability. Early in the sample, the winning speed is roughly 10 percent higher than the average speed for the race and qualifier. However, differences

[29]As noted in the main text of the paper, Indy 500 race and qualifying speeds are not exactly comparable. Data for race speeds are available only for the drivers still running when the winner crosses the finish line, which is often less than half the field and thus a truncated mean. In contrast, qualifying speeds are available for essentially all drivers and thus represent a true mean.

between the statististics for both Indy events declined steadily and similarly, reaching close to 0 percent by the end of the sample.

The dynamics of winning and average speeds shown in Figure 7 have important implications for inference about trends. Most importantly, their close correspondence means the trends of the two speed statistics are always the same sign (qualitatively similar) and about the same magnitude (quantitatively similar). If their difference remained about constant at 10 percent, their trends would be statistically and economically the same. However, the steady convergence of speed statistics induces a statistically significant difference, with average speed growing about 0.4 percentage point per year faster than winning speed in the full sample. Moreover, the lower trends in winning speed are statistically significantly different from the full-sample estimates of TFP trends in column (12) of Table 3, whereas the average speed trends are not.[30]

[30] In unreported regressions, the estimated coefficients (in percent per year) on the time trend (T) for winning race and qualifying speeds are 1.29 and 1.14, respectively; both are significant at the 1 percent level. These estimates are comparable to the analogous estimates for average speed of 1.67 and 1.54, respectively, from columns (4) and (8) of Table 3. In unreported SUR model regressions, the null hypothesis of equality between winning speed and TFP trend coefficients is rejected at less than the 1 percent level for both race and qualifying events.

Figure 7: Extreme and Central Moment Measures of Indy 500 Speeds

A.2 NHRA Drag Racing Data

The National Hot Rod Association (NHRA) Winternationals is one of the premier drag racing contests held annually in February since the 1961 at the In-N-Out Burger Pomona Dragstrip in Pomona, California. Drag racing differs dramatically from open wheel Indy car racing. Drag racing involves highly specialized automobiles competing on a short (0.25 mile) straight track. The automobile technology affecting drag racing outcomes could differ substantially and should showcase all technological progress in drag racing made from the previous year. The Winternationals significance in drag racing is comparable to the Indy 500 for Indy car racing.

A.2.1 Data Sources and Construction

The National Hot Rod Association (NHRA) publishes Winternationals results for a little less than two decades worth of observations, thus, to get the full time series for the entirety of the event, we turn towards identifying news excerpts and articles via ProQuest that document the winner's top speed and elapsed time. Despite this, we have three years in the sample that require linear interpolation. The full sample extends from 1961–2021. The NHRA maintains a database of annual results for the Winternationals event since 2015. These data contain driver-specific outcomes that include the elapsed time and miles per hour of each head-to-head race completed within the event's rounds.

Unlike the Indianapolis 500, however, the Winternationals event contains more than one contest. In drag racing, there numerous contests for different specializations and variations. The Winternationals event contains contests for different classifications of drag racing vehicles, including top fuel, funny car, pro stock, and top dragster categories. We only analyze results from the top fuel category in the Winternationals. We do this as top fuel events are the fastest sanctioned category of drag racing relative to other popular categories. As such, technological progress and the fastest observable speeds within the sport are best observed through this category. Throughout our study, when we refer to "elapsed times" or "top speeds" from the NHRA Winternationals event, we are referring specifically to elapsed times and top speeds from the top fuel race category.

In the data, head-to-head events for the Winternationals consist of four qualifying rounds, coded as "Q1" through "Q3," and four elimination rounds, coded as "E1," through "E4." The results of the qualifying rounds inform the order of each head-to-head race in the elimination rounds. The final elimination round, "E4," presents information on the race winner's outcomes, which we use for our study given the limited availability of complete data prior to 2015 on the NHRA's website.

To fill in the gaps from 1961 to 2015, digitized news articles referring to the event winner's time and speed in each year as well as archival footage retrieved on YouTube are used to supplement the NHRA's database, thereby giving us a "complete" history of Winternationals winner's elapsed times (in seconds) as well as speeds. For econometric analysis, we opt for using elapsed times of the Winternationals winner in year t as our outcome variable of interest. We utilize elapsed times over top speed, as pairwise winners in the elimination rounds can often produce lower marginal top speeds compared to their competition, but will hit their individual top speed sooner than their competitor resulting in a faster finish. As such, elapsed time (and their inverse) serve as the best representation of technological performance in this sport by comparison to top speeds on their own.

