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Common fixed points for two pairs of selfmaps satisfying certain contraction condition in b-metric spaces

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Abstract

This study introduces generalized contraction for two pairs of selfmaps in complete b-metric spaces, and it then establishes the existence of common fixed points under the presumptions that these two pairs of maps are weakly compatible and satisfy the condition for generalized contraction. A sequence of selfmaps is added as an extension of the same. Additionally, we demonstrate the same using various hypotheses on two pairs of selfmaps that satisfy the b-(E.A)-property. Some of the conclusions in the literature are extended /generalized to two pairs of self maps by our theorems.
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1 Introduction

Czerwik (10) introduced the notion of b-metric space which is a generalization of metric space. Following that, numerous authors looked into fixed point theorems for single-valued and multi-valued mappings in b-metric spaces, we refer (3; 8; 9; 11; 15; 16; 17; 22; 23; 24).

The concept of property (E.A) was introduced by Aamari and Moutawakil (1). Several authors then used this idea to demonstrate the presence of common fixed points, we refer (2; 4; 5; 6; 19; 20; 21).

Definition 1.1. (10) Let X_b be a non-empty set and $s \geq 1$ be a given real number. A function $d_b : X_b \times X_b \to [0, \infty)$ is said to be a b-metric if the following conditions are satisfied: for any $x_b, y_b, z_b \in X_b$

(i) $0 \leq d_b(x_b, y_b)$ and $d_b(x_b, y_b) = 0$ iff $x_b = y_b$,

(ii) $d_b(x_b, y_b) = d_b(y_b, x_b)$,

(iii) $d_b(x_b, z_b) \leq s[d_b(x_b, y_b) + d_b(y_b, z_b)]$.

The pair (X_b, d_b) is called a b-metric space with coefficient s.

Every metric space is a b-metric space with $s = 1$, but converse is need not be true.

Definition 1.2. (9) Let (X_b, d_b) be a b-metric space. Then a sequence $\{x_{b_n}\}$ inX_b *is said to be*

(i) b-convergent if there exists $x_b \in X_b$ such that $d_b(x_{b_n}, x_b) \to 0$ as $n \to \infty$. In this case, we write $\lim\limits_{n\to\infty} x_{b_n} = x_b$.

(ii) b-Cauchy if $d_b(x_{b_n}, x_{b_m}) \to 0$ as $n, m \to \infty$.

In general, a b-metric is not necessarily continuous (12).

Definition 1.3. (13) Let A and B be selfmaps of a metric space (X, d). The pair (A, B) is said to be a compatible pair on X, if $\lim\limits_{n\to\infty} d(ABx_n, BAx_n) = 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim\limits_{n\to\infty} Ax_n = \lim\limits_{n\to\infty} Bx_n = t$, for some $t \in X$.

Definition 1.4. (14) Let X be a nonempty set. Let $A : X \to X$ and $B : X \to X$ be two selfmaps. If $Ax = Bx$ implies that $ABx = BAx$ for x in X, then we say that the pair (A, B) is weakly compatible.

Definition 1.5. (19) Two selfmappings A_b and B_b of a b-metric space (X_b, d_b) are said to satisfy b-(E.A)-property if there exists a sequence $\{x_{b_n}\}$ in X_b $\ni$ $\lim_{n\to\infty} A_b x_{b_n} = \lim_{n\to\infty} B_b x_{b_n} = z_b$ for some $z_b \in X_b$.

Lemma 1.1. (2) Let (X_b, d_b) be a b-metric space with coefficient $s \geq 1$. Suppose that $\{x_{b_n}\}$ and $\{y_{b_n}\}$ are b-convergent to x_b and y_b respectively, then we have

$$\frac{1}{s^2} d_b(x_b, y_b) \leq \liminf_{n\to\infty} d_b(x_{b_n}, y_{b_n}) \leq \limsup_{n\to\infty} d_b(x_{b_n}, y_{b_n}) \leq s^2 d_b(x_b, y_b)$$

In particular, if $x_b = y_b$, then we have $\lim_{n\to\infty} d_b(x_{b_n}, y_{b_n}) = 0$. Moreover for each $z_b \in X_b$ we have

$$\frac{1}{s} d_b(x_b, z_b) \leq \liminf_{n\to\infty} d_b(x_{b_n}, z_b) \leq \limsup_{n\to\infty} d_b(x_{b_n}, z_b) \leq s d_b(x_b, z_b).$$

Lemma 1.2. (7) Let (X_b, d_b) be a b-metric space with coefficient $s \geq 1$ and $T_b : X_b \to X_b$ be a self map. Suppose that $\{x_{b_n}\}$ is a sequence in X_b induced by $x_{b_{n+1}} = T_b x_{b_n}$ such that $d_b(x_{b_n}, x_{b_{n+1}}) \leq \lambda d_b(x_{b_{n-1}}, x_{b_n}) for all n \in N$,where $\lambda \in (0, 1)$ is a constant.Then x_{b_n} is a b-cauchy sequence in X_b.

Recently, Nagaraju, Raju and Thirupathi (18) proved a theorem in metric spaces as follows:

Theorem 1.1. (18) Let E, F, G and H be self-mappings of a metric space (X, d) satisfying the following conditions:

(i) $E(X) \subseteq H(X)$ and $F(X) \subseteq G(X)$,

(ii) (E, G) and (F, H) are weakly compatible and

(iii) $[d(Ey, Fz)]^2 \leq \alpha \max\{[d(Gy, Ey)]^2, [d(Hz, Fz)]^2, [Gy, Hz]^2\}$
$+ \beta \max\{d_(Gy, Ey)d_(Gy, Fz), d_(Ey, Hz)d(Fz, Hz)\}$
$+ \delta d(Gy, Fz)d(Hz, Ey)$
for all $y, z \in X$, where $\alpha, \beta, \delta \geq 0, \alpha + 2\beta < 1$ and $\alpha + \delta < 1$.

(iv) Further, if the pair (E, G) satisfies (CLR_G)-property or the pair (F, H) satisfies (CLR_H)-property, then the self-maps E, F, G and H have a unique common fixed point.

We introduce generalized contraction for two pairs of selfmaps in b-metric spaces and prove the existence of common fixed points under the assumptions that these two pairs of maps are weakly compatible and satisfying a generalized contraction condition in complete b-metric spaces. Our work is inspired by works of Nagaraju, Raju and Thirupathi (18). A series of selfmaps is added as an extension of the same. Additionally, we demonstrate the same using various hypotheses on two pairs of selfmaps that satisfy the b-(E.A)-property. Some of the conclusions in the literature are extended or generalized to two pairs of self maps by our theorems. We present examples to corroborate our findings and draw some conclusions from them.

2 Main Results

We introduce generalized contraction maps in b-metric spaces as follows.

Definition 2.1. *Let (X_b, d_b) be a b-metric space with coefficient $s \geq 1$ and $A_b, B_b, S_b, T_b : X_b \to X_b$ be selfmaps. If there exist $\lambda_1, \lambda_2, \lambda_3 \geq 0$ with $\lambda_1 + s\lambda_2 + s^2\lambda_3 \leq 1$ such that*

$$
\begin{aligned}
s^4[d_b(A_b x_b, B_b y_b)]^2 \leq\ & \lambda_1 \max\{[d_b(S_b x_b, T_b y_b)]^2, [d_b(S_b x_b, A_b x_b)]^2, [d_b(T_b y_b, B_b y_b)]^2\} \\
& + \lambda_2 \max\{\tfrac{d_b(S_b x_b, A_b x_b) d_b(S_b x_b, B_b y_b)}{2}, \tfrac{d_b(T_b y_b, B_b y_b) d_b(T_b y_b, A_b x_b)}{2}\} \\
& + \lambda_3 \tfrac{d_b(S_b x_b, B_b y_b) d_b(T_b y_b, A_b x_b)}{2}.
\end{aligned} \tag{1}
$$

Then we call A_b, B_b, S_b and T_b are generalized contraction maps.

