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Octagonal Prime Graceful Labeling

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Abstract

The main objectives of this research work is to explore and detect some new types of graphs that exhibit octagonal prime graceful labeling. The methodology entails developing a mathematical formulation for labeling a given graph's vertices and demonstrating that these formulations result in octagonal prime graceful labeling. Here we describe octagonal prime graceful labeling which is a new version of octagonal graceful labeling. In the present paper, we establish octagonal prime graceful labeling for fan graph and friendship graph. Octagonal graceful labeling was introduced by S. Mahendran, K. Kovusalya1 and P. Namasivayam, here we find the octagonal prime graceful labeling for the join of two graphs, shell graph, generalised butterfly graph, prism and polar grid graph. This is the first attempt of its sort, involving the investigation of octagonal prime graceful labeling for special graphs.

Keywords: Octagonal numbers, Octagonal graceful labeling, Octagonal graceful graph, Octagonal prime graceful labeling
2020 AMS subject classifications: 05C78 [1]

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1 Introduction

The finite, straightforward, as well as undirected graphs are examined in this paper. The graph definition with q edges and p vertices is $G = (V, E)$. Labeling is the process of giving numbers to a graph's vertices, edges, or both. If the mapping's domain is a set of edge (vertices/both), this labeling is referred to as a vertex labeling (edge/both). Graph labeling strategies include graceful labeling, incidence labeling, gracious labeling, radio labeling, antimagic labeling and prime labeling. One of these most often-used graph labeling approaches is graceful labeling [15]. The concept of β-valuation of a graph is proposed by Rosa[1]. According to Golomb[12], it was a graceful labeling.

Let's assume that G is a (p, q) graph. The graceful G labeling is considered to be a one-to-one function if the induced edge labeling $f : V(G) \to \{0, 1, 2, ..., q\}$ represented by $f^*(e) = |f(u) - f(v)|$ for all edges $e = uv$ of G is likewise one-to-one. A 'G' graph with graceful labeling is referred to as a graceful graph. Several families of graceful graphs were built in [12]. A thorough examination of the many forms of graceful labeling can be found in [8].

Applications include creating an addressing system for communication networks, producing X-Ray crystallography, selecting the best circuit topologies and solving additive number theory problems and others, labelled graphs are becoming a more important family of mathematical models [7].

Numbers of the form $O_n = n(3n-2)$ for all $n \geq 1$ are called octagonal numbers. Let G be a graph with p vertices and q edges. Let $f : V(G) \to \{0, 1, 2, ..., O_m\}$ where O_m is the m^{th} octagonal number be an injective function. Define the function $f^* : E(G) \to \{1, 8, 21, ..., O_m\}$ such that $f^*(uv) = |f(u) - f(v)|$ for all edges $uv \in E(G)$. If $f^*(E(G))$ is a sequence of distinct consecutive octagonal numbers $1, 2, ..., O_q$, then the function f is said to be octagonal graceful labeling and the graph which admits such a labeling is called a octagonal graceful graph[7].

In this paper we have introduced octagonal prime graceful labeling and it is defined as follows. Let G be a graph with p vertices and q edges. Define a bijection $f : V(G) \to \{1, 8, ..., p(3p-2)\}$ by $f(v_i) = i(3i-2)$ for every i from 1 to p and define a $1-1$ mapping $f^*_{opgl} : E(G) \to$ set of natural number N such that $f^*(uv) = |f(u) - f(v)|$ for all edges $(uv) \in E(G)$. The induced function f is said to be octagonal prime graceful labeling if the gcin of each vertex of degree atleast 2 is one.

This research examines the octagonal prime graceful labeling fan graph and friendship graph and some special graphs namely join of two graphs, shell graph, generalised butterfly graph, prism and polar grid graph.

2 Preliminaries

Definition 2.1. *[9] A fan graph obtained by joining all vertices of F_n, $n \geq 2$ is a path P_n to a further vertex, called the centre. Thus F_n contains $n+1$ vertices say C and $(2n-1)$ edges, say $cv_i, 1 \leq i \leq n$ and $v_iv_{i+1}, 1 \leq i \leq n-1$.*

Definition 2.2. *[5] If G_1 and G_2 are disjoint graphs then the join of G_1 and G_2 written as $G_1 + G_2$ is the graph consisting of the union $G_1 \cup G_2$, together with all edges of the type v_1v_2 where $v_1 \in V(G_1)$ and $v_2 \in V(G_2)$.*

Definition 2.3. *[10] A shell graph is defined as a cycle C_n with $(n-3)$ chords sharing a common end point called the apex. Shell graph are denoted as $C_(n, n-3)$. A shell S_n is also called fan f_{n-1}.*

