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On Nano Generalized α^{**} Continuous and Nano Generalized α^{**} Irresolute Mappings

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Abstract

The purpose of this study is to present nano generalized α^{**}- continuous functions and nano generalized α^{**}- irresolute mappings in Nano Topological Spaces. We introduce the terminologies for the sets, Nano Supremum and Nano Infimum. The fundamental properties of these functions with nano interior, nano closure, nano supremum, and nano infimum are investigated. The terms nano generalized α^{**}- open mapping, nano generalized α^{**}-closed mapping and nano generalized α^{**}- homeomorphism are also defined and their associations with other continuous functions are investigated.

Keywords: Nano Supremum, Nano Infimum, $Ng\alpha^{**}$- continuous, $Ng\alpha^{**}$- irresolute, $Ng\alpha^{**}$- homeomorphism.

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1. Introduction

Levine [6] first proposed the idea of generalized closed sets in topological spaces. The concept of rough sets and rough set approximations was introduced by Zdzislaw Pawlak[7], which incorporates the properties of sets. Lellis Thivagar M and Carmel Richard[5] introduced the concepts of nano topological spaces using rough set approximations and defined nano continuous functions, nano open mappings, nano closed mappings and nano homeomorphism. Bhuvaneswari K and Mythili Gnanapriya K[3] introduced the concepts of nano generalized continuous functions in nano topological spaces. The definition of nano generalized α^{**} closed sets in nano topological spaces was made by Kalarani M et al. [1].

This research aims to extend the study of nano generalized α^{**}closed and nano generalized α^{**}open sets using properties of sets and to define the new sets called Nano infimum and Nano supremum of the subsets in nano topological spaces. These sets are based on the set inclusion principles and are defined using the concepts of upper bound and lower bound. The Nano Hasse diagram for the nano generalized α^{**}closed sets and the nano generalized α^{**}open sets have been constructed by utilizing the set inclusion principles and examining the distinctive features of the new sets. The definition and study of a new class of continuous function called nano generalized α^{**}- continuous function in continuation aim to investigate the behaviour of the new sets under the function. Bhuvaneswari K and Ezhilarasi K[2] introduced nano semi generalized irresolute maps in Nano topological spaces. The concepts of nano generalized α^{**}-irresolute mappings are introduced and investigated their relationship with other nano continuous functions. Further, nano generalized α^{**}- open mappings, nano generalized α^{**}- closed mappings and nano generalized α^{**}- homeomorphism are presented and analyzed for their characteristics.

2. Preliminaries

Definition 2.1[5]. Given U is a nonempty finite set of objects and R is an equivalence relation on U called the indiscernibility relation. The U is divided into disjoint equivalence classes. The elements of the same equivalence class are said to be indiscernible with each other. The pair (U, R) is called an approximation space. Let $X \subseteq U$, then

(i) The lower approximation of X with respect to R is $L_R(X) = \cup_{x\in X}\{R_X(x)/ R_X(x) \subseteq X\}$, where $R_X(x)$ is the equivalence class determined by $L_R(X)$.
(ii)The upper approximation of X with respect to R is $U_R(X) = \cup_{x\in X}\{R_X(x)/ R_X(x) \cap X \neq \phi\}$.
(iii) The boundary region of X is $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2[5]. Let U be the universe, R is an equivalence relation on U and $X \subseteq U$, $\tau_R(X) = \{\phi, U, L_R(X), U_R(X), B_R(X)\}$. Then $\tau_R(A)$ satisfies the axioms

(i) U and ϕ are in $\tau_R(X)$.
(ii) The arbitrary union of the elements of $\tau_R(X)$ is in $\tau_R(X)$.
(iii) The finite intersection of any finite sub collection of $\tau_R(X)$ is in$\tau_R(X)$.
Here $\tau_R(X)$is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called a nano topological space. The elements of $\tau_R(X)$ are called the nano open sets and the complement of the nano open sets are called nano closed sets.

Definition 2.3[5]. If $(U, \tau_R(X))$is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then
(i) The union of all nano open subsets contained in A is called the nano interior of A, denoted by $Nint(A)$. The $Nint(A)$) is the largest nano open subset of A.
(ii) The intersection of all nano closed sets containing A is called the nano closure of A denoted by $Ncl(A)$. The $Ncl(A)$ is the smallest nano closed set containing A.

Definition 2.4[1]. A subset A of a nano topological space $(U, \tau_R(X))$is called a nano generalized α^{**}- closed set ($Ng\alpha^{**}$- closed set) if $Ncl(A) \subseteq G$whenever $A \subseteq G$ and G is $Ng\alpha^{**}$-set. The complement of $Ng\alpha^{**}$- closed sets are called $Ng\alpha^{**}$- open sets.
Definition 2.5[4]. Let $(U, \tau_R(X))$ and $(V, \sigma_{R'}(Y))$ be two nano topological spaces. The function $f: (U, \tau_R(X)) \to (V, \sigma_{R'}(Y))$ is called nano continuous function on U, if the inverse image of every nano open set in V is nano open in U.
Definition 2.6[4]. A function $f: (U, \tau_R(X)) \to (V, \sigma_{R'}(Y))$ is nano open (nano closed) mapping if the image of every nano open (nano closed) set in $(U, \tau_R(X))$ is nano open (nano closed) set in $(V, \sigma_{R'}(Y))$.
Definition 2.7[3]. A function $f: (U, \tau_R(X)) \to (V, \sigma_{R'}(Y))$ is nano g-open (nano g-closed) mapping if the image of every nano g-open (nano g-closed) set in $(U, \tau_R(X))$ is nano g-open (Ng-closed) set in $(V, \sigma_{R'}(Y))$.
Definition 2.8[3]. A bijective function $f: (U, \tau_R(X)) \to (V, \sigma_{R'}(Y))$ is nano homeomorphism if f is both nano continuous and nano open.
Definition 2.9[3]. A bijective function $f: (U, \tau_R(X)) \to (V, \sigma_{R'}(Y))$ is nano g-homeomorphism if f is both nano g-continuous and nano g-open.
Definition 2.10[8]. A relation R on a set A is called a partially ordered set or poset if the relation R is reflexive, antisymmetric and transitive.
Definition 2.11[8]. A simplified form of the digraph of a partial ordering relation on a finite set that contains the information of the relation is called a Hasse diagram.
Definition 2.12[8]. Let A be subset of a partially ordered set $(P, \leq)$ and if u is an element of P such that $a \leq u$ for all elements $a \in A$, then u is called an upper bound of A.

Definition 2.13[8]. Let A be subset of a partially ordered set $(P, \leq)$ and if l is an element of P such that $l \leq a$ for all elements $a \in A$, then l is called the lower bound of A.

Definition 2.14[8]. The element x is called the least upper bound (LUB) or Supremum of the subset A of a poset $(\text{P}, \leq)$ if x is an upper bound that is less than every other upper bound of A.

Definition 2.15[8]. The element y is called the greatest lower bound (GLB) or Infimum of the subset A of a poset $(\text{P}, \leq)$ if y is a lower bound that is greater than every other lower bound of A.