Example 2.1. *Let $X_b = [0, 1]$ and let $d_b : X_b \times X_b \to [0, \infty)$ defined by*

$$
d_b(x_b, y_b) = \begin{cases} 0 & \text{if } x_b = y_b, \\ (x_b + y_b)^2 & \text{if } x_b \neq y_b. \end{cases}
$$

Then clearly (X_b, d_b) is a complete b-metric space with $s = 2$.
We define $A_b, B_b, S_b, T_b : X_b \to X_b$ by $A_b(x_b) = \frac{1-x_b}{5}$, $B_b(x_b) = \frac{\log_{10}(1+x_b)}{5}$, $S_b(x_b) = x_b^2, T_b(x_b) = x_b$ for all $x_b \in X_b$.
Take $\lambda_1 = \frac{1}{7}, \lambda_2 = \frac{1}{8}, \lambda_3 = \frac{1}{10}$.
Clearly, $\lambda_1 + s\lambda_2 + s^2\lambda_3 \leq 1$.
Then we have

$$
\begin{aligned}
s^4[d_b(A_b x_b, B_b y_b)]^2 &= 16(\tfrac{1-e^{x_b}}{5} + \tfrac{\log_{10}(1+y_b)}{5})^4 \\
&\leq \tfrac{1}{7} \max\{(x_b^2 + y_b)^4, (x_b^2 + \tfrac{1-e^{x_b}}{5})^4, (y_b + \tfrac{\log_{10}(1+y_b)}{5})^4\} \\
&\quad + \tfrac{1}{8} \max\{\tfrac{(x_b^2 + \frac{1-e^{x_b}}{5})^2 (x_b^2 + \frac{\log_{10}(1+y_b)}{5})^2}{2}, \tfrac{(y_b + \frac{\log_{10}(1+y_b)}{5})^2 (y_b + \frac{1-e^{x_b}}{5})^2}{2}\} \\
&\quad + \tfrac{1}{10} \tfrac{(x_b^2 + \frac{\log_{10}(1+y_b)}{5})^2 (y_b + \frac{1-e^{x_b}}{5})^2}{2} \\
&\leq \lambda_1 \max\{[d_b(S_b x_b, T_b y_b)]^2, [d_b(S_b x_b, A_b x_b)]^2, [d_b(T_b y_b, B_b y_b)]^2\} \\
&\quad + \lambda_2 \max\{\tfrac{d_b(S_b x_b, A_b x_b) d_b(S_b x_b, B_b y_b)}{2}, \tfrac{d_b(T_b y_b, B_b y_b) d_b(T_b y_b, A_b x_b)}{2}\} \\
&\quad + \lambda_3 \tfrac{d_b(S_b x_b, B_b y_b) d_b(T_b y_b, A_b x_b)}{2}.
\end{aligned}
$$

Therefore A_b, B_b, S_b and T_b are generalized contraction maps.

Let A_b, B_b, S_b and T_b be mappings from a b-metric space (X_b, d_b) into itself and satisfying

$$A_b(X_b) \subseteq T_b(X_b) \text{ and } B_b(X_b) \subseteq S_b(X_b) \tag{2}$$

Now, by (2), for any $x_{b_0} \in X_b$, there exists $x_{b_1} \in X_b$ such that
$y_{b_0} = A_b x_{b_0} = T_b x_{b_1}$.
In the same way for this x_{b_1}, we can choose a point $x_{b_2} \in X_b$ such that
$y_{b_1} = B_b x_{b_1} = S_b x_{b_2}$ and so on.
In general, we can define a sequence $\{y_{b_n}\} \in X_b$ such that

$$y_{b_{2n}} = A_b x_{b_{2n}} = T_b x_{b_{2n+1}} \text{ and } y_{b_{2n+1}} = B_b x_{b_{2n+1}} = S_b x_{b_{2n+2}} \text{ for } n = 0, 1, 2, \dots. \tag{3}$$

Proposition 2.1. *Let (X_b, d_b) be a b-metric space with coefficient $s \geq 1$. Suppose that A_b, B_b, S_b and T_b are generalized contraction maps. Then we have the following:*
(i) If $A_b(X_b) \subseteq T_b(X_b)$ and the pair (B_b, T_b) is weakly compatible, and if x_b is a common fixed point of A_b and S_b then x_b is a common fixed point of A_b, B_b, S_b and T_b and it is unique.
(ii) If $B_b(X_b) \subseteq S_b(X_b)$ and the pair (A_b, S_b) is weakly compatible, and if x_b is a common fixed point of B_b and T_b then x_b is a common fixed point of A_b, B_b, S_b and T_b and it is unique.

Proof. First, we assume that *(i)* holds. Let x_b be a common fixed point of A_b and S_b.
Then $A_b x_b = S_b x_b = x_b$.
Since $A_b(X_b) \subseteq T_b(X_b)$, there exists $y \in X_b$ such that $T_b y_b = x_b$.
Therefore $A_b x_b = S_b x_b = T_b y_b = x_b$. If $A_b x_b \neq B_b y_b$, then

$$\begin{aligned} s^4[d_b(A_b x_b, B_b y_b)]^2 \leq{} & \lambda_1 \max\{[d_b(S_b x_b, T_b y_b)]^2, [d_b(S_b x_b, A_b x_b)]^2, [d_b(T_b y_b, B_b y_b)]^2\} \\ & +\lambda_2 \max\{\tfrac{d_b(S_b x_b, A_b x_b) d_b(S_b x_b, B_b y_b)}{2}, \tfrac{d_b(T_b y_b, B_b y_b) d_b(T_b y_b, A_b x_b)}{2}\} \\ & + \lambda_3 \tfrac{d_b(S_b x_b, B_b y_b) d_b(T_b y_b, A_b x_b)}{2} \\ ={} & \lambda_1 [d_b(A_b x_b, B_b y_b)]^2 \end{aligned}$$

which implies that $(s^4 - \lambda_1)[d_b(A_b x_b, B_b y_b)]^2 \leq 0$.
Since $(s^4 - \lambda_1) \geq 0$, we have $d_b(A_b x_b, B_b y_b) \leq 0$ which implies that $A_b x_b = B_b y_b$.
Therefore $A_b x_b = B_b y_b = S_b x_b = T_b y_b = x_b$.
As (B_b, T_b) is weakly compatible and $T_b y_b = B_b y_b$, we have
$B_b T_b y_b = T_b B_b y_b$. i.e., $B_b x_b = T_b x_b$.
Now, we prove that $B_b x_b = x_b$. If $B_b x_b \neq x_b$, then

$$\begin{aligned} s^4[d_b(x_b, B_b x_b)]^2 ={} & s^4[d_b(A_b x_b, B_b x_b)]^2 \\ \leq{} & \lambda_1 \max\{[d_b(S_b x_b, T_b x_b)]^2, [d_b(S_b x_b, A_b x_b)]^2, [d_b(T_b x_b, B_b x_b)]^2\} \\ & +\lambda_2 \max\{\tfrac{d_b(S_b x_b, A_b x_b) d_b(S_b x_b, B_b x_b)}{2}, \tfrac{d_b(T_b x_b, B_b x_b) d_b(T_b x_b, A_b x_b)}{2}\} \\ & + \lambda_3 \tfrac{d_b(S_b x_b, B_b x_b) d_b(T_b x_b, A_b x_b)}{2} \\ ={} & \lambda_1 [d_b(x_b, B_b x_b)]^2 + \lambda_3 \tfrac{[d_b(x_b, B_b x_b)]^2}{2} \end{aligned}$$

$= (\lambda_1 + \frac{\lambda_3}{2})[d_b(x_b, B_b x_b)]^2$

which implies that $[s^4 - (\lambda_1 + \frac{\lambda_3}{2})][d_b(x_b, B_b x_b)]^2 \leq 0$.
Since $[s^4 - (\lambda_1 + \frac{\lambda_3}{2})] \geq 0$, we have $d_b(x_b, B_b x_b) \leq 0$.
Hence, $B_b x_b = x_b$.
Therefore $A_b x_b = B_b x_b = S_b x_b = T_b x_b = x_b$.
Therefore, x_b is a common fixed point of A_b, B_b, S_b and T_b.
If x'_b is also a common fixed point of A_b, B_b, S_b and T_b with $x_b \neq x'_b$, then

$$\begin{aligned}
s^4[d_b(x_b, x'_b)]^2 &= s^4[d_b(A_b x_b, B_b x'_b)]^2 \\
&\leq \lambda_1 \max\{[d_b(S_b x_b, T_b x'_b)]^2, [d_b(S_b x_b, A_b x_b)]^2, [d_b(T_b x'_b, B_b x'_b)]^2\} \\
&\quad +\lambda_2 \max\{\tfrac{d_b(S_b x_b, A_b x_b) d_b(S_b x_b, B_b x'_b)}{2}, \tfrac{d_b(T_b x'_b, B_b x'_b) d_b(T_b x'_b, A_b x_b)}{2}\} \\
&\quad +\lambda_3 \tfrac{d_b(S_b x_b, B_b x'_b) d_b(T_b x'_b, A_b x_b)}{2} \\
&= \lambda_1[d_b(x_b, x'_b)]^2 + \lambda_3 \tfrac{[d_b(x_b, x'_b)]^2}{2} \\
&= (\lambda_1 + \tfrac{\lambda_3}{2})[d_b(x_b, x'_b)]^2
\end{aligned}$$

which implies that $[s^4 - (\lambda_1 + \frac{\lambda_3}{2})][d_b(x_b, x'_b)]^2 \leq 0$.
Since $[s^4 - (\lambda_1 + \frac{\lambda_3}{2})] \geq 0$, we have $d_b(x_b, x'_b) \leq 0$.
Hence, $x'_b = x_b$.
Therefore x_b is the unique common fixed point of A_b, B_b, S_b and T_b.
The proof of (*ii*) is similar to (*i*) and hence is omitted. □

Lemma 2.1. Let A_b, B_b, S_b and T_b be selfmaps of a b-metric space (X_b, d_b) and satisfy (2) and are generalized contraction maps. Then for any $x_{b_0} \in X_b$, the sequence $\{y_{b_n}\}$ defined by (3) is b-Cauchy in X_b.