Definition 2.4. *[14] A generalized butterfly graph, BF_n , obtained by inserting vertices to every wing with assumption that sum of inserting vertices to every wing are same then it has $2n+1$ vertices and $4n-2$ edges. Let the vertex set of BF_n be $V(BF_n) = \{v_i | i = 0, 1, 2, ..., 2n\}$ and the edge set of BF_n be $E(BF_n) = \{(v_i, v_{i+1}) | i = 1, 2, ..., n-1, n+1, ..., 2n-1\} \cup \{(v_0, v_i) | i = 1, 2, ..., 2n\}$.*

Definition 2.5. *[3] The friendship graph $Fr_n^{(3)}$, is a planar undirected graph with $2n+1$ vertices and $3n$ edges constructed by joining n copies of the cycle graph C_3 with a common vertex*

Definition 2.6. *[16] For $n \geq 3$, let $\{v_0, v_1, v_2, ..., v_n\}$ be the vertices of W_n with hub vertex v_0 and $w_n^{'} = \{u_0, u_1, u_2, ..., u_n\}$ be a copy of W_n. Define the $Prism(W_n)$, called the prism of W_n by , $P(W_n) = K_2 \times W_n$ i.e., joining v_0 of W_n to the corresponding vertex u_0 of $w_n^{'}$ and each v_i of W_n to the corresponding vertex u_i of $W_n^{'}$ for all $i \in \{1, 2, ..., n\}$. Thus, $E(P(W_n)) = E(W_n) \cup E(W_n^{'}) \cup \{v_iu_i, i \in \{1, 2, ..., n\}\} \cup \{v_0u_0\}$.*

Definition 2.7. *[2] The polargrid graph $P_{m,n}$ is the graph consists of n copies of circles C_m which will be numbered from the inner most circle to the outer circle as $C_m^{(1)}, C_m^{(2)}, ..., C_m^{(n-1)}, C_m^{(n)}$ and m copies of paths P_{n+1} intersected at the center vertex v_0 which will be numbered as $P_{n+1}^{(1)}, P_{n+1}^{(2)}, ..., P_{n+1}^{(m-1)}, P_{n+1}^{(m)}$.*

3 Main Results

Theorem 3.1. *The Fan graph F_n admits octagonal prime graceful labeling.*

Proof. Let $G = F_n$ and let $v_1, v_2, ..., v_{n+1}$ are the vertices of G.
Let p and q be the total number of vertices and edges of G respectively.
The fan graph F_n has $n+1$ vertices and $(2n-1)$ edges.
$ie)$ $p = |V(F_n)| = n+1$ and $q = |E(F_n)| = 2n-1$.
Define a function $f : V(G) \to \{1, 8, ..., p(3p-2)\}$ by $f(v_i) = i(3i-2)$,
where $i = 1, 2, ..., n+1$ and the edge labelings are given as
$f^*_{opgl}(v_i v_{i+1}) = 6i + 1$, where $i = 2, 3, ..., n$
$f^*_{opgl}(v_1 v_{i+1}) = 3i^2 + 4i$, where $i = 1, 2, ..., n$
Clearly f^*_{opgl} is an injection and f induces the function f^*_{opgl} on $E(G)$ such that $f^*_{opgl}(uv) = |f(u) - f(v)|$.
Also the $gcin$ of (v_1) = gcd of edges incident on $v_1 = 1$
gcin of (v_{i+1}) = gcd of edges incident on $v_{i+1} = 1$, where $i = 1, 2, ..., n$
In general, $gcin$ of v_{n+1} = gcd of edges incident on v_{n+1}
Hence the $gcin$ of each vertex of degree atleast 2 is one.
Therefore f is said to be an octagonal prime graceful labeling.
Hence the graph F_n is an octagonal prime graceful graph.

Example 3.1. *Fan graph F_6 admits octagonal prime graceful labeling.*

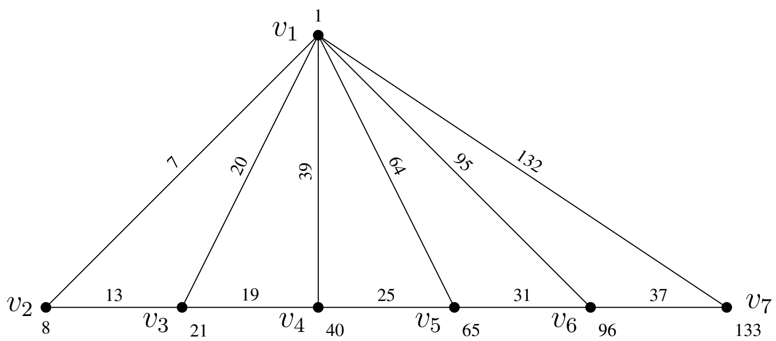

Figure 1: $Octagonal\ prime\ graceful\ labeling\ of\ F_7$

Theorem 3.2. *The graph $P_2 + mK_1$ is an octagonal prime graceful graph for $m \leq 6$.*