A.3 Foot Race Data

For foot racing, winning 100M dash times and one-mile run times are from the annual World Athletics Championships (WAC) occurring biannually since 1912 at essentially the same distances, except for a change from 100 meters from yards. Unlike the Indy 500, however, the WAC have been held at different geographic locations with different tracks at slightly different times in the calendar year. For example, in 2019, the WAC was held in Doha, Qatar from September 27 through October 6, while in 2023 it was held in Budapest, Hungary from August 19 through August 27.

This geographic heterogeneity is further complicated by changes in the average elevations of each WAC venue. For instance, Doha's average elevation is around 33 feet, while Budapest's elevation is on average around 430 feet. Changes in elevation are well-known to have implications for athletic performances in aerobic sports, including short- and long-distance running. Finally, both the timing of events, and the precision of measurement for record breaks have varied over time. Up until 1977, performance times for 100M dash participants were taken manually and rounded down to the nearest tenth of a second. Since 1977, however, times are taken "automatically" and measured out to the nearest hundredth of a second.

B Robustness Checks

B.1 Deterministic Trend Models: Elapsed Times

This section reports teh estimation results underlying the three deterministic trends plotted in Figure 1 of Section 4.1. Each trend line represents the fitted values of econometric models defined in Equation 1:

$$p^j = \beta_{0j} + \beta_{1j} f_i(T) + \varepsilon_{jt}$$

where p^j is the log of elapsed times in four races, $j = \{r, d, h, m\}$: 1) Indy 500, r (average time); 2) NHRA Winternationals, d (winning time); 3) 100M dash, h (world record time); and 4) the mile run, m (world record time). Trend models are defined by functions of a deterministic time dummy, T, indexed by, $i = \{1, 2, 3\}$: 1) $f_1(T) = \ln T$ (log-log); 2) $f_2(T) = T$ (semi-log); and 3) $f_3(T) = -tanh(T)$. All models are estimated with OLS.

Table 6 reports estimation results for the full sample. All coefficients are statistically significant at the 1-percent level. As expected from visual inspection of the data, the β_1 coefficients are all negative (elapsed times are declining, LBD in increasing). However, the magnitudes of the coefficient estimates are not comparable across model specifications, due to the different transformations in $f_i(T)$, or across races, due to heterogeneity in LBD across events. Instead, the literature has focused on comparisons of model fit measured by R^2. In this regard, the traditional log-log and semi-log models each fit two of the events better than the other, as judged by higher R^2. Overall, however, the *tanh* models fit the data best because it has consistently higher R^2 than the other two models.

Table 6: Deterministic Trend Model Estimates: Elapsed Times, Full Sample

	Semi-Log Models				Log-Log Models				tanh Models			
	p^r	p^d	p^h	p^m	p^r	p^d	p^h	p^m	p^r	p^d	p^h	p^m
β_0	172.9***	-551.4***	-583.3***	-265.3***	2.276***	-2.144***	-5.766***	-2.537***	1.945***	-4.633***	-5.829***	-2.636***
	(3.0)	(4.1)	(0.1)	(0.2)	(0.057)	(0.147)	(0.004)	(0.007)	(0.044)	(0.296)	(0.002)	(0.002)
β_1	-0.7***	-1.3***	-0.1***	-0.1***	-0.253***	-1.008***	-0.031***	-0.051***	-0.880***	-2.482***	0.155***	-0.180***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.015)	(0.034)	(0.001)	(0.002)	(0.043)	(0.209)	(0.019)	(0.004)
β_2									0.020***	0.013***	-0.007***	0.011***
									(0.002)	(0.002)	(0.001)	(0.001)
R^2	0.69	0.92	0.96	0.95	0.74	0.94	0.87	0.88	0.80	0.94	0.96	0.98

Note: $^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$. All semi-log model coefficients are scaled by $\times 100$.

Table 7 reports analogous estimation results for the two subsamples of data before and since 1973. Most coefficients are statistically significant at the 1-percent level, but several are not significantly different from zero. All statistically significant β_1 coefficients are negative. The subsample estimates in Figure 1 are from the semi-log model , whose β_1 coefficients are in percent (multiplied by 100). Again, the magnitudes of the coefficient estimates are not comparable across model specifications. In all but one case (log-log model for the mile race), the models fit (R^2) the trends before 1973 better than since 1973, perhaps due to the flattening of the trend. By the R^2 standard, the *tanh* models do not fit the data better in subsamples than the other two models.