Proof. Let $x_{b_0} \in X_b$ and let $\{y_{b_n}\}$ be a sequence defined by (3).
Assume that $y_{b_n} = y_{b_{n+1}}$ for some n.
Case (i): n even.
We write $n = 2m, m \in \mathbb{N}$.
Now, we consider

$$\begin{aligned}
s^4[d_b(y_{b_{n+1}}, y_{b_{n+2}})]^2 &= s^4[d_b(y_{b_{2m+1}}, y_{b_{2m+2}})]^2 \\
&= s^4[d_b(y_{b_{2m+2}}, y_{b_{2m+1}})]^2 \\
&= s^4[d_b(A_b x_{b_{2m+2}}, B_b x_{b_{2m+1}})]^2 \\
&\leq \lambda_1 \max\{[d_b(S_b x_{b_{2m+2}}, T_b x_{b_{2m+1}})]^2, [d_b(S_b x_{b_{2m+2}}, A_b x_{b_{2m+2}})]^2, \\
&\qquad [d_b(T_b x_{b_{2m+1}}, B_b x_{b_{2m+1}})]^2\} \\
&\quad + \lambda_2 \max\{\tfrac{d_b(S_b x_{b_{2m+2}}, A_b x_{b_{2m+2}}) d_b(S_b x_{b_{2m+2}}, B_b x_{b_{2m+1}})}{2}, \\
&\qquad \tfrac{d_b(T_b x_{b_{2m+1}}, B_b x_{b_{2m+1}}) d_b(T_b x_{b_{2m+1}}, A_b x_{b_{2m+2}})}{2}\} \\
&\quad + \lambda_3 \tfrac{d_b(S_b x_{b_{2m+2}}, B_b x_{b_{2m+1}}) d_b(T_b x_{b_{2m+1}}, A_b x_{b_{2m+2}})}{2} \\
&= \lambda_1 \max\{[d_b(y_{b_{2m+1}}, y_{b_{2m}})]^2, [d_b(y_{b_{2m+1}}, y_{b_{2m+2}})]^2, [d_b(y_{b_{2m}}, y_{b_{2m+1}})]^2\} \\
&\quad +\lambda_2 \max\{\tfrac{d_b(y_{b_{2m+1}}, y_{b_{2m+2}}) d_b(y_{b_{2m+1}}, y_{b_{2m+1}})}{2}, \tfrac{d_b(y_{b_{2m}}, y_{b_{2m+1}}) d_b(y_{b_{2m}}, y_{b_{2m+2}})}{2}\}
\end{aligned}$$

$$
\begin{aligned}
&+ \lambda_3 \tfrac{d_b(y_{b_{2m+1}}, y_{b_{2m+1}}) d_b(y_{b_{2m}}, y_{b_{2m+2}})}{2} \\
&= \lambda_1 \max\{[d_b(y_{b_{n+1}}, y_{b_n})]^2, [d_b(y_{b_{n+1}}, y_{b_{n+2}})]^2, [d_b(y_{b_n}, y_{b_{n+1}})]^2\} \\
&\quad + \lambda_2 \max\{\tfrac{d_b(y_{b_{n+1}}, y_{b_{n+2}}) d_b(y_{b_{n+1}}, y_{b_{n+1}})}{2}, \tfrac{d_b(y_{b_n}, y_{b_{n+1}}) d_b(y_{b_n}, y_{b_{n+2}})}{2}\} \\
&\quad + \lambda_3 \tfrac{d_b(y_{b_{n+1}}, y_{b_{n+1}}) d_b(y_{b_n}, y_{b_{n+2}})}{2} \\
&= \lambda_1 [d_b(y_{b_{n+1}}, y_{b_{n+2}})]^2
\end{aligned}
$$

which implies that $(s^4 - \lambda_1)[d_b(y_{b_{n+1}}, y_{b_{n+2}})]^2 \leq 0$.
Since $(s^4 - \lambda_1) \geq 0$, we have $d_b(y_{b_{n+1}}, y_{b_{n+2}}) \leq 0$
which implies that $y_{b_{n+2}} = y_{b_{n+1}} = y_{b_n}$.
In general, we have $y_{b_{n+k}} = y_{b_n}$ for $k = 0, 1, 2, \dots$.
Case (*ii*): n odd.
We write $n = 2m + 1$ for some $m \in \mathbb{N}$.
Now we consider

$$
\begin{aligned}
s^4[d_b(y_{b_{n+1}}, y_{b_{n+2}})]^2 &= s^4[d_b(y_{b_{2m+2}}, y_{b_{2m+3}})]^2 \\
&= s^4[d_b(A_b x_{b_{2m+2}}, B_b x_{b_{2m+3}})]^2 \\
&\leq \lambda_1 \max\{[d_b(S_b x_{b_{2m+2}}, T_b x_{b_{2m+3}})]^2, [d_b(S_b x_{b_{2m+2}}, A_b x_{b_{2m+2}})]^2, \\
&\qquad [d_b(T_b x_{b_{2m+3}}, B_b x_{b_{2m+3}})]^2\} \\
&\quad + \lambda_2 \max\{\tfrac{d_b(S_b x_{b_{2m+2}}, A_b x_{b_{2m+2}}) d_b(S_b x_{b_{2m+2}}, B_b x_{b_{2m+3}})}{2}, \\
&\qquad \tfrac{d_b(T_b x_{b_{2m+3}}, B_b x_{b_{2m+3}}) d_b(T_b x_{b_{2m+3}}, A_b x_{b_{2m+2}})}{2}\} \\
&\quad + \lambda_3 \tfrac{d_b(S_b x_{b_{2m+2}}, B_b x_{b_{2m+3}}) d_b(T_b x_{b_{2m+3}}, A_b x_{b_{2m+2}})}{2} \\
&= \lambda_1 \max\{[d_b(y_{b_{2m+1}}, y_{b_{2m+2}})]^2, [d_b(y_{b_{2m+1}}, y_{b_{2m+2}})]^2, [d_b(y_{b_{2m+2}}, y_{b_{2m+3}})]^2\} \\
&\quad + \lambda_2 \max\{\tfrac{d_b(y_{b_{2m+1}}, y_{b_{2m+2}}) d_b(y_{b_{2m+1}}, y_{b_{2m+3}})}{2}, \\
&\qquad \tfrac{d_b(y_{b_{2m+2}}, y_{b_{2m+3}}) d_b(y_{b_{2m+2}}, y_{b_{2m+2}})}{2}\} + \lambda_3 \tfrac{d_b(y_{b_{2m+1}}, y_{b_{2m+3}}) d_b(y_{b_{2m+2}}, y_{b_{2m+2}})}{2} \\
&= \lambda_1 \max\{[d_b(y_{b_n}, y_{b_{n+1}})]^2, [d_b(y_{b_n}, y_{b_{n+1}})]^2, [d_b(y_{b_{n+1}}, y_{b_{n+2}})]^2\} \\
&\quad + \lambda_2 \max\{\tfrac{d_b(y_{b_n}, y_{b_{n+1}}) d_b(y_{b_n}, y_{b_{n+2}})}{2}, \tfrac{d_b(y_{b_{n+1}}, y_{b_{n+2}}) d_b(y_{b_{n+1}}, y_{b_{n+1}})}{2}\} \\
&\quad + \lambda_3 \tfrac{d_b(y_{b_n}, y_{b_{n+2}}) d_b(y_{b_{n+1}}, y_{b_{n+1}})}{2} \\
&= \lambda_1 [d_b(y_{b_{n+1}}, y_{b_{n+2}})]^2
\end{aligned}
$$

which implies that $(s^4 - \lambda_1)[d_b(y_{b_{n+1}}, y_{b_{n+2}})]^2 \leq 0$.
Since $(s^4 - \lambda_1) \geq 0$, we have $d_b(y_{b_{n+1}}, y_{b_{n+2}}) \leq 0$
which implies that $y_{b_{n+2}} = y_{b_{n+1}} = y_{b_n}$.
In general, we have $y_{b_{n+k}} = y_{b_n}$ for $k = 1, 2, 3, \dots$.
From Case (*i*) and Case (*ii*), we have $y_{b_{n+k}} = y_{b_n}$ for $k = 0, 1, 2, \dots$.
Therefore, $\{y_{b_{n+k}}\}$ is a constant sequence and hence $\{y_{b_n}\}$ is b- Cauchy.
Now we assume that $y_{b_n} \neq y_{b_{n+1}}$ for all $n \in \mathbb{N}$.
If n is odd then $n = 2m + 1$ for some $m \in \mathbb{N}$.