Definition 2.16[4]. Let $(U, \tau_R(X))$ be a nano topological space and$A \subseteq U$. Then the set $N\text{ker}(A) = \bigcap\{B/A \subseteq B, B \in \tau_R(X)\}$ is called the nano kernel of A.

3. Nano Infimum and Nano Supremum of subsets

This section defines the nano supremum and nano infimum of a subset in nano topological space and examines its behaviour with nano interior and nano closure. Additionally, the discussions cover the characteristics of nano supremum and nano infimum under nano generalized α^{**}- continuous functions.

Definition 3.1. Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. The nano hasse diagram of the nano topological space is a digraph that contains the relation between its nano subsets.

Definition 3.2. Let $A \subseteq U$ be a subset of the nano topological space$(U, \tau_R(X))$. The nano generalized α^{**}- exterior of A or nano infimum of A is defined as the union of all $Ng\alpha^{**}$- closed sets of U contained in A and is denoted by $\mathcal{N}g\alpha^{**}\text{-} ext(A)$ or $\mathcal{N}inf(A)$.

$$Ng\alpha^{**}\text{-} ext(A) \ or \ Ninf(A) = \bigcup\{Q/Q \subseteq A, Q \ is \ Ng\alpha^{**}\text{- } closed\}.$$

Definition 3.3. Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. The nano generalized α^{**}- kernel of A or nano supremum of A is defined as the intersection of all $Ng\alpha^{**}$- open sets of U which contains A and is denoted by $\mathcal{N}g\alpha^{**}\text{-} ker\,(A)$ or $\mathcal{N}sup(A)$.

$$Ng\alpha^{**}\text{-} ker(A) \ or \ Nsup(A) = \bigcap\{P/A \subseteq P, P \ is \ Ng\alpha^{**}\text{-} open\ \}.$$

Example 3.4. Consider $U = \{a, b, c, d\}$ with $U/R = \{\{a\},\ \{c\}, \{b, d\}\}$ and $X = \{a, b\}$.The nano topology is $\tau_R(X) = \{\phi, U, \{a\},\ \{b, d\}, \{a, b, d\}\}$ and the complement is $\tau_R^c(X) = \{\phi, U, \{c\}, \{a, c\}, \{b, c, d\}\}$. The nano hasse diagram of $Ng\alpha^{**}$- closed and $Ng\alpha^{**}$-open sets of the above nano topology are given below.

$Ng\alpha^{**}$- $closed\ sets = \{\phi, U, \{c\}, \{a,c\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}\}$ and
$Ng\alpha^{**}$- $open\ sets = \{\phi, U, \{a\}, \{b\}, \{d\}, \{a,b\}, \{a,d\}, \{b,d\}, \{a,b,d\}\}$

Nano Hasse Diagram of of $Ng\alpha^{**}$- closed and $Ng\alpha^{**}$-open sets

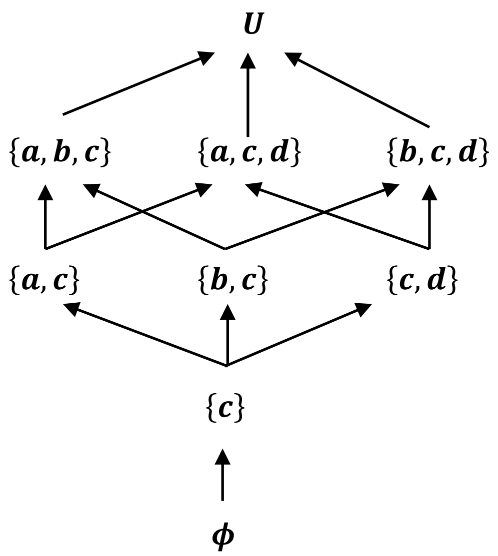

Figure.3.1

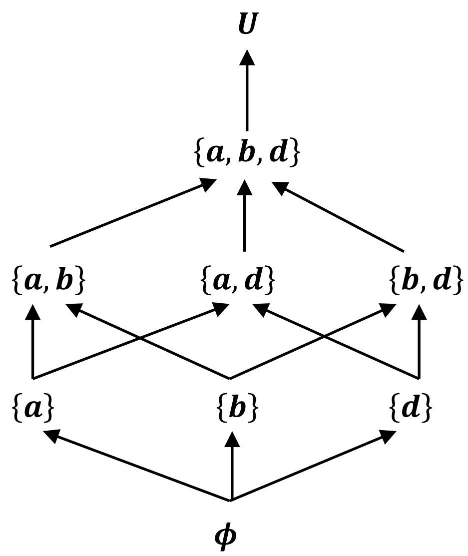

Figure. 3.2

In figure 3.1,
$Ninf(\{\phi\}) = \phi$
$Ninf(\{a\}) = Ninf(\{b\}) = Ninf(\{d\}) = \phi,\ Ninf(\{c\}) = \{c\}$
$Ninf(\{a,c\}) = \phi \cup \{c\} \cup \{a,c\} = \{a,c\}$
$Ninf(\{a,b,c\}) = \phi \cup \{c\} \cup \{a,c\} \cup \{b,c\} \cup \{a,b,c\} = \{a,b,c\}$
And from figure 3.2,
$Nsup(\{a\}) = \{a\} \cap \{a,b\} \cap \{a,d\} \cap \{a,b,d\} \cap U = \{a\}$
$Nsup(\{b\}) = \{b\} \cap \{a,b\} \cap \{b,d\} \cap \{a,b,d\} \cap U = \{b\}$
$Nsup(\{c\}) = U$
$Nsup(\{a,b\}) = \{a,b\} \cap \{a,b,d\} \cap U = \{a,b\}$

Corollary 3.5. The nano infimum of a set A is the largest $Ng\alpha^{**}$- closed subset of A or the greatest lower bound of A and the nano supremum of a set A is the smallest $Ng\alpha^{**}$-open subset of A or the least upper bound of A.

Remark 3.6. The subsets which are not in the hasse diagram have the nano infimum (GLB) as ϕ and the nano supremum (LUB) as U.

Definition 3.7. If A and B are any two subsets of the nano topological space$(U, \tau_R(X))$. The nano infimum of A and B is defined as the greatest lower bound of A and B.
The nano infimum is the largest $Ng\alpha^{**}$- closed subset of A and B.

$$Ninf(A, B) = GLB(A, B) = Largest\ \ Ng\alpha^{**}\text{-}\ closed\ subset\ of\ A\ and\ B$$

Definition 3.8. If A and B are any two subsets of the nano topological space$(U, \tau_R(X))$. The nano supremum of A and B is defined as the least upper bound of A and B. The nano supremum is the smallest $Ng\alpha^{**}$- open set that contains both A and B.

$$Nsup(A, B) = LUB(A, B)$$
$$= Smallest\ \ Ng\alpha^{**}\text{-}\ open\ set\ \text{that}\ contains\ \text{both}\ A\ and\ B$$

Example 3.9. Assume $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{b, d\}$.The nano topology is $\tau_R(X) = \{\phi, U, \{b\}, \{c, d\}, \{b, c, d\}\}$ and the complement is $\tau_R^c(X) = \{\phi, U, \{a\}, \{a, b\}, \{a, c, d\}\}$. The nano hasse diagram for the nano topology $(U, \tau_R(X))$ is given below.
$Ng\alpha^{**}\text{-}\ closed\ sets = \{\phi, U, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$ and
$Ng\alpha^{**}\text{-}\ open\ sets = \{\phi, U, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}\}$
The $Ng\alpha^{**}$ -closed sets are highlighted in the nano hasse diagram.