Table 7: Deterministic Trend Model Estimates: Elapsed Times, Subsamples

	Semi-Log Models							
	p^r		p^d		p^h		p^m	
β_0	187.7***	104.3***	-444.1***	-569.9***	-582.7***	-582.9***	-264.1***	-271.9***
	(2.7)	(1.15)	(15.7)	(4.9)	(0.1)	(0.5)	(0.1)	(0.4)
β_1	-1.2***	0.10	-3.1***	-1.1***	-0.1***	-0.1***	-0.2***	-0.1***
	(0.1)	(0.1)	(0.3)	(0.1)	(0.000)	(0.000)	(0.000)	(0.000)
R^2	0.82	0.012	0.92	0.89	0.93	0.86	0.98	0.79
Split	Pre-1973	Post-1973	Pre-1973	Post-1973	Pre-1973	Post-1973	Pre-1973	Post-1973
	Log-Log Models							
	p^r		p^d		p^h		p^m	
β_0	2.198***	0.675	0.848	-2.569***	-5.788***	-5.572***	-2.577***	-2.528***
	(0.062)	(0.492)	(0.625)	(0.220)	(0.004)	(0.023)	(0.008)	(0.015)
β_1	-0.219***	0.099	-1.745***	-0.913***	-0.023***	-0.075***	-0.037***	-0.055***
	(0.019)	(0.110)	(0.155)	(0.049)	(0.001)	(0.005)	(0.002)	(0.003)
R^2	0.71	0.004	0.92	0.88	0.86	0.82	0.80	0.85
Split	Pre-1973	Post-1973	Pre-1973	Post-1973	Pre-1973	Post-1973	Pre-1973	Post-1973

Note: $^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$. All semi-log model coefficients are scaled by $\times 100$.

B.2 SURE Estimation and Hypothesis Tests

To formalize the general conclusions from Table 3, we jointly estimated the speed and TFP models using seeming unrelated regression estimate (SURE) and tested the null hypothesis of equality for each speed-TFP pair of trend coefficients in the long run and each subsample. Table 8 reports the p-values for each pairwise test, with the column headings denoting the two regression models from Table 3 whose coefficients were compared. The first four columns compare the two speeds, the other columns compare the two speeds with TFP. Not surprisingly, equality of the long-run speed and TFP coefficients (on T) clearly is not rejected. In the full-sample models (4,8), equality of race and qualifying speed trends cannot be rejected for any period except the Productivity Slowdown (D_3). However, equality of each speed's trend with the TFP trend is rejected or marginally accepted for most subsample periods. Rejection of equality is more common for average speed (three of five coefficients) than for qualifying speed (only D_4, although two subsamples are rejected at the 12-13 percent level).

Table 8: SURE Tests of Trend Coefficient Equality

Paired Outcomes	s_t^r, s_t^q	s_t^r, tfp_t	s_t^q, tfp_t
Coefficient/Model	(1), (9)	(4), (12)	(8), (12)
T	0.62	0.90	0.53
$D_1 \times T$	0.50	0.26	0.32
$D_2 \times T$	0.29	0.00	0.12
$D_3 \times T$	0.00	0.06	0.32
$D_4 \times T$	0.68	0.00	0.00
$D_5 \times T$	0.87	0.42	0.13

B.3 Endogenous Break Points

This subsection relaxes the strong assumptions underlying the deterministic trend models in two ways. First, deterministic trends in auto racing speeds are estimated separately from trends in TFP. This changes allows the data to reveal rather than conform to trends, and helps determine whether the breaks in TFP and speeds align independently rather than being forced to be the same. Second, the number of trends and dates of trend breaks are determined endogenously for both TFP and speeds, rather than predetermined exogenously. This change allows the most recent data to reveal the optimal number and location of trend breaks for each variable without imposing any covariance structure.[31]

[31] Christiano (1992) first argued for endogenizing break points but acknowledged a tradeoff between precision of estimated break dates and gains in identification of the true number of actual breaks. See also Hansen (2001) for more recent advances and arguments for endogenous breakpoint estimation, especially the ability to distinguish estimated break dates from exogenous processes and random walks. However Byrne et al.