Proof. Consider a path P_2 with two vertices v_1, v_2.
Let $v_3, v_4, ..., v_{m+2}$ be the m isolated vertices.
Joining v_1, v_2 with v_i , $3 \leq i \leq m+2$ we get $P_2 + mK_1$

Let $G = P_2 + mK_1$
Let $V(G) = \{v_1, v_2, v_3, ..., v_{m+2}\}$ and $E(G) = \{v_1v_2\}$
$\cup \{v_1v_i/3 \leq i \leq m+2\} \cup \{v_2v_i/3 \leq i \leq m+2\}$
Then $|V(G)| = 2 + m$ and $|E(G)| = 2m + 1$
Let p and q denote the total number of vertices and edges of graph G respectively.
Define a function $f : V(G) \to \{1, 8, ..., p(3p-2)\}$ by $f(v_i) = i(3i-2)$, where $1 \leq i \leq m+2$ and the edge labels are as follows:
$f^*_{opgl}(v_1v_{i+1}) = 3i^2 + 4i$, where $i = 1, 2, ..., m+1$ and
$f^*_{opgl}(v_2v_{i+2}) = f(v_1v_{i+2}) - f(v_1v_2)$, where $i = 1, 2, ..., m$
Clearly , f^*_{opgl} is an injection and f induces the function f^*_{opgl} on $E(G)$ such that $f^*_{opgl}(uv) = |f(u) - f(v)|$
Also $gcin$ of v_1 = gcd of edges incident on v_1
= gcd of edges $(\{v_1v_{i+2}\} \cup \{v_1v_2\}) = 1$, where $i = 1, 2, ..., m$
$gcin$ of v_2 = gcd of edges incident on v_2
= gcd of edges $(\{v_2v_{i+2}\} \cup \{v_1v_2\}) = 1$, where $i = 1, 2, ..., m$ and
$gcin$ of v_{i+2} = gcd of edges incident on $v_{i+2} = 1$, where $i = 1, 2, ..., m$
In general, $gcin$ of v_{m+2} = gcd of edges incident on $v_{m+2} = 1$
Hence the $gcin$ of each vertex of degree greater than one is 1
Therefore f is said to be an octagonal prime graceful labeling.
Hence the graph $P_2 + mK_1$ is an octagonal prime graceful graph.

Example 3.2. *The octagonal prime graceful labeling of graph $P_2 + 6K_1$ is shown below.*

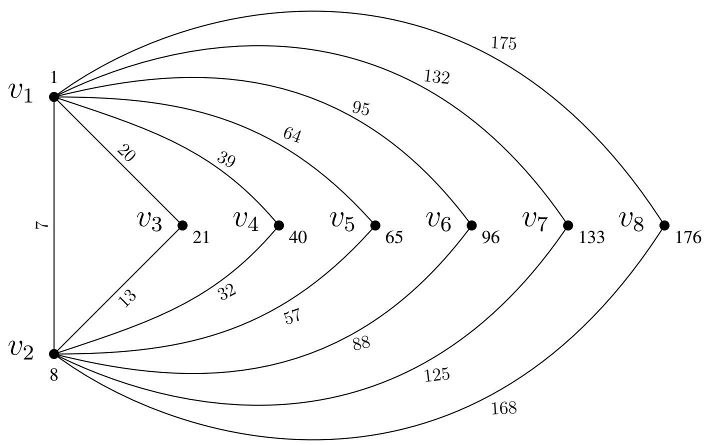

Figure 2: $Octagonal\ prime\ graceful\ labeling\ of\ P_2 + 6K_1$

3.1 Remark

The Graph $P_2 + mK_1$ is octagonal graceful but does not admits an octagonal prime graceful labeling for $m \geq 7$.

Proof. The Graph $P_2 + mK_1$ is octagonal graceful but not an octagonal prime graceful graph for $m \geq 7$ which is shown below by a figure $P_2 + 7K_1$ □

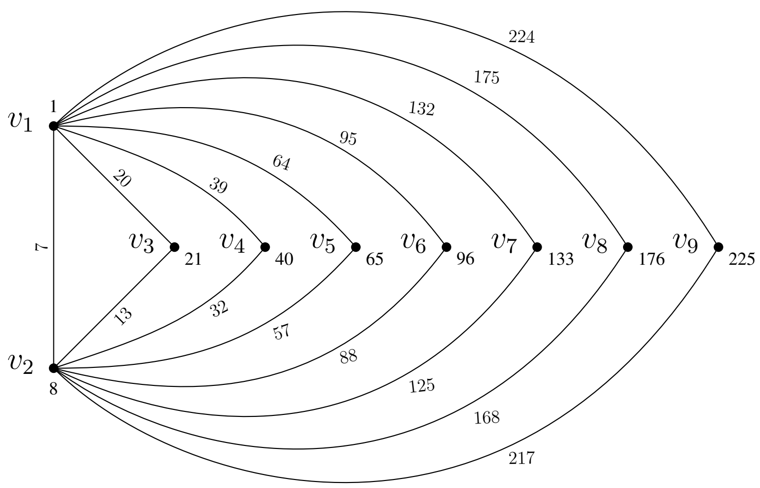

Figure 3: $P_2 + 7K_1$ *which is not an Octagonal prime graceful labeling.*
Here gcin of vertex v_9 $= gcd\{224, 217\} \neq 1$