Nano Hasse Diagram of Example.3.9

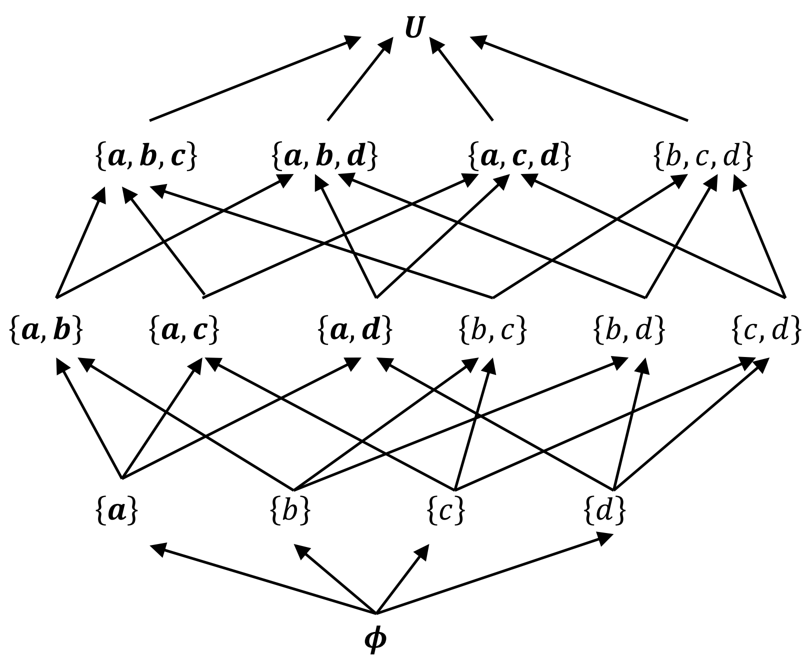

Figure.3.3

From figure.3.3,
$Ninf(\{a,c\},\{a,b,d\}) = GLB(\{a,c\},\{a,b,d\}) = \phi$
$Ninf(\{a,c\},\{b,c\}) = GLB(\{a,c\},\{b,c\}) = \{c\}$
$Ninf(\{a,c\},\{a,c,d\}) = GLB(\{a,c\},\{a,c,d\}) = \{a,c\}$
$Ninf(\{a,c,d\},\{b,c,d\}) = GLB(\{a,c,d\},\{b,c,d\}) = \{c,d\}$

$Nsup(\{a\},\{a,d\}) = LUB(\{a\},\{a,d\}) = \{a,d\}$
$Nsup(\{b\},\{a,b,c\}) = LUB(\{b\},\{a,b,c\}) = \{a,b,c\}$
$Nsup(\{a\},\{b,d\}) = LUB(\{a\},\{b,d\}) = \{a,b,d\}$

Theorem 3.10: Let P and Q be any two subsets of the nano topological space$(U,\tau_R(X))$. Then
(i). $Nsup(\phi) = \phi$, $Nsup(U) = U$.
(ii).$P \subseteq Nsup(P)$
(iii). If Q is a $Ng\alpha^{**}$- open subset of P, then $Q \subseteq Nsup(P)$
(iv). If $P \subseteq Q$, then $Nsup(P) \subseteq Nsup(Q)$
(v). $Nsup(Nsup(P)) = Nsup(P)$

Proof: (i). Assume P and Q are any two subsets of the nano topological space$(U,\tau_R(X))$.
(i) The sets ϕ and U are $Ng\alpha^{**}$- open sets. $Nsup(A)$ is the intersection of all $Ng\alpha^{**}$-open sets which contains A. So $Nsup(\phi) = \bigcap\{\phi\} = \phi$ and $Nsup(U) = \bigcap\{all\ Ng\alpha^{**} open\ sets\ which\ contains\ U\} = \bigcap\{U\} = U$
(ii) Let $P \subseteq U$, then $Nsup(P)$ is the smallest $Ng\alpha^{**}$- open set that contains P. Hence $P \subseteq Nsup(P)$.
(iii) Assume Q is a $Ng\alpha^{**}$- open subset and P is any subset with $Q \subseteq P$, then by (ii), $P \subseteq Nsup(P)$. But $Q \subseteq P \subseteq Nsup(P)$ which implies $Q \subseteq Nsup(P)$.
(iv) If $P \subseteq Q$. Let $x \in P$, then $x \in Nsup(P)$ by (ii) $P \subseteq Nsup(P)$ and $x \in P$ implies $x \in Q$. so $x \in Nsup(Q)$. Hence $Nsup(P) \subseteq Nsup(Q)$.
(v) Let $P \subseteq U$, then $Nsup(P) = \bigcap\{Ng\alpha^{**}$- open sets that contains P}
$Nsup(Nsup(P)) = Nsup$\{Intersection of $Ng\alpha^{**}$- open sets that contains P}
= Intersection \{$Ng\alpha^{**}$- open sets that contains P} = $Nsup(P)$
$Nsup(Nsup(P)) = Nsup(P)$

Theorem 3.11: Let R and S be any two subsets of the nano topological space $(U,\tau_R(X))$. Then
(i). $Ninf(\phi) = \phi$, $Ninf(U) = U$.
(ii). $Ninf(R) \subseteq R$

(iii). If S is any subset of a $Ng\alpha^{**}$- closed set R, then $S \subseteq Ninf(R)$
(iv). If $R \subseteq S$, then $Ninf(R) \subseteq Ninf(S)$
(v). $Ninf\big(Ninf(R)\big) = Ninf(R)$
Proof: Similar to theorem.3.10