Figure 8 portrays the results of endogenous breakpoint estimation with unknown number and dates of breaks in the two speeds and TFP for two samples.[32] The first column shows the full sample (1911-2019) and the second column the sample ending before the Dot Com/New Economy boom that temporarily increased productivity growth. Black symbols denote estimated breakpoints, and color segments illustrate estimated trend ranges. Comparing graphs within a column (across the three rows) reveals the extent to which estimated trend breaks and ranges align across the three variables.

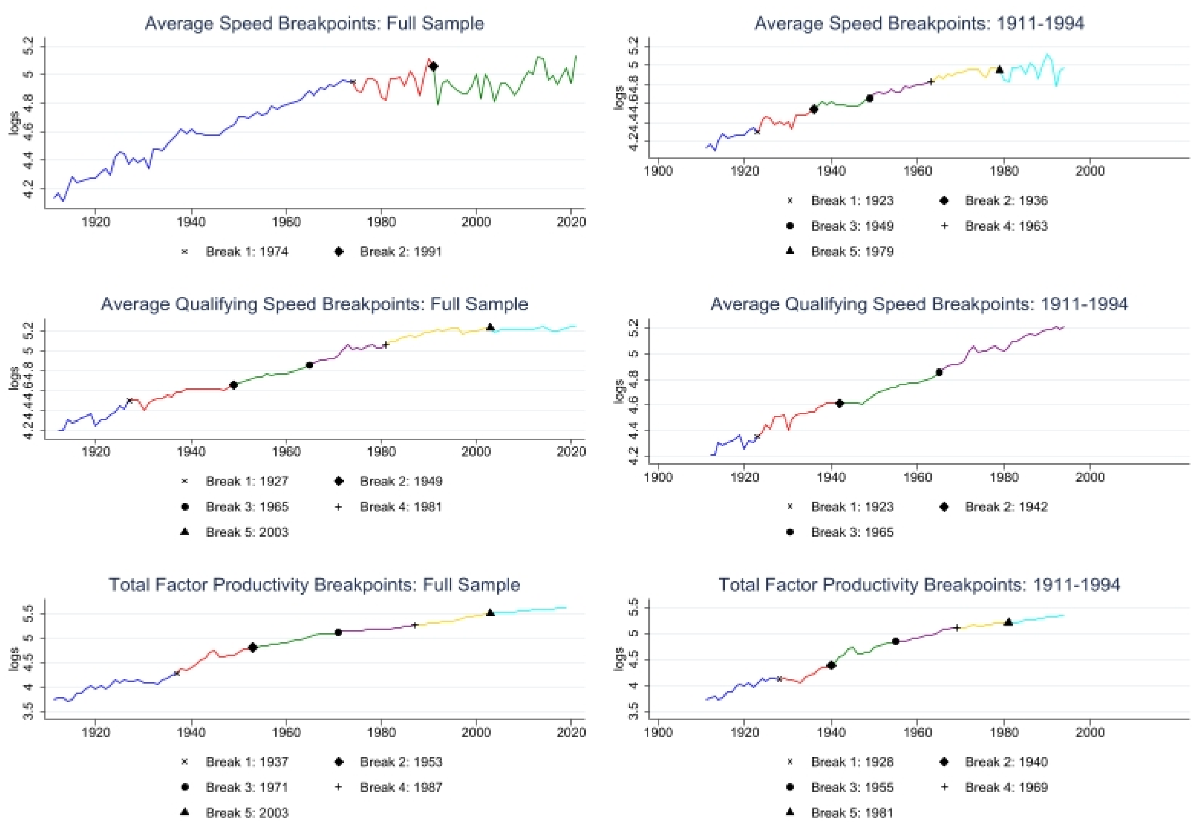

Figure 8: Endogenous Breakpoint Tests: Unknown Number of Breaks

Inspection of Figure 8 reveals that endogenously estimated breakpoints are not consistent across variables or sample periods. Both speeds have a different number of estimated breaks across samples, and wide differences in break years. TFP has five estimated breakpoints in both samples, but the break years vary almost as widely across samples. Most importantly

(2016) stress the limitations to endogenizing both the number and location of breaks, highlighting the uncertainty and imprecision of confidence intervals associated with endogenous breakpoint tests. Trend-break tests also rely on the presence of unit roots (Byrne and Perman, 2007), detection of which has low power in small samples. Addressing these limitations, Bai and Perron (2003) acknowledge that "true" simultaneity of estimating the number and location of breaks is unrealistic, so breaks should be tested sequentially.