Theorem 3.3. *The Shell graph S_n is an octagonal prime graceful graph.*

Proof. Let S_n be a shell graph and let $v_1, v_2, ..., v_n$ be the successive vertices of S_n.
Let $|V(S_n)| = n$ and $|E(S_n)| = 2n - 3$
Let p and q denote the total number of vertices and edges of graph S_n respectively.
Define a function $f : V(S_n) \rightarrow \{1, 8, ..., p(3p - 2)\}$ by $f(v_i) = i(3i - 2)$, where $i = 1, 2, ..., n$ and the edge labels are as follows:
$f^*_{opgl}(v_i v_{i+1}) = 6i + 1$, for $1 \leq i \leq n - 1$
$f^*_{opgl}(v_1 v_n) = O_n - 1$
Clearly f^*_{opgl} is an injection and f induces the function f^*_{opgl} on $E(G)$ such that $f^*_{opgl}(uv) = |f(u) - f(v)|$
Also $gcin$ of v_1 = gcd of edges incident on v_1

= gcd of edges $(v_1 v_{i+1}) = 1$, where $1 \leq i \leq n-1$
$gcin$ of v_2 = gcd of edges incident on v_2
= gcd of edges $\{f^*_{opgl}(v_1 v_2), f^*_{opgl}(v_2 v_3)\} = 1$
$gcin$ of v_i = gcd of edges incident on v_i , where $3 \leq i \leq n-1$
$gcin$ of v_i = gcd of edges $\{v_i v_{i-1}, v_i v_1, v_i v_{i+1}\}$, where $3 \leq i \leq n-1$ and
In general , $gcin$ of v_n= gcd of edges incident on v_n
= gcd of edges $\{v_n v_1, v_n v_{n-1}\} = 1$
Hence the $gcin$ of each vertex of degree atleast two is 1
Therefore f is said to be an octagonal prime graceful labeling.
Hence the graph S_n is an octagonal prime graceful graph

Example 3.3. *The octagonal prime graceful labeling of shell graph* S_8 *is shown in the figure.*

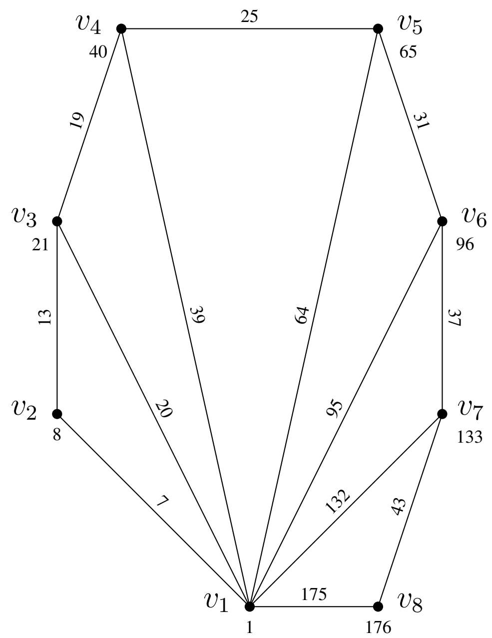

Figure 4: $Octagonal\ prime\ graceful\ labeling\ of\ S_8$

Theorem 3.4. *For* $n \geq 2$ *, a generalized butterfly graph* BF_n *admits octagonal prime graceful labeling.*

Proof. A generalized butterfly graph , BF_n is obtained by inserting vertices to every wing with the assumption that sum of inserting vertices to every wing are same then it has $2n+1$ vertices and $4n-2$ edges.
Let the vertex set of BF_n be $V(BF_n) = \{v_i / i = 0, 1, 2, ..., 2n\}$ and the edge set of BF_n be $E(BF_n) = \{(v_i v_{i+1}) / i = 1, 2, ..., n-1, n+1, ..., 2n-1\} \cup \{(v_0 v_i) / i = 1, 2, ..., 2n\}$