Theorem 3.12: Let P be a subset of a nano topological space $(U, \tau_R(X))$, then
(i). $Nint(P) \subseteq Ng\alpha^{**}int(P) \subseteq P \subseteq Nsup(P)$ for every subset P of U.
(ii). $Nint(P) = Ng\alpha^{**}int(P) = P = Nsup(P)$, if P is nano open in U.
Proof: (i) Let $x \in Nint(P)$, then x is in a nano open set in U. Every nano open set is a $Ng\alpha^{**}$-open set. $\Rightarrow$ $x \in Ng\alpha^{**}int(P)$. $Ng\alpha^{**}int(P)$ is the largest $Ng\alpha^{**}$- open subset contained in P, $\Rightarrow$ $Ng\alpha^{**}int(P) \subseteq P \Rightarrow Nint(P) \subseteq Ng\alpha^{**}int(P) \subseteq P$ ----(1)
If $x \in Ng\alpha^{**}int(P)$, then x is a $Ng\alpha^{**}$- open set of $U \Rightarrow x$ is in the intersection of $Ng\alpha^{**}$- open sets and intersection of any two $Ng\alpha^{**}$- open sets is again a $Ng\alpha^{**}$- open set, $\Rightarrow x \in Nsup(P) \Rightarrow Ng\alpha^{**}int(P) \subseteq Nsup(P)$ ----(2)
From (1) and (2), $Nint(A) \subseteq Ng\alpha^{**}int(A) \subseteq P \subseteq Nsup(P)$
(ii) Assume P is a nano open set and $x \in Nsup(P)$, then $x \in \bigcap\{B \,/\, P \subseteq B, B\ is\ Ng\alpha^{**} - open\}$. Every nano open set is a $Ng\alpha^{**}$- open, $x \in \bigcap\{B/P \subseteq B, B\ is\ Nano\ open\}$,
$\Rightarrow x \in Nint(P)$ $\Rightarrow Nsup(P) \subseteq Nint(P)$. From (i), $Nint(A) \subseteq Nsup(P)$. Hence, $Nint(A) = Nsup(P)$ and $Nint(A) = Ng\alpha^{**}int(A) = P = Nsup(P)$ for every nano open set P of U.

Theorem 3.13: If P is a subset of a nano topological space$(U, \tau_R(X))$, then
(i). $Ninf(P) \subseteq P \subseteq Ng\alpha^{**}cl(P) \subseteq Ncl(P)$ for every subset P of U.
(ii). $Ninf(P) = P = Ng\alpha^{**}cl(P) = Ncl(P)$ if P is a nano closed set of U.
Proof: (i). Assume P is a subset of U, then $Ninf(P)$ is the largest $Ng\alpha^{**}$- closed subset of P , $Ninf(P)$ is a subset of P $\Rightarrow$ $Ninf(P) \subseteq P$ -----(1)
$Ng\alpha^{**}cl(P)$ is the smallest $Ng\alpha^{**}$- closed set that contains P
$\Rightarrow$ $P \subseteq Ng\alpha^{**}cl(P)$ ----(2)
and $Ncl(P)$ is the smallest nano closed set that contains P$\Rightarrow P \subseteq Ncl(P)$ ----(3)
From (1) and (2), $\Rightarrow$ $Ninf(P) \subseteq P \subseteq Ng\alpha^{**}cl(P)$ ----(4)
Every nano closed set is a $Ng\alpha^{**}$ - closed set and $Ng\alpha^{**}cl(P) \subseteq Ncl(P)$ -----(5)
From (4) and (5), $Ninf(P) \subseteq P \subseteq Ng\alpha^{**}cl(P) \subseteq Ncl(P)$
Hence the proof.
(ii). Assume P is a nano closed set. Every nano closed set is a $Ng\alpha^{**}$ - closed set. Then $Ncl(P) = P = Ng\alpha^{**}cl(P)$. Let $x \in Ncl(P)$. Then $x \in Ng\alpha^{**}cl(P) \Rightarrow x \in \bigcup\{Ng\alpha^{**}$ closed sets of $P\}$ $\Rightarrow$ $x \in \bigcup\{Ng\alpha^{**}$ closed sets of P containing P$\}$ $\Rightarrow$ $x \in Ninf(P)$

$\Rightarrow Ncl(P) \subseteq Ninf(P)$. But from (i), $Ninf(P) \subseteq Ncl(P) \Rightarrow Ncl(P) = Ninf(P)$
Hence, $Ninf(P) = P = Ng\alpha^{**}cl(P) = Ncl(P)$

4. Nano generalized α**- continuous functions in Nano Topological Spaces

A new class functions called nano generalized α^{**}- continuous function is defined and its properties with nano interior, nano closure, nano infimum, nano supremum and other nano continuous functions are studied in this section.

Definition 4.1. Let $(U, \tau_R(X))$ and $(V, \sigma_{R'}(Y))$ be nano topological spaces, The mapping $f: (U, \tau_R(X)) \to (V, \sigma_{R'}(Y))$is said to be nano generalized α^{**}-continuous($Ng\alpha^{**}$- continuous) function on U if the inverse image of every nano open set in V is nano generalized α^{**}- open set in U.

Example 4.2: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$.
Then $\tau_R(X) = \{\phi, U, \{a\}, \{b, d\}, \{a, b, d\}\}$ is a nano topology with respect to X and the complement is $\tau_R^c(X) = \{\phi , U, \{c\}, \{a, c\}, \{b, c, d\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{w\}, \{y, z\}\}$ and $Y = \{x, z\}$. Then $\sigma_{R'}(Y) = \{\phi, V, \{x\}, \{y, z\}, \{x, y, z\}\}$ and $\sigma_{R'}^c(Y) = \{\phi, V, \{w\}, \{x, w\}, \{y, z, w\}\}$. Define $f: (U, \tau_R(X)) \to (V, \sigma_{R'}(Y))$ as $f(a) = y, f(b) = x, f(c) = w, f(d) = z$. Then $f^{-1}(\{x\}) = \{b\}, f^{-1}(\{y, z\}) = \{a, d\}$, $f^{-1}(\{x, y, z\}) = \{b, a, d\}$. The inverse image of every nano open set in V is $Ng\alpha^{**}$-open in U. Hence f is a $Ng\alpha^{**}$- continuous function.

Theorem 4.3: A function $f: (U, \tau_R(X)) \to (V, \sigma_{R'}(Y))$ is $Ng\alpha^{**}$- continuous if and only if the inverse image of every nano closed set in V is $Ng\alpha^{**}$- closed in U.
Proof: Assume f is a $Ng\alpha^{**}$- continuous and $Q \subseteq V$ is a nano closed set, then $V - Q$ is nano open in V. Here f is $Ng\alpha^{**}$- continuous, then $f^{-1}(V - Q)$ is $Ng\alpha^{**}$-open in U. $f^{-1}(V - Q) = f^{-1}(V) - f^{-1}(Q) = U - f^{-1}(Q)$ is $Ng\alpha^{**}$- open in U. So, $f^{-1}(Q)$ is $Ng\alpha^{**}$- closed in U. The inverse image of every nano closed set in V is $Ng\alpha^{**}$- closed in U.
Conversely, assume the inverse image of every nano closed set in V is $Ng\alpha^{**}$- closed in U. Let P be a nano closed set in V. Then $V - P$ is nano open in V ⇒by assumption, $f^{-1}(V - P)$ is $Ng\alpha^{**}$- open in U $\Rightarrow f^{-1}(V - P) = U - f^{-1}(P)$ is $Ng\alpha^{**}$- open in U $\Rightarrow f^{-1}(P)$ is $Ng\alpha^{**}$- closed in U. So if P is a nano closed set in V, then $f^{-1}(P)$ is $Ng\alpha^{**}$- closed in U. Hence f is $Ng\alpha^{**}$- continuous.