We now consider

$$\begin{aligned}
s^4[d_b(y_{b_{n+1}}, y_{b_{n+2}})]^2 &= s^4[d_b(y_{b_{2m+2}}, y_{b_{2m+3}})]^2 \\
&= s^4[d_b(A_b x_{b_{2m+2}}, B_b x_{b_{2m+3}})]^2 \\
&\leq \lambda_1 \max\{[d_b(S_b x_{b_{2m+2}}, T_b x_{b_{2m+3}})]^2, [d_b(S_b x_{b_{2m+2}}, A_b x_{b_{2m+2}})]^2, \\
&\qquad [d_b(T_b x_{b_{2m+3}}, B_b x_{b_{2m+3}})]^2\} \\
&\quad + \lambda_2 \max\{\tfrac{d_b(S_b x_{b_{2m+2}}, A_b x_{b_{2m+2}}) d_b(S_b x_{b_{2m+2}}, B_b x_{b_{2m+3}})}{2}, \\
&\qquad \tfrac{d_b(T_b x_{b_{2m+3}}, B_b x_{b_{2m+3}}) d_b(T_b x_{b_{2m+3}}, A_b x_{b_{2m+2}})}{2}\} \\
&\quad + \lambda_3 \tfrac{d_b(S_b x_{b_{2m+2}}, B_b x_{b_{2m+3}}) d_b(T_b x_{b_{2m+3}}, A_b x_{b_{2m+2}})}{2} \\
&= \lambda_1 \max\{[d_b(y_{b_{2m+1}}, y_{b_{2m+2}})]^2, [d_b(y_{b_{2m+1}}, y_{b_{2m+2}})]^2, [d_b(y_{b_{2m+2}}, y_{b_{2m+3}})]^2\} \\
&\quad + \lambda_2 \max\{\tfrac{d_b(y_{b_{2m+1}}, y_{b_{2m+2}}) d_b(y_{b_{2m+1}}, y_{b_{2m+3}})}{2}, \tfrac{d_b(y_{b_{2m+2}}, y_{b_{2m+3}}) d_b(y_{b_{2m+2}}, y_{b_{2m+2}})}{2}\} \\
&\quad + \lambda_3 \tfrac{d_b(y_{b_{2m+1}}, y_{b_{2m+3}}) d_b(y_{b_{2m+2}}, y_{b_{2m+2}})}{2} \\
&= \lambda_1 \max\{[d_b(y_{b_n}, y_{b_{n+1}})]^2, [d_b(y_{b_n}, y_{b_{n+1}})]^2, [d_b(y_{b_{n+1}}, y_{b_{n+2}})]^2\} \\
&\quad + \lambda_2 \max\{\tfrac{d_b(y_{b_n}, y_{b_{n+1}}) d_b(y_{b_n}, y_{b_{n+2}})}{2}, \tfrac{d_b(y_{b_{n+1}}, y_{b_{n+2}}) d_b(y_{b_{n+1}}, y_{b_{n+1}})}{2}\} \\
&\quad + \lambda_3 \tfrac{d_b(y_{b_n}, y_{b_{n+2}}) d_b(y_{b_{n+1}}, y_{b_{n+1}})}{2}
\end{aligned} \tag{4}$$

If $[d_b(y_{b_n}, y_{b_{n+1}})]^2 < [d_b(y_{b_{n+1}}, y_{b_{n+2}})]^2$ then from (4), we have
$s^4[d_b(y_{b_{n+1}}, y_{b_{n+2}})]^2 \leq \lambda_1[d_b(y_{b_{n+1}}, y_{b_{n+2}})]^2 + s\lambda_2[d_b(y_{b_{n+1}}, y_{b_{n+2}})]^2$
which implies that $(s^4 - \lambda_1 - s\lambda_2)[d_b(y_{b_{n+1}}, y_{b_{n+2}})]^2 \leq 0$.
Since $(s^4 - \lambda_1 - s\lambda_2) \geq 0$, we have $[d_b(y_{b_{n+1}}, y_{b_{n+2}})]^2 \leq 0$
which implies that $y_{b_{n+1}} = y_{b_{n+2}}$,
which is a contradiction.
Therefore $[d_b(y_{b_{n+1}}, y_{b_{n+2}})]^2 \leq [d_b(y_{b_n}, y_{b_{n+1}})]^2$.
From the inequality (4), we have $s^4[[d_b(y_{b_{n+1}}, y_{b_{n+2}})]^2] \leq (\lambda_1 + s\lambda_2)[d_b(y_{b_n}, y_{b_{n+1}})]^2$
which implies that $d_b(y_{b_{n+1}}, y_{b_{n+2}}) \leq k d_b(y_{b_n}, y_{b_{n+1}})$, where $k = \frac{\sqrt{(\lambda_1 + s\lambda_2)}}{s^2} < 1$.
Similarly, we can prove that $d_b(y_{b_{n+1}}, y_{b_{n+2}}) \leq k d_b(y_{b_n}, y_{b_{n+1}})$ whenever n is even.
By Lemma 1.2, we have $\{y_{b_n}\}$ is a b-Cauchy sequence in X_b. □

The following is the main result of this paper.

Theorem 2.1. *Let A_b, B_b, S_b and T_b be selfmaps on a complete b-metric space (X_b, d_b) and satisfy* (2) *and the maps are generalized contraction maps . If the pairs (A_b, S_b) and (B_b, T_b) are weakly compatible and one of the range sets $S_b(X_b), T_b(X_b), A_b(X_b)$ and $B_b(X_b)$ is closed, then for any $x_{b_0} \in X_b$, the sequence $\{y_{b_n}\}$ defined by* (3) *is Cauchy in X_b and $\lim_{n\to\infty} y_{b_n} = z_b$(say), $z_b \in X_b$ and z_b is the unique common fixed point of A_b, B_b, S_b and T_b.*

Proof. By Lemma 2.1, the sequence $\{y_{b_n}\}$ is b-Cauchy in X_b.
Since X_b is b-complete, $\exists\, z_b \in X_b \ni \lim_{n\to\infty} y_{b_n} = z_b$.

Then

$$\begin{cases} \lim\limits_{n\to\infty} y_{b_{2n}} = \lim\limits_{n\to\infty} A_b x_{b_{2n}} = \lim\limits_{n\to\infty} T_b x_{b_{2n+1}} = z_b \text{ and} \\ \lim\limits_{n\to\infty} y_{b_{2n+1}} = \lim\limits_{n\to\infty} B_b x_{b_{2n+1}} = \lim\limits_{n\to\infty} S_b x_{b_{2n+2}} = z_b. \end{cases} \tag{5}$$

We consider the below cases.

Case (i). $S_b(X_b)$ is closed.

In this case $z_b \in S_b(X_b)$ and there exists $t_b \in X_b$ such that $z_b = S_b t_b$.

If $A_b t_b \neq z_b$, then

$$\begin{aligned} s^4[d_b(A_b t_b, B_b x_{b_{2n+1}})]^2 \leq \lambda_1 \max\{&[d_b(S_b t_b, T_b x_{b_{2n+1}})]^2, [d_b(S_b t_b, A_b t_b)]^2, \\ &[d_b(T_b x_{b_{2n+1}}, B_b x_{b_{2n+1}})]^2\} \\ +\lambda_2 \max\{&\tfrac{d_b(S_b t_b, A_b t_b) d_b(S_b t_b, B_b x_{b_{2n+1}})}{2}, \\ &\tfrac{d_b(T_b x_{b_{2n+1}}, B_b x_{b_{2n+1}}) d_b(T_b x_{b_{2n+1}}, A_b t_b)}{2}\} \\ +\lambda_3 &\tfrac{d_b(S_b t_b, B_b x_{b_{2n+1}}) d_b(T_b x_{b_{2n+1}}, A_b t_b)}{2} \end{aligned} \tag{6}$$

On letting limit superior as $n \to \infty$ in the inequality (6) , using Lemma 1.1 and (5), we get

$\frac{1}{s^2}(s^4[d_b(A_b t_b, z_b)]^2) \leq \lambda_1 [d_b(A_b t_b, z_b)]^2$

which implies that $(s^2 - \lambda_1)[d_b(A_b t_b, z_b)]^2 \leq 0$.

Since $(s^2 - \lambda_1) \geq 0$, we have $A_b t_b = z_b$.

Therefore, $A_b t_b = z_b = S_b t_b$.

Since (A_b, S_b) is weakly compatible and $A_b t_b = S_b t_b$, we have

$A_b S_b t_b = S_b A_b t_b$. i.e., $A_b z_b = S_b z_b$.