Let BF_n has $\{v_0\}$ as an apex , $\{v_1, v_2, ..., v_{n-1}, v_n\}$ as vertices on right wing and $\{v_{n+1}, v_{n+2}, ..., v_{2n-1}, v_{2n}\}$ as vertices on left wing.
Let $f(v_o) = 1$
Define a function $f : V(BF_n) \rightarrow \{1, 8, ..., p(3p-2)\}$ by $f(v_{i-1}) = i(3i-2)$, where $i = 2, 3, ..., 2n+1$ and the edge labels are as follows:
$f^*_{opgl}(v_0 v_i) = 3i^2 + 4i$, for $1 \leq i \leq 2n+1$
$f^*_{opgl}(v_{i-1} v_i) = 6i + 1$, for $2, 3, ..., n$ and
$f^*_{opgl}(v_{i-1} v_i) = 6i + 1$, for $i = n+2, n+3, ..., 2n$
Clearly, f^*_{opgl} is an injection and f induces the function f^*_{opgl} on $E(G)$ such that $f^*_{opgl}(uv) = |f(u) - f(v)|$
Also the $gcin$ of each vertex of degree greater than one is 1
Therefore f is said to be an octagonal prime graceful labeling.
Hence the generalized butterfly graph BF_n is an octagonal prime graceful graph for $n \geq 2$

Example 3.4. *The Generalized butterfly graph BF_5 admits Octagonal prime graceful labeling.*

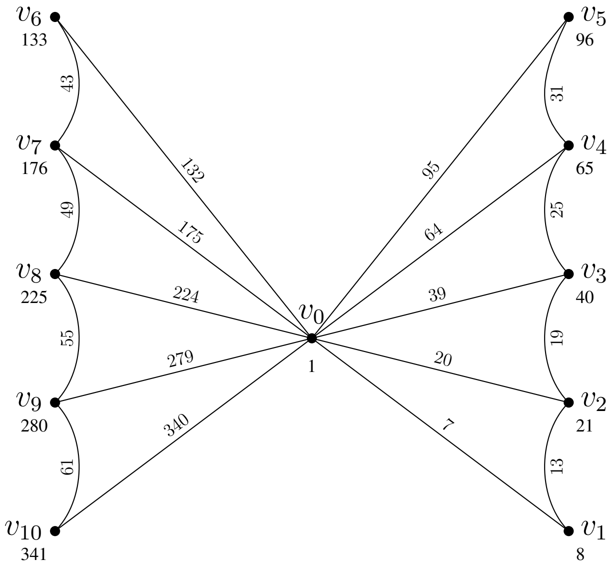

Figure 5: $Octagonal\ prime\ graceful\ labeling\ of\ BF_5$

Theorem 3.5. *The Friendship graph Fr_n is an octagonal prime graceful graph.*

Proof. The friendship graph Fr_n is a planar undirected graph constructed by joining n copies of cycle graph C_3 with common vertex.
Let $p = |V(Fr_n)| = 2n+1$ and $q = |E(Fr_n)| = 3n$
Let the vertex set of Fr_n be $V(Fr_n) = \{v_i / i = 0, 1, 2, ..., 2n\}$ with v_0 as the central vertex and edge set of Fr_n be $E(Fr_n) = \{v_0 v_i, v_i v_{i+1}, i = 1, 2, ..., 2n\}$

Define a function $f : V(Fr_n) \to \{1, 8, ..., p(3p-2)\}$ by $f(v_{i-1}) = i(3i-2)$,where $i = 2, 3, ..., 2n + 1$ and the edge labels are as follows:
$f^*_{opgl}(v_0v_i) = 3i^2 + 4i$, for $1 \leq i \leq 2n$
and $f^*_{opgl}(v_{i-1}v_i) = 6i + 1$, for $2 \leq i \leq 2n$
Clearly, f^*_{opgl} is an injection and f induces the function f^*_{opgl} on $E(Fr_n)$ such that $f^*_{opgl}(uv) = |f(u) - f(v)|$
Also the $gcin$ of each vertex of degree atleast two is 1
Therefore f is said to be an octagonal prime graceful labeling.
Hence the friendship graph Fr_n is an octagonal prime graceful graph.