Theorem 4.4: A function $f:(U,\tau_R(X)) \to (V,\sigma_{R'}(Y))$ is nano continuous if and only if f is $Ng\alpha^{**}$- continuous.
Proof: Assume $f:(U,\tau_R(X)) \to (V,\sigma_{R'}(Y))$ is a nano continuous function and let B be a nano closed set in V. Then $V-B$ is nano open in V ⇒ by assumption, $f^{-1}(V-B)$ $f^{-1}(V-B) = f^{-1}(V) - f^{-1}(B) = U - f^{-1}(B)$ is nano open in U ⇒$f^{-1}(B)$ is nano closed in U. We know every nano open set in V is a $Ng\alpha^{**}$- open set in U. $f^{-1}(B)$ is $Ng\alpha^{**}$- closed in U. Hence f is $Ng\alpha^{**}$- continuous.
Conversely, f is $Ng\alpha^{**}$- continuous, then every nano closed in V is $Ng\alpha^{**}$- closed in U. It contains all the closed sets of U. So, the image of a nano closed set in $(V,\sigma_{R'}(Y))$ is a nano closed set in $(U,\tau_R(X))$. So f is a nano continuous function.

Example 4.5: Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{c\},\{b,d\}\}$ and $X=\{a,b\}$. Then $\tau_R(X)=\{\phi,U,\{a\},\{b,d\},\{a,b,d\}\}$ and $\tau_R^c(X)=\{\phi,U,\{c\},\{a,c\},\{b,c,d\}\}$.
Let $V=\{x,y,z,w\}$ with $V/R'=\{\{x,w\},\{y\},\{z\}\}$ and $Y=\{x,z\}$.
Then $\sigma_{R'}(Y)=\{\phi,V,\{z\},\{x,w\},\{x,z,w\}\}$ and $\sigma_{R'}^c(Y)=\{\phi,V,\{y\},\{y,z\},\{x,y,w\}\}$.
Define $f:(U,\tau_R(X)) \to (V,\sigma_{R'}(Y))$ as $f(a)=z$, $f(b)=x$, $f(c)=y$, $f(d)=w$. Then $f^{-1}(\{z\})=\{a\}$, $f^{-1}(\{x,w\})=\{b,d\}$, $f^{-1}(\{x,z,w\})=\{a,b,d\}$. The inverse image of every nano open set in V is $Ng\alpha^{**}$- open in U. Here f is $Ng\alpha^{**}$- continuous function. Also $f^{-1}(\{y\})=\{c\}$, $f^{-1}(\{y,z\})=\{a,c\}$, $f^{-1}(\{x,y,w\})=\{b,c,d\}$ which are all nano closed sets in U. Hence f is a nano continuous function.

Theorem 4.6: Let $(U,\tau_R(X))$ and $(V,\sigma_{R'}(Y))$ are two nano topological spaces and let $f:(U,\tau_R(X)) \to (V,\sigma_{R'}(Y))$ be a mapping. Then the following statements are equivalent.
(i) f is $Ng\alpha^{**}$- continuous
(ii) $f(Ng\alpha^{**}cl(P) \subseteq Ncl(f(P))$ for every subset P of U
(iii) $Ng\alpha^{**}cl(f^{-1}(Q)) \subseteq f^{-1}(Ncl(Q))$ for every subset Q of V
Proof:
$(i) \Longrightarrow (ii)$
Assume$f$ is $Ng\alpha^{**}$- continuous and let $P \subseteq U$, then $Ncl(f(P))$ is nano closed in V. Now $f^{-1}(Ncl(f(P))$ is $Ng\alpha^{**}$- closed set containing P in U. Every nano closed set A is $Ng\alpha^{**}$- closed set in U and which is the smallest set containing A. $f(P) \subseteq Ncl(f(P)) \Rightarrow f^{-1}(f(P)) \subseteq f^{-1}(Ncl(f(P)) \Rightarrow P \subseteq f^{-1}(Ncl(f(P))$,
then $Ng\alpha^{**}cl(P) \subseteq Ng\alpha^{**}cl(f^{-1}(Ncl(f(P))) = f^{-1}(Ncl(f(P))$
$\Rightarrow f(Ng\alpha^{**}cl(P)) \subseteq Ncl(f(P)$ for every subset P of U.
$(ii) \Longrightarrow (i)$

Assume $f(Ng\alpha^{**}cl(P))) \subseteq Ncl(f(P))$. Let Q be a nano closed set in V, then $f^{-1}(Q)$ is nano closed in U, $f(Ng\alpha^{**}cl(Q) \subseteq Ncl\left(f\left(f^{-1}(Q)\right)\right) = Ncl(Q) = Q$
$\Rightarrow f(Ng\alpha^{**}cl(Q)) \subseteq Q \Rightarrow Ng\alpha^{**}cl(Q) \subseteq f^{-1}(Q)$. But $f^{-1}(Q) \subseteq Ng\alpha^{**}cl(Q)$
$\Rightarrow Ng\alpha^{**}cl(Q) = f^{-1}(Q)$. Hence $f^{-1}(Q)$ is $Ng\alpha^{**}$- closed in U. Therefore, f is $Ng\alpha^{**}$- continuous function.

$(ii) \implies (iii)$

Assume $f(Ng\alpha^{**}cl(C))) \subseteq Ncl(f(C))$. Let Q be a nano closed set in V, then $f^{-1}(Q)$ is nano closed, $f(Ng\alpha^{**}cl(f^{-1}(Q))) \subseteq Ncl\left(f\left(f^{-1}(Q)\right)\right) \subseteq Ncl(Q)$

$$\Rightarrow Ng\alpha^{**}cl\left(f^{-1}(Q)\right) \subseteq f^{-1}(Ncl(Q)$$

$(iii) \implies (i)$

Assume $Ng\alpha^{**}cl(f^{-1}(Q)) \subseteq f^{-1}(Ncl(Q))$ for any subset Q of V.

If Q is nano closed set in V, then $Ncl(Q) = Q$. From the assumption, $Ng\alpha^{**}cl(f^{-1}(Q)) \subseteq f^{-1}(Q)$. But for a nano closed set Q in V, $f^{-1}(Q) \subseteq Ng\alpha^{**}cl(f^{-1}(Q)) \Rightarrow Ng\alpha^{**}cl(f^{-1}(Q)) = f^{-1}(Q)$, a nano closed set $f^{-1}(Q)$ in V is a $Ng\alpha^{**}$- closed in U. That is the inverse image of a nano closed set is a $Ng\alpha^{**}$- closed set in U. So, f is $Ng\alpha^{**}$- continuous.

Theorem 4.7: Let $f:(U,\tau_R(X)) \to (V,\sigma_{R'}(Y))$ be a function. Then (i) f is $Ng\alpha^{**}$-continuous if and only if

(ii) $Nint(f(P)) \subseteq f(Ng\alpha^{**}int(P))$ for every subset P of U

(iii) $f^{-1}\left(Nint(Q)\right) \subseteq Ng\alpha^{**}int(f^{-1}(Q))$ for every subset Q of V

Proof:

$(i) \implies (ii)$

Assume f is $Ng\alpha^{**}$ continuous and let $P \subseteq U$, then $Nint(f(P))$ is a nano open set in V. By the definition of f, $f^{-1}(Nint(f(P))$ is $Ng\alpha^{**}$- open in U. We know, $Nint(A) \subseteq A \Rightarrow f^{-1}(Nint(f(P)) \subseteq f^{-1}((f(P)) \subseteq P$, but $P \subseteq Ng\alpha^{**}int(P)$.