[32]We use the STATA *xtbreak* command.

for the research question of this paper, the breaks in speeds do not align consistently with TFP for either sample. In the full sample, qualifying speed also have five breaks that are less than 10 years different from the TFP breaks, but race speed only has three breaks in entirely different years. In the shorter sample, race speeds have five breaks that are roughly aligned with TFP, but qualifying speeds only have four breaks in markedly different years.

Results are more consistent when the number of breaks (but not year) is fixed *a priori*, as shown in Figure 9 for the full sample only. Here columns represent the number of endogenous breaks (2-5). The estimated endogenous breakpoints and subsamples line up across the three variables better as the number of predetermined breakpoints increases. With only two breakpoints, the estimates range almost a half century (1927-1974 for the first and 1963-1991 for the second). With five breakpoints, however, the majority of estimated breakpoints are within a decade of each other across all three variables. Although rarely an exact year match, the five endogenous breakpoints align similarly across variables.

Figure 9: Endogenous Break Point Tests: Fixed Number of Breaks

B.4 Robustness to Alternative Productivity Data

For robustness, this section reports estimates of semi-log deterministic trend models of productivity (equation 3) using two alternative measures of productivity: 1) labor productivity (Y_t/L_t) from the Bank of France's Long-Term Productivity Database (LTPD), the same data source as TFP; and 2) multifactor productivity (A_t) from the U.S. Bureau of Labor Statistics (BLS), which uses different data and methods to construct productivity.

Estimates of the deterministic trend models with labor productivity and multifactor productivity are consistent with estimates for TFP, as shown in Table 9. The estimated long-term trends (coefficients on T) are all significant and not sensitive to the addition of subsample trend breaks. As expected in theory, the long-run trend in labor productivity (2.4 percent per year) is larger than for TFP (1.7 percent). The multifactor productivity trend (1.8 percent) is essentially the same as for TFP, which suggests that the results are not sensitive to methodology of data construction. The only significant subsample estimates for either productivity variable are for post-1973 (D_3) and post-New Economy Boom (D_5); the relative magnitudes also are quite similar to those for TFP. There is less residual serial correlation in labor productivity (Durbin-Watson statistics) than in TFP or multifactor productivity, which have very similar persistence. Normality of most residuals can be rejected (Jarque-Bera statistics), same as with TFP (and auto racing speeds).

Table 9: Estimates of Deterministic Trend Models, Alternative Productivity

	Labor Productivity (LTPD)				Multifactor Productivity (BLS)			
T	2.40***	2.40***	2.40***	2.40***	1.80***	1.80***	1.80***	1.80***
	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)
$D_1 \times T$	0.20	0.20	0.20	0.20				
	(1.00)	(0.80)	(0.80)	(0.70)				
$D_2 \times T$	0.30	0.20	0.20	0.20				
	(0.20)	(0.10)	(0.10)	(0.10)				
$D_3 \times T$		-1.10***	-1.10***	-1.10***		-1.20***	-1.20***	-1.20***
		(0.20)	(0.20)	(0.10)		(0.10)	(0.10)	(0.10)
$D_4 \times T$			-0.10	-0.10			-0.30	-0.30
			(0.30)	(0.30)			(0.20)	(0.20)
$D_5 \times T$				-1.40***				-1.30***
				(0.20)				(0.10)
Adj. R^2	0.99	0.99	0.99	0.96	0.97	0.99	0.99	0.99
Date Range	1911–1969	1911–1995	1911–2006	1911–2021	1948–1969	1948–1995	1948–2006	1948–2021
Durbin-Watson	0.75	1.49	1.49	1.48	0.78	0.79	0.80	0.88
Jarque-Bera	0.00	0.09	0.00	0.00	0.04	0.08	0.00	0.22

Note: $^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$. Labor productivity data is from the Long-Term Productivity Database (LTPD) by the Bank of France (BoF), while multifactor productivity is from the U.S. Bureau of Labor Statistics (BLS). Coefficients and standard errors are multiplied ×100.

B.5 VECM Pre-Estimation Results

This section reports the results of three types of pre-estimation tests for the VECM specification. The first test is for non-stationarity in the model variables. The VECM requires a unit root in log levels and stationary first differences (growth rates). Auto racing speeds and TFP are clearly $I(1)$ in levels and $I(0)$ in growth rates, as shown by the augmented Dickey-Fuller (ADF) tests in Table 10.