Example 3.5. *The Friendship graph Fr_7 is an Octagonal prime graceful graph.*

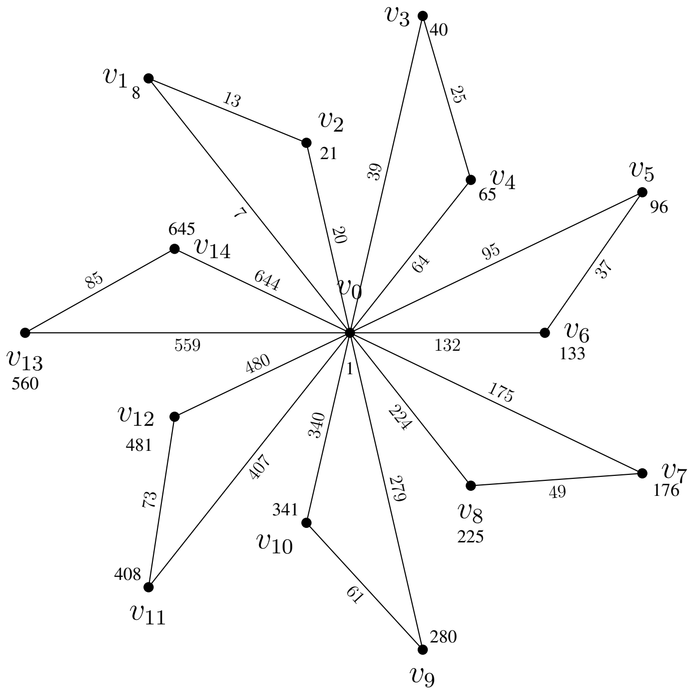

Figure 6: $Octagonal\ prime\ graceful\ labeling\ of\ Fr_7$

Theorem 3.6. *The Prism of the wheel $P(W_n)$ is an Octagonal prime graceful graph for $n \geq 3$.*

Proof. Let $P(W_n)$ be the prism of the wheel graph and let the vertices of $P(W_n)$ be $\{v_0, v_0^{'}, v_i/1 \leq i \leq 2n\}$ and the corresponding edges of $P(W_n)$ be $E(P(W_n) = E(W_n) \cup E(W_n^{'}) \cup \{v_iv_j/i = 1, 2, ..., n, j = n+1, n+2, ..., 2n\} \cup \{v_0v_0^{'}\}$ where $W_n^{'}$ is the copy of W_n
Let $p = 2n + 2$ and $q = 5n + 1$ be the total number of vertices and edges of the prism of wheel $P(W_n)$ respectively.

Define a function $f : V(P(W_n)) \to \{1, 8, ..., p(3p-2)\}$ by $f(v_{i-1}) = i(3i-2)$, where $i = 2, 3, ..., 2n+1$ and let
$f(v_0) = 1$ and
$f(v_0^{'}) = O_{2n+2}$
Let the edge labels are $f^*_{opgl}(v_0^{'}v_i) = 3i^2 + 4i$, for $1 \le i \le n$
Also $f^*_{opgl}(v_0^{'}v_i) = f(v_0^{'}) - O_{i+1}$, for $n+1 \le i \le 2n$
$f^*_{opgl}(v_{i-1}v_i) = 6i + 1$, for $i = 2, 3, ..., n, n+2, n+3, ..., 2n$
$f^*_{opgl}(v_0v_0^{'}) = O_{2n+2} - O_1$
$f^*_{opgl}(v_iv_{n+i}) = O_{n+1+i} - O_{i+1}$, for $1 \le i \le n$
$f^*_{opgl}(v_nv_1) = O_{n+1} - f(v_1)$
$f^*_{opgl}(v_{2n}v_{n+1}) = O_{2n+1} - f(v_{n+1})$
Clearly, f^*_{opgl} is an injection and f induces the function f^*_{opgl} on $E(P(W_n))$ such that $f^*_{opgl}(uv) = |f(u) - f(v)|$
Also the $gcin$ of each vertex of degree atleast two is 1
Therefore f is said to be an octagonal prime graceful labeling.
Hence the prism of the wheel $P(W_n)$ is an octagonal prime graceful graph for $n \ge 3$.

Example 3.6. *The prism graph $P(W_5)$ is an octagonal prime graceful graph .*

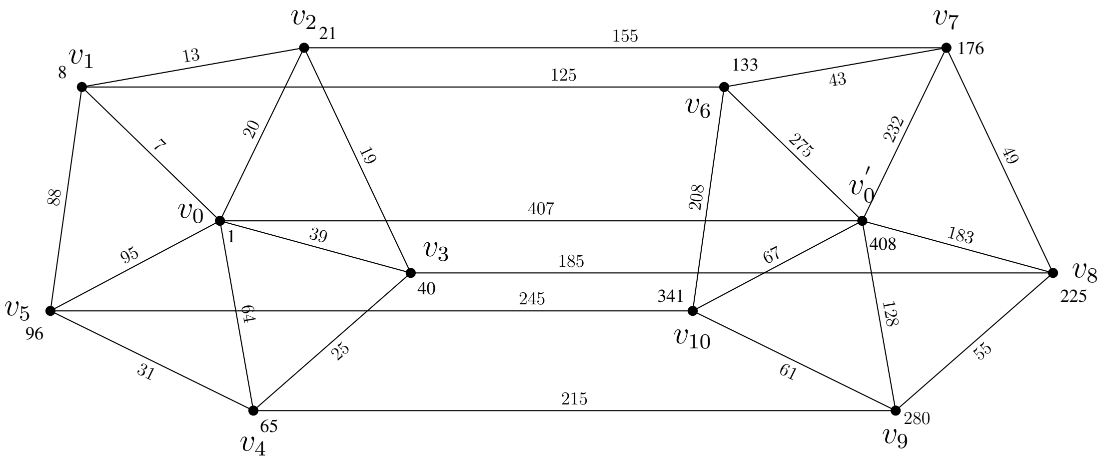

Figure 7: $Octagonal\ prime\ graceful\ labeling\ of\ P(W_5)$

Theorem 3.7. *If m and n are positive integers such that $m \ge 3$ and $n \ge 2$, then the polar grid graph $P_{m,n}$ is an octagonal prime graceful graph.*