Suppose $A_b z_b \neq z_b$. We now consider

$$\begin{aligned} &s^4[d_b(A_b z_b, B_b x_{b_{2n+1}})]^2 \\ &\leq \lambda_1 \max\{[d_b(S_b z_b, T_b x_{b_{2n+1}})]^2, [d_b(S_b z_b, A_b z_b)]^2, [d_b(T_b x_{b_{2n+1}}, B_b x_{b_{2n+1}})]^2\} \\ &\quad +\lambda_2 \max\{\tfrac{d_b(S_b z_b, A_b z_b) d_b(S_b z_b, B_b x_{b_{2n+1}})}{2}, \tfrac{d_b(T_b x_{b_{2n+1}}, B_b x_{b_{2n+1}}) d_b(T_b x_{b_{2n+1}}, A_b z_b)}{2}\} \\ &\quad +\lambda_3 \tfrac{d_b(S_b z_b, B_b x_{b_{2n+1}}) d_b(T_b x_{b_{2n+1}}, A_b z_b)}{2} \end{aligned} \tag{7}$$

On letting limit superior as $n \to \infty$ in the inequality (7) , using Lemma 1.1 and (5), we get

$\frac{1}{s^2}(s^4[d_b(A_b z_b, z_b)]^2) \leq (\lambda_1 + \frac{s^2\lambda_3}{2})[d_b(A_b z_b, z_b)]^2$

which implies that $(s^2 - \lambda_1 - \frac{s^2\lambda_3}{2})[d_b(A_b z_b, z_b)]^2 \leq 0$.

Since $(s^2 - \lambda_1 - \frac{s^2\lambda_3}{2}) \geq 0$, we have $A_b z_b = z_b$.

Therefore $A_b z_b = S_b z_b = z_b$.

Hence, z_b is a common fixed point of A_b and S_b.

By Proposition 2.1, we get that z_b is a unique common fixed point of A_b, B_b, S_b and T_b.

Case (ii). $T_b(X_b)$ is closed.

In this case $z_b \in T_b(X_b)$ and there exists $u_b \in X_b \ni z_b = T_b u_b$.

If $B_bu_b \neq z_b$, then

$$\begin{aligned} & s^4[d_b(A_bx_{b_{2n+2}}, B_bu_b)]^2 \\ & \leq \lambda_1 \max\{[d_b(S_bx_{b_{2n+2}}, T_bu_b)]^2, [d_b(S_bx_{b_{2n+2}}, A_bx_{b_{2n+2}})]^2, [d_b(T_bu_b, B_bu_b)]^2\} \\ & \quad +\lambda_2 \max\{\tfrac{d_b(S_bx_{b_{2n+2}}, A_bx_{b_{2n+2}})d_b(S_bx_{b_{2n+2}}, B_bu_b)}{2}, \tfrac{d_b(T_bu_b, B_bu_b)d_b(T_bu_b, A_bx_{b_{2n+2}})}{2}\} \\ & \quad +\lambda_3 \tfrac{d_b(S_bx_{b_{2n+2}}, B_bu_b)d_b(T_bu_b, A_bx_{b_{2n+2}})}{2} \end{aligned} \tag{8}$$

On letting limit superior as $n \to \infty$ in (8) , using Lemma 1.1 and (5), we get $\frac{1}{s^2}(s^4[d_b(B_bu_b, z_b)]^2) \leq \lambda_1[d_b(B_bu_b, z_b)]^2$ which implies that
$(s^2 - \lambda_1)[d_b(B_bu_b, z_b)]^2 \leq 0$.
Since $(s^2 - \lambda_1) \geq 0$, we have $B_bu_b = z_b$.
Therefore, $B_bu_b = z_b = T_bu_b$.
Since the pair (B_b, T_b) is weakly compatible and $B_bu_b = T_bu_b$, we have
$B_bT_bu_b = T_bB_bu_b$. i.e., $B_bz_b = T_bz_b$.
We now prove that $B_bz_b = z_b$. Suppose that $B_bz_b \neq z_b$. We now consider

$$\begin{aligned} & s^4[d_b(A_bx_{b_{2n+2}}, B_bz_b)]^2 \\ & \leq \lambda_1 \max\{[d_b(S_bx_{b_{2n+2}}, T_bz_b)]^2, [d_b(S_bx_{b_{2n+2}}, A_bx_{b_{2n+2}})]^2, [d_b(T_bz_b, B_bz_b)]^2\} \\ & \quad +\lambda_2 \max\{\tfrac{d_b(S_bx_{b_{2n+2}}, A_bx_{b_{2n+2}})d_b(S_bx_{b_{2n+2}}, B_bz_b)}{2}, \tfrac{d_b(T_bz_b, B_bz_b)d_b(T_bz_b, A_bx_{b_{2n+2}})}{2}\} \\ & \quad +\lambda_3 \tfrac{d_b(S_bx_{b_{2n+2}}, B_bz_b)d_b(T_bz_b, A_bx_{b_{2n+2}})}{2} \end{aligned} \tag{9}$$

On letting limit superior as $n \to \infty$ in (9) , using Lemma 1.1 and (5), we get
$\frac{1}{s^2}(s^4[d_b(B_bz_b, z_b)]^2) \leq (\lambda_1 + \frac{s^2\lambda_3}{2})[d_b(B_bz_b, z_b)]^2$ which implies that
$(s^2 - \lambda_1 - \frac{s^2\lambda_3}{2})[d_b(B_bz_b, z_b)]^2 \leq 0$.
Since $(s^2 - \lambda_1 - \frac{s^2\lambda_3}{2}) \geq 0$, we have $B_bz_b = z_b$.
Therefore $B_bz_b = T_bz_b = z_b$.
Therefore, z_b is a common fixed point of B and T.
By Proposition 2.1, we get that z_b is the unique common fixed point of A_b, B_b, S_b and T_b.
Case (iii). $A_b(X_b)$ is closed.
From the inequality (2) and Case (ii), the conclusion follows.
Case (iv). $B_b(X_b)$ is closed.
From the inequality (2) and Case (i), the Proof follows. □

Theorem 2.2. *Let (X_b, d_b) be a b-metric space with coefficient $s \geq 1$. Assume that $A_b, B_b, S_b, T_b : X_b \to X_b$ are generalized contraction maps and satisfy (2). Suppose that one of the pairs (A_b, S_b) and (B_b, T_b) satisfies the b-(E.A)-property and that one of the subspace $A_b(X_b), B_b(X_b), S_b(X_b)$ and $T_b(X_b)$ is b-closed in X_b. Then the pairs (A_b, S_b) and (B_b, T_b) have a point of coincidence in X_b. Moreover, if the pairs (A_b, S_b) and (B_b, T_b) are weakly compatible, then A_b, B_b, S_b and T_b have a unique common fixed point in X_b.*

Proof. We first assume that the pair (A_b, S_b) satisfies the b-(E.A)-property. So there exists a sequence $\{x_{b_n}\}$ in X_b such that

$$\lim_{n\to\infty} A_b x_{b_n} = \lim_{n\to\infty} S_b x_{b_n} = q_b \text{ for some } q_b \in X_b \tag{10}$$

Since $A_b(X_b) \subseteq T_b(X_b)$, there exists a sequence $\{y_{b_n}\}$ in X_b such that $A_b x_{b_n} = T_b y_{b_n}$, and hence

$$\lim_{n\to\infty} T_b y_{b_n} = q_b. \tag{11}$$

We now show that $\lim_{n\to\infty} B_b y_{b_n} = q_b$. Suppose that $\lim_{n\to\infty} B_b y_{b_n} \neq q_b$.
From (1), we have

$$\begin{aligned} s^4[d_b(A_b x_{b_n}, B_b y_{b_n})]^2 \leq \lambda_1 \max\{&[d_b(S_b x_{b_n}, T_b y_{b_n})]^2, [d_b(S_b x_{b_n}, A_b x_{b_n})]^2,\\ &[d_b(T_b y_{b_n}, B_b y_{b_n})]^2\}\\ &+\lambda_2 \max\{\tfrac{d_b(S_b x_{b_n}, A_b x_{b_n}) d_b(S_b x_{b_n}, B_b y_{b_n})}{2},\\ &\tfrac{d_b(T_b y_{b_n}, B_b y_{b_n}) d_b(T_b y_{b_n}, A_b x_{b_n})}{2}\}\\ &+\lambda_3 \tfrac{d_b(S_b x_{b_n}, B_b y_{b_n}) d_b(T_b y_{b_n}, A_b x_{b_n})}{2} \end{aligned} \tag{12}$$

By taking limit superior as $n \to \infty$ in (12), and using (10) and (11), we obtain

$$\begin{aligned} \tfrac{1}{s^2} s^4 \liminf_{n\to\infty}[d_b(q_b, B_b y_{b_n})]^2 &\leq s^4 \limsup_{n\to\infty}[d_b(A_b x_{b_n}, B_b y_{b_n})]^2\\ &\leq \limsup_{n\to\infty}(\lambda_1 \max\{[d_b(S_b x_{b_n}, T_b y_{b_n})]^2, [d_b(S_b x_{b_n}, A_b x_{b_n})]^2,\\ &\qquad [d_b(T_b y_{b_n}, B_b y_{b_n})]^2\}\\ &\quad +\lambda_2 \max\{\tfrac{d_b(S_b x_{b_n}, A_b x_{b_n}) d_b(S_b x_{b_n}, B_b y_{b_n})}{2}, \tfrac{d_b(T_b y_{b_n}, B_b y_{b_n}) d_b(T_b y_{b_n}, A_b x_{b_n})}{2}\}\\ &\quad + \lambda_3 \tfrac{d_b(S_b x_{b_n}, B_b y_{b_n}) d_b(T_b y_{b_n}, A_b x_{b_n})}{2})\\ &\leq s^2 \lambda_1 \limsup_{n\to\infty}[d_b(q_b, B_b y_{b_n})]^2. \end{aligned}$$