Table 10: ADF Tests for Stationarity

	Levels		Growth Rates	
Variable	T-Statistic	p-Value	T-Statistic	p-Value
s^r	-1.96	.30	-13.79	$< .001$
s^q	-1.81	.37	-13.56	$< .001$
s^d	-1.63	.47	-10.94	$< .001$
a	-1.92	.32	-10.76	$< .001$

Note: 5% critical values are -2.922 for s^d and -2.889 for s^r, s^q, a

The second pre-estimation test is for the optimal lag length of the VECM specification. Table 11 reports the results of three common information criterion tests applied to the three model vectors for race speeds of up to four lagged levels of data ($L = 4$). The HQIC and SIC (aka SBIC) tests are consistent estimators and uniformly show one lag ($L = 1$) as optimal (hence zero lagged changes in the VECM). The traditional AIC test, which is not consistent and tends to overestimate lag lengths especially in small samples like ours, also indicates an optimal lag length of one ($L = 1$) for the NHRA drag race speeds but zero ($L = 0$) and four ($L = 4$) for the Indy 500 race and qualifying speeds, respectively. Test statistics across all models and lag lengths are quantitatively similar. On balance, one lag ($L = 1$) is most likely the best model specification.

Table 11: VECM Lag Length Selection Tests

Vector	Lag Length (L)	AIC	HQIC	SIC
Z_t^r	$L=1$	-6.60	-6.54*	-6.45*
	$L=2$	-6.55	-6.44	-6.29
	$L=3$	-6.57	-6.43	-6.22
	$L=4$	-6.56	-6.37	-6.10
Z_t^q	$L=1$	-8.12	-8.10*	-7.97*
	$L=2$	-8.12	-8.02	-7.86
	$L=3$	-8.11	-7.97	-7.76
	$L=4$	-8.15*	-7.97	-7.97
Z_t^d	$L=1$	-11.09*	-11.01*	-10.87*
	$L=2$	-11.02	-10.88	-10.66
	$L=3$	-10.99	-10.80	-10.48
	$L=4$	-10.86	-10.60	-10.20

Note: * indicates the optimum lag length as per the STATA *varsoc* command. For the Z_t^r vector, the AIC criteria prescribes an lag length of $L=0$ (unreported).

The third pre-estimation test is for the number of cointegrating vectors in the model. Table 12 reports the Johansen rank tests for cointegration.[33] At the most likely optimal lag length ($L=1$), the null hypothesis of no cointegrating vector ($r=0$) can be rejected at the 5-percent level or better for the Z_t^r and Z_t^d models. The absence of a cointegrating vector ($r=0$) for the Z_t^q model cannot be rejected with the same high confidence but is close to rejection at the 10 percent level (not reported). Nevertheless, the sequential test for $r \geq 1$ can be rejected at the 5 percent level, so cointegration is less likely for the Z_t^q model by this measure. However, unreported tests of cointegrating residuals from two-step estimation of equation (9) test $I(0)$ at the 10 percent level (or better) for all three models, and the ADF statistics are nearly identical to those from the VECM estimated residuals.

Table 12: Johansen Test Results for $1 \leq L \leq 4$ Lags

Vector	H_0	Trace Statistics			
		$L=1$	$L=2$	$L=3$	$L=4$
Z_t^r	$r=0$	29.31**	23.10**	17.17*	15.59*
	$r=1$	4.40*	4.92*	5.76*	5.75*
Z_t^q	$r=0$	12.02	12.13	12.36	12.28
	$r=1$	5.02*	4.81*	4.95*	4.66*
Z_t^d	$r=0$	18.56*	12.76	14.97	11.63
	$r=1$	4.43*	2.54	1.65	1.20

Note: * and ** indicate rejection of the null hypothesis (H_0) at levels of 5% (critical values of 15.41 for $r=0$, and 3.76 for $r=1$) and 1% (critical values of 20.04 for $r=0$, and 6.65 for $r=1$), respectively.

[33] We use the *vecrank* command in STATA.

Because estimation of error-correction frameworks like the Johansen rank tests can be sensitive to model lag length (Kilian and Lütkepohl, 2017), Table 12 includes results for up to four model lags for robustness. Overall, the results for the Z_t^r and Z_t^q models are qualitatively similar to those for the optimal lag length ($L = 1$). However, the results for the Z_t^d vectors with $L > 1$ cannot reject the null hypotheses of either zero or one cointegrating vector, a qualitatively different result that demonstrates sensitivity to lag length. On balance, the standard cointegration tests provide reasonable support for a single cointegrating vector for speed and TFP.