Hence $f^{-1}(Nint(f(P)) \subseteq Ng\alpha^{**}int(P) \Rightarrow Nint(f(P) \subseteq f(Ng\alpha^{**}int(P))$.

$(ii) \implies (i)$

Conversely, let $Nint(f(C)) \subseteq f(Ng\alpha^{**}int(C))$ for every subset C in U. Let D be a nano open set in V, then $Nint(D) = D \Rightarrow Nint\left(f(D)\right) = f(D)$. Using this in the assumption, $f(D) \subseteq f(Ng\alpha^{**}int(D)) \Rightarrow f^{-1}((f(D)) \subseteq Ng\alpha^{**}int(D)$
$\Rightarrow D \subseteq Ng\alpha^{**}int(D)$

But for any open set D in V, $Ng\alpha^{**}int(D) \subseteq D$. Hence, $Ng\alpha^{**}int(D) = D$. Here D is $Ng\alpha^{**}$- open in U, $\Rightarrow f$ is $Ng\alpha^{**}$ continuous.

$(i) \implies (iii)$

Let f is a $Ng\alpha^{**}$ continuous function and let $Q \subseteq V$, then $Nint(Q)$ is nano open set in V and $Nint(Q) \subseteq Q$.Then $f^{-1}(Nint(Q))$ is $Ng\alpha^{**}$open in U.
$Ng\alpha^{**}int(f^{-1}(Nint(Q))) = f^{-1}(Nint(Q))$, also $f^{-1}(Nint(Q) \subseteq f^{-1}(Q)$
Hence, $Ng\alpha^{**}int(f^{-1}(Nint(Q))) \subseteq f^{-1}(Q) \Rightarrow f^{-1}(Nint(Q)) \subseteq Ng\alpha^{**}int(f^{-1}(Q))$.
$(iii) \Longrightarrow (i)$
Conversely, let us take F is a nano open set in V, then $Nint(F) = F$.
Using this in the assumption, $f^{-1}(Nint(F)) \subseteq Ng\alpha^{**}int(f^{-1}(F))$,
$f^{-1}(F) \subseteq Ng\alpha^{**}int(f^{-1}(F))$.
But for any open set F in V, $Ng\alpha^{**}int(f^{-1}(F)) \subseteq f^{-1}(F)$.
So, $Ng\alpha^{**}int(f^{-1}(F)) = f^{-1}(F)$. Hence, $f^{-1}(F)$ is $Ng\alpha^{**}$- open in U, for any open set F in V, $f^{-1}(F)$ is $Ng\alpha^{**}$ - open in U $\Rightarrow f$is $Ng\alpha^{**}$ continuous.

Theorem 4.8: If $f: (U, \tau_R(X)) \to (V, \sigma_{R'}(Y))$ is a $Ng\alpha^{**}$- continuous function, then
(i) $Nsup(f(P)) \subseteq f(Nsup(P))$
(ii) $Ninf(f(P)) \subseteq f(Ninf(P))$
Proof:
(i) Assume $f: (U, \tau_R(X)) \to (V, \sigma_{R'}(Y))$ is a $Ng\alpha^{**}$- continuous function. Let $P \subseteq U$,then $f(P) \subseteq V$ and let $x \in Nsup(f(P))$, x is a $Ng\alpha^{**}$-open set in V. For $P \subseteq U$, $Nsup(P)$ is the smallest $Ng\alpha^{**}$-open set that contains P in U $\Longrightarrow f(Nsup(P))$ is a $Ng\alpha^{**}$-open set in V $\Longrightarrow x \in f(Nsup(P)) \Longrightarrow Nsup(f(P)) \subseteq f(Nsup(P))$.
(ii) Let $x \in Ninf(f(P))$, then x is a $Ng\alpha^{**}$-closed set in V. For $Q \subseteq U$, $Ninf(f(Q))$ is the largest $Ng\alpha^{**}$-closed subset of $f(Q)$ in V. Given $Q \subseteq U$, $f(Ninf(Q)) \subseteq V$is also a $Ng\alpha^{**}$-closed set. Given f is a $Ng\alpha^{**}$- continuous, every nano closed set in V is a $Ng\alpha^{**}$-closed set $\Longrightarrow x \in f(Ninf(P)) \Longrightarrow Ninf(f(P)) \subseteq f(Ninf(P))$.

Theorem 4.9: If $f: (U, \tau_R(X)) \to (V, \sigma_{R'}(Y))$ is a $Ng\alpha^{**}$- continuous function, then
(i) $Ng\alpha^{**}int(f(P)) \subseteq f(Nsup(P))$
(ii) $f(Ninf(P)) \subseteq Ng\alpha^{**}cl(f(P)))$
Proof:
(i) Assume f is a $Ng\alpha^{**}$- continuous function. Let $P \subseteq U$, and $x \in Ng\alpha^{**}int(f(P))$, then x is a $Ng\alpha^{**}$-open set in V. $Nsup(P)$ is the smallest $Ng\alpha^{**}$-open set that contains P in U $\Longrightarrow f(Nsup(P))$ is either nano open or $Ng\alpha^{**}$-open set in V. If it is nano open in V, then it is also $Ng\alpha^{**}$-open set in V $\Longrightarrow x \in f(Nsup(P)) \Longrightarrow Ng\alpha^{**}int(f(P)) \subseteq f(Nsup(P))$
(ii) Let $x \in Ninf(P) \Longrightarrow Ninf(P)$ is the largest $Ng\alpha^{**}$-closed subset of P in V. So $f(Ninf(P)) \subseteq V$is also a $Ng\alpha^{**}$-closed set. For $P \subseteq U, f(P) \subseteq V$ and $Ng\alpha^{**}cl(f(P))$

is the largest $Ng\alpha^{**}$-closed set in V. The $Ng\alpha^{**}$- closed sets of $f(Ninf(P))$ is contained in $Ng\alpha^{**}cl(f(P))$. Hence $f(Ninf(P)) \subseteq Ng\alpha^{**}cl(f(P)))$.

Corollary 4.10: If f is a $Ng\alpha^{**}$- continuous function, then
(i) $Nint(f(P)) \subseteq f(Nsup(P))$
(ii) $f(Ninf(P)) \subseteq Ncl(f(P)))$
Proof: Similar as theorem.4.9

Theorem 4.11: If $f:(\mathrm{U},\tau_{\mathrm{R}}(\mathrm{X})) \rightarrow (\mathrm{V},\sigma_{\mathrm{R'}}(\mathrm{Y}))$ is a Ngα**- continuous function, then $f^{-1}(Ng\alpha^{**}int(P)) \subseteq Nsup(f^{-1}(P))$
Proof: Let $P \subseteq V$ is a subset of V and $f^{-1}(P) \subseteq U$. $Nsup(f^{-1}(P))$ is the smallest $Ng\alpha^{**}$-open set contains $f^{-1}(P)$ in U. That is $f^{-1}(P) \subseteq Nsup(f^{-1}(P))$ ----(1)
But $Ng\alpha^{**}int(P) \subseteq P \Longrightarrow f^{-1}(Ng\alpha^{**}int(P)) \subseteq f^{-1}(P)$ ----(2)
From(1) and (2), $f^{-1}(Ng\alpha^{**}int(P)) \subseteq Nsup(f^{-1}(P))$.