Since $(1 - \lambda_1) > 0$, we have

$$\lim_{n\to\infty} B_b y_{b_n} = q_b. \tag{13}$$

Case (i). Assume $T_b(X_b)$ is a b-closed subset of X_b.
In this case $q_b \in T_b(X_b)$, we can choose $r_b \in X_b \ni T_b r_b = q_b$.
Now, our claim is $B_b r_b = q_b$. Suppose $d_b(B_b r_b, q_b) > 0$. From (1), we have

$$\begin{aligned} &s^4[d_b(A_b x_{b_{2n+2}}, B_b r_b)]^2\\ &\leq \lambda_1 \max\{[d_b(S_b x_{b_{2n+2}}, T_b r_b)]^2, [d_b(S_b x_{b_{2n+2}}, A_b x_{b_{2n+2}})]^2, [d_b(T_b r_b, B_b r_b)]^2\}\\ &\quad +\lambda_2 \max\{\tfrac{d_b(S_b x_{b_{2n+2}}, A_b x_{b_{2n+2}}) d_b(S_b x_{b_{2n+2}}, B_b r_b)}{2}, \tfrac{d_b(T_b r_b, B_b r_b) d_b(T_b r_b, A_b x_{b_{2n+2}})}{2}\}\\ &\quad +\lambda_3 \tfrac{d_b(S_b x_{b_{2n+2}}, B_b r_b) d_b(T_b r_b, A_b x_{b_{2n+2}})}{2} \end{aligned} \tag{14}$$

On letting limit superior as $n \to \infty$ in (14), using (10), (11), (12) and Lemma 1.1, we have $\frac{1}{s^2} s^4 d_b(q_b, B_b r_b) \leq \lambda_1 [d_b(q_b, B_b r_b)]^2$ which implies that

$(s^2 - \lambda_1)[d_b(q_b, B_b r_b)]^2 \leq 0.$
Since $(s^2 - \lambda_1) \geq 0$, we have $B_b r_b = q_b$.
Hence $B_b r_b = T_b r_b = q_b$, so that q_b is a coincidence point of B_b and T_b.
Since $B_b(X_b) \subseteq S_b(X_b)$, we have $q_b \in S_b(X_b)$, there exists $z_b \in X_b$ such that $S_b z_b = q_b = B_b r_b$.
Now we show that $A_b z_b = q_b$. Suppose $A_b z_b \neq q_b$. From the inequality (1), we have

$$\begin{aligned} s^4[d_b(A_b z_b, q_b)]^2 &= s^4[d_b(A_b z_b, B_b r_b)]^2 \\ &\leq \lambda_1 \max\{[d_b(S_b z_b, T_b r_b)]^2, [d_b(S_b z_b, A_b z_b)]^2, [d_b(T_b r_b, B_b r_b)]^2\} \\ &\quad +\lambda_2 \max\{\tfrac{d_b(S_b z_b, A_b z_b) d_b(S_b z_b, B_b r_b)}{2}, \tfrac{d_b(T_b r_b, B_b r_b) d_b(T_b r_b, A_b z_b)}{2}\} \\ &\quad +\lambda_3 \tfrac{d_b(S_b z_b, B_b r_b) d_b(T_b r_b, A_b z_b)}{2} \end{aligned}$$

which implies that $(s^4 - \lambda_1)[d_b(q_b, A_b z_b)]^2 \leq 0$.
Since $(s^4 - \lambda_1) \geq 0$, we have $A_b z_b = q_b$.
Therefore $A_b z_b = S_b z_b = q_b$ so that z_b is a coincidence point of A_b and S_b.
Since the pairs (A_b, S_b) and (B_b, T_b) are weakly compatible, we have $A_b q_b = S_b q_b$ and $B_b q_b = T_b q_b$.
Therefore q_b is also a coincidence point of the pairs (A_b, S_b) and (B_b, T_b).
We now show that q_b is a common fixed point of A_b, B_b, S_b and T_b.
Suppose $A_b q_b \neq q_b$.
From the inequality (1), we have

$$\begin{aligned} s^4[d_b(A_b q_b, q_b)]^2 &= s^4[d_b(A_b q_b, B_b r_b)]^2 \\ &\leq \lambda_1 \max\{[d_b(S_b q_b, T_b r_b)]^2, [d_b(S_b q_b, A_b q_b)]^2, [d_b(T_b r_b, B_b r_b)]^2\} \\ &\quad +\lambda_2 \max\{\tfrac{d_b(S_b q_b, A_b q_b) d_b(S_b q_b, B_b r_b)}{2}, \tfrac{d_b(T_b r_b, B_b r_b) d_b(T_b r_b, A_b q_b)}{2}\} \\ &\quad +\lambda_3 \tfrac{d_b(S_b q_b, B_b r_b) d_b(T_b r_b, A_b q_b)}{2} \end{aligned}$$

which implies that $[s^4 - (\lambda_1 + \frac{\lambda_3}{2})][d_b(q_b, A_b q_b)]^2 \leq 0$.
Since $(s^4 - (\lambda_1 + \frac{\lambda_3}{2})) \geq 0$, we have $A_b q_b = q_b$.
Therefore $A_b q_b = S_b q_b = q_b$ so that q_b is a common fixed point of A_b and S_b.
By Proposition 2.1, q_b is a unique common fixed point of A_b, B_b, S_b and T_b.
Case (ii). Suppose $A_b(X_b)$ is b-closed.
In this case, we have $q_b \in A_b(X_b)$ and $A_b(X_b) \subseteq T_b(X_b)$,
we choose $r_b \in X_b \ni \ q_b = T_b r_b$.
Rest of the proof follows as in Case (i).
Case (iii). Suppose $S_b(X_b)$ is b-closed.
We follow the argument similar as Case (i) and we get conclusion.
Case (iv). Suppose $B_b(X_b)$ is b-closed. As in Case (ii), we get the conclusion.

For the case of (B_b, T_b) satisfies the b-(E.A)-property, we follow the argument similar to the case (A_b, S_b) satisfies the b-(E.A)-property. □

3 Corollaries and Examples

The following is an example in support of Theorem 2.1.

Example 3.1. *Let* $X_b = [0, \infty)$ *and let* $d_b : X_b \times X_b \to \mathbb{R}^+$ *defined by*

$$d_b(x_b, y_b) = \begin{cases} 0 & \text{if } x_b = y_b, \\ 4 & \text{if } x_b, y_b \in (0,1), \\ \frac{9}{2} + \frac{1}{x_b + y_b} & \text{if } x_b, y_b \in [1, \infty), \\ \frac{12}{5} & \text{otherwise}. \end{cases}$$

Then clearly (X_b, d_b) *is a complete* b*-metric space with coefficient* $s = \frac{25}{24}$.
We define $A_b, B_b, S_b, T_b : X_b \to X_b$ *by*
$A_b(x_b) = 1$ *if* $x_b \in [0, \infty)$, $B_b(x_b) = \begin{cases} x_b & \text{if } x_b \in [0,1) \\ \frac{1}{x_b} & \text{if } x_b \in [1, \infty), \end{cases}$
$S_b(x_b) = \begin{cases} x_b & \text{if } x_b \in [0,1) \\ \frac{1+x_b}{2} & \text{if } x_b \in [1, \infty), \end{cases}$ *and* $T_b(x_b) = \begin{cases} 2 & \text{if } x_b \in [0,1) \\ 2x_b^2 - 1 & \text{if } x_b \in [1, \infty). \end{cases}$
Clearly $A_b(X_b) \subseteq T_b(X_b)$, $B_b(X_b) \subseteq S_b(X_b)$ *and* $A_b(X_b)$ *is closed.*
Clearly the pairs (A_b, S_b) *and* (B_b, T_b) *are weakly compatible.*
We take $\lambda_1 = \frac{10}{51}, \lambda_2 = \frac{1}{4}, \lambda_3 = \frac{1}{2}$.
Then clearly $\lambda_1 + s\lambda_2 + s^2\lambda_3 \le 1$.
With out loss generality, we assume that $x \ge y$.
Case (i). $x_b, y_b \in [0, 1)$.
$d_b(A_bx_b, B_by_b) = \frac{12}{5}, d_b(S_bx_b, T_by_b) = \frac{12}{5}, d_b(S_bx_b, A_bx_b) = \frac{12}{5}$,
$d_b(T_by_b, B_by_b) = \frac{12}{5}, d_b(S_bx_b, B_by_b) = 4, d_b(T_by_b, A_bx_b) = \frac{9}{2} + \frac{1}{x_b+y_b}$.
We now consider

$$\begin{aligned} s^4[d_b(A_bx_b, B_by_b)]^2 &= (\tfrac{25}{24})^4(\tfrac{12}{5})^2 \\ &\le \tfrac{10}{51}(\tfrac{12}{5})^2 + \tfrac{1}{8}((\tfrac{12}{5})(\tfrac{9}{2} + \tfrac{1}{x_b+y_b})) + \tfrac{1}{4}((4)(\tfrac{9}{2} + \tfrac{1}{x_b+y_b})) \\ &\le \lambda_1 \max\{[d_b(S_bx_b, T_by_b)]^2, [d_b(S_bx_b, A_bx_b)]^2, [d_b(T_by_b, B_by_b)]^2\} \\ &\quad + \lambda_2 \max\{\tfrac{d_b(S_bx_b, A_bx_b)d_b(S_bx_b, B_by_b)}{2}, \tfrac{d_b(T_by_b, B_by_b)d_b(T_by_b, A_bx_b)}{2}\} \\ &\quad + \lambda_3 \tfrac{d_b(S_bx_b, B_by_b)d_b(T_by_b, A_bx_b)}{2}. \end{aligned}$$