Proof. Let $P_{m,n}$ be the polar grid graph and let the vertices and edges of $P_{m,n}$ be respectively
$p = |V(P_{m,n})| = mn + 1$ and $q = |E(P_{m,n})| = 2mn$
Let v_0 be the central vertex and v_i where $1 \le i \le mn$ be the remaining vertices.

Define a function $f : V(P_{m,n}) \to \{1, 8, ..., p(3p-2)\}$ by $f(v_{i-1}) = i(3i-2)$, where $i = 2, 3, ..., mn+1$.
The edges of $P_{m,n}$ are labeled in such a way that
$f^*_{opgl}(v_0 v_i) = 3i^2 + 4i$, for $1 \leq i \leq m$
$f^*_{opgl}(v_{i-1} v_i) = 6i + 1$, where $i = 2, 3, ..., m, m+2, ..., 2m, 2m+2, ..., 3m, 3m+2, ..., mn$
$f^*_{opgl}(v_i v_j) = f(v_j) - f(v_i)$, for $1 \leq i \leq m(n-1)$ and $m+1 \leq j \leq mn$ and
$f^*_{opgl}(v_i v_j) = f(v_i) - f(v_j)$, where $i = mk$ and $j = mk - (m-1)$ for $1 \leq k \leq n$
Here all the edge labels are distinct.
Clearly , f^*_{opgl} is an injection and f induces the function f^*_{opgl} on $E(P_{m,n})$ such that $f^*_{opgl}(uv) = |f(u) - f(v)|$
Also the $gcin$ of each vertex of degree atleast two is 1
Therefore f is said to be an octagonal prime graceful labeling.
Hence the polar grid graph $P_{m,n}$ is an octagonal prime graceful graph for $m \geq 3$ and $n \geq 2$.

Example 3.7. *An octagonal prime graceful labeling of polar grid $P_{m,n}$ where both m and n are even is given below*

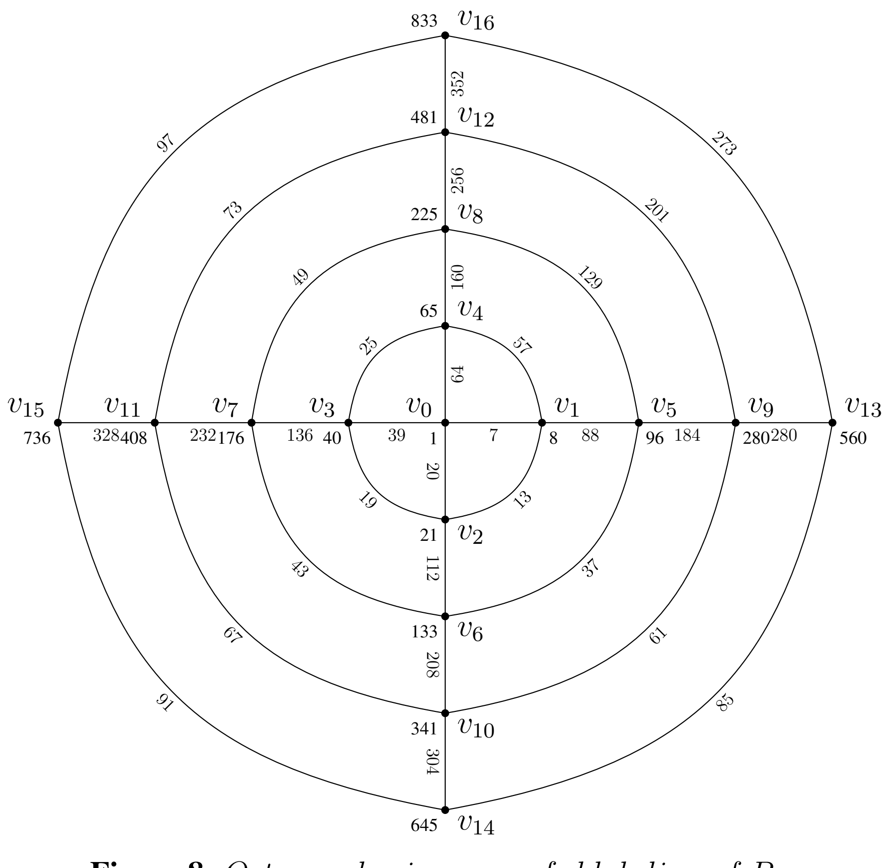

Figure 8: $Octagonal\ prime\ graceful\ labeling\ of\ P_{4,4}$

3.2 Remark.

If m and n are positive integers such that $m \leq 3$ and $n \leq 2$, then the polar grid graph $P_{m,n}$ is not an octagonal prime graceful graph.