5. Nano generalized α**- open and Nano generalized α**- closed mappings

In this section, nano generalized α^{**}- open and nano generalized α^{**}- closed mappings and its properties are described.

Definition 5.1. A function $f:(U,\tau_R(X)) \rightarrow (V,\sigma_{R'}(Y))$ is said to be nano $g\alpha^{**}$- open (or nano $g\alpha^{**}$- closed) function if the image of every nano open (or nano closed) set in $\left(U,\tau_R(X)\right)$ is nano $g\alpha^{**}$- open (or nano $g\alpha^{**}$- closed) set in $(V,\sigma_{R'}(Y))$.

Definition 5.2. A function $f:(U,\tau_R(X)) \rightarrow (V,\sigma_{R'}(Y))$ is said to be nano $g\alpha^{**}$- closed function if the image of every nano closed set in $\left(U,\tau_R(X)\right)$ is nano $g\alpha^{**}$- closed set in $(V,\sigma_{R'}(Y))$.

Example 5.3. Let $U = \{a,b,c,d\}$ with $U/R = \left\{\{a\},\{b,c\},\{d\}\right\}$ and $X = \{b,d\}$. Then $\tau_R(X) = \left\{\phi,U,\{d\},\{b,c\},\{b,c,d\}\right\}$ is a nano topology with respect to X and its complement is $\tau_R^c(X) = \{\phi,U,\{a\},\{a,d\},\{a,b,c\}\}$. Let $V = \{x,y,z,w\}$ with $V/R' = \{\{z\},\ \{y\},\{x,w\}\}$ and $Y = \{y,w\}$. Then $\sigma_{R'}(Y) = \left\{\phi,V,\{y\},\{x,w\},\{x,y,w\}\right\}$ and $\sigma_{R'}^c(Y) = \{\phi,V,\{z\},\{y,z\},\{x,z,w\}\}$.
Define $f:(U,\tau_R(X)) \rightarrow (V,\sigma_{R'}(Y))$ as $f(a) = z$, $f(b) = x$, $f(c) = w$, $f(d) = y$. Then $f^{-1}(\{d\}) = \{y\}$, $f^{-1}(\{b,c\}) = \{x,w\}$, $f^{-1}(\{b,c,d\}) = \{x,y,w\}$. Thus, the image of every nano open set in U is $Ng\alpha^{**}$- open in V. Hence f is a $Ng\alpha^{**}$- open function.

Theorem 5.4. Let $f: (U, \tau_R(X)) \rightarrow (V, \sigma_{R'}(Y))$ be a function and if f is nano open, then f is $Ng\alpha^{**}$- open function.
Proof: Assume $f: (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is a nano open function and let $A \subseteq U$ is a nano open set. By definition, $f(A)$ is nano open in V. Every nano open set in V is a nano $g\alpha^{**}$-open in V. So $f(A)$ is nano $Ng\alpha^{**}$- open in V. Hence f is $Ng\alpha^{**}$- open function.

Theorem 5.5. Let $f: (U, \tau_R(X)) \rightarrow (V, \sigma_{R'}(Y))$ be a nano closed function, then f is $Ng\alpha^{**}$- closed function.
Proof: Similar to Theorem. 5.4

Theorem 5.6. Every nano closed function is $Ng\alpha^{**}$- closed function but not conversely.
Proof: Let $f: (U, \tau_R(X)) \rightarrow (V, \sigma_{R'}(Y))$ is a nano closed function and let A is a nano closed Set in U, then $f(A)$ is nano closed in V. Every nano closed set in V is a nano $g\alpha^{**}$- closed. Hence $f(A)$ is nano $g\alpha^{**}$- closed in V. Therefore f is $Ng\alpha^{**}$- closed function.

Example 5.7. In example. 5.3, if f is defined as $f(a) = z$, $f(b) = x$, $f(c) = y$, $f(d) = w$. Then $f(\{a\}) = \{z\}$, $f(\{a, d\}) = \{x, w\}$, $f(\{a, b, c\}) = \{x, y, z\}$. Thus, the image of the nano closed sets are $Ng\alpha^{**}$- closed but f is not nano closed.

Theorem 5.8. Every $Ng\alpha^{**}$- closed function is nano g-closed function but not conversely.
Proof: Assume $f: (U, \tau_R(X)) \rightarrow (V, \sigma_{R'}(Y))$ is a $Ng\alpha^{**}$- closed function and let A is a nano closed set in U, then the image of A under f is $Ng\alpha^{**}$- closed in V. Every $Ng\alpha^{**}$- closed set is nano g-closed. Hence $f(A)$ is nano g-closed in V. The function f is Ng-closed function.
Conversely, a nano g-closed set need not be a $Ng\alpha^{**}$- closed set. So, the converse need not be true.

Remark 5.9. The function f defined in example.5.7 is $Ng\alpha^{**}$- closed but not nano g-closed.

Theorem 5.10.I $f: (U, \tau_R(X)) \rightarrow (V, \sigma_{R'}(Y))$ is a $Ng\alpha^{**}$- closed function, then f is a
(i) nano $g\alpha$- closed function.
(ii) nano αg - closed function.
(iii) nano g^*- closed function.

(iv) nano rg - closed function.
Proof: Similar to the above theorem.

6. Nano generalized α**- irresolute mappings

This section discusses nano generalized α^{**}- irresolute mappings and their characterizations.

Definition 6.1. A function $f:(U,\tau_R(X)) \to (V,\sigma_{R'}(Y))$ is called nano $g\alpha^{**}$- irresolute ($Ng\alpha^{**}$- irresolute) function if the image of every $Ng\alpha^{**}$- closed set in V is nano $g\alpha^{**}$- closed set in U.