Case (ii). $x_b, y_b \in (1, \infty)$.
$d_b(A_bx_b, B_by_b) = \frac{12}{5}, d_b(S_bx_b, T_by_b) = \frac{9}{2} + \frac{1}{x_b+y_b}, d_b(S_bx_b, A_bx_b) = \frac{9}{2} + \frac{1}{x_b+y_b}$,
$d_b(T_by_b, B_by_b) = \frac{12}{5}, d_b(S_bx_b, B_by_b) = \frac{12}{5}, d_b(T_by_b, A_bx_b) = \frac{9}{2} + \frac{1}{x_b+y_b}$.
We now consider

$$\begin{aligned} s^4[d_b(A_bx_b, B_by_b)]^2 &= (\tfrac{25}{24})^4(\tfrac{12}{5})^2 \\ &\le \tfrac{10}{51}(\tfrac{9}{2} + \tfrac{1}{x_b+y_b})^2 + \tfrac{1}{8}((\tfrac{9}{2} + \tfrac{1}{x_b+y_b})(\tfrac{12}{5})) + \tfrac{1}{4}((\tfrac{12}{5})(\tfrac{9}{2} + \tfrac{1}{x_b+y_b})) \\ &\le \lambda_1 \max\{[d_b(S_bx_b, T_by_b)]^2, [d_b(S_bx_b, A_bx_b)]^2, [d_b(T_by_b, B_by_b)]^2\} \\ &\quad + \lambda_2 \max\{\tfrac{d_b(S_bx_b, A_bx_b)d_b(S_bx_b, B_by_b)}{2}, \tfrac{d_b(T_by_b, B_by_b)d_b(T_by_b, A_bx_b)}{2}\} \\ &\quad + \lambda_3 \tfrac{d_b(S_bx_b, B_by_b)d_b(T_by_b, A_bx_b)}{2}. \end{aligned}$$

Case (iii). $x_b \in (1, \infty), y_b \in (0, 1)$.
$d_b(A_bx_b, B_by_b) = \frac{12}{5}, d_b(S_bx_b, T_by_b) = \frac{9}{2} + \frac{1}{x_b+y_b}, d_b(S_bx_b, A_bx_b) = \frac{9}{2} + \frac{1}{x_b+y_b}$,

$d_b(T_by_b, B_by_b) = \frac{12}{5}, d_b(S_bx_b, B_by_b) = \frac{12}{5}, d_b(T_by_b, A_bx_b) = \frac{9}{2} + \frac{1}{x_b+y_b},$

We now consider

$$\begin{aligned} s^4[d_b(A_bx_b, B_by_b)]^2 &= (\tfrac{25}{24})^4(\tfrac{12}{5})^2 \\ &\leq \tfrac{10}{51}(\tfrac{9}{2} + \tfrac{1}{x_b+y_b})^2 + \tfrac{1}{8}((\tfrac{12}{5})(\tfrac{9}{2} + \tfrac{1}{x_b+y_b})) + \tfrac{1}{4}((\tfrac{12}{5})(\tfrac{9}{2} + \tfrac{1}{x_b+y_b})) \\ &\leq \lambda_1 \max\{[d_b(S_bx_b, T_by_b)]^2, [d_b(S_bx_b, A_bx_b)]^2, [d_b(T_by_b, B_by_b)]^2\} \\ &\quad + \lambda_2 \max\{\tfrac{d_b(S_bx_b,A_bx_b)d_b(S_bx_b,B_by_b)}{2}, \tfrac{d_b(T_by_b,B_by_b)d_b(T_by_b,A_bx_b)}{2}\} \\ &\quad + \lambda_3 \tfrac{d_b(S_bx_b,B_by_b)d_b(T_by_b,A_bx_b)}{2}. \end{aligned}$$

Therefore A_b, B_b, S_b and T_b satisfy all the hypotheses of Theorem 2.1 and 1 is the unique common fixed point in X_b.

The following is an example in support of Theorem 2.2.

Example 3.2. *Let $X_b = [0, 1]$ and let $d_b : X_b \times X_b \to \mathbb{R}^+$ defined by*

$$d_b(x_b, y_b) = \begin{cases} 0 & \text{if } x_b = y_b, \\ \frac{11}{15} & \text{if } x_b, y_b \in [0, \frac{2}{3}), \\ \frac{99}{100} + \frac{x_b+y_b}{200} & \text{if } x_b, y_b \in [\frac{2}{3}, 1], \\ \frac{12}{25} & \text{otherwise.} \end{cases}$$

Then clearly (X_b, d_b) is a complete b-metric space with coefficient $s = \frac{25}{24}$.
We define $A_b, B_b, S_b, T_b : X_b \to X_b$ by

$A_b(x_b) = \frac{2}{3}$ *if* $x_b \in [0, 1]$, $B_b(x_b) = \begin{cases} \frac{1}{2} & \text{if } x_b \in [0, \frac{2}{3}) \\ \frac{2}{3} & \text{if } x_b \in [\frac{2}{3}, 1], \end{cases}$

$S_b(x_b) = \begin{cases} \frac{1}{2} & \text{if } x_b \in [0, \frac{2}{3}) \\ \frac{2+5x_b}{8} & \text{if } x_b \in [\frac{2}{3}, 1], \end{cases}$ *and* $T_b(x_b) = \begin{cases} 1 & \text{if } x_b \in [0, \frac{2}{3}) \\ \frac{4+x_b}{7} & \text{if } x_b \in [\frac{2}{3}, 1]. \end{cases}$

Clearly $A_b(X_b) \subseteq T_b(X_b)$ and $B_b(X_b) \subseteq S_b(X_b)$. $A_b(X_b) = \{\frac{2}{3}\}$ is b-closed.
We choose a sequence $\{x_{b_n}\}$ with $\{x_{b_n}\} = \frac{2}{3} + \frac{1}{n}, n \geq 4$ with
$\lim_{n\to\infty} A_bx_{b_n} = \lim_{n\to\infty} S_bx_{b_n} = \frac{2}{3}$*, hence the pair (A_b, S_b) satisfies the b-(E.A)-property.*
Clearly the pairs (A_b, S_b) and (B_b, T_b) are weakly compatible.
We take $\lambda_1 = \frac{10}{51}, \lambda_2 = \frac{1}{4}, \lambda_3 = \frac{1}{2}$. Then clearly $\lambda_1 + s\lambda_2 + s^2\lambda_3 \leq 1$.
With out loss generality, we assume that $x \geq y$.

Case (i). $x_b, y_b \in (0, \frac{2}{3})$.

$d_b(A_bx_b, B_by_b) = \frac{12}{25}, d_b(S_bx_b, T_by_b) = \frac{12}{25}, d_b(S_bx_b, A_bx_b) = \frac{12}{25},$
$d_b(T_by_b, B_by_b) = \frac{12}{25}, d_b(S_bx_b, B_by_b) = \frac{11}{15}, d_b(T_by_b, A_bx_b) = \frac{99}{100} + \frac{x_b+y_b}{200},$

We now consider

$$\begin{aligned} s^4[d_b(A_bx_b, B_by_b)]^2 &= (\tfrac{25}{24})^4(\tfrac{12}{25})^2 \\ &\leq \tfrac{10}{51}(\tfrac{12}{25})^2 + \tfrac{1}{8}((\tfrac{12}{25})(\tfrac{99}{100} + \tfrac{x_b+y_b}{200})) + \tfrac{1}{4}((\tfrac{11}{15})(\tfrac{99}{100} + \tfrac{x_b+y_b}{200})) \\ &\leq \lambda_1 \max\{[d_b(S_bx_b, T_by_b)]^2, [d_b(S_bx_b, A_bx_b)]^2, [d_b(T_by_b, B_by_b)]^2\} \\ &\quad + \lambda_2 \max\{\tfrac{d_b(S_bx_b,A_bx_b)d_b(S_bx_b,B_by_b)}{2}, \tfrac{d_b(T_by_b,B_by_b)d_b(T_by_b,A_bx_b)}{2}\} \\ &\quad + \lambda_3 \tfrac{d_b(S_bx_b,B_by_b)d_b(T_by_b,A_bx_b)}{2}. \end{aligned}$$

Case (ii). $x_b, y_b \in (\frac{2}{3}, 1]$.