Proof. We prove this by an example , that is the graph $P_{2,1}$ is not an octagonal prime graceful graph.

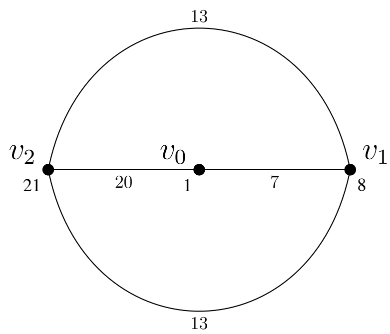

Figure 9: $P_{2,1}$
In the above figure the edge labels are not distinct
Hence $P_{2,1}$ is not an octagonal prime graceful graph

4 Conclusion

The authors of this study investigated the octagonal prime graceful labeling of fan F_n and friendship graph Fr_n. Further octagonal prime graceful labeling of some special graphs namely join of two graphs $P_2 + mK_1$, shell graph S_n, generalised butterfly graph BF_n, prism $P(W_n)$ and polar grid graph $P_{m,n}$ have been studied. We have also proved $P_2 + mK_1$ for $m \geq 7$ and polar grid graph $P_{m,n}$ for $m \leq 3$ & $n \leq 2$ are not octagonal prime graceful. Future development requires combination of the prime graceful labeling with some other polygonal numbers such as triangular, pentagonal, tetrahedral etc with reference from this study.

References

[1] A. Rosa, On certain valuation of the vertices of the graph, *in Theory of graphs (International Symposium, Rome*, July 1963), (1964), 349 –355.

[2] Daoud, Salama Nagy,Edge even graceful labeling of polar grid graphs, *Symmetry*, **11(1)** (2019), 38, doi:10.3390/sym11010038

[3] Elsonbaty, Amr and Daoud, Salama Nagy, Edge even graceful labeling of some path and cycle related graphs, *Ars combinatoria*, **130** (2017), 79–96.

[4] Kani, C and Asha, S, 4-Square Sum E-Cordial Labeling for Some Graphs, *Mathematical Statistician and Engineering Applications*, **71(4)** (2022), 1475–1480, doi:10.17762/msea.v71i4.643.

[5] Lina, S Selestin and Asha , On Sequential Graphs, *Bulletin of Pure & Applied Sciences-Mathematics and Statistics*,**37(1)** (2018), 54–62, doi:10.1002/jgt.3190070208.

[6] Lina, S Selestin and Asha , Odd triangular graceful labeling on simple graphs, *Malaya Journal of Matematik*, **8(4)** (2020), 1574–1576, doi:10.26637/MJM0804/0040.

[7] Mahendran, S, Octagonal Graceful Labeling of Some Special Graphs, *World Scientific News*, 2021, 156.

[8] Murugesan, S and Jayaraman, D and Shiama, J, Tetrahedral and pentatopic sum labeling of graphs, *International Journal of Applied Information Systems*, **5** (2013),31–35.

[9] Meena, S and Vaithilingam, K,Prime labeling for some fan related graphs, *International Journal of Engineering Research & Technology (IJERT)*, **1(9)** (2012).

[10] Sivasakthi, S Meena M Renugha M, Cordial Labeling For Different Types of Shell Graph, *International Journal of Scientific & Engineering Research*, **6** (2015).

[11] Sunoj, BS and Varkey, Mathew TK,Hexagonal Difference Prime Labeling of Some Path Graphs, *Mapana Journal of Sciences*, **16(3)**(2017), 41–46, doi:4110.12723/mjs.42.4.

[12] S. W. Golomb, How to number a graph, *in Graph Theory and Computing (edited by R. C. Read), Academic Press, New York*, (1972), 23–37, doi:10.1016/B978-1-4832-3187-7.50008-8.

[13] T.Tharmaraj and PB Sarasija,Square graceful graphs, *iternational journal of Mathematics and Soft Computing*, **4** (2014), 129–137.

[14] Wahyuna, Hafidhyah Dwi and Indriati, Diari, On the total edge irregularity strength of generalized butterfly graph, *Journal of Physics: Conference Series*, **1008** (2018), 012027, doi:10.1088/1742-6596/1008/1/012027.

[15] W.K.M. Indunil and A.A.I. Perera, k-Graceful labeling of triangular type grid graphs, *Proceedings of FARS2022*, **51** (2022), doi:123456789/628.

[16] Zeen El Deen, Mohamed R,Edge δ- graceful labeling for some cyclic-related graphs,*Advances in Mathematical Physics*, (2020), 1–18, doi:10.1155/2020/6273245.