Example 6.2. Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\}, \{b\}, \{c,d\}\}$ and $X = \{b,d\}$. Then $\tau_R(X) = \{\phi, U, \{b\}, \{c,d\}, \{b,c,d\}\}$ and the closed sets are $\{\phi, U, \{a\}, \{a,b\}, \{a,c,d\}\}$. The $Ng\alpha^{**}$- closed sets are $\phi, U, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}$.
Let $V = \{x,y,z,w\}$ with $V/R' = \{x, \{y,w\}, \{z\}\}$ and $Y = \{x,y\}$, then $\sigma_{R'}(Y) = \{\phi, V, \{x\}, \{y,w\}, \{x,y,w\}\}$ and $\sigma^c_{R'}(Y) = \{\phi, V, \{z\}, \{x,z\}, \{y,z,w\}\}$.
Define $f:(U,\tau_R(X)) \to (V,\sigma_{R'}(Y))$ as $f(a) = z$, $f(b) = x$, $f(c) = y$, $f(d) = w$. Then inverse image of $Ng\alpha^{**}$- closed sets in V is $f^{-1}(\{z\}) = \{a\}$, $f^{-1}(\{x,z\}) = \{a,b\}$, $f^{-1}(\{y,z\}) = \{a,c\}$, $f^{-1}(\{z,w\}) = \{a,d\}$, $f^{-1}(\{x,y,z\}) = \{a,b,c\}$, $f^{-1}(\{x,z,w\}) = \{a,b,d\}$, $f^{-1}(\{y,z,w\}) = \{a,c,d\}$. Hence f is a $Ng\alpha^{**}$- irresolute function.

Theorem 6.3. A function $f:(U,\tau_R(X)) \to (V,\sigma_{R'}(Y))$ is $Ng\alpha^{**}$- irresolute if and only if the inverse image of every $Ng\alpha^{**}$- open set in V is $Ng\alpha^{**}$- open set U.
Proof: Assume $f:(U,\tau_R(X)) \to (V,\sigma_{R'}(Y))$ is a $Ng\alpha^{**}$- irresolute map and let P be any $Ng\alpha^{**}$- open set in V. Then $V - P$ is $Ng\alpha^{**}$- closed in V ⇒ by assumption, every $Ng\alpha^{**}$- closed set in V is $Ng\alpha^{**}$- closed in U. Then $f^{-1}(V - B)$ is $Ng\alpha^{**}$- closed in U ⇒$f^{-1}(V - P) = f^{-1}(V) - f^{-1}(P) = U - f^{-1}(P)$ is $Ng\alpha^{**}$- closed in U ⇒$f^{-1}(P)$ is $Ng\alpha^{**}$- open in U. Thus the inverse image of every $Ng\alpha^{**}$- open set in V is a $Ng\alpha^{**}$- open set in U.
Conversely, assume the inverse image of any $Ng\alpha^{**}$- open set $Q \subseteq V, f^{-1}(Q)$ is $Ng\alpha^{**}$- open set in U. Let $Q \subseteq V$ is $Ng\alpha^{**}$- closed, Then $V - Q$ is $Ng\alpha^{**}$- open in V ⇒ by assumption, $f^{-1}(V - Q) = U - f^{-1}(Q)$ is $Ng\alpha^{**}$- open in U ⇒$f^{-1}(P)$ is $Ng\alpha^{**}$- closed in U. Thus f is $Ng\alpha^{**}$- irresolute.

Theorem 6.4. Every $Ng\alpha^{**}$- irresolute map is $Ng\alpha^{**}$- continuous and conversely.
Proof: Assume $f:(U,\tau_R(X)) \to (V,\sigma_{R'}(Y))$ is an $Ng\alpha^{**}$- irresolute map, then every $Ng\alpha^{**}$- closed set in V is $Ng\alpha^{**}$- closed in U. Let Q be any nano closed set in V. The

inverse image of $Q \subseteq V, f^{-1}(Q)$ is $Ng\alpha^{**}$- closed in U, since every nano closed set is a $Ng\alpha^{**}$- closed. So f is a nano continuous function.
Conversely, assume f is $Ng\alpha^{**}$- continuous, then every nano closed set in V is $Ng\alpha^{**}$- closed in U. But every nano closed set in V is $Ng\alpha^{**}$- closed in V. So the inverse image of every $Ng\alpha^{**}$- closed set in V is $Ng\alpha^{**}$- closed in U.

7. Nano generalized α**- Homeomorphism in Nano Topological

In this section, nano generalized α^{**}- homeomorphism in nano topological spaces is defined and its properties are discussed

Definition 7.1. A function $f: (U, \tau_R(X)) \to (V, \sigma_{R'}(Y))$ is called nano generalized $g\alpha^{**}$-homeomorphism (or $Ng\alpha^{**}$- homeomorphism) if
(i) f is one to one and onto
(ii) f is $Ng\alpha^{**}$- continuous
(iii) f is $Ng\alpha^{**}$- open

Theorem 7.2. If $f: (U, \tau_R(X)) \to (V, \sigma_{R'}(Y))$ is a bijective mapping. Then f is a $Ng\alpha^{**}$-homeomorphism if and only if f is $Ng\alpha^{**}$- closed and $Ng\alpha^{**}$- continuous.
Proof: Assume $f: (U, \tau_R(X)) \to (V, \sigma_{R'}(Y))$ is a $Ng\alpha^{**}$- homeomorphism. Then by definition, f is bijective, $Ng\alpha^{**}$-continuous and $Ng\alpha^{**}$- open. Let A be a nano closed set in U, then $U - A$ is a nano open set in U. $f(U - A)$ is $Ng\alpha^{**}$- open in V and $V - f(A)$ is $Ng\alpha^{**}$- open in V. This implies $f(A)$ is $Ng\alpha^{**}$- closed in V for the nano closed set A in U. So f is $Ng\alpha^{**}$- closed function.
Conversely, assume f is $Ng\alpha^{**}$- closed and $Ng\alpha^{**}$ continuous function. Let B be a nano open set in U, then $U - B$ is a nano closed set in U. $f(U - B)$ is $Ng\alpha^{**}$- closed in V and $V - f(B)$ is $Ng\alpha^{**}$- closed. That is $f(B)$ is $Ng\alpha^{**}$- open, hence f is $Ng\alpha^{**}$- open mapping. Also, f is $Ng\alpha^{**}$ continuous. So f is $Ng\alpha^{**}$ homeomorphism. Hence the proof.

8. Discussion

This study uses $Ng\alpha^{**}$- closed and $Ng\alpha^{**}$-open sets to define two new classes of subsets called Nano Infimum and Nano Supremum of a nano topological space. These two sets are independent with $Ng\alpha^{**}$-closure and $Ng\alpha^{**}$-interior. The characteristics of nano infimum and nano supremum have been identified. Next, the new class of continuous function called the nano generalized α^{**} continuous function is defined and explored together with other nano continuous functions. Additionally, nano generalized

α^{**}- open mappings, nano generalized α^{**}- closed mappings, nano generalized α^{**}-irresolute mappings, and nano generalized α^{**} homeomorphism are defined and their characteristics are examined.

In conclusion, the nano infimum of a subset is the inferior set which has the largest $Ng\alpha^{**}$- closed set contained in it and the nano supremum of a subset is the superior set in which it is the smallest $Ng\alpha^{**}$- open set that contains it. By drawing the nano hasse diagram for the $Ng\alpha^{**}$- closed and $Ng\alpha^{**}$-open sets, one can discover the nano infimum and nano supremum of a subset by finding the GLB and LUB of the set. Moreover, the ideas of $Ng\alpha^{**}$- closed and $Ng\alpha^{**}$-open sets are broadened to define $Ng\alpha^{**}$-continuous functions and the behaviours of nano infimum and nano supremum under this function are discovered.

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