$d_b(A_bx_b, B_by_b) = 0$. *In this case the inequality (1) trivially holds.*

Case (iii). $x_b \in (\frac{2}{3}, 1], y_b \in (0, \frac{2}{3})$.
$d_b(A_b x_b, B_b y_b) = \frac{12}{25}, d_b(S_b x_b, T_b y_b) = \frac{99}{100} + \frac{x_b+y_b}{200}, d_b(S_b x_b, A_b x_b) = \frac{99+x_b}{100}$,
$d_b(T_b y_b, B_b y_b) = \frac{12}{25}, d_b(S_b x_b, B_b y_b) = \frac{12}{25}, d_b(T_b y_b, A_b x_b) = \frac{99}{100} + \frac{x_b+y_b}{200}$,
We now consider

$$\begin{aligned} s^4[d_b(A_b x_b, B_b y_b)]^2 &= (\tfrac{25}{24})^4(\tfrac{12}{25})^2 \\ &\leq \tfrac{10}{51}(\tfrac{99+x_b}{100})^2 + \tfrac{1}{8}((\tfrac{99+x_b}{100})(\tfrac{12}{25})) + \tfrac{1}{4}(\tfrac{12}{25}))((\tfrac{99}{100} + \tfrac{x_b+y_b}{200}) \\ &\leq \lambda_1 \max\{[d_b(S_b x_b, T_b y_b)]^2, [d_b(S_b x_b, A_b x_b)]^2, [d_b(T_b y_b, B_b y_b)]^2\} \\ &\quad + \lambda_2 \max\{\tfrac{d_b(S_b x_b, A_b x_b) d_b(S_b x_b, B_b y_b)}{2}, \tfrac{d_b(T_b y_b, B_b y_b) d_b(T_b y_b, A_b x_b)}{2}\} \\ &\quad + \lambda_3 \tfrac{d_b(S_b x_b, B_b y_b) d_b(T_b y_b, A_b x_b)}{2}. \end{aligned}$$

Therefore A_b, B_b, S_b and T_b satisfy all the hypotheses of Theorem 2.2 and $\frac{2}{3}$ is the unique common fixed point in X_b.

Corolary 3.1. *Let $\{A_n\}_{n=1}^{\infty}, S_b$ and T_b be selfmaps on a complete b-metric space (X_b, d_b) satisfying $A_1 \subseteq S_b(X_b)$ and $A_1 \subseteq T_b(X_b)$. Assume that there exist positive reals $\lambda_1, \lambda_2, \lambda_3$ with $\lambda_1 + s\lambda_2 + s^2\lambda_3 \leq 1$ such that*

$$\begin{aligned} s^4[d_b(A_1 x_b, A_j y_b)]^2 \leq{} & \lambda_1 \max\{[d_b(S_b x_b, T_b y_b)]^2, [d_b(S_b x_b, A_1 x_b)]^2, [d_b(T_b y_b, A_j y_b)]^2\} \\ & + \lambda_2 \max\{\tfrac{d_b(S_b x_b, A_1 x) d_b(S_b x_b, A_j y_b)}{2}, \tfrac{d_b(T_b y_b, A_j y) d_b(T_b y_b, A_1 x_b)}{2}\} \\ & + \lambda_3 \tfrac{d_b(S_b x_b, A_j y_b) d_b(T_b y_b, A_1 x_b)}{2}. \end{aligned} \tag{15}$$

for all $x_b, y_b \in X_b$ and $j = 1, 2, 3, \ldots$ .. If the pairs (A_1, S_b) and (A_1, T_b) are weakly compatible and one of the range sets $A_1(X_b), S_b(X_b)$ and $T_b(X_b)$ is closed, then $\{A_n\}_{n=1}^{\infty}, S_b$ and T_b have a unique common fixed point in X_b.

Proof. Under the assumptions on A_1, S_b and T_b, the existence of common fixed point z_b of A_1, S_b and T_b follows by choosing $A_b = B_b = A_1$ in Theorem 2.1.
Therefore $A_1 z_b = S_b z_b = T_b z_b = z_b$.
Now, let $j \in \mathbb{N}$ with $j \neq 1$.
We now consider

$$\begin{aligned} s^4[d_b(z_b, A_j z_b)]^2 &= s^4[d_b(A_1 z_b, A_j z_b)]^2 \\ &\leq \lambda_1 \max\{[d_b(S_b z_b, T_b z_b)]^2, [d_b(S_b z_b, A_1 z_b)]^2, [d_b(T_b z_b, A_j z_b)]^2\} \\ &\quad + \lambda_2 \max\{\tfrac{d_b(S_b z_b, A_1 z_b) d_b(S_b z_b, A_j z_b)}{2}, \tfrac{d_b(T_b z_b, A_j z_b) d_b(T_b z_b, A_1 z_b)}{2}\} \\ &\quad + \lambda_3 \tfrac{d_b(S_b z_b, A_j z_b) d_b(T_b z_b, A_1 z_b)}{2}. \end{aligned} \tag{16}$$

From the inequality (16), we have
$s^4[d_b(z_b, A_j z_b)]^2 \leq \lambda_1[d_b(z_b, A_j z_b)]^2$
which implies that $(s^4 - \lambda_1)[d_b(z_b, A_j z_b)]^2 \leq 0$.
Since $(s^4 - \lambda_1) \geq 0$, we have $A_j z_b = z_b$ for $j = 1, 2, 3, \ldots$ and uniqueness of common fixed point follows from the inequality (15).

Therefore $\{A_n\}_{n=1}^{\infty}, S_b$ and T_b have a unique common fixed point in X_b. □

Corolary 3.2. *Let* $\{A_n\}_{n=1}^{\infty}, S_b$ *and* T_b *be selfmaps on a b-metric space* (X_b, d_b) *satisfy the conditions* $A_1 \subseteq S_b(X_b), A_1 \subseteq T_b(X_b)$ *and (15). If one of the pairs* (A_1, S_b) *and* (A_1, T_b) *satisfies the b-(E.A)-property and that one of the subspace* $A_1(X), S_b(X_b)$ *or* $T_b(X_b)$ *is b-closed in* X_b*. Then the pairs* (A_1, S_b) *and* (A_1, T_b) *have a point of coincidence in* X_b*. Moreover, if the pairs* (A_1, S_b) *and* (A_1, T_b) *are weakly compatible, then* $\{A_n\}_{n=1}^{\infty}, S_b$ *and* T_b *have a unique common fixed point in* X_b*.*

Proof. Under the assumptions on A_1, S_b and T_b, the existence of common fixed point z_b of A_1, S_b and T_b follows by choosing $A_b = B_b = A_1$ in Theorem 2.2.
Therefore $A_1 z_b = S_b z_b = T_b z_b = z_b$.
Now, let $j \in \mathbb{N}$ with $j \neq 1$.
We now consider

$$\begin{aligned} s^4[d_b(z_b, A_j z_b)]^2 &= s^4[d_b(A_1 z_b, A_j z_b)]^2 \\ &\leq \lambda_1 \max\{[d_b(S_b z_b, T_b z_b)]^2, [d_b(S_b z_b, A_1 z_b)]^2, [d_b(T_b z_b, A_j z_b)]^2\} \\ &\quad + \lambda_2 \max\{\tfrac{d_b(S_b z_b, A_1 z_b) d_b(S_b z_b, A_j z_b)}{2}, \tfrac{d_b(T_b z_b, A_j z_b) d_b(T_b z_b, A_1 z_b)}{2}\} \\ &\quad + \lambda_3 \tfrac{d_b(S_b z_b, A_j z_b) d_b(T_b z_b, A_1 z_b)}{2}. \end{aligned} \tag{17}$$

From the inequality (17), we have
$s^4[d_b(z_b, A_j z_b)]^2 \leq \lambda_1 [d_b(z_b, A_j z_b)]^2$ which implies that
$(s^4 - \lambda_1)[d_b(z_b, A_j z_b)]^2 \leq 0$.
Since $(s^4 - \lambda_1) \geq 0$, we have $A_j z_b = z_b$ for $j = 1, 2, 3, \dots$ and uniqueness of common fixed point follows from the inequality (15).

Therefore $\{A_n\}_{n=1}^{\infty}, S_b$ and T_b have a unique common fixed point in X_b. □

4 Conclusion

In this paper, we introduced generalized contraction for two pairs of selfmaps in complete b-metric spaces and proved the existence and of common fixed points. Our results extend/generalize the known results that are available in the literature. A sequence of selfmaps is added as an extension of the same. We provided examples in support of our results and some corollaries to our results are presented.

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